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A multi-level Taguchi-factorial two-stage stochastic programming approach for characterization of parameter uncertainties and their interactions: An application to water resources management

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ABSTRACT

This paper presents a multi-level Taguchi-factorial two-stage stochastic programming (MTTSP) approach for supporting water resources management under parameter uncertainties and their interactions. MTTSP is capable of performing uncertainty analysis, policy analysis, factor screening, and interaction detection in a comprehensive and systematic way. A water resources management problem is used to demonstrate the applicability of the proposed approach. The results indicate that interval solutions can be generated for the objective function and decision variables, and a variety of decision alternatives can be obtained under different policy scenarios. The experimental data obtained from the Taguchi's orthogonal array design are helpful in identifying the significant factors affecting the total net benefit. Then the findings from the multi-level factorial experiment reveal the latent interactions among those important factors and their curvature effects on the model response. Such a sequential strategy of experimental designs is useful in analyzing the interactions for a large number of factors in a computationally efficient manner.

1. Introduction

The world has been turning its attention to the increasingly critical issue of water scarcity. According to the United Nations, approximately 700 million people in 43 countries are now suffering from water scarcity, and it is projected that 1.8 billion people will be living in countries or regions with absolute water scarcity by 2025 (UN-Water, 2006). The limited availability of water leads to a growing competition for water use among municipality, industry and agriculture in many countries. As rapid population growth and economic development, the competition for limited supplies will intensify, resulting in tensions and conflicts among water users. Therefore, wise decisions are desired to make best use of limited water resources. Optimization techniques have played an important role in helping decision makers (DMs) allocate and manage water resources in an effective and efficient way. However, a variety of uncertainties exist in water resources management systems and their latent interactions may further intensify the complexity in the decision-making process. As a result, conventional optimization methods such as linear programming, quadratic programming and integer programming would become ineffective when a variety of uncertainties exist in system components.

Over the past few years, a number of optimization methods have been proposed for dealing with uncertainties in water resources management (Abdelaziz, 2012; Bravo & Gonzalez, 2009; Chung, Lansey, & Bayraksan, 2009; Gaivoronski, Sechi, & Zuddas, 2012; Guo, Huang, Zhu, & Wang, 2010; Li, Huang, Nie, & Liu, 2008, 2009; Qin, Huang, Zeng, Chakma, & Huang, 2007; Teegavarapu, 2010; Wang & Huang, 2011, 2012). Among these methods, two-stage stochastic programming (TSP) has the ability to take corrective actions after a random event occurs (Birge & Louveaux, 1988, 1997). In a TSP model, two groups of decision variables can be distinguished. The first-stage decision must be made prior to the realization of random variables, and then the second-stage decision can be determined after a random event takes place. The recourse action in the second stage is effective in minimizing the risk of infeasibility as a result of the first-stage decision. TSP can thus be used to tackle uncertain information presented as probability distributions and make decisions in a two-stage fashion. However, TSP has difficulty in dealing with uncertainties when the sample size is too small to generate distribution functions. Even if such distributions are available, addressing them in large-scale optimization models can be challenging.

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Interval-parameter linear programming (ILP) is efficient in coping with uncertain information expressed as interval numbers with known lower and upper bounds but unknown distribution functions (Huang, Baetz, & Patry, 1992). Moreover, ILP can reflect interval information in the coefficients of the objective function and constraints, as well as in the solutions of the objective-function value and decision variables, which is helpful for DMs to interpret and adjust decision schemes according to practical situations. Consequently, an integration of TSP and ILP is desired to enhance the capability of addressing uncertainties in different formats (Huang & Loucks, 2000).

The aforementioned optimization methods mainly focus on addressing parameter uncertainties that exist in various formats such as intervals, fuzzy sets and probability distributions. However, they can hardly reveal the potential interactions among model parameters in the optimization model. It is thus necessary to explore the correlated parameters and their contributions to the variability of the model output. Factorial designs have been widely used to study the interaction effects of two or more factors on a response variable (Lewis & Dean, 2001; Lin, Huang, Lu, & He, 2008; Mabilia, Scipioni, Vegliò, & Tomasi Scianò, 2010; Onsekizoglu, Bahceci, & Acar, 2010; Qin, Huang, & Chakma, 2008; Wang & Huang, 2013; Wang, Huang, & Veawab, 2013; Zhou, Huang, & Yang, 2013). All these studies used the most popular two-level factorial design which assumed that the response was linear over the range of the factor levels chosen. However, many real-world problems involve the nonlinear relationships between the factors and the response. The two-level factorial experiment cannot address the nonlinear effects. Thus, the multi-level factorial design is proposed to detect the curvature in the response function (Box & Behnken, 1960; Wu & Hamada, 2009; Xu, Chen, & Wu, 2004). As the number of factors increases, the multi-level factorial design would become infeasible from a time and resource viewpoint due to a large number of experimental runs required.

To reduce the number of experiments to a practical level when there are many factors to be studied, factor screening is necessary to identify a few factors that have significant effects on the response and remove those insignificant ones at the early stage of the factorial experiment. The concept of Taguchi's orthogonal arrays is an effective and efficient means of identifying the importance of factors through performing only a small subset of the experimental runs (Adenso-Díaz & Laguna, 2006). Nevertheless, it can hardly provide information on how these factors interact. Thus, Taguchi's orthogonal arrays can be employed to screen out the important factors from a large number of potential factors in a computationally efficient way. Then the multi-level factorial design can be used to analyze the interactions among those important factors. Combining the Taguchi's orthogonal arrays with the multi-level factorial design is thus a sound strategy to study the potential interactions for a large number of factors at multiple levels.

The objective of this study is to develop a multi-level Taguchi-factorial two-stage stochastic programming (MTTSP) approach through incorporating ILP, TSP, Taguchi's orthogonal arrays, and the multi-level factorial design within a general framework. MTTSP is capable of analyzing parameter uncertainties and their interactions in a comprehensive and systematic manner. A water resources management problem will be used to illustrate the applicability of the proposed method.

2. Methodology

2.1. Interval-parameter two-stage stochastic programming

Consider a problem wherein a water manager is responsible for allocating water to multiple users, with the objective of maximizing the total net benefit through identifying optimized water-allocation schemes. As these users need to know how much water they can expect so as to make sound plans for their activities and investments, a prescribed amount of water is promised to each user according to local water management policies. If the promised water is delivered, it will bring net benefits to the local economy; otherwise, the users will have to obtain water from other sources or curtail their expansion plans, resulting in economic penalties (Maqsood, Huang, & Yeomans, 2005).

In this problem, a first-stage decision on the water-allocation targets must be made before unknown seasonal flows are realized. When the uncertainty of seasonal flows is uncovered, a second-stage recourse decision can be made to compensate for any adverse effects that may have been experienced as a result of the first-stage decision. Thus, this problem under consideration can be formulated as a TSP model (Huang & Loucks, 2000):

$$\text{Max } f = \sum_{i=1}^{m} NB_i T_i - E\left[\sum_{i=1}^{m} C_i S_{iQ}\right] \tag{1a}$$

subject to:

$$\sum_{i=1}^{m}(T_i - S_{iQ}) \leqslant Q, \tag{1b}$$

$$S_{iQ} \leqslant T_i \leqslant T_{i\max}, \ \forall i, \tag{1c}$$

$$S_{iQ} \geqslant 0, \ \forall i. \tag{1d}$$

where f is total net benefit (\$); NB_i is net benefit to user i per meter^3 of water allocated (\$/meter^3); T_i (first-stage decision variable) is allocation target for water that is promised to user i (meter^3); $E[\cdot]$ is expected value of a random variable; C_i is loss to user i per meter^3 of water not delivered, $C_i > NB_i$ (\$/meter^3); S_{iQ} (second-stage decision variable) is shortage of water to user i when the seasonal flow is Q (meter^3); Q (random variable) is total amount of the seasonal flow (meter^3); $T_{i\max}$ is maximum allowable allocation amount for user i (meter^3); m is number of water users; i is index of water users, $i = 1, 2, 3$, with $i = 1$ for the municipality, $i = 2$ for the industrial sector, and $i = 3$ for the agricultural sector.

