Optimal Placement and Sizing of Distributed Generation via an Improved Nondominated Sorting Genetic Algorithm II

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***Abstract*—An improved nondominated sorting genetic algorithm–II (INSGA-II) has been proposed for optimal planning of multiple distributed generation (DG) units in this paper. First, multiobjective functions that take minimum line loss, minimum voltage deviation, and maximal voltage stability margin into consideration have been formed. Then, using the proposed INSGA-II algorithm to solve the multiobjective planning problem has been described in detail. The improved sorting strategy and the novel truncation strategy based on hierarchical agglomerative clustering are utilized to keep the diversity of population. In order to strengthen the global optimal searching capability, the mutation and recombination strategies in differential evolution are introduced to replace the original one. In addition, a tradeoff method based on fuzzy set theory is used to obtain the best compromise solution from the Pareto-optimal set. Finally, several experiments have been made on the IEEE 33-bus test case and multiple actual test cases with the consideration of multiple DG units. The feasibility and effectiveness of the proposed algorithm for optimal placement and sizing of DG in distribution systems have been proved.**

***Index Terms*—Distributed generation (DG), distribution system planning, multiobjective optimization (MOO), nondominated sorting genetic algorithm–II (NSGA-II).**

I. Introduction

IN RECENT years, distributed-generation (DG) technology has become a heavily researched topic, given increasing global concerns for environmental protection, energy-saving issues, increasing complexities of wind power, photovoltaic power generation, and other renewable energy technologies. After DG is connected to a distribution network, the structure, operation, and control mode of the distribution network will be changed tremendously. It is difficult to estimate how many DG capacities will be needed to be connected to distribution systems. Undoubtedly, it is certain that increasing penetration levels require robust tools and methods that help assess capabilities and requirements of networks in order to produce the best planning and control strategy [1]. The challenge from DG planning is that the planning needs to solve the optimization problem with many objectives and constraint factors. It should be recognized that modern distribution business will see many market players pursuing contrasting and different objectives. Among many methods and tools, the multiobjective evolutionary algorithm provided a powerful searching method in multiple objective components to obtain an even Pareto-optimal set.

From the perspective of mathematical optimization, DG unit injection is also a complex multiobjective optimization issue. The objectives include optimal energy consumption, the minimum power consumer's electricity purchasing cost, and the minimum power loss based on constraints of power grid security and DG power output. Multiobjective economic/emission dispatch algorithms were investigated in [2] and [3]. Among the research about optimization methods, multiobjective models of DG planning were optimized by various methods, such as the simulated annealing technique, Tabu search method integrated with the genetic algorithm (GA) [4], and Fuzzy optimization method [5].

Recent studies about the DG planning model and various algorithms are surveyed as follows. Several intelligent optimization algorithms, such as GA [6]; particle swarm optimization (PSO) [7], [8]; differential evolution (DE) [9]; and artificial bee colony (ABC) [10] are used to solve the optimization problem considering minimum costs for network upgrading, operation, maintenance, and losses for handling the load growth and maximum DG penetration level. Besides, several sensitivity analysis methods of DG allocation were proposed in [11]–[13]. In case of multiple conflicting objectives, there may not be a solution which is the best compromise for all objectives. Therefore, a "tradeoff" solution is needed instead of a single solution in multiobjective optimization. Based on GA, Farouk *et al.* [14] proposed a multiobjective optimization approach to maximize savings in system-upgrade investment deferral, cost of annual energy losses, and cost of interruption. Jin *et al.* [15] established a multicriteria planning model for minimizing the cost and maximizing the reliability of generating units. A multiobjective performance index-based size and location determination of DG

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with different load models was presented in [16]. Nekooei *et al.* [17] studied a new approach using an improved harmony-search (HS) algorithm.

Although the attention of previous works focused on power loss and costs for network upgrading, relatively few efforts were involved with voltage deviation and voltage stability improvement. The inherent relations among multiple objectives were not investigated yet. In this paper, we first establish a mathematical model of the optimal DG planning problem, and then solve this optimization problem by the proposed improved nondominated sorting genetic algorithm II (INSGA-II) with the consideration of line loss, voltage deviation, and voltage stability margin.

