A Fuzzy Petri Net-Based Expert System and Its Application to Damage Assessment of Bridges

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***Abstract*— In this paper, a fuzzy Petri net approach to modeling fuzzy rule-based reasoning is proposed to bring together the possibilistic entailment and the fuzzy reasoning to handle uncertain and imprecise information. The three key components in our fuzzy rule-based reasoning—fuzzy propositions, truth-qualified fuzzy rules, and truth-qualified fuzzy facts—can be formulated as fuzzy places, uncertain transitions, and uncertain fuzzy tokens, respectively. Four types of uncertain transitions—inference, aggregation, duplication, and aggregation-duplication transitions—are introduced to fulfill the mechanism of fuzzy rule-based reasoning. A framework of integrated expert systems based on our fuzzy Petri net, called fuzzy Petri net-based expert system (FPNES), is implemented in Java. Major features of FPNES include knowledge representation through the use of hierarchical fuzzy Petri nets, a reasoning mechanism based on fuzzy Petri nets, and transformation of modularized fuzzy rule bases into hierarchical fuzzy Petri nets. An application to the damage assessment of the Da-Shi bridge in Taiwan is used as an illustrative example of FPNES.**

***Index Terms*— Damage assessment, fuzzy Petri net-based expert systems, fuzzy truth value, hierarchical fuzzy Petri nets, possibilistic entailment.**

I. INTRODUCTION

IT IS widely recognized that the trend of integrating expert systems with other technologies will continue to the next generation of expert systems [17], [24], [27], [32], [33]. A number of researchers have reported progress toward the integration of expert systems with Petri nets. Petri nets with a powerful modeling and analysis ability are capable of providing a basis for variant purposes, such as knowledge representation [38], [47], reasoning mechanisms [3], [46], knowledge acquisition [6], and knowledge verification [50], [58]. There are several rationales behind which to base a computational paradigm for expert systems on Petri net theory.

- Petri nets achieve the structuring of knowledge within rule bases, which can express the relationships among rules and help experts construct and modify rule bases [12].
- The Petri net's graphic nature provides the visualization of the dynamic behavior of rule-based reasoning.
- Petri nets make it easier to design an efficient reasoning algorithm.
- The Petri net's analytic capability provides a basis for developing a knowledge verification technique.
- The underlying relationship of concurrency among rules activation can be modeled by Petri nets, which is an important aspect where real-time performance is crucial [5].

To model fuzzy rule-based reasoning through the use of fuzzy Petri nets, several important issues need to be addressed.

- Is partial matching considered?
- Does the Petri net's firing rule that tokens will be removed from the input places of a transition after the transition fired remain unchanged? It should be noted that the firing rule in Petri nets is a basis for controlling the evolution of markings in the execution process. To modify the firing rule is to change the evolution of markings.
- Is the proposed algorithm consistent with the rule-based reasoning?
- Is the proposed algorithm consistent with the execution of Petri nets?

We have examined a variety of related literature based on these issues [30]. Looney's approach [34] did not allow partial matching and changed the firing rule: after firing an enabled transition, the tokens in all input places of this transition are not removed, and new tokens are generated and deposited in all output places of this transition. Chen *et al.*'s approach [7] takes care of not only fuzziness but also uncertainty (i.e., modeled as certainty factors) for representing a fuzzy rule base. However, only exact matching is allowed. One of the problems arising from their algorithm is in the case that the intermediate places have more than one input arc. Therefore, the algorithm cannot have two or more rules that will result in a same conclusion. Bugarin *et al.*'s approach [3] is based on compositional rule of inference. It is not appropriate for large systems since the arrangement of the linking transitions in a net and applied algorithm depend on the initial markings. Konar *et al.*'s approach [25] has improved Chen *et al.*'s [7] algorithm to deal with the case that there exist intermediate places with multi-input arcs. However, adopting Looney's [34] modifications on the firing rule makes their algorithm inconsistent with the execution of the Petri net. Scarpelli *et al.* [45], [46] have proposed high-level fuzzy Petri nets for modeling fuzzy reasoning based on the compositional rule of inference. After carrying out their proposed algorithm to extract the subnet from the entire net, the subnet with

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concurrency cannot be executed as a Petri net since only one path is shown.

In this paper, a fuzzy Petri nets approach to modeling fuzzy rule-based reasoning is proposed to bring together the possibilistic entailment and the fuzzy reasoning in order to handle uncertain and imprecise information. The three key components in our fuzzy rule-based reasoning (fuzzy propositions, truth-qualified fuzzy rules, and truth-qualified fuzzy facts) can be formulated as fuzzy places, uncertain transitions, and uncertain fuzzy tokens, respectively. Four types of uncertain transitions (inference, aggregation, duplication, and aggregation-duplication transitions) are introduced to fulfill the mechanism of fuzzy rule-based reasoning.

A framework of integrated expert systems based on our fuzzy Petri net, called fuzzy Petri net-based expert system (FPNES), is implemented in Java with a client–server architecture. Major features of FPNES include knowledge representation through the use of hierarchical fuzzy Petri nets, a reasoning mechanism based on fuzzy Petri nets, and transformation of modularized fuzzy rule bases into hierarchical fuzzy Petri nets. An application to the damage assessment of the Da-Shi bridge in Taiwan is used as an illustrative example of FPNES.

The organization of this paper is as follows. Background work on our fuzzy rule-based reasoning is described in the next section. In Section III, a fuzzy Petri nets approach to modeling fuzzy rule-based reasoning is introduced. In Section IV, a framework of FPNES is proposed. In Section V, an application of FPNES to the damage assessment of the Da-Shi bridge in Taiwan is used as an illustration. Related work is described in Section VI. Last, a summary of our approach and its potential benefits are given in the Section VII.

II. Background Work on Fuzzy Rule-Based Reasoning

The distinction between imprecise and uncertain information can be best explained by the canonical form representation (i.e., a quadruple of attribute, object, value, confidence) proposed by Dubois and Prade [10], [43]. Imprecision implies the absence of a sharp boundary of the value component of the quadruple, whereas uncertainty is related to the confidence component of the quadruple, which is an indication of our reliance on the information. To perform reasoning for both imprecise and uncertain information, two important issues need to be addressed.

- Any improvement of the confidence level for a piece of information can only be achieved at the expense of the specificity of the information, and vise versa [51], [56].
- The matching between a fact and the premise of a rule is not exact, but only partial [2], [56].

We have roughly classified the existing approaches in dealing with both imprecise and uncertain information into three categories based on their treatments for the two issues [30], [31].

1) An uncertainty-qualified fuzzy proposition is translated into a proposition whose confidence level is certain but with less specific information, while partial matching is used to modify the intended meaning of conclusions. This approach was advocated by Yager [51] and Zadeh [56]. Zadeh proposed three uncertainty qualifications for fuzzy propositions: probability, possibility, and truth qualifiers; Yager focused on the certainty qualifier.
2) The degree of partial matching is used to influence the confidence level of conclusions, which was adopted by researchers such as Martin-Clouaire *et al.* [35], Ogawa *et al.* [40], and Umano [49]. Ogawa *et al.* combined certainty factors and fuzzy sets to represent uncertain and imprecise information in an expert system, SPERIL-2. Martin-Clouaire *et al.* attached possibility and necessity degrees to fuzzy propositions. Umano employed the fuzzy truth value for the uncertainty qualifier of fuzzy propositions.
3) No partial matching is allowed in Godo *et al.* [16] and Ishizuka *et al.* [21]. Ishizuka *et al.* extended Dempster–Shafer's evidence theory to a fuzzy set in the expert system SPERIL-1. Godo *et al.* used the fuzzy truth value as an uncertainty qualifier of fuzzy propositions.

Note that the first kind of research results in a completely certain conclusion whose intended meaning has been changed. On the other hand, the second one produces a new confidence level for a conclusion without modifying its intended meaning. The third one can be viewed as a special case of the second one. It is obvious that these inference strategies are somewhat limited due to the fact that either the intended meaning is required to be unchanged or the confidence level has to be completely certain.

We have proposed the use of truth-qualified fuzzy propositions as the representation of imprecise and uncertain information for its capability to express the possibility of the degree of truth [30], [31]. The inference rule for the truth-qualified fuzzy propositions has been developed based on our proposed possibilistic entailment. It is not only a generalization of Zadeh's generalized *modus ponens* [56] but also an uncertain reasoning for classical propositions with necessity and possibility pairs.

A. Possibilistic Entailment

A possibilistic reasoning has been proposed for classical propositions r_i weighted by the lower bounds N_{r_i} of necessity measures and the upper bounds Π_{r_i} of possibility measures [i.e., $N(r_i) \geq N_{r_i}$ and $\Pi(r_i) \leq \Pi_{r_i}$] [30], [31], which is expressed as shown in (1) at the bottom of the next page, where r_i $(i = 1 \sim n)$ and q are classical propositions and N_{r_i}, N_q, and $N_{(r_1 \wedge r_2 \wedge \cdots \wedge r_n) \rightarrow q}$ are the lower bounds of necessity measures. Π_{r_i}, Π_q, and $\Pi_{(r_1 \wedge r_2 \wedge \cdots \wedge r_n) \rightarrow q}$ are the upper bounds of possibility measures.

To infer N_q and Π_q, we have proposed an approach called possibilistic entailment, inspired by Nilsson's probabilistic entailment [39]. After performing the possibilistic entailment, we can derive the conclusions

$$N_q = \min\{\max[N_{(r_1 \wedge r_2 \wedge \cdots \wedge r_n) \rightarrow q},\, 1 - \Pi_h],\ \max[1 - \Pi_{(r_1 \wedge r_2 \wedge \cdots \wedge r_n) \rightarrow q},\, N_h]\}$$

$$\Pi_q = \max\{\min[\Pi_{(r_1 \wedge r_2 \wedge \cdots \wedge r_n) \rightarrow q},\, \Pi_h],\ \min[\Pi_{(r_1 \wedge r_2 \wedge \cdots \wedge r_n) \rightarrow q},\, 1 - N_h]\}$$

where $N_h = \min[N_{r_1}, N_{r_2}, \cdots, N_{r_n}]$ and $\Pi_h = \min[\Pi_{r_1}, \Pi_{r_2}, \cdots, \Pi_{r_n}]$. In the case that $\Pi_h < 1$ and

$\Pi_{(r_1 \wedge r_2 \wedge \cdots \wedge r_n) \to q} < 1$ (called partially inconsistent [8], [29]) do not exist simultaneously, conclusions then become $N_q = \min\{N_{(r_1 \wedge r_2 \wedge \cdots \wedge r_n) \to q}, N_h\}$ and $\Pi_q = \Pi_{(r_1 \wedge r_2 \wedge \cdots \wedge r_n) \to q}$ (see [30] and [31] for details).

When several rules having a same conclusion are fired, these inferred conclusions with different confidence levels, for example $(q, (N_q^i, \Pi_q^i))$ $(i = 1 \sim n)$, should be aggregated as a conclusion $(q, (N_q^{n+1}, \Pi_q^{n+1}))$. This aggregation can be viewed as a disjunction; we then obtain $N_q^{n+1} = \max[N_q^1, N_q^2, \cdots, N_q^n]$ and $\Pi_q^{n+1} = \max[\Pi_q^1, \Pi_q^2, \cdots, \Pi_q^n]$.

B. Rule-Based Systems with Uncertainty and Fuzziness

The truth-qualified fuzzy propositions are chosen as the representation of imprecise and uncertain information for its capability to express the possibility of the degree of truth [1], [11], [55]. There are three steps involved in the inference mechanism for truth-qualified fuzzy propositions.

- The fuzzy rules and fuzzy facts with fuzzy truth values are transformed into a set of uncertain classical propositions with necessity and possibility measures.
- The possibilistic entailment is performed on the set of uncertain classical propositions.
- We reverse the process in the first step to synthesize all the classical sets obtained in the second step into a fuzzy set and to compose necessity and possibility pairs to form a fuzzy truth value.

1) Representation: To represent uncertain imprecise information, we have chosen a fuzzy proposition with a fuzzy valuation [30], [31], denoted as $(\tilde{r}, \tau)$, where $\tilde{r}$ is a fuzzy proposition of the form "X *is* $\tilde{F}$" [51] (i.e., X is a linguistic variable [57] and $\tilde{F}$ is a fuzzy set in a universe of discourse U) and τ is a fuzzy valuation. It should be noted that for every formula $(\tilde{r}, \tau)$ (called a truth-qualified fuzzy proposition), we assume $\tau \geq \tau(\tilde{r}|\pi)$ [i.e., $\tau(\tilde{r}|\pi)$ is the real fuzzy truth value derived from $\tilde{r}$ and a possibility distribution π], which means $\mu_\tau(t)$ is the upper bound of the possibility that $\tilde{r}$ is true to a degree t. The fuzzy set is to represent the intended meaning of imprecise information, while the fuzzy truth value serves as the representation of uncertainty for its capability to express the possibility of the degree of truth.

To develop inference rules for truth-qualified fuzzy propositions, we treat a truth-qualified fuzzy proposition $(\tilde{r}, \tau)$ as a set of weighted classical propositions $\{(\tilde{r}_\lambda, (N_{\tilde{r}_\lambda}, \Pi_{\tilde{r}_\lambda})), \lambda \in (0, 1]\}$, where $N_{\tilde{r}_\lambda}$ denotes the lower bound of the necessity measure that $\tilde{r}_\lambda$ is true, whereas $\Pi_{\tilde{r}_\lambda}$ denotes the upper bound of the possibility measure that $\tilde{r}_\lambda$ is true, defined as $N_{\tilde{r}_\lambda} = 1 - \max\{\mu_\tau(t) | t \in [0, \lambda)\}$ and $\Pi_{\tilde{r}_\lambda} = \max\{\mu_\tau(t) | t \in [\lambda, 1]\}$.