To solve the above problem through linear programming, the distribution of Q must be approximated by a set of discrete values (i.e. random seasonal flow can be discretized into three interval numbers representing low, medium and high flows with each having a probability of occurrence). Letting Q take values q_j with probabilities p_j ($j = 1, 2, \ldots, n$), we have:

$$E\left[\sum_{i=1}^{m} C_i S_{iQ}\right] = \sum_{i=1}^{m} C_i \left(\sum_{j=1}^{n} p_j S_{ij}\right) \tag{2}$$

Thus, model (1) can be reformulated as follows:

$$\text{Max } f = \sum_{i=1}^{m} NB_i T_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j C_i S_{ij} \tag{3a}$$

subject to:

$$\sum_{i=1}^{m}(T_i - S_{ij}) \leqslant q_j, \ \forall j, \tag{3b}$$

$$S_{ij} \leqslant T_i \leqslant T_{i\max}, \ \forall i,j, \tag{3c}$$

$$S_{ij} \geqslant 0, \ \forall i,j. \tag{3d}$$

where S_{ij} denotes the amount by which the water-allocation target (T_i) is not met when the seasonal flow is q_j with probability p_j.

Model (3) is effective in tackling uncertainty in water availability (q_j) presented as probability distributions. However, uncertainties may also exist in other parameters such as net benefits (NB_i), penalties (C_i), and water-allocation targets (T_i). In real-world problems, it is difficult to generate probability distributions for

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these parameters with small sample sizes. Thus, ILP can be integrated within the TSP framework to communicate uncertainties in NB_i, C_i, and T_i into the optimization process. This leads to an interval-parameter two-stage stochastic programming (ITSP) model as follows:

$$\text{Max } f^{\pm} = \sum_{i=1}^{m} NB_i^{\pm} T_i^{\pm} - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j C_i^{\pm} S_{ij}^{\pm} \tag{4a}$$

subject to:

$$\sum_{i=1}^{m}\left(T_i^{\pm} - S_{ij}^{\pm}\right) \leqslant q_j^{\pm}, \ \forall j, \tag{4b}$$

$$S_{ij}^{\pm} \leqslant T_i^{\pm} \leqslant T_{i\max}, \ \forall i,j, \tag{4c}$$

$$S_{ij}^{\pm} \geqslant 0, \ \forall i,j. \tag{4d}$$

where $NB_i^{\pm}$, $T_i^{\pm}$, $C_i^{\pm}$, $S_{ij}^{\pm}$, and $q_j^{\pm}$ are interval parameters/variables. An interval number is defined as a range with known lower and upper bounds (Huang, 1998). For example, letting a^- and a^+ be the lower and upper bounds of $a^{\pm}$, respectively, we have $a^{\pm} = [a^-, a^+] = \{t \in a | a^- \leqslant t \leqslant a^+\}$.

2.2. Robust two-step method

To solve model (4), a robust two-step method can be used to convert the interval-parameter linear programming problem into two submodels that correspond to the lower and upper bounds of the objective-function value (Fan & Huang, 2012). In model (4), since target values $(T_i^{\pm})$ are considered as uncertain inputs, it is difficult to determine whether their lower bounds (T_i^-) or upper bounds (T_i^+) correspond to the upper bound of the total net benefit (Huang & Loucks, 2000). Therefore, an optimized set of target values will be identified to achieve a maximized total net benefit. Accordingly, let $T_i^{\pm} = T_i^- + \Delta T_i y_i$, where $\Delta T_i = T_i^+ - T_i^-$, and $y_i (0 \leqslant y_i \leqslant 1)$ are decision variables that are used for identifying the optimized target values. By introducing decision variables (y_i), model (4) can thus be reformulated to:

$$\text{Max } f^{\pm} = \sum_{i=1}^{m} NB_i^{\pm}(T_i^- + \Delta T_i y_i) - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j C_i^{\pm} S_{ij}^{\pm} \tag{5a}$$

subject to:

$$\sum_{i=1}^{m}\left(T_i^- + \Delta T_i y_i - S_{ij}^{\pm}\right) \leqslant q_j^{\pm}, \ \forall j, \tag{5b}$$

$$S_{ij}^{\pm} \leqslant T_i^- + \Delta T_i y_i \leqslant T_{i\max}, \ \forall i,j, \tag{5c}$$

$$S_{ij}^{\pm} \geqslant 0, \ \forall i,j, \tag{5d}$$

$$0 \leqslant y_i \leqslant 1, \ \forall i. \tag{5e}$$

In model (5), since the objective is to maximize the total net benefit, the submodel corresponding to the lower bound of the objective-function value (f^-) can be first formulated as follows:

$$\text{Max } f^{-} = \sum_{i=1}^{m} NB_i^{-}(T_i^- + \Delta T_i y_i) - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j C_i^{+} S_{ij}^{+} \tag{6a}$$

subject to:

$$\sum_{i=1}^{m}\left(T_i^- + \Delta T_i y_i - S_{ij}^{+}\right) \leqslant q_j^{-}, \ \forall j, \tag{6b}$$

$$S_{ij}^{+} \leqslant T_i^- + \Delta T_i y_i \leqslant T_{i\max}, \ \forall i,j, \tag{6c}$$

$$S_{ij}^{+} \geqslant 0, \ \forall i,j, \tag{6d}$$

$$0 \leqslant y_i \leqslant 1, \ \forall i. \tag{6e}$$

where S_{ij}^{+} and y_i are decision variables, and their solutions of S_{ijopt}^{+} and y_{iopt} can be obtained through solving submodel (6). The optimized water-allocation targets can then be determined by calculating $T_{iopt}^{\pm} = T_i^- + \Delta T_i y_{iopt}$. Based on the solutions of submodel (6), the submodel corresponding to the upper bound of the objective-function value (f^+) can be formulated as follows:

$$\text{Max } f^{+} = \sum_{i=1}^{m} NB_i^{+}(T_i^- + \Delta T_i y_{iopt}) - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j C_i^{-} S_{ij}^{-} \tag{7a}$$

subject to:

$$\sum_{i=1}^{m}\left(T_i^- + \Delta T_i y_{iopt} - S_{ij}^{-}\right) \leqslant q_j^{+}, \ \forall j, \tag{7b}$$