The remainder of this paper is organized as follows. Section II formulates the proposed multiobjective optimization issue for DG planning. Section III describes the proposed INSGA-II method to solve the optimization problem. Section IV provides numerical results and comparisons with the proposed approach using various test systems with DG units, and Section V summarizes the main contributions and conclusions of this paper.

II. Problem Formulation

A. Objective Functions

Three objectives are considered in the optimization model, which include: 1) reducing system line losses; 2) reducing voltage deviation; and 3) increasing voltage stability margin when DG units are considered in the distribution network (DN).

1) Minimization of Line Losses: The first objective is to minimize system line losses after DG injection into the distribution network. This objective function is as

$$\min\ f_1(x) = \min \sum_{(i,j)\in B} g_{ij}\left(V_i^2 + V_j^2 - 2V_iV_j\cos\theta_{ij}\right) \quad (1)$$

where B is the set of branches of network, and $(i,j)\in B$ denotes that (i,j) are two nodes of a branch, and V_i and V_j are voltage magnitudes of nodes i and j, respectively. g_{ij} is the conductance between nodes i and j. And θ_{ij} is the difference between nodal phase angles θ_i and θ_j.

2) Minimization of Voltage Deviation: The second objective is to minimize the voltage deviation between nodal voltage and specified voltage magnitude. Nodal voltage magnitude is an important indicator to evaluate system security and power quality (PQ). The minimization of voltage deviation can help guarantee a better voltage level in distribution power systems. The function can be written as

$$\min f_2(x) = \min \sum_{i=0}^{N}\left(\frac{V_i - V_i^{\text{spec}}}{V_i^{\max} - V_i^{\min}}\right)^2 \quad (2)$$

where V_i is the voltage magnitude at the ith bus, and V_i^{spec} is the specified voltage magnitude. $V_i^{\max}$ and $V_i^{\min}$ are the upper and lower limits at the ith bus, respectively. N is the number of buses. The exponent in (2) is set to 2 in order to make the difference between the voltage in the ith node and the specified voltage non-negative.

3) Maximization of Voltage Stability Margin: The third objective is to maximize steady-state voltage stability margin. Voltage stability margin is the measure of the security level of the distribution system. Among different indices for voltage stability, a fast indicator of voltage stability, L-index, is chosen as the indicator for voltage stability index. L-index was presented by Kessel and Glavitsch [18], and developed by Jasmon and Lee [19]. The L-index of branch ij can be expressed as follows:

$$L_{ij} = \frac{4\left[(P_jX_{ij} - Q_jR_{ij})^2 + (P_jR_{ij} + Q_jX_{ij})V_i^2\right]}{V_i^4} \quad (3)$$

where L_{ij} indicates the extent of branch voltage stability. The branch voltage will be instable if the value of L_{ij} is large. Obviously, the voltage instability of the network is determined by the most instable branch, and its expression is as

$$L = \max(L_1, L_2, \ldots, L_{N-1}) \quad (4)$$

where the L-index ranges from 0 (no load of system) to 1 (voltage collapse). The bus with the highest L-index will be the most vulnerable bus and, hence, this method helps identify the weak areas needing critical reactive power support in the system. In order to maximize the voltage stability margin, the corresponding function is as

$$\min f_3(x) = \min L. \quad (5)$$

B. Constraints

For DN with the integrated DG units, three types of constraints, which include power-flow equality, nodal voltage, and DG capacity are considered in the optimization model.

1) Equality Constraints: The constraint of power-flow equations is as follows:

$$\begin{aligned} P_{\text{DGi}} - P_{di} &= V_i\sum_{j=1}^{N} V_j(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) \\ Q_{\text{DGi}} - Q_{di} &= V_i\sum_{j=1}^{N} V_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) \end{aligned} \quad (6)$$

where P_{DGi} and Q_{DGi} are active and reactive generation outputs, and P_{di} and Q_{di} are the active and reactive loads at node i, respectively. G_{ij} and B_{ij} are the real and imaginary parts of the nodal admittance matrix, respectively.