The membership function of $\tilde{F}$ can be reconstructed in terms of the set of the characteristic functions $\mu_{\tilde{F}_\lambda}$ of its λ-level sets $\tilde{F}_\lambda$, i.e.,

$$\mu_{\tilde{F}}(u) = \text{Sup}\{\lambda \cdot \mu_{\tilde{F}_\lambda}(u) | \lambda \in (0, 1]\} \qquad u \in U. \tag{2}$$

Reconstruction of τ from the set of $(N_{\tilde{r}_\lambda}, \Pi_{\tilde{r}_\lambda})$ pairs is through the use of the principle of minimum specificity [9]

$$\mu_\tau(t) = \text{Inf}\{\mu_{\tau(\lambda)}(t) | \lambda \in (0, 1]\} \qquad t \in [0, 1] \tag{3}$$

where

$$\mu_{\tau(\lambda)}(t) = \begin{cases} \Pi_{\tilde{r}_\lambda}, & \text{if } t \geq \lambda \\ 1 - N_{\tilde{r}_\lambda}, & \text{if } t < \lambda. \end{cases} \tag{4}$$

2) Inference: An inference rule for truth-qualified fuzzy propositions is expressed as follows:

$$\begin{array}{cc} (\tilde{r}_1 \wedge \tilde{r}_2 \wedge \cdots \wedge \tilde{r}_n) \to \tilde{q}, & \tau_1 \\ \tilde{r}'_1, & \tau_2 \\ \tilde{r}'_2, & \tau_3 \\ \vdots & \vdots \\ \tilde{r}'_n, & \tau_{n+1} \\ \hline \tilde{q}', & \tau_{n+2} \end{array} \tag{5}$$

where $\tilde{r}_i$, $\tilde{r}'_i (i = 1 \sim n)$, $\tilde{q}$, and $\tilde{q}'$ are fuzzy propositions and are characterized by "X_i *is* $\tilde{F}_i$," "X_i *is* $\tilde{F}'_i$," "Y *is* $\tilde{G}$," and "Y *is* $\tilde{G}'$," respectively; and $\tau_j (j = 1 \sim n + 2)$ are fuzzy valuations for truth values and are defined by $\mu_{\tau_j}(t)$. $\tilde{F}_i$ and $\tilde{F}'_i$ are the subsets of U_i, while $\tilde{G}$ and $\tilde{G}'$ are the subsets of V. There are three major steps for deriving $\tilde{q}'$ and τ_{n+2} of (5).

Step 1—Transformation: The truth-qualified fuzzy propositions in (5) can be transformed into a set of classical propositions with necessity and possibility pairs as shown in (6) at the bottom of the next page, where $N_{((\tilde{r}_1 \wedge \tilde{r}_2 \wedge \cdots \wedge \tilde{r}_n) \to \tilde{q})_\lambda} = 1 - \max\{\mu_{\tau_1}(t) | t \in [0, \lambda)\}$, $\Pi_{((\tilde{r}_1 \wedge \tilde{r}_2 \wedge \cdots \wedge \tilde{r}_n) \to \tilde{q})_\lambda} = \max\{\mu_{\tau_1}(t) | t \in [\lambda, 1]\}$, $N_{\tilde{r}'_{i_\lambda}} = 1 - \max\{\mu_{\tau_{i+1}}(t) | t \in [0, \lambda)\}$, and $\Pi_{\tilde{r}'_{i_\lambda}} = \max\{\mu_{\tau_{i+1}}(t) | t \in [\lambda, 1]\}$.

Step 2—Inference: Computing $\tilde{q}'_\lambda$. $\tilde{G}'_\lambda$ is computed through the use of compositional rule of inference, that is

$$\tilde{G}'_\lambda = \left(\tilde{F}'_1 \wedge \tilde{F}'_2 \wedge \cdots \wedge \tilde{F}'_n\right)_\lambda \circ \left(\left(\tilde{F}_1 \wedge \tilde{F}_2 \wedge \cdots \wedge \tilde{F}_n\right) \to \tilde{G}\right)_\lambda \tag{7}$$

where $\circ$ is a composition operator and $\to$ denotes an implication operator. In our approach, "Sup-min" [54] and Gödel are chosen as the composition operator and the implication operator, respectively.

$$\begin{array}{cl} (r_1 \wedge r_2 \wedge \cdots \wedge r_n) \to q, & (N_{(r_1 \wedge r_2 \wedge \cdots \wedge r_n) \to q}, \Pi_{(r_1 \wedge r_2 \wedge \cdots \wedge r_n) \to q}) \\ r_1, & (N_{r_1}, \Pi_{r_1}) \\ r_2, & (N_{r_2}, \Pi_{r_2}) \\ \vdots & \vdots \\ r_n, & (N_{r_n}, \Pi_{r_n}) \\ \hline q, & (N_q, \Pi_q) \end{array} \tag{1}$$

Computing $N_{\tilde{q}'_\lambda}$ and $\Pi_{\tilde{q}'_\lambda}$. With the help of the principle of minimum specificity [9], (6) can be transformed into a set of classical propositions with necessity and possibility pairs as shown in (8) at the bottom of the page, where $\pi(u_1, u_2, \cdots, u_n, v)$ denotes a possibility distribution over $U_1 \times U_2 \times, \cdots, \times U_n \times V$, derived by means of the principle of minimum specificity $\pi(u_1, u_2, \cdots, u_n, v) = \text{Inf}\{\pi_\lambda(u_1, u_2, \cdots, u_n, v) | \lambda \in (0, 1]\}$, as shown in (9) at the bottom of the page.

The possibilistic reasoning in Section II-A is then applied to (8) to obtain the upper bound of the possibility measure and the lower bound of the necessity measure of $\tilde{q}'_\lambda$

$$N_{\tilde{q}'_\lambda} = \min\left\{ N_{\left(\tilde{r}_{1'_\lambda} \wedge \tilde{r}_{2'_\lambda} \wedge \cdots \wedge \tilde{r}_{n'_\lambda}\right) \to \tilde{q}'_\lambda} \; N_{\tilde{r}_{1'_\lambda} \wedge \tilde{r}_{2'_\lambda} \wedge \cdots \wedge \tilde{r}_{n'_\lambda}} \right\}$$

$$\Pi_{\tilde{q}'_\lambda} = \Pi_{\left(\tilde{r}_{1'_\lambda} \wedge \tilde{r}_{2'_\lambda} \wedge \cdots \wedge \tilde{r}_{n'_\lambda}\right) \to \tilde{q}'_\lambda}$$

where

$$N_{\tilde{r}_{1'_\lambda} \wedge \tilde{r}_{2'_\lambda} \wedge \cdots \wedge \tilde{r}_{n'_\lambda}} = \min\left[N_{\tilde{r}_{1'_\lambda}}, N_{\tilde{r}_{2'_\lambda}}, \cdots, N_{\tilde{r}_{n'_\lambda}} \right]$$

and

$$\Pi_{\tilde{r}_{1'_\lambda} \wedge \tilde{r}_{2'_\lambda} \wedge \cdots \wedge \tilde{r}_{n'_\lambda}} = \min\left[\Pi_{\tilde{r}_{1'_\lambda}}, \Pi_{\tilde{r}_{2'_\lambda}}, \cdots, \Pi_{\tilde{r}_{n'_\lambda}} \right].$$

Step 3—Composition: Based on (2), the construction of the membership function of $\tilde{G}'$ is performed by the following equation: $\mu_{\tilde{G}'}(v) = \text{Sup}\{\lambda \cdot \mu_{\tilde{G}'_\lambda}(v) | \lambda \in (0, 1]\}$. Meanwhile, the construction of τ_{n+2} is calculated by (3), that is, $\mu_{\tau_{n+2}}(t) = \text{Inf}\{\mu_{\tau_{n+2}(\lambda)}(t) | \lambda \in (0, 1]\}$, where

$$\mu_{\tau_{n+2}(\lambda)}(t) = \begin{cases} \Pi_{\tilde{q}'_\lambda}, & \text{if } t \geq \lambda \\ 1 - N_{\tilde{q}'_\lambda}, & \text{if } t < \lambda. \end{cases} \tag{10}$$

3) Aggregation of Conclusions: Several inferred conclusions having a same linguistic variable should be aggregated. For example, there are m inferred conclusions having a same linguistic variable, represented as

$$\begin{array}{ll} \tilde{q}'_{11}, & \tau_1 \\ \tilde{q}'_{12}, & \tau_2 \\ \vdots & \vdots \\ \tilde{q}'_{1m}, & \tau_m \\ \hline \tilde{q}'_{1(m+1)}, & \tau_{m+1} \end{array} \tag{11}$$

where $\tilde{q}'_{1i} (i = 1 \sim m+1)$ are fuzzy conclusions having the form of "Y *is* $\tilde{G}'_{1i}$." There are three major steps for deriving $\tilde{q}'_{1(m+1)}$ and τ_{m+1}.

Step 1—Transformation: The inferred conclusions in (11) can be transformed into a set of classical propositions

$$\begin{array}{ll} ((\tilde{r}_1 \wedge \tilde{r}_2 \wedge \cdots \wedge \tilde{r}_n) \to \tilde{q})_\lambda, & (N_{((\tilde{r}_1 \wedge \tilde{r}_2 \wedge \cdots \wedge \tilde{r}_n) \to \tilde{q})_\lambda}, \Pi_{((\tilde{r}_1 \wedge \tilde{r}_2 \wedge \cdots \wedge \tilde{r}_n) \to \tilde{q})_\lambda}) \\ \tilde{r}_{1'_\lambda}, & (N_{\tilde{r}_{1'_\lambda}}, \Pi_{\tilde{r}_{1'_\lambda}}) \\ \tilde{r}_{2'_\lambda}, & (N_{\tilde{r}_{2'_\lambda}}, \Pi_{\tilde{r}_{2'_\lambda}}) \\ \vdots & \vdots \\ \tilde{r}_{n'_\lambda}, & (N_{\tilde{r}_{n'_\lambda}}, \Pi_{\tilde{r}_{n'_\lambda}}) \\ \hline \tilde{q}'_\lambda, & (N_{\tilde{q}'_\lambda}, \Pi_{\tilde{q}'_\lambda}) \end{array} \tag{6}$$

$$\begin{array}{ll} (\tilde{r}_{1'_\lambda} \wedge \tilde{r}_{2'_\lambda} \wedge \cdots \wedge \tilde{r}_{n'_\lambda}) \to \tilde{q}'_\lambda, & (N_{(\tilde{r}_{1'_\lambda} \wedge \tilde{r}_{2'_\lambda} \wedge \cdots \wedge \tilde{r}_{n'_\lambda}) \to \tilde{q}'_\lambda}, \Pi_{(\tilde{r}_{1'_\lambda} \wedge \tilde{r}_{2'_\lambda} \wedge \cdots \wedge \tilde{r}_{n'_\lambda}) \to \tilde{q}'_\lambda}) \\ \tilde{r}_{1'_\lambda}, & (N_{\tilde{r}_{1'_\lambda}}, \Pi_{\tilde{r}_{1'_\lambda}}) \\ \tilde{r}_{2'_\lambda}, & (N_{\tilde{r}_{2'_\lambda}}, \Pi_{\tilde{r}_{2'_\lambda}}) \\ \vdots & \vdots \\ \tilde{r}_{n'_\lambda}, & (N_{\tilde{r}_{n'_\lambda}}, \Pi_{\tilde{r}_{n'_\lambda}}) \\ \hline \tilde{q}'_\lambda, & (N_{\tilde{q}'_\lambda}, \Pi_{\tilde{q}'_\lambda}) \end{array} \tag{8}$$

$$N_{(\tilde{r}_{1'_\lambda} \wedge \tilde{r}_{2'_\lambda} \wedge \cdots \wedge \tilde{r}_{n'_\lambda}) \to \tilde{q}'_\lambda} = 1 - \max\{\pi(u_1, u_2, \cdots, u_n, v) | (u_1, u_2, \cdots, u_n, v) \notin (\tilde{F}'_{1_\lambda} \wedge \tilde{F}'_{2_\lambda} \wedge \cdots \wedge \tilde{F}'_{n_\lambda}) \to \tilde{G}'_\lambda\}$$

$$\Pi_{(\tilde{r}_{1'_\lambda} \wedge \tilde{r}_{2'_\lambda} \wedge \cdots \wedge \tilde{r}_{n'_\lambda}) \to \tilde{q}'_\lambda} = \max\{\pi(u_1, u_2, \cdots, u_n, v) | (u_1, u_2, \cdots, u_n, v) \in (\tilde{F}'_{1_\lambda} \wedge \tilde{F}'_{2_\lambda} \wedge \cdots \wedge \tilde{F}'_{n_\lambda}) \to \tilde{G}'_\lambda\}$$

$$\pi_\lambda(u_1, u_2, \cdots, u_n, v) = \begin{cases} \Pi_{((\tilde{r}_1 \wedge \tilde{r}_2 \wedge \cdots \wedge \tilde{r}_n) \to \tilde{q})_\lambda} & (u_1, u_2, \cdots, u_n, v) \in ((\tilde{F}_1 \wedge \tilde{F}_2 \wedge \cdots \wedge \tilde{F}_n) \to \tilde{G})_\lambda \\ 1 - N_{((\tilde{r}_1 \wedge \tilde{r}_2 \wedge \cdots \wedge \tilde{r}_n) \to \tilde{q})_\lambda} & (u_1, u_2, \cdots, u_n, v) \notin ((\tilde{F}_1 \wedge \tilde{F}_2 \wedge \cdots \wedge \tilde{F}_n) \to \tilde{G})_\lambda \end{cases} \tag{9}$$

with necessity and possibility pairs as follows:

$$\begin{array}{ll} \tilde{q}_{11'_\lambda}, & (N_{\tilde{q}_{11'_\lambda}}, \Pi_{\tilde{q}_{11'_\lambda}}) \\ \tilde{q}_{12'_\lambda}, & (N_{\tilde{q}_{12'_\lambda}}, \Pi_{\tilde{q}_{12'_\lambda}}) \\ \vdots & \vdots \\ \tilde{q}_{1m'_\lambda}, & (N_{\tilde{q}_{1m'_\lambda}}, \Pi_{\tilde{q}_{1m'_\lambda}}) \\ \hline \tilde{q}_{1(m+1)'_\lambda}, & (N_{\tilde{q}_{1(m+1)'_\lambda}}, \Pi_{\tilde{q}_{1(m+1)'_\lambda}}) \end{array} \tag{12}$$

where $\lambda \in (0, 1]$, $N_{\tilde{q}_{1i'_\lambda}} = 1 - \max\{\mu_{\tau_i}(t) | t \in [0, \lambda)\}$, and $\Pi_{\tilde{q}_{1i'_\lambda}} = \max\{\mu_{\tau_i}(t) | t \in [\lambda, 1]\}$ $(i = 1 \sim m)$.

Step 2—Aggregation: Computing $\tilde{G}_{1(m+1)'_\lambda}$

$$\begin{aligned} \tilde{G}_{1(m+1)'_\lambda} &= (\tilde{G}'_{11} \wedge \tilde{G}'_{12} \wedge \cdots \wedge \tilde{G}'_{1m})_\lambda \\ &= \tilde{G}_{11'_\lambda} \wedge \tilde{G}_{12'_\lambda} \wedge \cdots \wedge \tilde{G}_{1m'_\lambda} \end{aligned}$$

is computed through the use of T-norm.