$$S_{ij}^{-} \leqslant T_i^- + \Delta T_i y_{iopt}, \ \forall i,j, \tag{7c}$$

$$S_{ij}^{-} \leqslant S_{ijopt}^{+}, \ \forall i,j. \tag{7d}$$

$$S_{ij}^{-} \geqslant 0, \ \forall i,j. \tag{7e}$$

where S_{ij}^{-} are decision variables, and the solutions of S_{ijopt}^{-} can be generated through solving submodel (7). Model (5) attempts to obtain the lower and upper bounds on the maximum total net benefit $f_{opt}^{\pm} = \left[f_{opt}^{-}, f_{opt}^{+}\right]$. Submodel (6) obtains the lower bound f_{opt}^{-}. Submodel (7) obtains the upper bound f_{opt}^{+} based on the solutions of submodel (6). Consequently, the value of f_{opt}^{+} is dependent on the value of f_{opt}^{-}. By combining the solutions from two submodels, the final solutions of model (5) under the optimized water-allocation targets can thus be obtained as follows:

$$S_{ijopt}^{\pm} = \left[S_{ijopt}^{-}, S_{ijopt}^{+}\right], \ \forall i,j, \tag{8a}$$

$$f_{opt}^{\pm} = \left[f_{opt}^{-}, f_{opt}^{+}\right] \tag{8b}$$

where S_{ijopt}^{+} and f_{opt}^{-} are the solutions of submodel (6), and S_{ijopt}^{-} and f_{opt}^{+} are the solutions of submodel (7). Thus, the optimized water-allocation schemes are:

$$A_{ijopt}^{\pm} = T_{iopt}^{\pm} - S_{ijopt}^{\pm}, \ \forall i,j. \tag{9}$$

To facilitate more informed decision making in water resources management, sensitivity analysis is an indispensable tool to investigate the importance of uncertainties in model parameters. Conventional sensitivity analysis examines the effects of changes in a single parameter over its range assuming no changes in all the other parameters. Such a one-parameter-at-a-time strategy only reveals the individual impacts of parameters on the model response, but it has trouble detecting their latent interaction effects. Therefore, sensitivity analysis using statistical methods is desired for conducting a more comprehensive investigation of the importance of model parameters affecting system performance.

2.3. Multi-level Taguchi-factorial design

Factorial designs are the cornerstone of industrial experimentation and used extensively in industrial research and development for process improvement, among which the multi-level factorial design is a powerful statistical technique to study the effects of several independent variables (factors) with multiple levels on a dependent variable (response). As an extension of the most common two-level factorial design, the multi-level factorial design is particularly useful when there is a curvilinear relationship between the design factors and the response.

The most important case of the multi-level factorial design is the 3^k factorial design which consists of k factors with each at three levels. The three levels of factors are represented as low, medium, and high; they are often denoted by −1, 0, and +1, respectively. In the 3^k system of designs, there are 3^k treatment combinations with $3^k - 1$ degrees of freedom between them. These treatment combinations allow sums of squares to be computed for k main effects with each having two degrees of freedom; $\binom{k}{2}$ two-factor interactions with each having four degrees of freedom;...; and one k-factor interaction with 2^k degrees of freedom. In general, an h-factor interaction has 2^hdegrees of freedom. Furthermore, any h-factor interaction can be partitioned into 2^{h-1} orthogonal two-degrees-of-freedom components (Montgomery, 2001). For example, the three-factor interaction ABC can be subdivided into four orthogonal two-degrees-of-freedom components, denoted by ABC, ABC^2, AB^2C, and AB^2C^2, respectively. These components are useful in constructing complex designs. Since the number of experimental runs increases exponentially with the number of factors, the 3^k factorial design is too expensive to implement when there are a large number of factors under consideration.

Therefore, factor screening is necessary to identify a short list of important factors affecting the response when there is a long list of possibly influential factors to be investigated. Such a screening process generally tests only a fraction of the experimental runs of a full factorial design, leading to a considerable reduction in the computational effort. Taguchi's orthogonal arrays are highly fractional orthogonal designs proposed by Taguchi (1987), which can help study the effects of factors on the response mean and variations in a fast and economic way. These designs can be used to determine the main effects of factors using only a few experimental runs instead of having to test all possible combinations of the levels of the factors in the factorial design (Taguchi, 1986). Such a statistical technique allows for the maximum number of main effects to be estimated in an orthogonal manner, with a minimum number of experiments, resulting in a significant saving in the experimental time and resources. Nevertheless, the main limitation of the Taguchi method is the difficulty in detecting the potential interactions among factors due to its assumption that the interaction effects are unimportant and can be ignored. Thus, the concept of Taguchi's orthogonal arrays can be employed to identify important factors with an economic run size. Then the multi-level factorial design involving those important factors can be used to study their interactions. Such a sequential strategy of experimental designs can help ease the computational burden when there are many factors to be studied.

Fig. 1 provides an outline of the proposed methodology that incorporates ILP, TSP, Taguchi's orthogonal arrays, and the multi-level factorial design within a general framework. These methods can be classified into two categories: optimization techniques and statistical experimental designs. The optimization techniques can be used to conduct uncertainty analysis and policy analysis; the statistical experimental designs can be employed for factor screening and interaction detection. Such an integrated approach is capable of addressing parameter uncertainties and their interactions in a systematic manner.

3. Case study

3.1. Statement of problems

Uncertainty is inherent in water resources planning and management; it arises from a variety of sources, such as inadequate information, incomplete knowledge of parameter values, incorrect assumptions, and hydrologic variability (e.g., precipitation, stream flow, water quality). Decisions have to be made in the face of an uncertain future, complicating the decision-making process. Thus, water resources planning has always required an implicit handling of uncertainty. Over the past decades, a number of optimization methods have been developed for dealing with uncertainties in water resources management. These methods are effective in addressing parameter uncertainties that exist in the objective function and constraints; however, they cannot reflect the potential interactions among parameters and their effects on system performance. Actually, model parameters do not exist independently; they may interact in significant ways, intensifying the complexity in the decision-making process. It is thus necessary to perform a comprehensive analysis of parameter uncertainties and their interactions for supporting water resources management in an uncertain and complex environment.

3.2. Overview of the study system

The following case will be used to demonstrate the applicability of the developed approach. A water manager is responsible for allocating water from an unregulated reservoir to three users: municipality, industry, and agriculture. The problem under consideration is how to effectively allocate limited water to multiple users in order to achieve a maximized total net benefit. Table 1 provides maximum allowable water allocations and prescribed water-allocation targets, as well as the related economic data acquired from governmental reports and public surveys. The seasonal flows and the associated probabilities are shown in Table 2.

3.3. Result analysis

Interval solutions could be first obtained through the ITSP model introduced in Section 2. As shown in Fig. 2, the optimized water-allocation schemes would be obtained in the format of intervals with the lower and upper bounds. These interval solutions stem from uncertainty in input parameters. Water scarcity would occur if the amount of the available water is insufficient to satisfy the promised water-allocation targets, resulting in an increasing competition among municipality, industry and agriculture for the limited water supply. Thus, the identification of the appropriate water-allocation targets plays a key role in the decision-making process. The optimized water-allocation targets to three water users could be obtained by letting $T_{iopt}^{\pm} = T_i^- + \Delta T_i y_{iopt}$. The solutions of $T_{iopt}^{\pm}$ indicate that the optimized water-allocation targets would be 3.5×10^6 meter^3 for the municipal use, 3.2×10^6 meter^3 for the industrial use, and 4.7×10^6 meter^3 for the agricultural use, respectively. These prescribed targets would help achieve an optimized total net benefit of \$[324.4, 638.0] $\times 10^6$. The water shortage would be the difference between the water-allocation target and the actual water allocation (i.e. water shortage = promised target – water allocation) under a given stream flow condition with a probability of occurrence. Thus, the results reveal that there would be a water shortage [2.2, 3.2] $\times 10^6$ meter^3 for the industrial sector when the stream flow is low with a probability of 20% and a water shortage of [0, 3.4] $\times 10^6$ meter^3 for the agriculture sector when the stream flow is medium with a probability of 60%. Consequently, the water allocation would firstly be guaranteed for the municipal use, secondly for the industrial use, and lastly for the agricultural use when the water scarcity occurs. This is because the municipal water use could bring the highest profit when its water demand is satisfied; contrarily, it would be subject to the highest penalty if the promised water is not delivered.