2) Inequality Constraints: Generation limits

$$P_{\text{DGi}}^{\min} \le P_{\text{DGi}} \le P_{\text{DGi}}^{\max} \quad (7)$$

$$Q_{\text{DGi}}^{\min} \le Q_{\text{DGi}} \le Q_{\text{DGi}}^{\max}. \quad (8)$$

Load bus voltage constraints

$$V_i^{\min} \le V_i \le V_i^{\max}. \quad (9)$$

Thermal limits

$$|S_{ij}| = \left|V_i^2G_{ij} - V_iV_j(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij})\right| \le S_{ij}^{\max} \quad (10)$$

where $P_{\mathrm{DGi}}^{\min}$, $P_{\mathrm{DGi}}^{\max}$, $Q_{\mathrm{DGi}}^{\min}$, and $Q_{\mathrm{DGi}}^{\max}$ are the lower/upper active and reactive generating unit limits of DG, respectively. $S_{ij}^{\max}$ is the apparent power thermal limit of the circuit between bus i and j.

3) DG Capacity Constraint: Every country has a limit on the penetration of DG for a distribution system to ensure reliability. If we assume the maximum DG penetration factor is 25%, then the maximum injected DG capacity should be less than 25% of the total active power load in DN, that is

$$\sum_{i=1}^{N_{\mathrm{DG}}} P_{\mathrm{DGi}} \leq 0.25 \sum_{i=1}^{N} P_{\mathrm{Loadi}} \tag{11}$$

where $\sum_{i=1}^{N} P_{\mathrm{Loadi}}$ is the total active power load of DN.

C. Overview Formulation

Aggregating objectives and constraints, the problem can be formulated as a nonlinear MOO problem as follows:

$$\min \left[f_1(x_{\mathrm{s}}, x_{\mathrm{c}}), f_2(x_{\mathrm{s}}, x_{\mathrm{c}}), \ldots, f_{N_{\mathrm{obj}}}(x_{\mathrm{s}}, x_{\mathrm{c}})\right] \tag{12}$$

$$\text{s.t.} \quad h_i(x_{\mathrm{s}}, x_{\mathrm{c}}) = 0,\ i = 1, \ldots, p \tag{13}$$

$$g_i(x_{\mathrm{s}}, x_{\mathrm{c}}) \leq 0,\ i = 1, \ldots, q \tag{14}$$

where N_{obj} is the number of objectives, and x_{s} and x_{c} are the state vector and the control vector, respectively.

x_{c} is composed of independent adjustable variables of injected DG units. If each DG has a prespecified power factor, then each installed DG has two variables: injecting position and active power output of DG. For multiple DG units to be installed in a system, x_{c} can be illustrated as follows:

$$x_{\mathrm{c}}^T = [Loc_{\mathrm{DG}_1}, P_{\mathrm{DG}_1}, \cdots, Loc_{\mathrm{DG}_i}, P_{\mathrm{DG}_i}, \ldots, Loc_{\mathrm{DG}_{N\_DG}}, P_{\mathrm{DG}_{N\_\mathrm{DG}}}] \tag{15}$$

where N_DG is the maximal allowable injection number of DG. Loc_{DG_i} is the allocation position of the ith DG, and x_{s} is the state vector to be composed of system nodal voltage magnitude and phase angle as follows:

$$x_{\mathrm{s}}^T = [V_1, \theta_1, \cdots, V_N, \theta_N]. \tag{16}$$

D. Treatment for Equality and Inequality Constraints

Power-flow equations can be satisfied during the process of power-flow calculation. The inequality constraints (7) and (8) can be satisfied in the encoding period. Through penalizing inequality constraints (9)–(11) to the objective function, the constrained optimization problem can be transformed to the unconstrained form, which can be expressed as follows:

$$\begin{aligned} \min f_k'(x)|_{k=1,2,3} &= f_k(x) + w_1 \sum_{i=1}^{N} \left[\frac{\min\left(V_i - V_i^{\min}, V_i^{\max} - V_i, 0\right)}{V_i^{\max} - V_i^{\min}}\right]^2 \\ &+ w_2 \sum_{i=1}^{Nb} \left[\frac{\min\left(S_i^{\max} - S_i, 0\right)}{S_i^{\max}}\right]^2 \\ &+ w_3 \left[\frac{\min\left(0.25 \sum P_{\mathrm{Loadi}} - \sum P_{\mathrm{DGi}}, 0\right)}{0.25 \sum P_{\mathrm{Loadi}}}\right]^2 \end{aligned} \tag{17}$$

where w_1, w_2, and w_3 are penalty factors of voltage constraint, line thermal constraint, and DG penetration, respectively.