Computing $N_{\tilde{q}_{1(m+1)'_\lambda}}$ and $\Pi_{\tilde{q}_{1(m+1)'_\lambda}}$. With the help of the principle of minimum specificity [9], (12) can be transformed into a set of classical propositions with necessity and possibility pairs

$$\begin{array}{ll} \tilde{q}_{1(m+1)'_\lambda}, & (N^1_{\tilde{q}_{1(m+1)'_\lambda}}, \Pi^1_{\tilde{q}_{1(m+1)'_\lambda}}) \\ \tilde{q}_{1(m+1)'_\lambda}, & (N^2_{\tilde{q}_{1(m+1)'_\lambda}}, \Pi^2_{\tilde{q}_{1(m+1)'_\lambda}}) \\ \vdots & \vdots \\ \tilde{q}_{1(m+1)'_\lambda}, & (N^m_{\tilde{q}_{1(m+1)'_\lambda}}, \Pi^m_{\tilde{q}_{1(m+1)'_\lambda}}) \\ \hline \tilde{q}_{1(m+1)'_\lambda}, & (N_{\tilde{q}_{1(m+1)'_\lambda}}, \Pi_{\tilde{q}_{1(m+1)'_\lambda}}) \end{array} \tag{13}$$

$N^i_{\tilde{q}_{1(m+1)'_\lambda}} = 1 - \max\{\pi_i(v) | v \notin \tilde{G}_{1(m+1)'_\lambda}\}$ and $\Pi^i_{\tilde{q}_{1(m+1)'_\lambda}} = \max\{\pi_i(v) | v \in \tilde{G}_{1(m+1)'_\lambda}\}$ $(i = 1 \sim m)$, where $\pi_i(v)$ denotes a possibility distribution over V, derived by the principle of minimum specificity: $\pi_i(v) = \mathrm{Inf}\{\pi_{i\lambda}(v) | \lambda \in (0, 1]\}$, where

$$\pi_{i\lambda}(v) = \begin{cases} \Pi_{\tilde{q}_{1i'_\lambda}} & v \in \tilde{G}_{1i'_\lambda} \\ 1 - N_{\tilde{q}_{1i'_\lambda}} & v \notin \tilde{G}_{1i'_\lambda}. \end{cases} \tag{14}$$

The possibilistic aggregation in Section II-A is then applied to (13) to obtain the upper bound of the possibility measure and the lower bound of the necessity measure of $N_{\tilde{q}_{1(m+1)'_\lambda}}$ and $\Pi_{\tilde{q}_{1(m+1)'_\lambda}}$

$$\begin{aligned} N_{\tilde{q}_{1(m+1)'_\lambda}} &= \max\left[N^1_{\tilde{q}_{1(m+1)'_\lambda}}, N^2_{\tilde{q}_{1(m+1)'_\lambda}}, \cdots, N^m_{\tilde{q}_{1(m+1)'_\lambda}}\right] \\ \Pi_{\tilde{q}_{1(m+1)'_\lambda}} &= \max\left[\Pi^1_{\tilde{q}_{1(m+1)'_\lambda}}, \Pi^2_{\tilde{q}_{1(m+1)'_\lambda}}, \cdots, \Pi^m_{\tilde{q}_{1(m+1)'_\lambda}}\right]. \end{aligned}$$

Step 3—Composition: Based on (2), the construction of the membership function of $\tilde{G}'_{1(m+1)}$ is performed by the following equation:

$$\mu_{\tilde{G}'_{1(m+1)}}(v) = \mathrm{Sup}\left\{\lambda \cdot \mu_{\tilde{G}'_{1(m+1)_\lambda}}(v) | \lambda \in (0, 1]\right\}.$$

Meanwhile, the construction of τ_{m+1} is calculated by (3): $\mu_{\tau_{m+1}}(t) = \mathrm{Inf}\{\mu_{\tau_{m+1}(\lambda)}(t) | \lambda \in (0, 1]\}$, where

$$\mu_{\tau_{m+1}(\lambda)}(t) = \begin{cases} \Pi_{\tilde{q}_{1(m+1)'_\lambda}}, & \text{if } t \geq \lambda \\ 1 - N_{\tilde{q}_{1(m+1)'_\lambda}}, & \text{if } t < \lambda. \end{cases} \tag{15}$$

III. Fuzzy Petri Nets

Petri nets are a graphical and mathematical modeling tool applicable to many systems. In this section, fuzzy Petri nets are defined for modeling fuzzy systems and used as knowledge representation for fuzzy rules [30].

A. Petri Nets

A Petri net is a directed, weighted, bipartite graph consisting of two kinds of nodes, called places (p_i) and transitions (t_j), where arcs are either from a place to a transition or from a transition to a place [42]. Murata has formally defined Petri nets as a five-tuple [37]: $PN = (P, T, F, W, M_0)$, where $P = \{p_1, p_2, \cdots, p_m\}$ is a finite set of places, $T = \{t_1, t_2, \cdots, t_n\}$ is a finite set of transitions, $F \subseteq (P \times T) \cup (T \times P)$ is a set of arcs, $W\colon F \to \{1, 2, 3, \cdots\}$ is a weight function, and $M_0\colon P \to \{0, 1, 2, 3, \cdots\}$ is the initial marking. A marking M is an m-vector, $\langle M(p_1), \cdots, M(p_m)\rangle$, where $M(p_i)$ denotes the number of the tokens in place p_i. The incidence matrix $A = [a_{\mathrm{ij}}]$ is an $n \times m$ matrix of integers, and its typical entry is defined by $a_{\mathrm{ij}} = a^+_{\mathrm{ij}} - a^-_{\mathrm{ij}}$, where a^+_{ij} is the weight of the arc from a transition t_i to its output place p_j and a^-_{ij} is the weight of the arc to a transition t_i from its input place p_j. The reachability set $R(M_0)$ of a Petri net is defined as the set of all possible markings reachable from M_0. A place having two or more output transitions is referred to as a *conflict*. Two transitions are said to be concurrent if they are causally independent. The evolution of markings, used to simulate the dynamic behavior of a system, is based on the firing rule, such as: a transition t is enabled if each input place t is marked with at least $w(p, t)$ tokens, where $w(p, t)$ is the weight of the arc from p to t; an enabled transition may or may not be enabled. A firing of an enabled transition t removes $w(p, t)$ tokens from each input place p of t and adds $w(t, p)$ tokens to each output place p of t, where $w(t, p)$ is the weight of the arc from t to p. Some notations are introduced as follows: $\bullet t_j$ denotes the input places of t_j, $t_j\bullet$ denotes the output places of t_j, $\bullet p_i$ denotes the input transitions of p_i, and $p_i\bullet$ denotes the output transitions of p_i.

B. Fuzzy Petri Nets

A typical interpretation of Petri nets is to view a place as a condition, a transition as the causal relationship of conditions, and a token in a place as a fact used to claim the truth of the condition associated with the place. However, fuzzy systems include the following situations.

- The conditions are fuzzy.
- The causal relationships of fuzzy conditions are uncertain.
- The values of facts are fuzzy, and may partially match the value of the associated fuzzy condition.
- The confidence about the truths of the facts is uncertain.

To take the above situations into account, we formally define our version of fuzzy Petri nets below.

Definition 1—Fuzzy Petri Nets: A fuzzy Petri net FPN is defined as a five-tuple

$$FPN = (FP, UT, F, W, M_0)$$

$FP = \{(p_1, \tilde{F}_1), (p_2, \tilde{F}_2), \cdots, (p_m, \tilde{F}_m)\}$ is a finite set of fuzzy places, where p_i represents a fuzzy condition and $\tilde{F}_i$ is a fuzzy subset of U_i that represents the fuzzy set of the condition. $UT = \{(t_1, \tau_1), (t_2, \tau_2), \cdots, (t_n, \tau_n)\}$ is a finite set of uncertain transitions, where t_j represents the causal relationship of fuzzy conditions and τ_j is a fuzzy truth value to represent the uncertainty about the causal relationship of fuzzy conditions. $F \subseteq (FP \times UT) \cup (UT \times FP)$ is a set of arcs. $W: F \to \{1, 2, 3, \cdots\}$ is a weight function. $M_0 = \langle M(p_1), M(p_2), \cdots, M(p_m)\rangle$ is the initial marking, where $M(p_i)$ is the number of tokens in p_i.

The fuzzy truth value serves as the representation of uncertainty for its capability to express the possibility of the degree of truth. In Definition 1, we assume that $\mu_{\tau_i}(t)$ $(t \in [0, 1])$ of each τ_i $(i = 1 \sim m + n)$ means the upper bound of the possibility measure that for which degree of truth is t.

Each token is associated with a pair of fuzzy sets $(\tilde{F}_i', \tau_i)$ (called an uncertain fuzzy token). Fuzzy places with uncertain fuzzy tokens can be interpreted as uncertain fuzzy facts related to the fuzzy conditions modeled by the fuzzy places. An example is illustrated in Fig. 1(a): three fuzzy conditions are modeled as three fuzzy places; their uncertain causal relationship is modeled as an uncertain transition. Two truth-qualified fuzzy facts concerning the preconditions are modeled as two uncertain fuzzy tokens.

To simulate the dynamic behavior of a fuzzy system, a marking in a fuzzy Petri net is changed according to the firing rule: a firing of an enabled uncertain transition t_j removes the uncertain fuzzy token from each input place p_i of t_j and adds a new token to each output place p_k of t_j. The fuzzy set and fuzzy truth value attached to the new token will be computed based on the mechanism in fuzzy reasoning. Fig. 1 illustrates the evolution of markings by the firing rule.

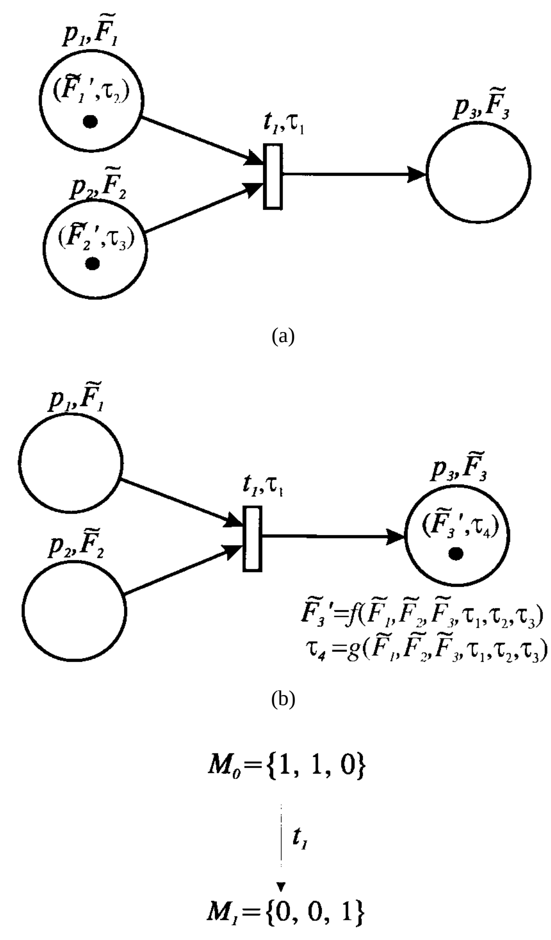


Fig. 1. Illustration of a fuzzy Petri net: (a) before firing t_1, (b) after firing t_1, and (c) the reachability tree.

C. Analysis of Fuzzy Petri Nets

This section describes how fuzzy Petri nets can be analyzed. Two major Petri net analysis methods, the coverability tree and state equation, are used to analyze fuzzy Petri nets.

1) The Coverability Tree: The coverability tree represents the reachability set of a fuzzy Petri net. Given a fuzzy Petri net, a tree representation of the markings can be constructed [37]. In this tree, a symbol ω is used to represent "infinity," nodes represent markings reachable from M_0, and each arc represents an uncertain transition firing that transforms one marking to another. Some of the behavioral properties that can be studied by using the coverability tree are boundedness, safeness, and deadlock in uncertain transitions. For a bounded fuzzy Petri net, the coverability tree is called the reachability tree. Fig. 1(c) illustrates the reachability tree of a fuzzy Petri net.

2) State Equation: The state equation that governs the dynamic behavior of concurrent fuzzy systems modeled by fuzzy Petri nets is represented by $A^T x = \Delta M$, where $\Delta M = M_n - M_0$, A is the incidence matrix, and x is an $n\times$ 1 column vector called the firing count vector. The ith entry of x denoted the number of times that uncertain transition t_i must fire to transform M_0 to M_n. The state equation is used to solve the reachability problem, that is, the problem of finding if $M_n \in R(M_0)$ for a given M_n. If M_n is reachable from M_0, then the state equation has a solution in nonnegative integers. If the state equation has no solution, then M_n is not reachable from M_0.

D. Fuzzy Rule-Based Reasoning and Fuzzy Petri Nets

It is widely recognized that fuzzy Petri nets is a promising modeling mechanism for formulating fuzzy rule-based reasoning [3], [7], [23], [25], [34], [45], [46], [52]. The three key components in fuzzy rule-based reasoning—fuzzy propositions, fuzzy rules, and fuzzy facts—can be formulated as places, transitions, and tokens, respectively. However, there is still one main issue that needs to be addressed: conflict. In fuzzy rule-based reasoning, several fuzzy rules having a same antecedent will be fired if a fuzzy fact matches the antecedent of those rules. In Petri nets, these fuzzy rules and the fuzzy fact are modeled as several transitions departing from a place and a token in the place, respectively. However, only one of these transitions will be fired since they are in conflict. As is illustrated in Fig. 2, two fuzzy conclusions will be inferred, if fact 1 partially matches rules 1 and 2. But, only one transition will be fired since transitions t_1 and t_2 are in conflict.

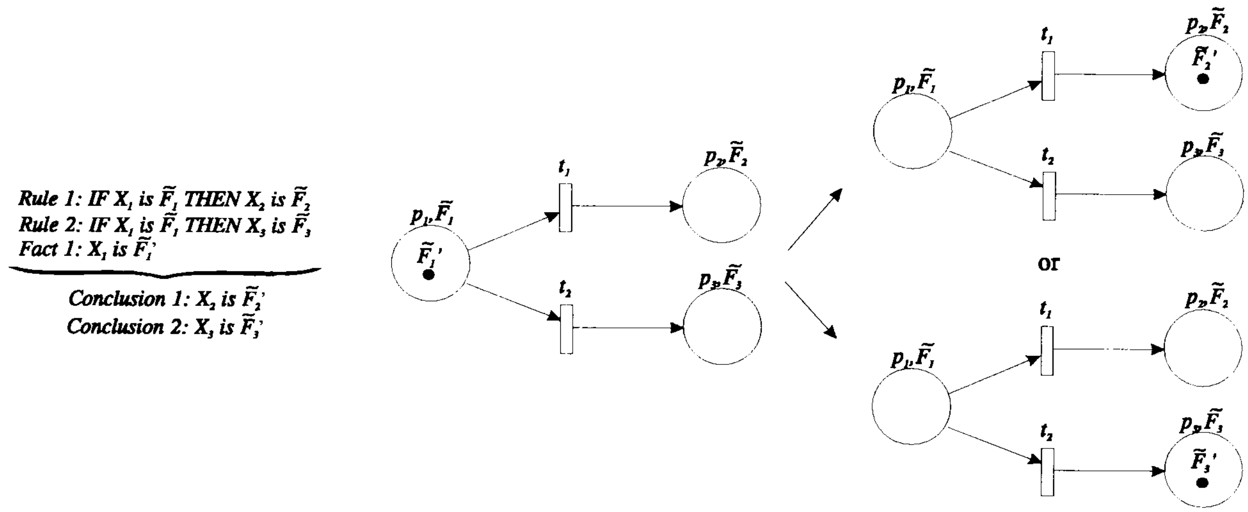


Fig. 2. The problem of modeling fuzzy rule-based reasoning by fuzzy Petri nets: conflict.