In real-world problems, policy making is crucial to the sustainable water resources systems planning. In this study, variations in water-allocation targets correspond to different water resources management policies. The ITSP framework is capable of establishing an effective linkage between the water-allocation policies and the associated economic implications. Solutions under various policy scenarios could thus be obtained by letting the water-allocation targets $(T_i^{\pm})$ have different deterministic values. As shown in Table 3, $T_i^{\pm} = T_i^-$ $(i = 1, 2, 3)$ implies that the water-allocation targets (expressed as intervals) for the municipality, industrial sector,

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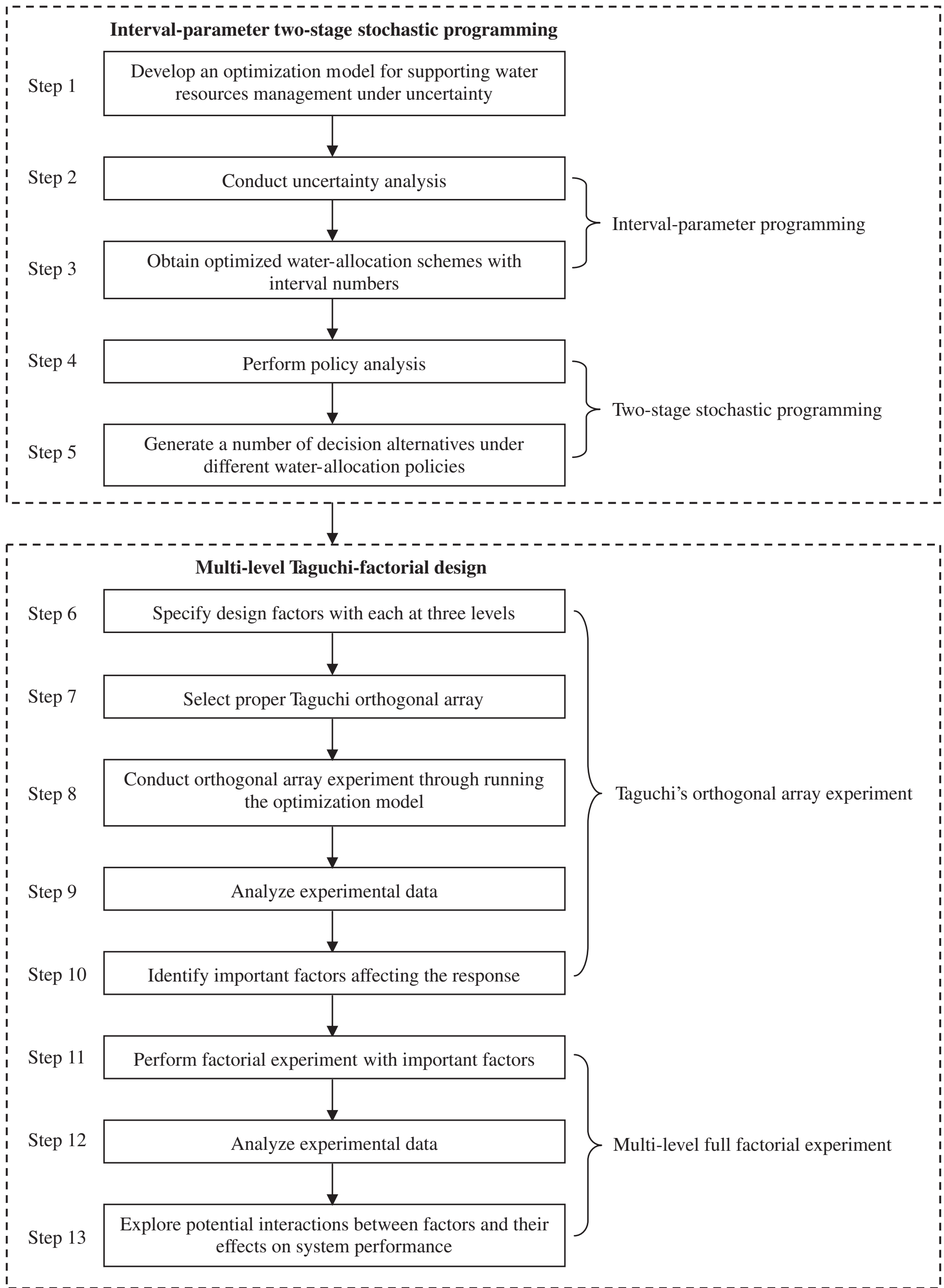

Fig. 1. Outline of the proposed methodology.

Table 1
Water-allocation targets (10^6 meter^3) and the related economic data (\$/meter^3).

	User		
	Municipal ($i = 1$)	Industrial ($i = 2$)	Agricultural ($i = 3$)
Maximum allowable allocation (T_{imax})	8	8	8
Water allocation target ($T_i^{\pm}$)	[2.7, 3.7]	[3.2, 5.2]	[4.7, 7.7]
Net benefit when water demand is satisfied ($NB_i^{\pm}$)	[90, 110]	[45, 55]	[28, 32]
Reduction of net benefit when demand is not delivered ($C_i^{\pm}$)	[220, 280]	[60, 90]	[50, 70]

Table 2
Seasonal flows (10^6 meter^3) and the associated probabilities.

	Seasonal flow $\left(q_j^{\pm}\right)$	Probability (p_j) (%)
Low flow ($j = 1$)	[3.5, 4.5]	20
Medium flow ($j = 2$)	[8.0, 12.0]	60
High flow ($j = 3$)	[15.0, 19.0]	20

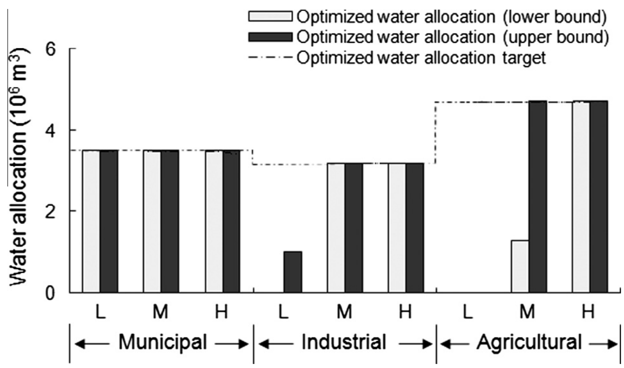

Fig. 2. Optimized water-allocation patterns under low (L), medium (M), and high (H) flows.

and agricultural sector reach their lower bounds. Such a conservative policy would generate both less water shortage and less water allocation, but a higher risk of wasting available water resources. Contrarily, $T_i^{\pm} = T_i^{+} (i = 1, 2, 3)$ implies that the water-allocation targets reach their upper bounds. Despite such an optimistic policy would lead to more water allocation, it would face a higher risk of system failure when the promised water is not delivered due to the inadequate water supply. $T_i^{\pm} = T_i^{\text{mid}}$ $(i = 1, 2, 3)$ implies that the water-allocation targets reach their mid-values, representing a neutral water-allocation policy. From the economic point of view, the optimistic policy would bring the highest total net benefit of $\$671.6 \times 10^6$ under advantageous conditions (e.g., when the stream flow is high), but at the same time the lowest total net benefit of $\$175.6 \times 10^6$ under demanding conditions (e.g., when the stream flow is low). Conversely, the conservative policy would generate the lowest upper-bound total net benefit of $\$559.6 \times 10^6$ and the highest lower-bound total net benefit of $\$300.4 \times 10^6$, indicating a relatively low system risk. As the water-allocation policies are directly associated with economic benefits and system-failure risks, it is indispensable to perform the policy analysis for supporting water resources management under uncertainty.