III. Improved NSGA-II Algorithm

A. Overview of NSGA-II

NSGA-II uses nondominated sorting and sharing to search a compromising solution for MOO, and it is an efficient algorithm for a large number of benchmark problems [20].

B. Dominated, Nondominated, and Pareto-Optimal Set

MOO can be expressed as follows:

$$\min f_i(x), \quad i = 1, 2, \ldots, N_{\mathrm{obj}}, \quad x \in \chi \tag{18}$$

where $f_i(x)$ denotes the ith objective function, and χ is the feasible searching space.

Definition 1: A solution x_1 is said to dominate x_2 (denoted by $x_1 \prec x_2$) if and only if

$$\forall i, j \in \{1, 2, \ldots, N_{\mathrm{obj}}\}, \exists f_i(x_1) \leq f_i(x_2) \wedge f_j(x_1) < f_j(x_2) \quad \text{for } j \neq i. \tag{19}$$

Definition 2: For $S = \{x_i, i = 1, \ldots, n\}$, solution x is said to be a nondominated solution (Pareto solution) of set S if $x \in S$, and there is no solution $x' \in S$ for which x' dominates x.

Definition 3: Assume that set P contains all nondominated solutions of S, then $\mathrm{PF} = \{v | v = [f_1(x), f_2(x), \ldots, f_{N_{\mathrm{obj}}}(x)]^T, x \in P\}$ is a Pareto front of set S.

C. INSGA-II: Improved Nondominated Sorting Strategy

The improved sorting strategy simultaneously considers the nondominated sorting and density information for each individual. Suppose that NP is the population size, it first computes the nondominated rank for each individual in population using the fast nondominated sorting strategy to be introduced by NSGA-II; then, it adds its nondominated rank and the number of individuals that dominate it. The procedure is as follows:

$$m(X_i) = R(X_i) + n(X_i) \tag{20}$$

where X_i is the ith individual, $R(X_i)$ is the nondominated rank of X_i, and $n(X_i)$ is the number of individuals to dominate X_i. Finally, for $i = 1, 2, \cdots, \mathrm{NP}$, sorts $m(X_i)$ in ascending order, and assigns the order to X_i as the improved rank, i.e.,

$$R^{new}(X_i) = \text{ascendingOrder}\left\{m(X_1), m(X_2), \cdots, m(X_i)\right\} \tag{21}$$

where $R^{new}(X_i)$ is the improved rank of X_i. It will be as a rank result of improved nondominated sorting.

In order to demonstrate the effect of the INSGA-II, a 2-D objectives optimization problem is taken as an example. It assumes that after fast nondominated sorting, individuals $a \sim e$ are at the

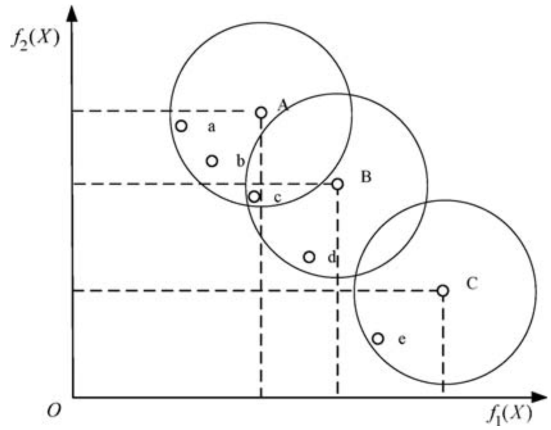

Fig. 1. Individual distribution chart.