To overcome this problem, a subclass of Petri nets—marked graphs—is used in this paper since each place in a marked graph has exactly one input transition and exactly one output transition, i.e., $|\bullet p_j| = |p_j \bullet| = 1$. Furthermore, among models that can represent concurrent activities, marked graphs are the most amenable to analysis [37]. The mapping between fuzzy rule-based reasoning and fuzzy Petri nets is fully described below.

- *Fuzzy Places:* Fuzzy places correspond to fuzzy propositions. The fuzzy sets, attached to the fuzzy places, represent the values of fuzzy propositions. Fuzzy input and fuzzy output places of a truth-qualified transition are used to represent the antecedent and conclusion parts of a truth-qualified fuzzy rule, respectively.
- *Uncertain Fuzzy Tokens:* An uncertain fuzzy token represents a truth-qualified fuzzy fact. The fuzzy sets and fuzzy truth values are attached to uncertain fuzzy tokens to represent the values and our confidence level about the observed facts, respectively.
- *Uncertain Transitions:* Uncertain transitions are classified into four types: inference, aggregation, duplication, and aggregation-duplication transitions. The inference transitions represent the truth-qualified fuzzy rules, the aggregation transitions are designed to aggregate the conclusion parts of rules that have the same linguistic variables, the duplication transitions are used to duplicate uncertain fuzzy tokens to avoid the conflict problem, and the aggregation-duplication transitions link the fuzzy propositions with the same linguistic variables. These are formally defined below.

Type 1—Inference Transition (t^i)*:* An inference transition serves as a modeling of a truth-qualified fuzzy rule. A truth-qualified fuzzy rule having multiple antecedents is represented as

$$(\tilde{r}_1 \wedge \tilde{r}_2 \wedge \cdots \wedge \tilde{r}_n) \rightarrow \tilde{q},\, \tau_1$$

where $\tilde{r}_i$ and $\tilde{q}$ are of the forms of "X_i *is* $\tilde{F}_i$" and "Y *is* $\tilde{G}$," respectively.

In Fig. 3, after firing the inference transition t_1^i, the tokens will be removed from the input places of t_1^i, a new token will be deposited into the output place of t_1^i, and the fuzzy set and the fuzzy truth value attached to the new token are derived by three steps (see Section II-B2).

1) *Transformation:* The fuzzy facts and fuzzy rules with fuzzy truth values are transformed into a set of uncertain classical propositions with necessity and possibility measures by means of λ-cut.
2) *Inference:* The possibilistic entailment is performed on the set of uncertain classical propositions.
3) *Composition:* We reverse the process in the first step to synthesize all the λ-level sets obtained in the second step into a fuzzy set and to compose necessity and possibility pairs to form a fuzzy truth value.

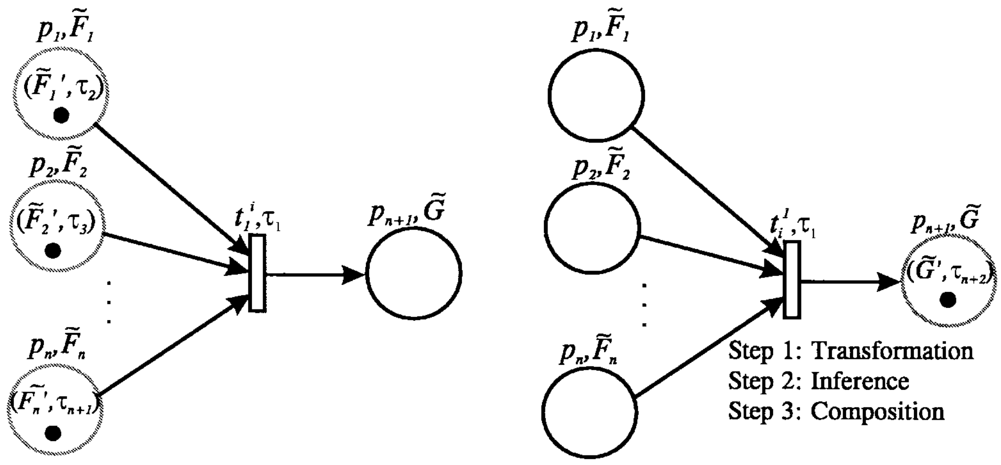
Type 2—Aggregation Transition (t^a)*:* An aggregation transition is used to aggregate the conclusions of several truth-qualified fuzzy rules that have a same linguistic variable and to link the antecedent of a truth-qualified fuzzy rule that also has the same linguistic variable. For example, there are m truth-qualified fuzzy rules having a same linguistic variable in the conclusions, denoted as

$$(\tilde{r}_1 \rightarrow \tilde{q}_{11},\, \tau_1),\, (\tilde{r}_2 \rightarrow \tilde{q}_{12},\, \tau_2),\, \cdots,\, (\tilde{r}_m \rightarrow \tilde{q}_{1m},\, \tau_m)$$

where $\tilde{q}_{1i}$ is "Y *is* $\tilde{G}_{1i}$."

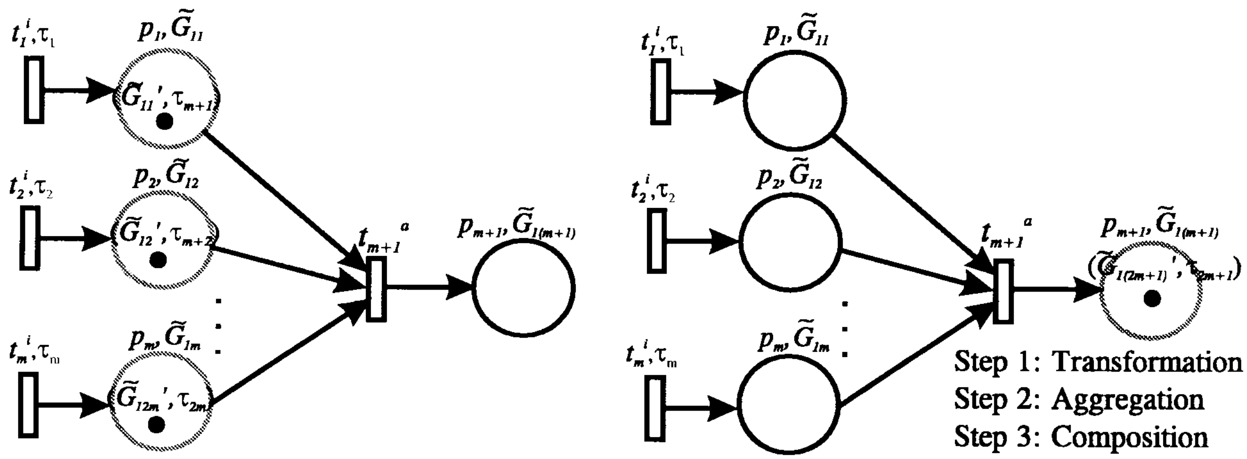
In Fig. 4, after firing the aggregation transition t_{m+1}^a, the tokens in the input places of t_{m+1}^a will be removed, a new token will be deposited into the output place of t_{m+1}^a, and the fuzzy set and the fuzzy truth value attached to the new token are derived by three steps (see Section II-B3).

1) *Transformation:* The fuzzy facts with fuzzy truth values are transformed into a set of uncertain classical propositions with necessity and possibility measures by means of λ-cut.
2) *Aggregation:* The aggregation is performed on the set of uncertain classical propositions.
3) *Composition:* We reverse the process in the first step to synthesize all the λ-level sets obtained in the second step



(a) (b)

Fig. 3. Modeling fuzzy rule-based reasoning through fuzzy Petri nets: (a) before and (b) after firing t_1^i.



(a) (b)

Fig. 4. Modeling the aggregation of conclusions by an aggregation transition: (a) before and (b) after firing t_{m+1}^a.

into a fuzzy set and to compose necessity and possibility pairs to form a fuzzy truth value.

It should be noted that t_{m+1}^a is dead if one of its input places never received a token. To avoid deadlock in aggregation transitions, we assume that for each source place p_i, a token will be inserted into p_i, and that the fuzzy set $\tilde{F}_i'$ and the fuzzy truth value τ_i attached to the token are assigned to be their universe of discourse if no fact matches the fuzzy proposition in the place p_i. That is, $\tilde{F}_i' = U_i$ and $\tau_i = T$ are assigned.

Type 3—Duplication Transition (t^d)*:* The purpose of duplication transitions is to avoid the conflict by duplicating the token. For example, there are m truth-qualified fuzzy rules having a same linguistic variable in the antecedents, denoted as

$$(\tilde{r}_{11} \rightarrow \tilde{q}_1, \tau_1), (\tilde{r}_{12} \rightarrow \tilde{q}_2, \tau_2), \cdots, (\tilde{r}_{1l} \rightarrow \tilde{q}_l, \tau_l)$$

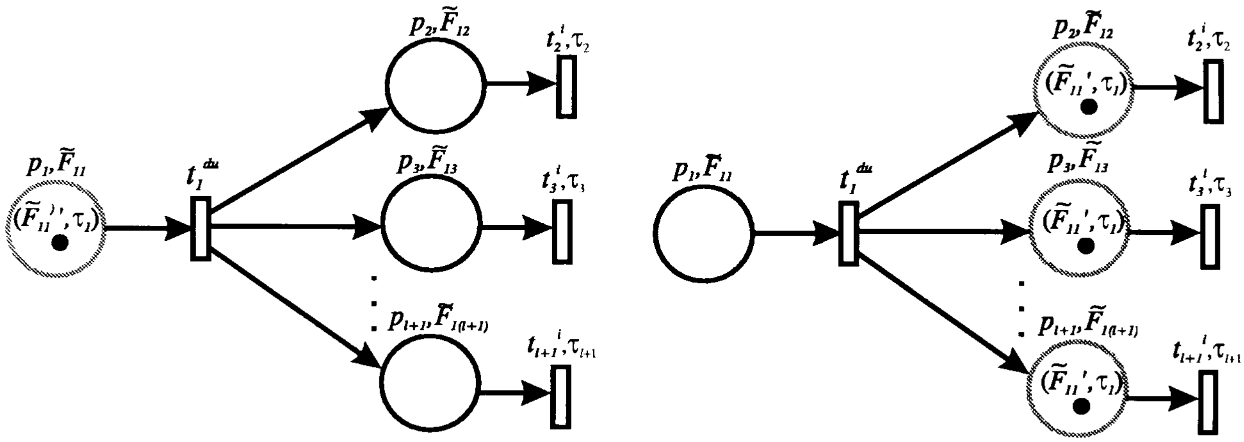
where r_{1i} means "X_i *is* $\tilde{F}_{1i}$." They are linked by a duplication transition shown in Fig. 5. After firing the duplication transition t_1^d, the tokens in the input place of t_1^d will be removed, new tokens will be added into the output places of t_1^d, and the fuzzy sets and the fuzzy truth values attached to the new tokens are not changed.

Type 4—Aggregation-duplication Transition (t^{ad})*:* An aggregation-duplication transition is a combination of an aggregation transition and a duplication transition (see Fig. 6). It is used to link all fuzzy propositions that have a same linguistic variable. For example, there are m truth-qualified fuzzy rules having a same linguistic variable in the conclusions and l truth-qualified fuzzy rules having the same linguistic variable in the antecedents, denoted as

$$(\tilde{r}_1 \rightarrow \tilde{q}_{11}, \tau_1), (\tilde{r}_2 \rightarrow \tilde{q}_{12}, \tau_2), \cdots, (\tilde{r}_m \rightarrow \tilde{q}_{1m}, \tau_m)$$
$$(\tilde{q}_{1(m+1)} \rightarrow \tilde{s}_1, \tau_{m+1}), (\tilde{q}_{1(m+2)} \rightarrow \tilde{s}_2, \tau_{m+2}),$$
$$\cdots, (\tilde{q}_{1(m+l)} \rightarrow \tilde{s}_l, \tau_{m+l})$$

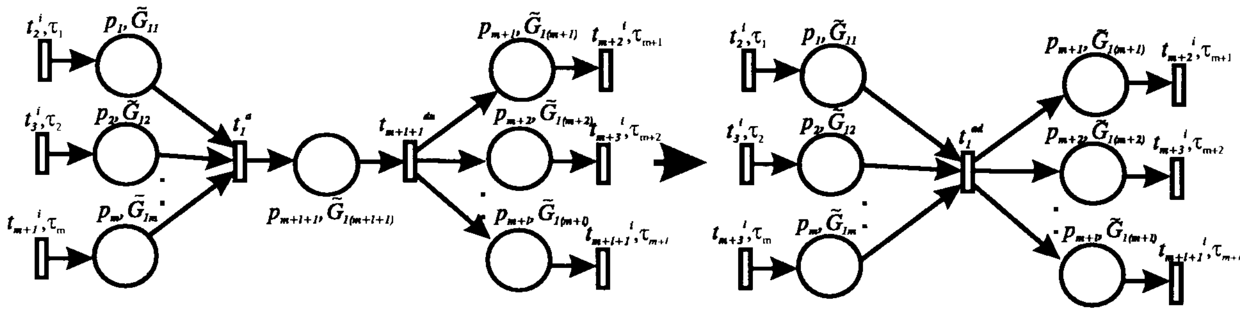
where $\tilde{q}_{1i}$ is of the form of "Y_1 *is* $\tilde{G}_{1i}$." They are linked by an aggregation-duplication transition shown in Fig. 7.