To address parameter uncertainties in a thorough manner, not only the uncertain parameters need to be incorporated into the optimization process, but also their potential interactions and the consequent effects on system performance should be analyzed. Table 4 shows all uncertain parameters in the ITSP model; they are chosen as the factors of interest. To carry out the factorial experiment, these factors are denoted as A, B, C, D, E, F, G, H, and J, respectively. As all the factors are present at three levels, such a three-level factorial design with nine factors would require 19,683 experimental runs, resulting in a tremendous computational effort. At the initial stage of the factorial experiment, an efficient screening procedure is thus necessary to identify a subset of the factors that have a significant effect on the model response and eliminate those unimportant factors from further analysis. A full factorial experiment can then be performed on the smaller subset of factors. Such a sequential strategy can help achieve a remarkable savings of the computational resources.

The concept of Taguchi's orthogonal arrays is thus proposed as an effective and efficient method for investigating the main effects of the nine factors at three levels. Table 5 provides the Taguchi's L_{27} (3^9) orthogonal array used for the multi-level factorial experiment, as well as the corresponding optimization results. Such an orthogonal array design only requires 27 experimental runs for estimating the main effects of the nine factors. The results indicate that variations of these factors would cause a noticeable difference in total net benefits. It is thus necessary to examine their effects and analyze those dominant factors as well as their potential interactions.

Fig. 3 presents the main effects plot for the nine factors at three levels, which is helpful in visualizing the magnitudes of main effects of factors. In the main effects plot, the points are the means of total net benefits at the various levels of each factor, with a reference line drawn at the grand mean of total net benefits. This plot reveals that factor H has the greatest magnitude of the main effect upon the total net benefit. The total net benefit would increase from \$443.7 to $\$516.7 \times 10^6$ and then from \$516.7 to $\$588.5 \times 10^6$, if the amount of the medium flow varies from its low level of 8 to its mid-level of 10×10^6 meter^3 and then from its mid-level of 10 to its high level of 12×10^6 meter^3, respectively. This is because the medium flow has the highest probability of occurrence (60%); any change in the amount for the medium flow would cause a considerable variation in the total net benefit. Contrarily, factor J with a near-zero slope has the smallest contribution to the variability of the total net benefit, since no water shortage would occur and the water demands of all users would be always satisfied when the stream flow is high.

As shown in Table 6, the effects of factors are estimated based on the means (averages) of total net benefits. The results indicate that factor H has the largest delta value of $\$144.8 \times 10^6$ in means of total net benefits (delta value is calculated as the difference between maximum and minimum means of total net benefits), implying that factor H has the most significant effect on the model

Table 3
Solutions (10^6 meter^3) under different scenarios of the water-allocation targets.

	$T_i^{\pm} = T_i^{-}$			$T_i^{\pm} = T_i^{+}$			$T_i^{\pm} = T_i^{(\text{mid})}$		
	$i = 1$	$i = 2$	$i = 3$	$i = 1$	$i = 2$	$i = 3$	$i = 1$	$i = 2$	$i = 3$
Target $(T_i^{\pm})$	2.7	3.2	4.7	3.7	5.2	7.7	3.2	4.2	6.2
Shortage $\left(S_{ij}^{\pm}\right)$:									
$j = 1$	0	[1.4, 2.4]	4.7	[0, 0.2]	[4.4, 5.2]	7.7	0	[2.9, 3.9]	6.2
$j = 2$	0	0	[0, 2.6]	0	[0, 0.9]	[4.6, 7.7]	0	0	[1.6, 5.6]
$j = 3$	0	0	0	0	0	[0, 1.6]	0	0	0
Allocation $\left(A_{ij}^{\pm}\right)$:									
$j = 1$	2.7	[0.8, 1.8]	0	[3.5, 3.7]	[0, 0.8]	0	3.2	[0.3, 1.3]	0
$j = 2$	2.7	3.2	[2.1, 4.7]	3.7	[4.3, 5.2]	[0, 3.1]	3.2	4.2	[0.6, 4.6]
$j = 3$	2.7	3.2	4.7	3.7	5.2	[6.1, 7.7]	3.2	4.2	6.2
Total net benefit	$f^{\pm}$ = \$[300.4, 559.6] × 10^6			$f^{\pm}$ = \$[175.6, 671.6] × 10^6			$f^{\pm}$ = \$[258.4, 636.6] × 10^6		

Table 4
Investigated factors at three levels.

Symbol	Factor	Level		
		Low (−1)	Medium (0)	High (+1)
A	Net benefit to municipal user per meter^3 of water allocated (\$/meter^3)	90	100	110
B	Net benefit to industrial user per meter^3 of water allocated (\$/meter^3)	45	50	55
C	Net benefit to agricultural user per meter^3 of water allocated (\$/meter^3)	28	30	32
D	Loss to municipal user per meter^3 of water not delivered (\$/meter^3)	220	250	280
E	Loss to industrial user per meter^3 of water not delivered (\$/meter^3)	60	75	90
F	Loss to agricultural user per meter^3 of water not delivered (\$/meter^3)	50	60	70
G	Amount of low flow (10^6 meter^3)	3.5	4.0	4.5
H	Amount of medium flow (10^6 meter^3)	8	10	12
J	Amount of high flow (10^6 meter^3)	15	17	19

Table 5
Taguchi's L_{27} (3^9) orthogonal array and the corresponding optimization results.