After firing the aggregation-duplication transition t_1^{ad}, the tokens in the input places of t_1^{ad} will be removed and new tokens will be deposited into the output places of t_1^{ad}. The



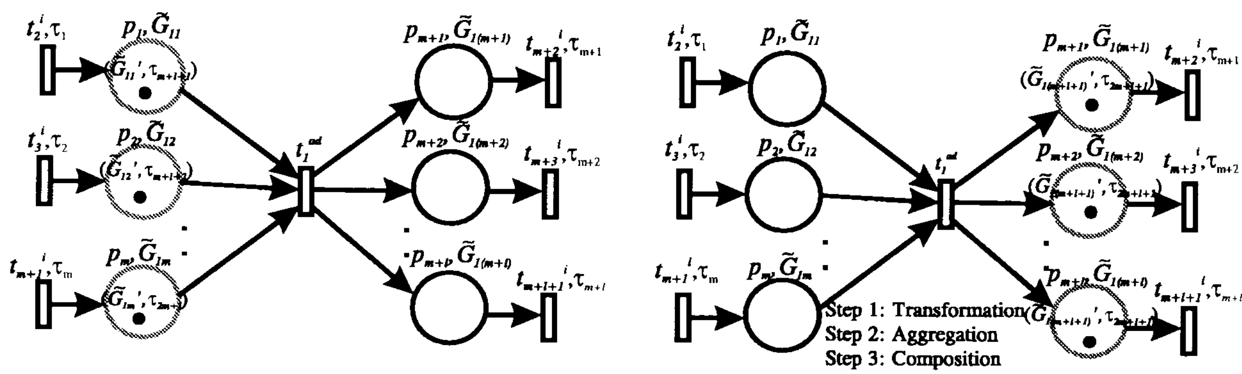
(a) (b)

Fig. 5. Modeling the duplication of an uncertain fuzzy token through fuzzy Petri nets: (a) before and (b) after firing t_1^d.



(a) (b)

Fig. 6. An aggregation-duplication transition is a combination of an aggregation transition and a duplication transition.



(a) (b)

Fig. 7. Modeling the aggregation-duplication of uncertain fuzzy tokens through proposed FPN: (a) before and (b) after firing t_1^{ad}.

fuzzy sets and the fuzzy truth values attached to the new tokens are derived by three steps (see Section II-B3).

1) *Transformation:* The fuzzy facts with fuzzy truth values are transformed into a set of uncertain classical propositions with necessity and possibility measures by means of λ-cut.
2) *Aggregation:* The aggregation is performed on the set of uncertain classical propositions.
3) *Composition:* We reverse the process in the first step to synthesize all the λ-level sets obtained in the second step into a fuzzy set and to compose necessity and possibility pairs to form a fuzzy truth value.

IV. Fuzzy Petri Net-Based Expert System

A framework of integrated expert systems based on our fuzzy Petri net, called fuzzy Petri net-based expert system, is described in this section. Major features of FPNES include knowledge representation through the use of hierarchical

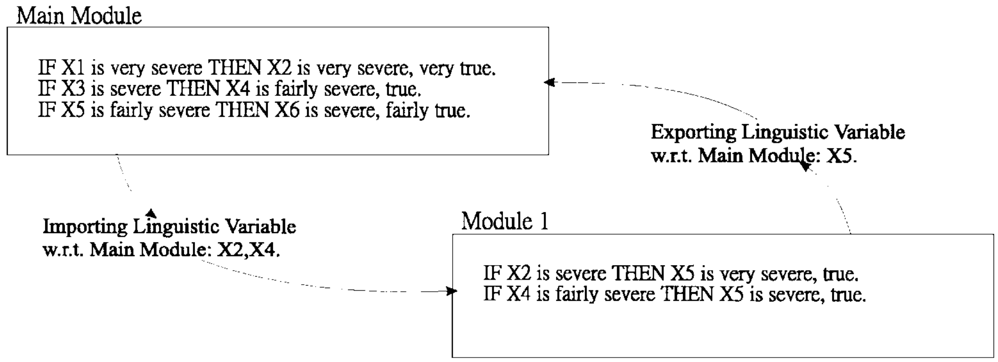


Fig. 8. Modules have importing and exporting linguistic variables.

fuzzy Petri nets, a reasoning mechanism based on fuzzy Petri nets, and transformation of modularized fuzzy rule bases into hierarchical fuzzy Petri nets.

A. Knowledge Representation: Hierarchical Fuzzy Petri Nets

Our fuzzy Petri nets are used as the knowledge representation to formulate fuzzy propositions, truth-qualified fuzzy rules, truth-qualified fuzzy facts as fuzzy places, uncertain transitions, and uncertain fuzzy tokens, respectively. Four types of uncertain transitions—inference, aggregation, duplication, and aggregation-duplication transitions—are introduced to fulfill the mechanism of a fuzzy rule-based reasoning.

To overcome the complexity arising from large sizes of rule bases and fuzzy Petri nets, two important features, modularized rule bases and hierarchical fuzzy Petri nets, are adopted in FPNES. Modularization to partition rule bases into smaller parts is a well-known method useful for organizing rules. Each module may have importing linguistic variables and exporting linguistic variables with respect to some specific modules. As illustrated in Fig. 8, in module 1, the importing linguistic variables $X2$ and $X4$ with respect to the main module ($M0$) receive facts from the main module, and the exporting linguistic variable $X5$ with respect to the main module exports facts to the main module after receiving facts.

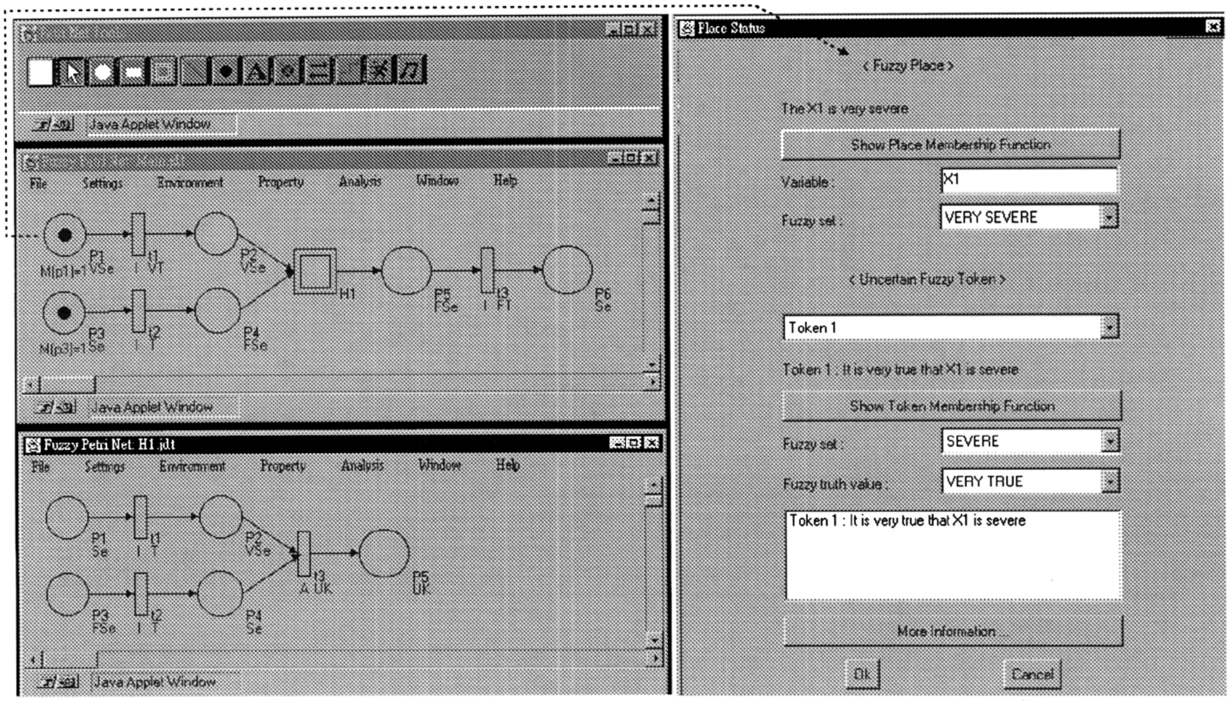
In a hierarchical fuzzy Petri net, each hierarchy contains a fuzzy Petri net, which may or may not contain other hierarchies. The connections between hierarchies are achieved by defining importing and exporting fuzzy places. That is, an exporting fuzzy place with respect to a hierarchy is defined as a fuzzy place that is connected to the hierarchy by an arc from the fuzzy place to the hierarchy; meanwhile, an importing fuzzy place with respect to a hierarchy is defined as a fuzzy place connected to the hierarchy by an arc from the hierarchy to the fuzzy place. In a graphical representation, a hierarchy is drawn as a double-lined square to connect the importing or exporting fuzzy places. A hierarchical fuzzy Petri net that contains a main hierarchy $H0$ and hierarchy $H1$ is illustrated in Fig. 9(a). The status of the fuzzy place $P1$ in Fig. 9(a) is shown in Fig. 9(b). In this figure, the fuzzy Petri net in the middle window is the main hierarchy at the top level of the hierarchical structure, and the fuzzy Petri net in the bottom window is hierarchy $H1$ at the second level. In $H0$, fuzzy places $P2$ and $P4$ are the exporting fuzzy place with respect to hierarchy $H1$, and fuzzy place $P5$ is the importing fuzzy place with respect to hierarchy $H1$. In the hierarchy $H1$, fuzzy places $P1$ and $P3$ are the importing fuzzy places with respect to $H0$, and fuzzy place $P5$ is the exporting fuzzy place with respect to $H0$. When a token is inserted into the fuzzy place $P2$ in $H0$, it will be transited into hierarchy $H1$ and added to place $P1$ in hierarchy $H1$. Similarly, once a token enters into place $P4$ in $H0$, it will be sent into hierarchy $H1$ and reach the fuzzy place $P3$ in $H1$. After firing transitions $t1$, $t2$, and $t3$ in hierarchy $H1$, the token arrives at the fuzzy place $P5$ in $H1$ and then enters the fuzzy place $P5$ in $H0$.

Hierarchical incidence matrices are introduced to solve the complexity problem arising from the large size of fuzzy Petri nets. A hierarchical incidence matrix is defined as an algebraic form of a hierarchical fuzzy Petri net. For example, the hierarchical incidence matrices of the main hierarchy $H0$ and hierarchy $H1$ in Fig. 9(a) are presented in Fig. 10(a) and (b), respectively. The symbol $-1/P1$ at $(H1, P2)$ shows that there is an arc from the fuzzy place $P2$ in $H0$ to the fuzzy place $P1$ in hierarchy $H1$, and the symbol $1/P5$ at $(H1, P5)$ means that there is an arc from the fuzzy place $P5$ in $H1$ to the fuzzy place $P5$ in hierarchy $H0$. By defining this symbol, the connections between hierarchies are identified in an algebraic form.

There are two main benefits of having a hierarchical structure in our system: 1) the notion of hierarchy makes the handling of complex systems easy through decomposition and 2) a hierarchical Petri net facilitates the reusability, namely, each hierarchy can be considered as a reuse unit.

B. Reasoning Mechanism

To improve the efficiency of a fuzzy rule-based reasoning, it is crucial that fuzzy facts (input or inferred) find the matched fuzzy rules efficiently, rather than scanning all of the fuzzy rules. Fuzzy Petri nets offer an opportunity to achieve this goal by using transitions and arcs to connect fuzzy rules as a net-based structure [15]. A data-driven reasoning algorithm is developed by defining an extended fuzzy marking, denoted by FM^E. Each hierarchy has an extended fuzzy marking. The elements of FM^E, denoted by $FM^E(p_i)$, are called extended fuzzy places, which are defined



(a) (b)

Fig. 9. (a) A hierarchical fuzzy Petri net and (b) place status for $P1$ in $H0$.

	(P1,VSe)	(P2,FSl)	(P3,VSe)	(P4,Se)	(P5,FSe)	(P6,Se)
(t_1^i,VT)	-1	1	0	0	0	0
(t_2^i,T)	0	0	-1	1	0	0
(t_3^i,FT)	0	0	0	0	-1	1
H1	0	-1/P1	0	-1/P3	1/P5	0

(a)

	(P1,FSl)	(P2,VSe)	(P3,Se)	(P4,VSe)	(P5,UK)
(t_1^i,T)	-1	1	0	0	0
(t_2^i,T)	0	0	-1	1	0
(t_3^{cd},UK)	0	-1	0	-1	1

(b)

Fig. 10. Hierarchical incidence matrices for the hierarchical fuzzy Petri net in Fig. 9(a): (a) for the main hierarchy $H0$ and (b) for the hierarchy $H1$.

as $FM^E(p_i) = [p_i, \tilde{F}'_i, \tau_i, p_i\bullet, \bullet(p_i\bullet)\backslash\{p_i\}, (p_i\bullet)\bullet]$. From an extended fuzzy place $FM^E(p_i)$, we know:

1) the fuzzy set and the fuzzy truth value are attached to the token in p_i (i.e., $\tilde{F}'_i$ and τ_i);
2) the other tokens need to fire $p_i\bullet$ [i.e., $\bullet(p_i\bullet)\backslash\{p_i\}$];
3) the kind of computation to carry out after the firing (i.e., the type of $p_i\bullet$);
4) where to go for the new tokens after the firing [i.e., $(p_i\bullet)\bullet$].

For details about the reasoning algorithm, see Appendix A.

C. Transforming Modularized Fuzzy Rule Bases into Hierarchical Fuzzy Petri Nets

To bridge the gap between fuzzy rule-based expert systems and fuzzy Petri nets, it is important to have a mechanism to automatically transform modularized fuzzy rule bases into hierarchical fuzzy Petri nets. In our approach, two algorithms are involved in the transformation. One is to transform modularized fuzzy rule bases into a hierarchical incidence matrix. The other is to transform the hierarchical incidence matrix into a hierarchical fuzzy Petri net (see Appendix B).

D. An Overview of FPNES Tool

FPNES is implemented in Java with a client–server architecture, encompassing four main parts: fuzzy Petri net system (FPNS), user interface, transformation engine, and knowledge bases (see Fig. 11). Java is adopted as the programming language for the FPNES tool for its capability of running on multiple platforms and on the Internet.

FPNS is a modeling and analysis tool for fuzzy Petri nets and serves as an inference engine and explanation facility in FPNES. FPNS mainly contains the simulator and analyzer for fuzzy Petri nets. It provides the basic constructs for hierarchical fuzzy Petri nets (e.g., hierarchies, fuzzy places, uncertain transitions, arcs, and uncertain fuzzy tokens). After judging the firing conditions, the simulator will compute the fuzzy sets and move tokens. The analyzer performs the tasks of analyzing the properties of fuzzy Petri nets, such as incidence matrix, reachability trees, and state equations.

Users can edit modularized fuzzy Petri rule bases in the client site, including the assignments of linguistic variables, truth-qualified fuzzy rules, the relationship of modules, and modularized structures of input facts. When users finish editing the modularized fuzzy rule bases and the corresponding facts,

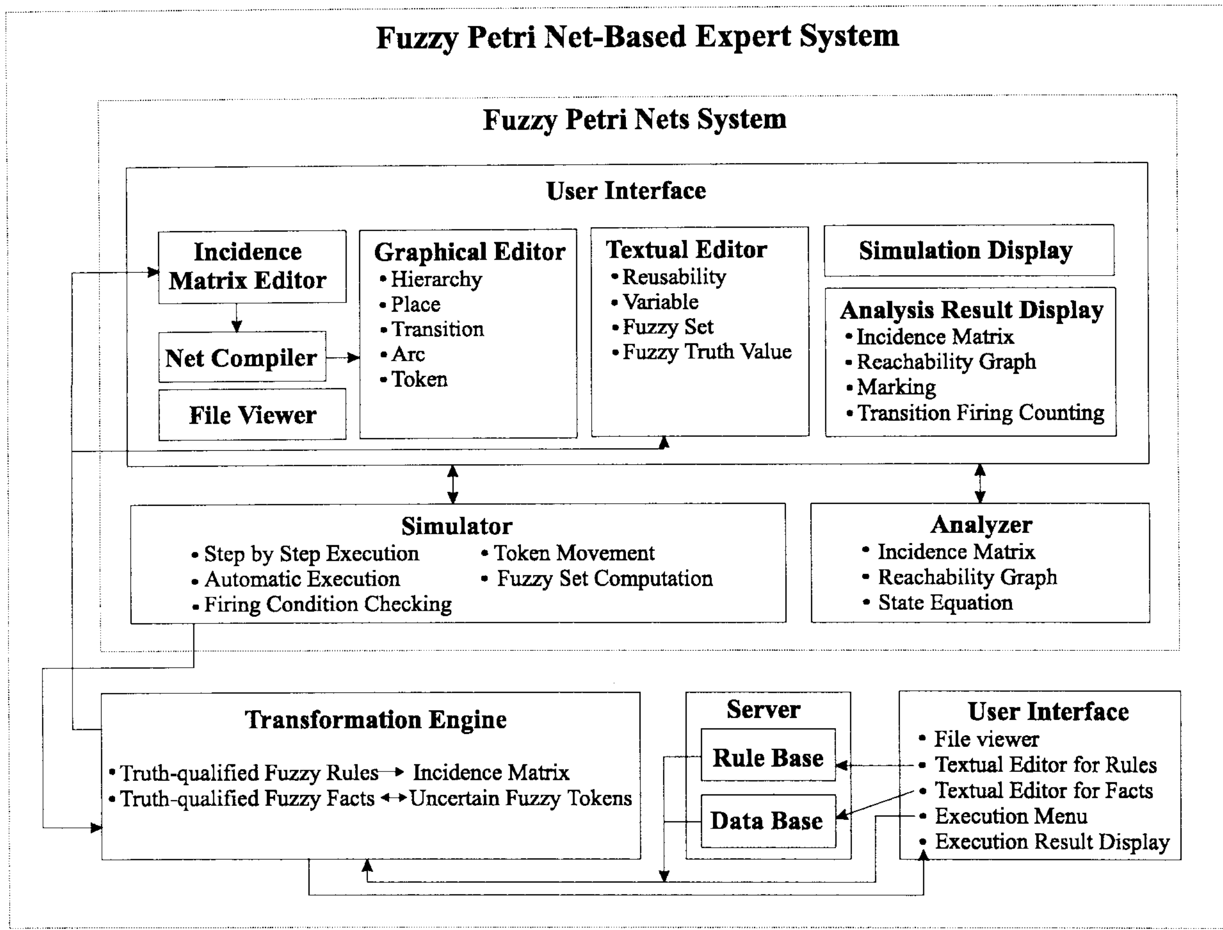


Fig. 11. An overview of FPNES tool.

and decide to run them, the data are then sent to the transformation engine and transformed to a hierarchical fuzzy Petri net in FPNS. After FPNS processes the hierarchical fuzzy Petri net with the aid of our reasoning algorithm, it sends the results back to the users. The results are presented in a hierarchical fashion to provide a flexible explanation facility (see Fig. 18).

V. Application to Damage Assessment of Bridges

In recent years, many countries have been aware of bridge problems and initiated the development of bridge management systems (BMS's) to assist their decision makers in establishing efficient repair and maintenance programs [19]. A key to success in BMS's relies heavily on the reliability of the technique adopted for damage assessment. Damage assessment for a bridge is defined as the process for evaluating the damage state of the bridge based on visual inspection and empirical testing on it.

A. Using FPNES for Damage Assessment

Damage assessment of a bridge is a difficult task due to a lack of complete understanding of the mechanism of bridge deterioration. Bridge structures are too complex to analyze completely, and therefore numerical simulations require a host of simplified assumptions. Nevertheless, an experienced engineer who has closely studied these problems over years could use his heuristic knowledge to achieve the task by linking the observed defects with causes, evaluating the impacts of these causes on bridge safety, assessing the damage level, and proposing recommendations for a bridge. However, there are far too few experts who can correctly inspect and assess deficient bridges. Recently, researchers have begun to investigate the use of expert systems to perform damage assessment, for example, [13], [14], [18], [20], [26], [28], [36], [40], [41], [44], [48], and [53]. Since heuristic knowledge plays an important role in the process of damage assessment, exploiting expert systems to capture the expertise and mimic the reasoning patterns of experts for damage assessment is a promising direction.

The descriptions of heuristic damage-assessment knowledge from bridge engineers usually take the form of natural language that contains intrinsic imprecision and uncertainty. For example, a bridge engineer may make an imprecise statement for assessing a crack observed on a prestressed I-girder, such as "If a shear crack has large extent, wide width and deep depth, severe corrosion accompanied with rust stain occurs in the crack, serious efflorescence comes out of the crack, and water leaches from the crack, then the damage level of this

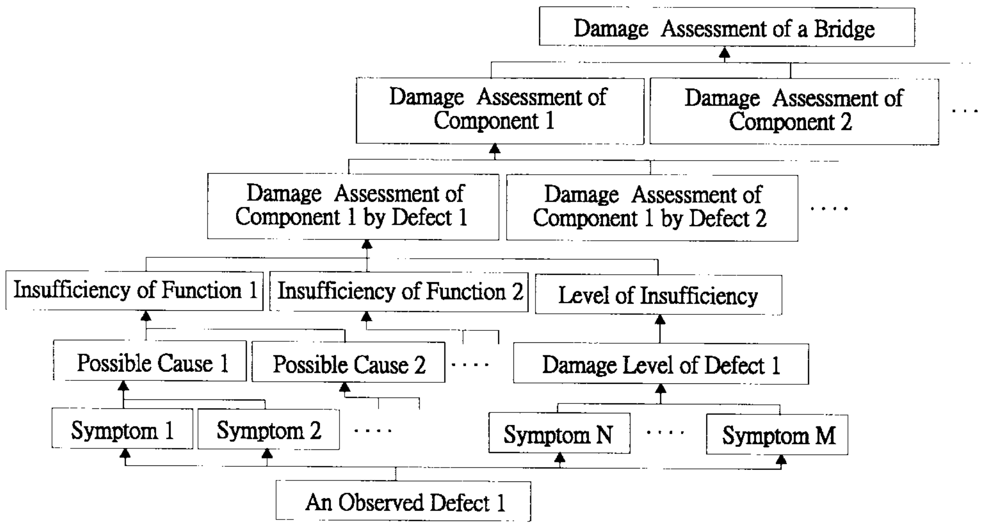


Fig. 12. The inference procedure for damage assessment.

shear crack is very severe." Furthermore, sometimes bridge engineers are not completely confident about their imprecise statements since various exceptions may occur due to the complexity in damage assessment. Besides, descriptions on the observed defects by bridge inspectors are often imprecise and uncertain. For example, a statement "We are not that confident that the delamination within a beam is extensive" made by an inspector contains imprecision and uncertainty. Therefore, a reasoning mechanism that can deal with uncertain and imprecise information is expected for damage assessment. In addition, in order to increase the confidence about the assessment results, an explanation facility that can describe how the conclusions are derived is crucial for a computer-aided tool designed especially for damage assessment.

B. Development of Modularized Rule Bases

Damage assessment is based on the notion of functionality. Each component of a bridge structure carries out several functions simultaneously to keep the bridge working. A bridge is considered damaged if some of its components are not functioning correctly. The damage level will depend upon how many functions are impaired.

The inference procedure for damage assessment of a bridge is described as follows (see Fig. 12).

1) A group of inspectors visually investigates each component of a bridge to record the observed defects, such as scaling, cracks, delamination, spalls, honeycomb, efflorescence, corrosions, leaching, etc., and their symptoms.
2) Based on defect symptoms such as defect positions, defect patterns, etc., experienced bridge engineers can identify the possible causes of the defects.
3) The damage level of each defect is evaluated according to the symptoms, which contain quantitative descriptions.
4) The possible causes induce what kinds of functions are eliminated due to the defects.
5) The levels of functional derogation are inferred based on the damage levels of the defects.
6) The assessment of damage can be obtained by aggregating both the functional derogation and its levels.

Fig. 13 shows the factors that are involved in the evaluation of a shear crack in an I-girder. Fig. 15 shows the fuzzy Petri nets after the transformation of the rule bases of shear crack in an I-girder (see Fig. 14 for an example of fuzzy rules). Based on the inference procedure, we construct the modularized rule bases that contain 100 truth-qualified fuzzy rules and 133 recommendations (see Fig. 16).

C. Case Study

The Da-Shi bridge in north Taiwan is used to demonstrate the use of FPNES. It was rebuilt in 1960 as a simply supported and 12-spanned bridge that is of 550 m long and of 7.8 m wide to cross the Da-Han river. This bridge consists of 12 decks, 36 prestressed I-girders, 120 diaphragms, 11 piers, and two abutments. In 1997, this bridge was inspected by the Center of Bridge Engineering Research at National Central University. Through visual inspection, many minor cracks accompanied with efflorescence spread over eight panels within deck 7. The I-girders S9G1, S9G2, S9G3 in span 9 and S10G1, S10G2, S10G3 in span 10 had severe flexure, shear cracks, and some spalls. There were two diaphragms where several spalls were found. The detailed descriptions on these defects can be found in [4].

After executing FPNES for damage assessment of the Da-Shi bridge, the hierarchical fuzzy Petri nets are constructed based on the modularized rule bases, and uncertain fuzzy tokens are transformed into these nets in order to fire transitions and perform the reasoning mechanism (see Fig. 17). The results of damage assessment using FPNES for the Da-Shi bridge are expressed in a hierarchical fashion to serve as an explanation mechanism to facilitate the retrieval of detailed information on damaged components from the top down to

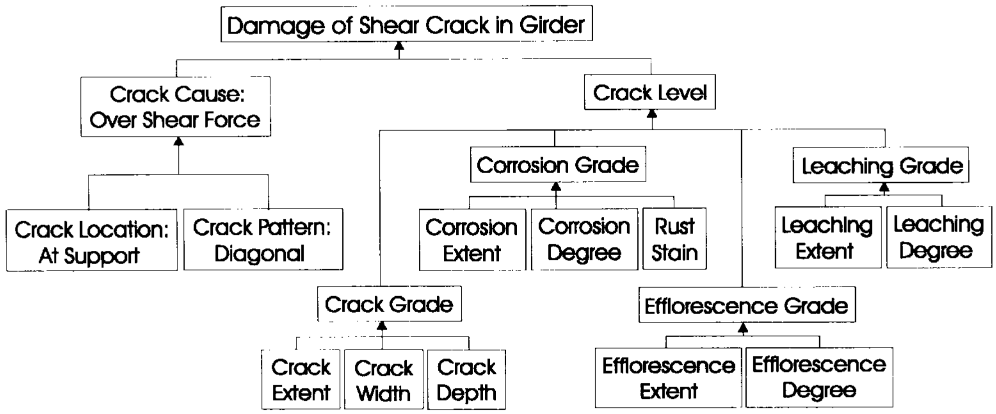


Fig. 13. Factors related to shear crack in a girder.

⋮

Rule 3: IF crack extent is very severe AND crack width is very severe AND crack depth is very severe
THEN crack measure very severe, very true
Rule 4: IF corrosion extent is very severe AND corrosion degree is very severe AND rust stain is very severe
THEN corrosion grade is very severe, very true
Rule 5: IF efflorescence extent is very severe AND efflorescence degree is very severe
THEN efflorescence grade is very severe, very true
Rule 6: IF leaching extent is very severe AND leaching degree is very severe THEN leaching grade is very severe, very true

⋮

Fig. 14. Part of rule bases for shear crack in an I-girder.

lower levels (see Fig. 18). The recommendations embedded in rule bases are also provided on an if-needed basis. As a result, the damage of the superstructure of the Da-Shi bridge is evaluated to be severe with common confidence (i.e., true) since the overall damage of the decks is fairly slight with fair confidence (i.e., fairly true), the overall damage of the I-girders is severe with common confidence, and the overall damage of the diaphragms is very slight with strong confidence (i.e., very true). This result matches the experts' judgments in the report; moreover, it is more informative than the report itself because the explanation provided in the system and the confidence level associated with the conclusions can be used as a way of justification on whether to take the recommendations into account or not.

VI. Related Work

A number of researchers have addressed the use of expert systems for damage assessment for a variety of structures. We examine their studies below.

Ishizuka *et al.* developed a rule-based expert system (SPERIL-I) to assess damage states of existing buildings [20]. They advocated that the damage states of structures had the nature of fuzziness; therefore, a fuzzy degree of damage state was evaluated for a building based on the accelerometer record and visual inspection after it suffered earthquake excitation. The fuzzy rules with certainty factors were employed jointly in their inexact inference to cope with the continuous nature of the damage state. They also developed a fuzzy extension of Dempster's rule of combination to aggregate similar conclusions [21]. Ogawa *et al.* intended to represent damage states by not only fuzzy degrees but also certainty factors in the new system (SPERIL-II). Therefore, they improved the previous inference mechanism to handle fuzzy facts with certainty factors.

A damage-assessment technique for protective structures was proposed by Hadipriono and Ross [18]. The overall damage level of a protective structure was evaluated to a fuzzy degree after visual inspection. Fuzzy rules were constructed based on three damage criteria: functionality, repairability, and the structural integrity of the structure. Different from Zadeh's fuzzy reasoning, their inference mechanism for fuzzy rules and facts was achieved by the notion of truth functional modification.

Rather than giving a single fuzzy degree, Shiraishi *et al.* assessed reinforced concrete bridge decks by three items: damage pattern, damage propagation, and damage cause [48]. Although fuzzy sets and certainty facts were included in their inference mechanism, partial matching was not allowed. Meanwhile, they also used fuzzy truth values instead of certainty factors as an uncertainty model to develop another inference mechanism [14]. Recently, they made joint use of genetic algorithms and neural networks to support a knowledge-acquisition method [13].