Run	Factor									Total net benefit (\$$10^6$)
	A	B	C	D	E	F	G	H	J	
1	−1	−1	−1	−1	−1	−1	−1	−1	−1	409.7
2	−1	−1	−1	−1	0	0	0	0	0	451.1
3	−1	−1	−1	−1	+1	+1	+1	+1	+1	510.4
4	−1	0	0	0	−1	−1	−1	0	0	506.6
5	−1	0	0	0	0	0	0	+1	+1	548.1
6	−1	0	0	0	+1	+1	+1	−1	−1	373.8
7	−1	+1	+1	+1	−1	−1	−1	+1	+1	600.8
8	−1	+1	+1	+1	0	0	0	−1	−1	434.3
9	−1	+1	+1	+1	+1	+1	+1	0	0	483.2
10	0	−1	0	+1	−1	0	+1	−1	0	440.2
11	0	−1	0	+1	0	+1	−1	0	+1	462.8
12	0	−1	0	+1	+1	−1	0	+1	−1	566.6
13	0	0	+1	−1	−1	0	+1	0	+1	541.6
14	0	0	+1	−1	0	+1	−1	+1	−1	571.8
15	0	0	+1	−1	+1	−1	0	−1	0	475.4
16	0	+1	−1	0	−1	0	+1	+1	−1	620.8
17	0	+1	−1	0	0	+1	−1	−1	0	402.6
18	0	+1	−1	0	+1	−1	0	0	+1	544.4
19	+1	−1	+1	0	−1	+1	0	−1	+1	469.4
20	+1	−1	+1	0	0	−1	+1	0	−1	570.4
21	+1	−1	+1	0	+1	0	−1	+1	0	588.2
22	+1	0	−1	+1	−1	+1	0	0	−1	545.0
23	+1	0	−1	+1	0	−1	+1	+1	0	637.6
24	+1	0	−1	+1	+1	0	−1	−1	+1	443.8
25	+1	+1	0	−1	−1	+1	0	+1	0	652.4
26	+1	+1	0	−1	0	−1	+1	−1	+1	544.0
27	+1	+1	0	−1	+1	0	−1	0	−1	545.6

response. Contrarily, factor J has the smallest delta value of \$$3.1 \times 10^6$ in means of total net benefits and thus has little influence on the response. The significance of all the factors affecting the economic objective is determined according to the delta values. Accordingly, factors A, B, C, E, F, G, and H are identified as the dominant factors, while factors D and J are unimportant factors and thus removed from further factorial experiments.

Based on the results of the Taguchi's orthogonal array experiment, a full factorial experiment involving those important factors was performed to analyze their interaction effects on system performance. Such a three-level factorial design with seven factors requires 2187 experimental runs for estimating the joint effects of factors. The half-normal plot is a graphical technique used to help distinguish between important and unimportant effects of factors; it is particularly useful for analyzing the unreplicated factorial experiments. Fig. 4 presents the half-normal plot of effects, which is a plot of the absolute values of effect estimates against their cumulative normal probabilities. Effects that lie along the straight line are deemed to be insignificant, whereas prominent effects lie away from the line. Accordingly, the important effects that emerge from this analysis are the main effects of factors H, A, F, B, E, G, and C, as well as the interaction effects of factors F and H, E and F, B and F, and B and E.

The interaction plot for factors F and H at three levels is presented in Fig. 5. This plot shows the total net benefit versus the amount of the medium flow for each of the three different agricultural costs. It reveals that the change in the total net benefit differs across the three levels of factor H depending on the level of factor F, implying that an interaction between these factors occurs that their effects are dependent upon each other. The highest total net benefit of \$$600.7 \times 10^6$ would be obtained when factor F is at its low level and factor H is at its high level. Fig. 6 presents the full interactions plot matrix for factors E, F, and H at three levels, in which each pair of factors provides two panels. This plot reveals that the three lines of factor F would decline as factor E varies across its low, medium, and high levels, whereas the dashed line representing the high level of factor F decreases faster than the other two, implying an interaction between this pair of factors. Although all the three lines of factor E would go up at different rates when factor H increases across its low, medium, and high levels, their interaction does not seem as strong as it does for factors E and F.

4. Discussion

In this study, the water allocation problem was also solved using the factorial two-stage stochastic programming (FTSP) method presented by Zhou and Huang (2011). Table 7 shows the effects of significant factors and their interactions identified through FTSP. The results reveal that there are more significant two-factor interactions (e.g., factors A and C and factors E and H) identified by using FTSP compared to using the MTTSP approach. For example, the results of FTSP indicate that the interaction between the loss to industrial user per meter^3 of water not delivered and the amount of medium flow has a significant contribution to the variability of the total net benefit, while such an interaction has little effect on the economic objective in the light of the results of MTTSP. This is because FTSP used a 2^{9-3} fractional factorial design in which two-factor interaction effects might be confounded with other two-factor interactions, resulting in difficulty in separating two-factor interactions from one another. In comparison, MTTSP used a full factorial design that allowed a clear estimation of all two-factor interactions, avoiding the misleading information. Moreover, the Taguchi's orthogonal array used in MTTSP is a highly fractional orthogonal design that can identify the main effects of factors using only a small number of experimental runs (27 runs in this study), leading to a remarkable saving in the experimental time and resources.

Besides, FTSP used the most popular two-level factorial design. Fig. 7 presents the full interaction plot matrix for factors E, F, and H at two levels. This plot reveals that the model response is linear over the range of the factor levels. However, the nonlinear relationships between the factors and the response inherently exist in

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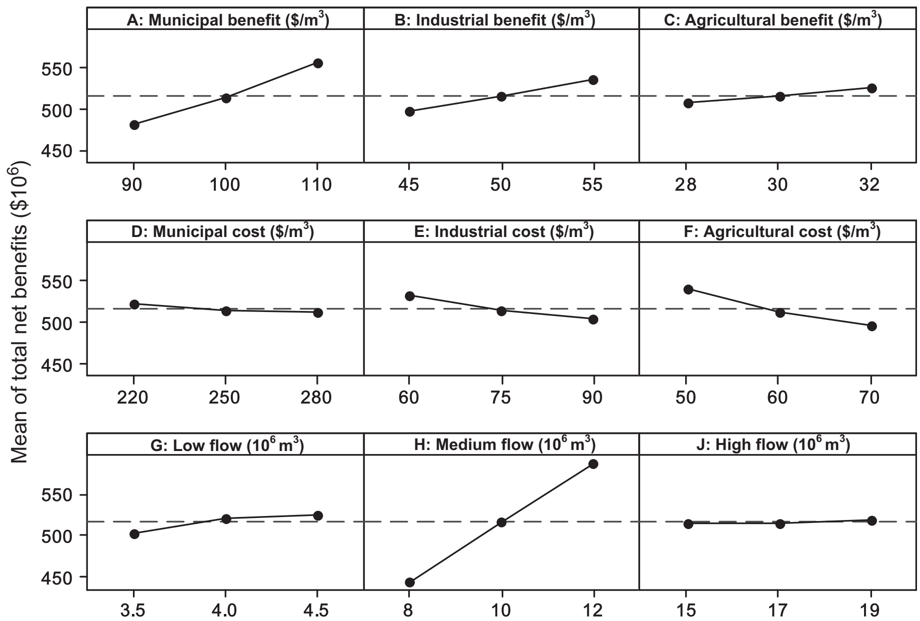

Fig. 3. Main effects plot.

Table 6
Response table for means of total net benefits ($10^6).

Level	Factor								
	A	**B**	**C**	D	**E**	**F**	**G**	**H**	J
1	479.8	496.5	507.3	522.4	531.8	539.5	503.5	443.7	515.3
2	514.0	516.0	515.6	513.8	513.6	512.6	520.7	516.7	515.3
3	555.2	536.5	526.1	512.7	503.5	496.8	524.7	588.5	518.4
Delta (Max–Min)	75.4	39.9	18.9	9.7	28.3	42.7	21.1	144.8	3.1
Rank	2	4	7	8	5	3	6	1	9

Note: Factors in bold are identified as the significant factors.