Besides applying rule-based reasoning, most researchers used numerical computations for damage assessment. Some who implemented their techniques into expert systems are described briefly as follows. Ross *et al.* used a fuzzy weight average technique to assess reinforced concrete protective

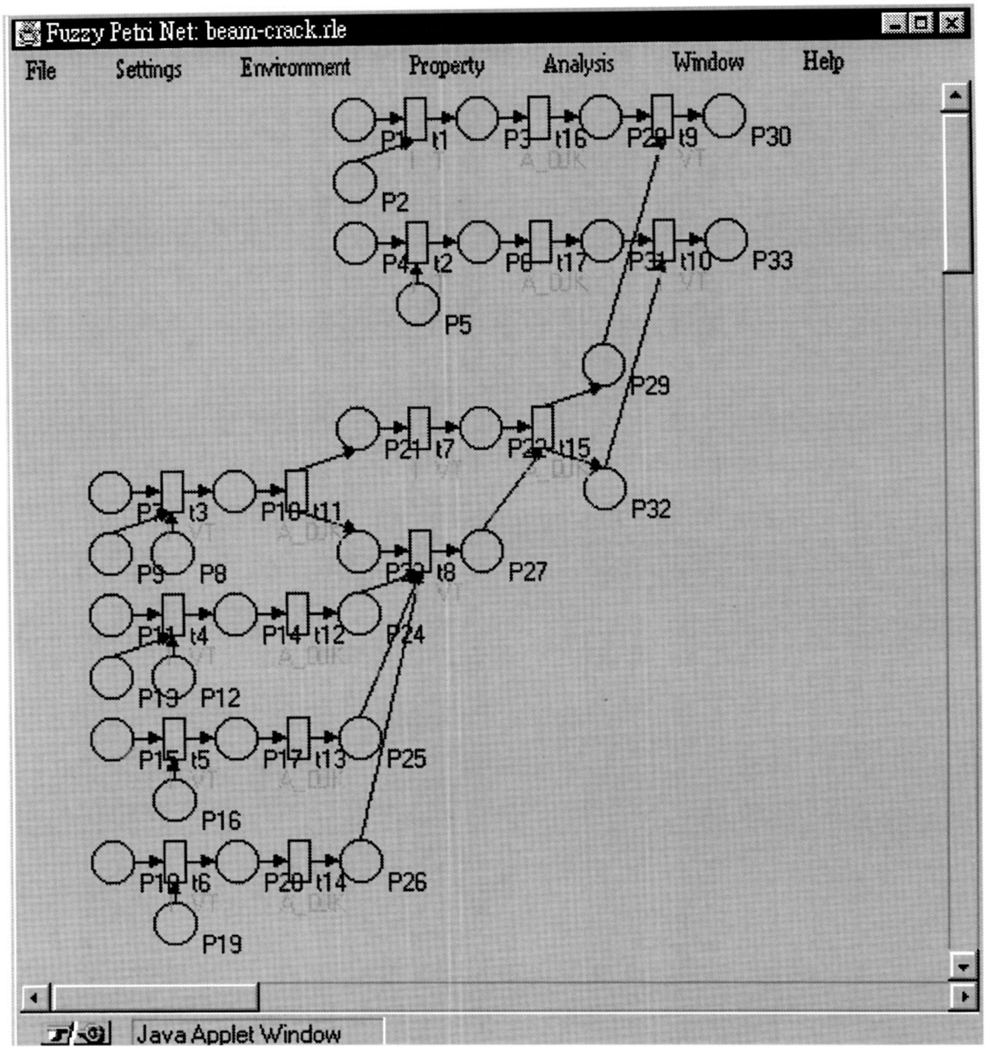


Fig. 15. The fuzzy rule Petri net transformed from the rule bases of a shear crack in an I-girder.

structures by providing damage modes and fuzzy degrees [44]. Miyamoto *et al.* proposed a fuzzy mapping formalism to evaluate the remaining life and soundness degrees for concrete bridges [36]. They further improved their system by substituting neural networks for their fuzzy mapping formalism [28]. Issa *et al.* established a bridge rating expert system, in which the rating methods considered were inventory rating, operating rating, rating factor rating, and sufficiency rating [22]. A strength rating was based on the evaluation of existing prestressed concrete bridges in accordance with the American Association of State Highway Transportation Officials specification, and inventory rating for all bridges according to the Federal Highway Administration guide for the Structure Inventory and Appraisal of the Nation's Bridges.

Unlike other researchers, our approach does not impose any restriction on the inference mechanism, that is, the intended meaning is not required to be intact; meanwhile, the confidence level can be partially certain. Furthermore, our approach offers more informative results because the explanation provided in the system and the confidence level of the conclusions can be used as a way of justification on whether to take the recommendations into account or not (see Table I).

VII. Conclusion

A fuzzy Petri nets approach to modeling fuzzy rule-based reasoning is proposed to bring together the possibilistic entailment and the fuzzy reasoning to handle uncertain and imprecise information. The three key components in our fuzzy rule-based reasoning—fuzzy propositions, truth-qualified fuzzy rules, and truth-qualified fuzzy facts—can be formulated as fuzzy places, uncertain transitions, and uncertain fuzzy tokens, respectively. Four types of uncertain transitions—inference, aggregation, duplication, and aggregation-duplication transitions—are introduced to fulfill the mechanism of fuzzy rule-based reasoning. We also propose a framework of integrated expert systems based on our fuzzy Petri net, called fuzzy Petri net-based expert system. Major features of FPNES include: knowledge representation through the use of hierarchical fuzzy Petri nets, a reasoning mechanism based on fuzzy Petri nets, and transforming modularized fuzzy rule bases into hierarchical fuzzy Petri nets. An application to the damage assessment of the Da-Shi bridge in Taiwan is used as an illustrative example of FPNES. FPNES offers several benefits.

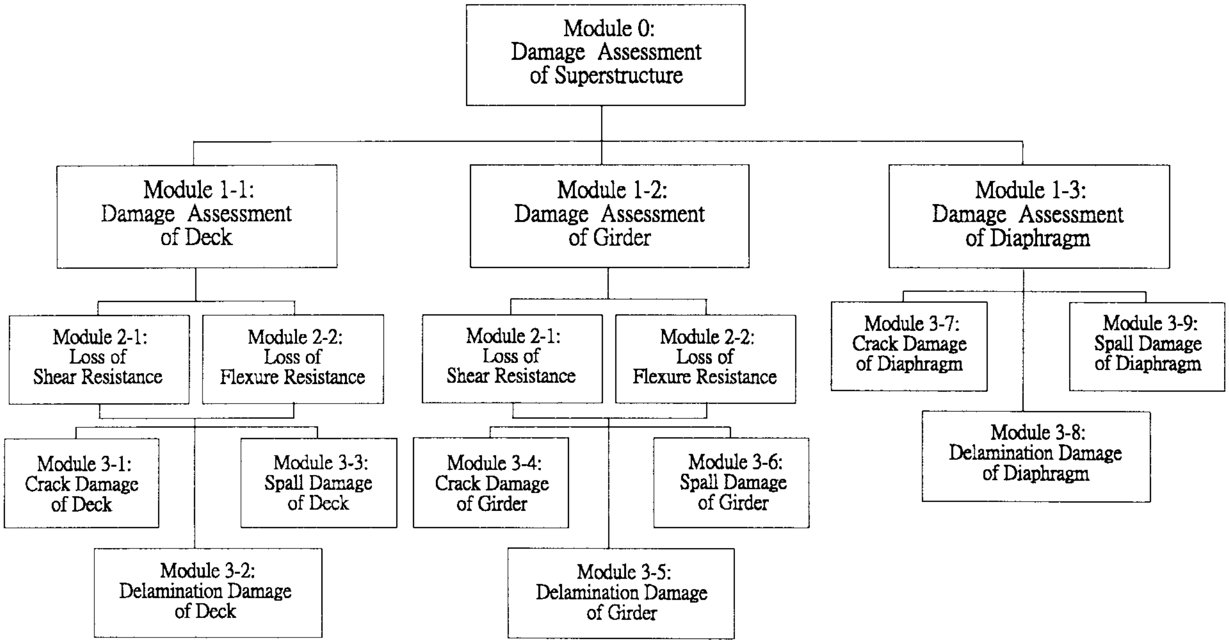


Fig. 16. The modularized rule bases for damage assessment of superstructure.

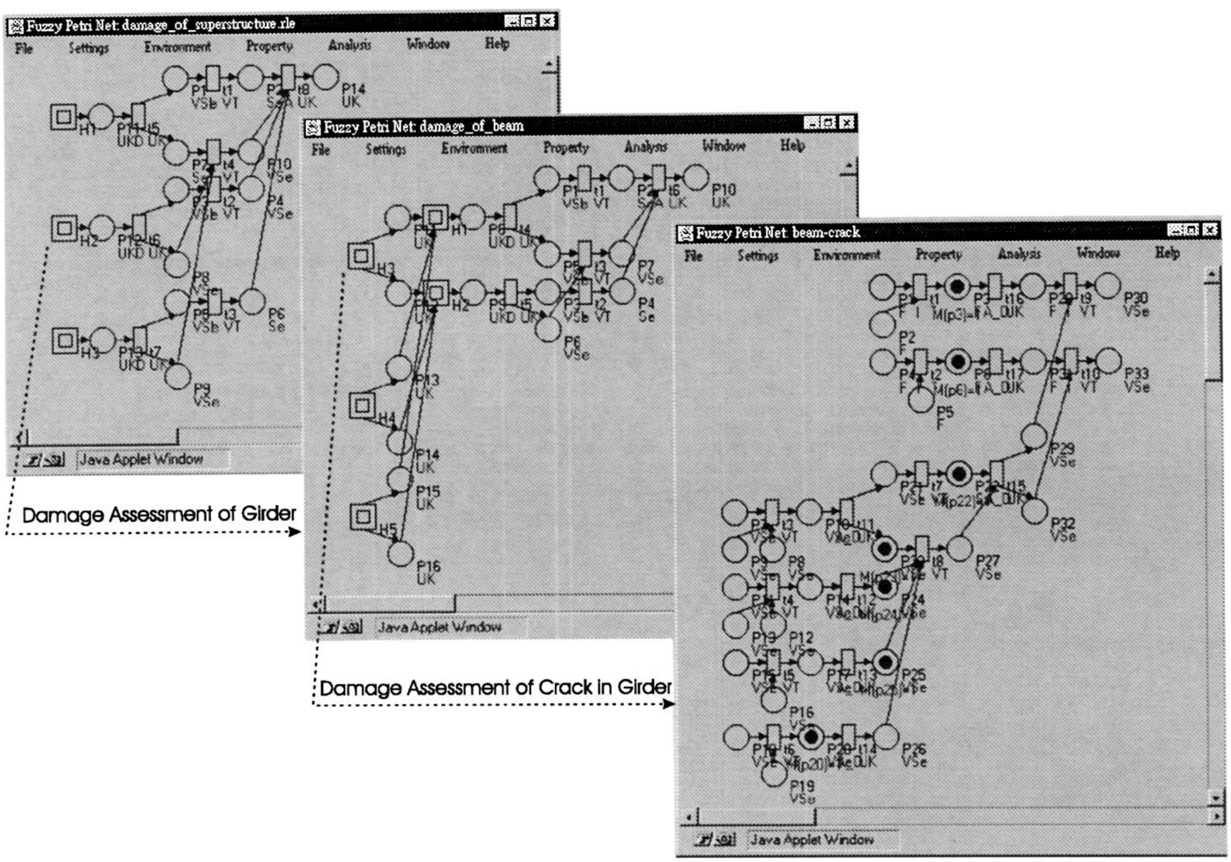


Fig. 17. Simulation of damage assessment for the Da-Shi bridge.

- The efficiency of rule-based reasoning is improved by designing an efficient reasoning algorithm based on fuzzy Petri nets.
- The explanation of how to reach conclusions is expressed through the movements of tokens in fuzzy Petri nets.
- The hierarchical fuzzy Petri nets make the handling of complex systems easy and facilitate reusability.

Our future work consists of two tasks: 1) to develop a knowledge verification scheme based on our fuzzy Petri nets and 2) to apply the proposed approach to other applications.

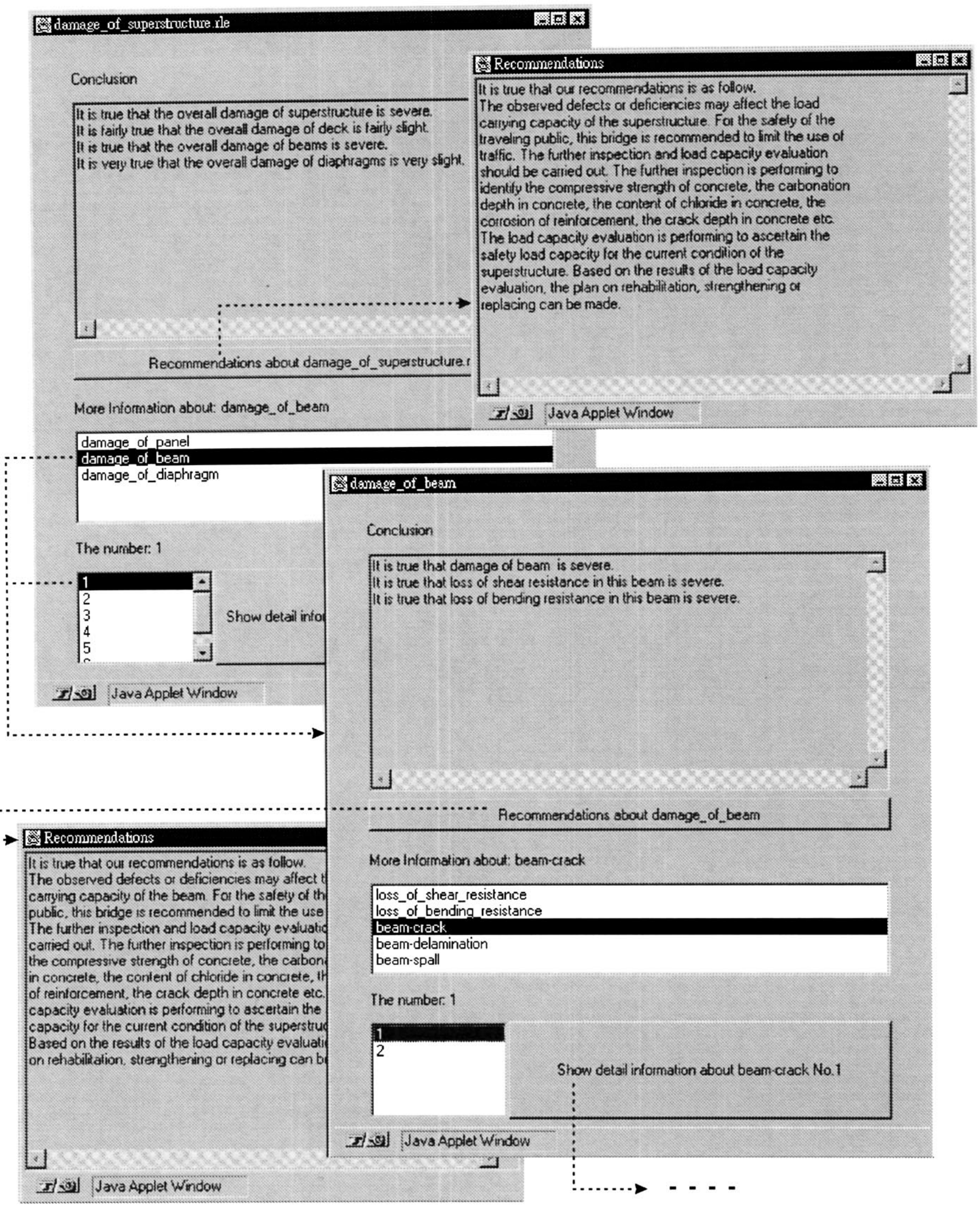


Fig. 18. Results of damage assessment for the Da-Shi bridge using FPNES.