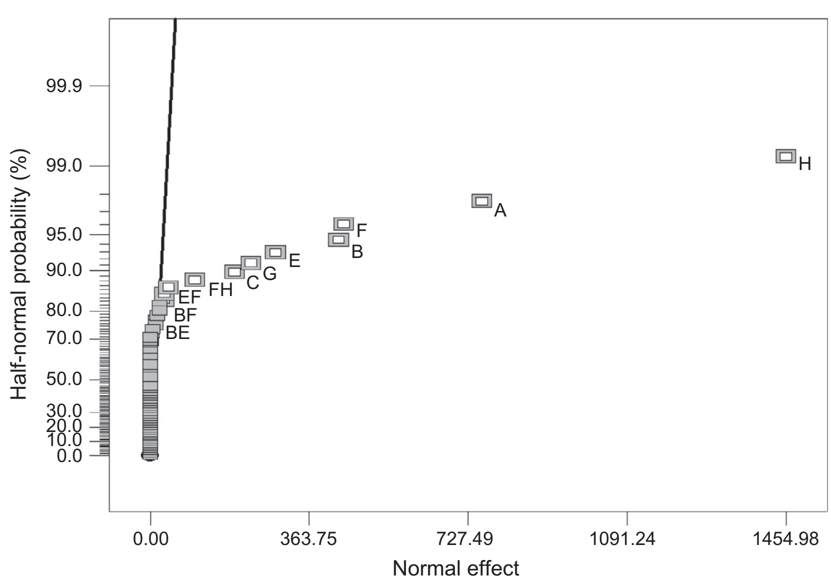

Fig. 4. Half-normal plot of effects.

many practical applications. The two-level factorial experiment can hardly reveal the nonlinear effects. In comparison, MTTSP is able to detect the curvature in the response function (see Fig. 6).

In this study, MTTSP is capable not only of communicating uncertainties presented in the formats of intervals/probability distributions into the optimization process and reflecting them in the resulting solutions, but also of establishing an effective linkage between the prescribed water-allocation policies and the associated economic implications, providing an in-depth policy analysis. On the other hand, MTTSP employed the Taguchi' orthogonal array

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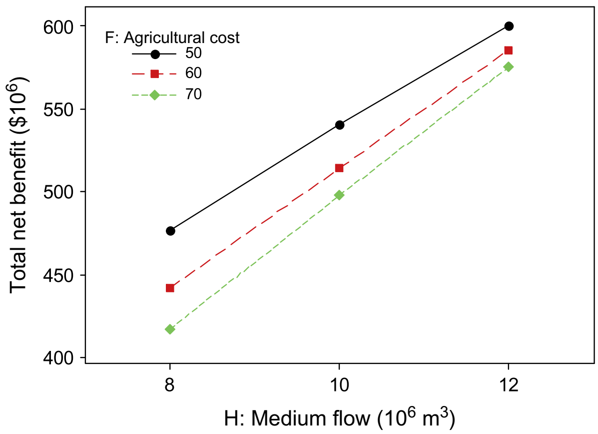

Fig. 5. Interaction plot for factors F and H at three levels.

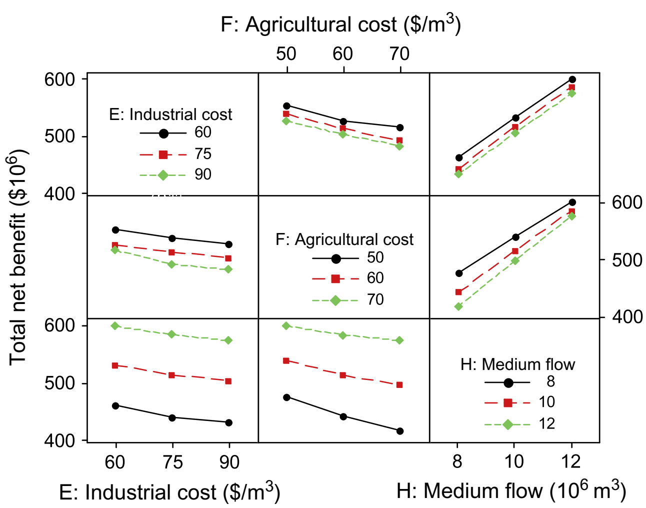

Fig. 6. Full interaction plot matrix for factors E, F, and H at three levels.

Table 7
Effects of significant factors and their interactions.

Factor	Standardized effect	Sum of squares	Contribution (%)
A	73.87	87320.25	17.92
B	43.70	30555.04	6.27
C	18.83	5670.09	1.16
E	−30.20	14592.64	3.00
F	−40.50	26244.00	5.39
G	22.27	7938.81	1.63
H	139.00	309100.00	63.46
AC	4.90	384.16	0.079
BE	−4.87	380.25	0.078
BF	−3.37	182.25	0.037
EF	−4.52	327.61	0.067
EH	4.17	278.89	0.057
FH	15.72	3956.41	0.81

as a screening technique to identify important factors with an economic number of experimental runs, and then performed a full factorial experiment involving those important factors to investigate their potential interactions. Such a sequential strategy of experimental designs is useful in analyzing the interactions for a large number of factors of interest in a computationally efficient manner. Moreover, the multi-level factorial design used in MTTSP is capable of detecting the curvature in the factor-response relationship, while it is impossible to reflect such a nonlinear effect

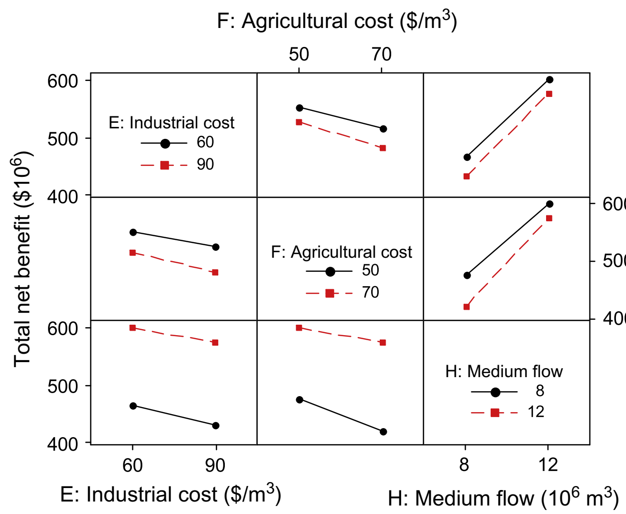

Fig. 7. Full interaction plot matrix for factors E, F, and H at two levels.

with the two-level factorial design due to its assumption of linearity over the range of factor levels. In real-world problems, MTTSP is thus helpful for decision makers to identify the dominant factors and their latent interactions in the decision-making process, as well as to make sound decisions of water allocation under compound and interactive uncertainties.

5. Conclusions

In this study, a MTTSP approach was developed for the analysis of parameter uncertainties and their interactions. MTTSP incorporated ILP, TSP, Taguchi's orthogonal arrays, and the multi-level factorial design within a general framework. Such an integrated approach was capable of performing uncertainty analysis, policy analysis, factor screening, and interaction detection in a systematic and computational efficient manner.

A water resources management problem was used to demonstrate the applicability of the proposed method. Interval solutions were generated for the objective function and decision variables so that decision makers could identify desired water-allocation schemes with maximized total net benefits. A variety of decision alternatives were also generated under different scenarios of water-allocation targets, which could help decision makers to formulate appropriate water resources management policies according to practical situations. The results obtained from the Taguchi's orthogonal array experiment were helpful in identifying the significant factors affecting the means of total net benefits. Then the findings from the factorial experiment revealed the potential interactions among those important factors at three levels and their curvature effects on the model response, as well as the valuable information hidden beneath their interrelationships.