Appendix A
Reasoning Algorithm

Our reasoning algorithm is used to manage the evolution of extended fuzzy marking. We describe it as follows:

Algorithm 1—Implementing Fuzzy Petri Nets:

1) Get the initial extended fuzzy marking FM_0^E, which consists of all source fuzzy places.

2) For each i, set a current extended fuzzy marking $FM_c^E = FM_i^E$ and the next extended fuzzy marking $FM_{i+1}^E = \{\}$.

3) Select an element of the current extended fuzzy marking $FM_c^E(p_j) = [p_j,\, \tilde{F}_j{}',\, \tau_j,\, p_j\bullet,\, \bullet(p_j\bullet)\backslash\{p_j\},\, (p_j\bullet)\bullet]$.

4) a) If the output transition of p_j is a duplication transition, then infer the extended fuzzy place $FM_{i+1}^E(p_k)$ of each $p_k \in (p_j\bullet)\bullet$ by duplication.

TABLE I
A SUMMARY OF RELATED WORK ON EXPERT SYSTEMS FOR DAMAGE ASSESSMENT

	DAMAGE ASSESSMENT		SYSTEM				
			KNOWLEDGE BASE		INFERENCE ENGINE		IMPLEMENTATION
	OUTPUT	INPUT	KNOWLEDGE REPRESENTATION	UNCERTAINTY MODEL	INFERENCE MECHANISM	CONTROL STRATEGY	
M. ISHIZUKA ET AL. (SPERIL-I) 1981 (R.C. STRUCTURES)	DAMAGE STATE: (A FUZZY DEGREE)	VISUAL INSPECTION ACCELEROMETER RECORD	FUZZY RULES + C.F.	CERTAINTY FACTOR	F -> G, CF F1 —— G1	FOLLOW THE ORDERS OF RULES	C
H. OGAWA ET AL. (SPERIL-II) 1985 (R.C. STRUCTURES)	DAMAGE STATE: (A FUZZY DEGREE + C.F.)	VISUAL INSPECTION ACCELEROMETER RECORD	FUZZY RULES + C.F.	CERTAINTY FACTOR	F -> G, CF1 F1 —— , CF2 G , CF3	METARULES	PROLOG
F. HADIPRIONO ET AL. 1991 (R.C. PROTECTIVE STRUCTURES)	DAMAGE LEVEL: (A FUZZY DEGREE)	VISUAL INSPECTION	FUZZY RULES	NONE	F -> G F1 —— G1	UNKNOWN	UNKNOWN
N. SHIRAISHI ET AL. 1991 (R.C. BRIDGE DECK)	1.DAMAGE PATTERN: (A PATTERN + C.F.) 2.DAMAGE PROPAGATION: (A PATTERN + C.F.) 3.DAMAGE CAUSE: (A CAUSE + C.F.)	VISUAL INSPECTION	FUZZY RULES + C.F.	CERTAINTY FACTOR	F -> G, CF1 F —— , CF2 G, CF3	METARULES	LISP
H. FURUTA ET AL. 1991,1996 (R.C. BRIDGE DECK)	1.REMAINING LIFE: (YEARS + T.V.) 2.DAMAGE DEGREE : (A FUZZY DEGREE + T.V.) 3.DAMAGE PROPAGATION: (A FUZZY DEGREE + T.V.) 4.DAMAGE CAUSE: (A CAUSE + T.V.)	PAST RECORD VISUAL INSPECTION	FUZZY RULES + T.V.	FUZZY TRUTH VALUE	F -> G, TV1 F1 —— , TV2 G, TV3	METARULES	FRANZ LISP
T.J. ROSS ET AL. (DAPS) 1990 (R.C. PROTECTIVE STRUCTURES)	1.DAMAGE MODE 2.DAMAGE LEVEL: (A FUZZY DEGREE)	TESTING DATA VISUAL INSPECTION	HIERARCHICAL STRUCTURE	NONE	FUZZY WEIGHTED AVERAGE	METARULES	EXSYS
A. MIYAMOTO ET AL. 1993 (R.C. BRIDGE)	1.REMAINING LIFE: YEAR 2.SOUNDNESS: (0~100) 3.PROBABILITIES OF FUZZY DEGREES	INSPECTION DATA ENVIRONMENT CONDITION TRAFFIC VOLUME	HIERARCHICAL STRUCTURE	NONE	FUZZY MAPPING	METARULES	PROLOG
M. KUSHIDA ET AL. 1997 (R.C. BRIDGE)	1.REMAINING LIFE: YEAR 2.SOUNDNESS: (0~100) 3.PROBABILITIES OF FUZZY DEGREES	INSPECTION DATA ENVIRONMENT CONDITION TRAFFIC VOLUME	HIERARCHICAL STRUCTURE + NEURAL NETWORKS	NONE	NEURAL NETWORKS	METARULES	PROLOG
M.A. ISSA ET AL. (BRES) 1995 (P.C. BRIDGES)	1.INVENTORY RATING 2.OPERATION RATING 3. RATING FACTOR 4.SUFFICIENCY RATING	NUMERICAL DATA	EQUATIONS	NONE	EQUATIONS	METARULES	EXSYS
OUR APPROACH FPNES 1997 (P.C.I-GIRDER BRIDGES)	ALL INFORMATION IS SHOWN HIERARCHICALLY FROM OVERALL DAMAGE LEVEL TO INSPECTION DETAILS, SUCH AS: 1.DAMAGE LEVEL: (A FUZZY DEGREE + T.V.) 2.DAMAGE CAUSE: (A CAUSE + T.V.) 3.RECOMMENDATIONS	VISUAL INSPECTION	FUZZY RULES + T.V.	FUZZY TRUTH VALUE	F -> G, TV1 F1 —— , TV2 G1, TV3	FUZZY PETRI NETS	FPNES (IN JAVA)

I.E.: C.F.: CERTAINTY FACTOR; T.V.: FUZZY TRUTH VALUE; F1 AND G1 ARE CLOSE TO F AND G, RESPECTIVELY.

b) Else if the output transition of p_j is an inference transition, and the extended fuzzy place of each $p_l \in \bullet(p_j\bullet)\backslash\{p_j\}$ exists in FM_c^E, then infer the extended fuzzy place $FM_{i+1}^E(p_k)$ of $p_k = (p_j\bullet)\bullet$ by i) transformation, ii) inference, and iii) composition.

c) Else if the output transition of p_j is an aggregation transition, and the extended fuzzy place of each $p_l \in \bullet(p_j\bullet)\backslash\{p_j\}$ exists in FM_c^E, then infer the extended fuzzy place $FM_{i+1}^E(p_k)$ of $p_k = (p_j\bullet)\bullet$ by i) transformation, ii) aggregation, and iii) composition.

d) Else if the output transition of p_j is an aggregation-duplication transition, and the extended fuzzy place of each $p_l \in \bullet(p_j\bullet)\backslash\{p_j\}$ exists in FM_c^E, then infer the extended fuzzy place $FM_{i+1}^E(p_k)$ of each $p_k \in (p_j\bullet)\bullet$ by i) transformation, ii) aggregation, and iii) composition.

5) a) If the output transition of p_j is fired, then insert the inferred extended fuzzy place $FM_{i+1}^E(p_k)$ into the next extended fuzzy marking FM_{i+1}^E.

b) If the output transition of p_j is a hierarchy, then insert the extended fuzzy place $FM_c^E(p_j)$ into the hierarchy and wait for the final extended fuzzy marking of the hierarchy to be inserted into the current extended fuzzy marking.

c) Else insert this element $FM_c^E(p_j)$ and each $FM_c^E(p_l)(p_l \in \bullet(p_j\bullet)\backslash\{p_j\})$ into the next extended fuzzy marking FM_{i+1}^E.

6) Delete the element $FM_c^E(p_j)$ and each $FM_c^E(p_l)(p_l \in \bullet(p_j\bullet)\backslash\{p_j\})$ from the current extended fuzzy marking.
7) Repeat steps 3)–6) until no element is in the current extended fuzzy marking.
8) Repeat steps 2)–7) until all output transitions in the current extended fuzzy marking are not fired.
9) Send the final extended fuzzy marking to the upper level hierarchy.

APPENDIX B
TRANSFORMATION ALGORITHMS

A labeling system for fuzzy propositions in rule bases is defined first. Each fuzzy proposition in a module is labeled by $L_j(a_j, b_j, c_j, d_j, e_j)$, where a_j denotes the rule number in this module, b_j denotes right-hand side (RHS) or left-hand side (LHS) of this rule (1 for LHS and 2 for RHS), c_j denotes the index of linguistic variable, d_j denotes the index of fuzzy set, and e_j refers to the module number for which c_j is an importing or exporting linguistic variable. It should be noted that $e_j = 0$ means that this fuzzy proposition has neither importing nor exporting linguistic variables. For example, in Fig. 8, "$X2$ *is* very severe" in the first rule in main module $M0$ is labeled as $L_2(1, 2, 2, 1, 1)$, where F_1 denotes "very severe."

Algorithm 2—Transforming Modularized Fuzzy Rules into Hierarchical Incidence Matrix:

1) *Labeling:* Label each fuzzy proposition in each truth-qualified fuzzy rule in sequence as $L_j(a_j, b_j, c_j, d_j, e_j)$ $(j = 1 \sim m)$.
2) *Inference Transition Part:*
   a) Create the row of fuzzy places (FP), whose elements are defined as (p_j, F_{d_j}) $(j = 1 \sim m)$.
   b) Create the column of uncertain transition (UT), whose elements are defined as (t_i, τ_i) $(i = 1 \sim n,\ \tau_i$ means the fuzzy truth value of the ith rule$)$.
   c) Create the $n \times m$ incidence matrix A, where a_{ij} is -1 if i) L_j's a_j is i, ii) its b_j is 1, and iii) its e_j is 0; a_{ij} is $+1$ if i) L_j's a_j is i, ii) its b_j is two, and iii) its e_j is zero. a_{ij} is zero if L_j's a_j is not i.
3) *Aggregation-Duplication Transition Part*: If 1) some L_j's have the same c_j and e_j is zero and 2) parts of 1) have $b_j = 1$ (called group 1) and the other parts of 1) have $b_j = 2$ (called group 2), then:
   a) insert an aggregation-duplication transition as the last element in the column of uncertain transition UT;
   b) add a new row at the bottom of the incidence matrix A, where a_{ij} is -1 if L_j is in group 2; a_{ij} is $+1$ if L_j is in group 1. a_{ij} is zero for the rest.
4) Repeat step 3) until no L_j is satisfied.
5) *Duplication Transition Part:* If 1) some L_j's have the same c_j and e_j is zero and 2) all of 1) have $b_j = 1$ (called group 1), then:
   a) insert a duplication transition as the last element in the column of uncertain transition UT;
   b) insert a fuzzy place as the last element in the row of fuzzy place FP;
   c) add a new row at the bottom and a new column at the left end of the incidence matrix A, where a_{ij} is -1 for the last element of the new row (or column), a_{ij} is $+1$ if L_j is in group 1, and a_j is zero for the rest.
6) Repeat step 5) until no L_j is satisfied.
7) *Aggregation Transition Part:* If 1) some L_j's have the same c_j and e_j is zero and 2) all of 1) have $b_j = 2$ (called group 1), then:
   a) insert an aggregation transition as the last element in the column of uncertain transition UT;
   b) insert a fuzzy place as the last element in the row of fuzzy place FP;
   c) add a new row at the bottom and a new column at the left end of the incidence matrix A, where a_{ij} is $+1$ for the last element of the new row (or column), a_{ij} is -1 if L_j is in group 1, a_j is zero for the rest.
8) Repeat step 7) until no L_j is satisfied.
9) *Hierarchy Part:* If 1) some L_j's have the same c_j and e_j is not zero and 2) part of 1) has $b_j = 1$ (called group 1) and the other part of 1) has $b_j = 2$ (called group 2), then:
   a) insert a hierarchy H_a as the last element in the column of uncertain transition UT;
   b) add a new row at the bottom of the incidence matrix A, where a_{ij} is $-1/P_k$ if L_j is in group 2 and P_k, an importing fuzzy place with respect to this hierarchy in H_a, has the same c_j in related L; and a_{ij} is $1/P_l$ if L_j is in group 1 and P_l, an exporting fuzzy place with respect to this hierarchy in H_a, has the same c_j in related L. a_{ij} is zero for the rest.
10) Repeat step 9) until no L_j is satisfied.

Based on the hierarchical incidence matrix, hierarchical fuzzy Petri nets are constructed by an algorithm which is described below.

Algorithm 3—Transforming Hierarchical Incidence Matrix into Hierarchical Fuzzy Petri Nets:

1) *Fuzzy Places:* Draw fuzzy places based on the row of fuzzy places FP.
2) *Uncertain Transitions:* Draw uncertain transitions based on the column of uncertain transitions UT.
3) *Arcs:* Link fuzzy places and uncertain transitions based on the incidence matrix A.
   a) If a_{ij} is -1, then draw an arc from fuzzy place p_j to uncertain transition t_i.
   b) Else If a_{ij} is $+1$, then draw an arc from uncertain transition t_i to fuzzy place p_j.
   c) If a_{ij} is $-1/P_k$, then draw an arc from fuzzy place p_j to hierarchy H_l and an arc from hierarchy $H1$ to fuzzy place P_k in $H1$.
   d) Else if a_{ij} is $1/P_k$, then draw an arc from fuzzy place P_k in $H1$ to hierarchy $H1$ and an arc from hierarchy $H1$ to fuzzy place p_j.
   e) Else if a_{ij} is zero, then there is no arc between uncertain transition t_i and fuzzy place p_j.

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