This study is a first attempt to support water resources management by using the proposed MTTSP approach. This approach would also be applicable to other environmental management problems in the presence of correlated parameters. The two-stage stochastic program in this study was solved based on approximating the underlying probability distribution by a discrete set of representative scenarios, and decisions were then made in two stages. Such a two-stage decision procedure is thus incapable of dealing with large-scale optimization problems that often involve a multi-stage decision process. Therefore, one potential extension of this research is to develop a multi-stage stochastic program for tackling large-scale dynamic decision problems. Nevertheless, the

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computational complexity for solving the stochastic program would be getting worse with an increasing number of stages in combination with a large number of possible random outcomes at each stage. It is thus desired to integrate multi-stage stochastic programming with other optimization techniques such as Benders Decomposition for solving large-scale stochastic optimization problems in a computationally efficient manner.

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References

Abdelaziz, F. B. (2012). Solution approaches for the multiobjective stochastic programming. *European Journal of Operational Research, 216*, 1–16.

Adenso-Díaz, B., & Laguna, M. (2006). Fine-tuning of algorithms using fractional experimental designs and local search. *Operations Research, 54*, 99–114.

Birge, J. R., & Louveaux, F. V. (1988). A multicut algorithm for two-stage stochastic linear programs. *European Journal of Operational Research, 34*, 384–392.

Birge, J. R., & Louveaux, F. V. (1997). *Introduction to stochastic programming*. New York: Springer.

Box, G. E. P., & Behnken, D. W. (1960). Some new three level designs for the study of quantitative variables. *Technometrics, 2*, 455–475.

Bravo, M., & Gonzalez, I. (2009). Applying stochastic goal programming: A case study on water use planning. *European Journal of Operational Research, 196*, 1123–1129.

Chung, G., Lansey, K., & Bayraksan, G. (2009). Reliable water supply system design under uncertainty. *Environmental Modelling & Software, 24*, 449–462.

Fan, Y. R., & Huang, G. H. (2012). A robust two-step method for solving interval linear programming problems within an environmental management context. *Journal of Environmental Informatics, 19*, 1–9.

Gaivoronski, A., Sechi, G. M., & Zuddas, P. (2012). Cost/risk balanced management of scarce resources using stochastic programming. *European Journal of Operational Research, 216*, 214–224.

Guo, P., Huang, G. H., Zhu, H., & Wang, X. L. (2010). A two-stage programming approach for water resources management under randomness and fuzziness. *Environmental Modelling & Software, 25*, 1573–1581.

Huang, G. H. (1998). A hybrid inexact-stochastic water management model. *European Journal of Operational Research, 107*, 137–158.

Huang, G. H., Baetz, B. W., & Patry, G. G. (1992). A grey linear programming approach for municipal solid waste management planning under uncertainty. *Civil Engineering and Environmental Systems, 9*, 319–335.

Huang, G. H., & Loucks, D. P. (2000). An inexact two-stage stochastic programming model for water resources management under uncertainty. *Civil Engineering and Environmental Systems, 17*, 95–118.

Lewis, S. M., & Dean, A. M. (2001). Detection of interactions in experiments on large numbers of factors. *Journal of the Royal Statistical Society: Series B (Statistical Methodology), 63*, 633–672.

Li, Y. P., Huang, G. H., Huang, Y. F., & Zhou, H. D. (2009). A multistage fuzzy-stochastic programming model for supporting sustainable water-resources allocation and management. *Environmental Modelling & Software, 24*, 786–797.

Li, Y. P., Huang, G. H., Nie, S. L., & Liu, L. (2008). Inexact multistage stochastic integer programming for water resources management under uncertainty. *Journal of Environmental Management, 88*, 93–107.

Lin, Y. P., Huang, G. H., Lu, H. W., & He, L. (2008). A simulation-aided factorial analysis approach for characterizing interactive effects of system factors on composting processes. *Science of the Total Environment, 402*, 268–277.

Mabilia, R., Scipioni, C., Vegliò, F., & Tomasi Scianò, M. C. (2010). Fractional factorial experiments using a test atmosphere to assess the accuracy and precision of a new passive sampler for the determination of formaldehyde in the atmosphere. *Atmospheric Environment, 44*, 3942–3951.

Maqsood, I., Huang, G. H., & Yeomans, J. S. (2005). An interval-parameter fuzzy two-stage stochastic program for water resources management under uncertainty. *European Journal of Operational Research, 167*, 208–225.

Montgomery, D. C. (2001). *Design and analysis of experiments* (5th ed.). New York: John Wiley & Sons Inc.

Onsekizoglu, P., Bahceci, K. S., & Acar, J. (2010). The use of factorial design for modeling membrane distillation. *Journal of Membrane Science, 349*, 225–230.

Qin, X. S., Huang, G. H., & Chakma, A. (2008). Modeling groundwater contamination under uncertainty: A factorial-design-based stochastic approach. *Journal of Environmental Informatics, 11*, 11–20.

Qin, X. S., Huang, G. H., Zeng, G. M., Chakma, A., & Huang, Y. F. (2007). An interval-parameter fuzzy nonlinear optimization model for stream water quality management under uncertainty. *European Journal of Operational Research, 180*, 1331–1357.

Taguchi, G. (1986). *Introduction to quality engineering: Designing quality into products and processes*. Asian Productivity Organization, Tokyo.

Taguchi, G. (1987). *System of experimental design: Engineering methods to optimize quality and minimize costs* (Vols. 1 & 2). New York: UNIPUB/Kraus International Publications.

Teegavarapu, R. S. V. (2010). Modeling climate change uncertainties in water resources management models. *Environmental Modelling & Software, 25*, 1261–1265.

UN-Water (2006). *Coping with water scarcity: A strategic issue and priority for system-wide action.*

Wang, S., & Huang, G. H. (2011). Interactive two-stage stochastic fuzzy programming for water resources management. *Journal of Environmental Management, 92*, 1986–1995.

Wang, S., & Huang, G. H. (2012). Identifying optimal water resources allocation strategies through an interactive multi-stage stochastic fuzzy programming approach. *Water Resources Management, 26*, 2015–2038.

Wang, S., & Huang, G. H. (2013). A coupled factorial-analysis-based interval programming approach and its application to air quality management. *Journal of the Air & Waste Management Association, 63*, 179–189.

Wang, S., Huang, G. H., & Veawab, A. (2013). A sequential factorial analysis approach to characterize the effects of uncertainties for supporting air quality management. *Atmospheric Environment, 67*, 304–312.

Wu, C. F. J., & Hamada, M. S. (2009). *Experiments: Planning, analysis, and optimization* (2nd ed.). New Jersey: John Wiley & Sons Inc.

Xu, H., Chen, S. W., & Wu, C. F. J. (2004). Optimal projective three-level designs for factor screening and interaction detection. *Technometrics, 46*, 280–292.

Zhou, Y., & Huang, G. H. (2011). Factorial two-stage stochastic programming for water resources management. *Stochastic Environmental Research and Risk Assessment, 25*, 67–78.

Zhou, Y., Huang, G. H., & Yang, B. (2013). Water resources management under multi-parameter interactions: A factorial multi-stage stochastic programming approach. *Omega, 41*, 559–573.