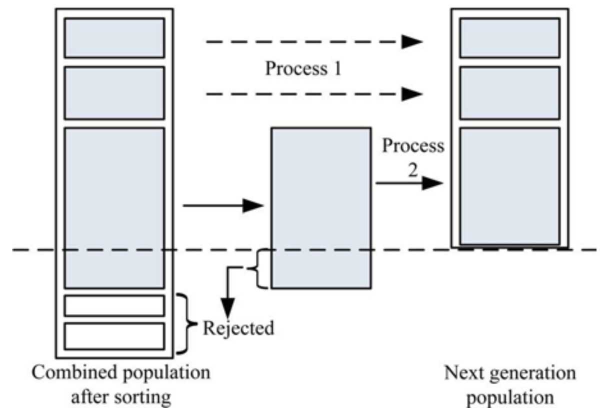

Fig. 2. Schematic chart of the truncation strategy.

first rank, and individuals $A \sim C$ are at the second rank. It also assumed that individual A is dominated by individual a, b, and c; individual B is dominated by individual c and d; and individual C is dominated by individual e, respectively, which can be shown in Fig. 1. Using the traditional sorting strategy, the individuals $A \sim C$ have the same rank. However, the number of their respective adjacent dominating solutions is different. To illustrate the different density corresponding to A, B, and C, three equal radius circles are displayed to cover the adjacent individuals. As shown in Fig. 1, individual A is more crowded than B, and individual B is more crowded than C. Through the improved sorting strategy, the rank of individual A, B and C are 4, 3, and 2, respectively. Therefore, the individual C that has less density holds the lower rank, which means it has more superiority in the selection process. In this way, the improved sorting strategy is beneficial to maintain better diversity in population.

D. INSGA-II: HAC-Based Truncation Strategy

The truncation strategy in NSGA-II is shown in Fig. 2. Here, individuals with lower rank can be conserved directly to the next-generation population (see process 1), until the size of the next-generation population overflows if all individuals in certain rank are maintained. According to NSGA-II, individuals in that rank should be sorted using the crowded comparison operator [20] in descending order, and then individuals needed to fill all population slots are chosen (see process 2).

However, the truncation strategy may destroy the diversity of solutions; namely, it may lead to the uneven distribution. In order to better conserve the diversity and evenly distributed performance in Process 2, a method based on hierarchical agglomerative clustering (HAC) [21] is introduced. In the proposed method, individuals in the truncated rank are clustered into an appropriate number of clusters, and then those with the largest crowding distance in each cluster are conserved to fill population slots. The space distance of solutions is used to measure the similarity of individuals in pair, and the Euclidean distance is utilized as the distance metric.

Taking 2-D objectives optimization problem as one example, as shown in Fig. 3, five individuals need be extracted from candidates $a \sim i$ which are all in the same rank. Two steps should be followed to complete the truncation:

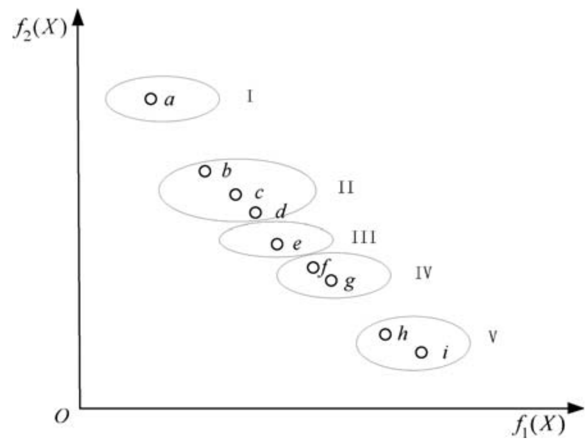

Fig. 3. Schematic chart of the truncation strategy based on HAC.

Step 1) Use the HAC algorithm to separate all individuals into five clusters ($\mathrm{I} \sim \mathrm{V}$).

Step 2) Execute the crowded-comparison operator for each individual and choose the one with the largest crowding distance in each cluster. Then, the individuals a, b, e, g, and i are extracted.

According to the traditional truncation strategy, individuals a, b, g, h, and i will be selected. Comparing two groups of results, the difference can be noticed that the HAC-based truncation strategy chooses individual e but not h into population slots, which is shown in Fig. 4. In order to demonstrate the evenly distributed performance of the proposed strategy, the coordinates of each individual are illustrated in Fig. 4. And the variance of distances of adjacent individuals is calculated to measure the even degree. Based on the HAC-based truncation strategy, the variance is 0.3528. For the traditional truncated approach, the variance is 2.1580. So the proposed improved truncation strategy can make the obtained solutions be evenly distributed with more diversity.

E. INSGA-II: Improved Mutation and Crossover Strategy

The mutation and recombination strategy [22] in DE optimization is integrated in the proposed INSGA-II algorithm. Detailed operations are described as follows.

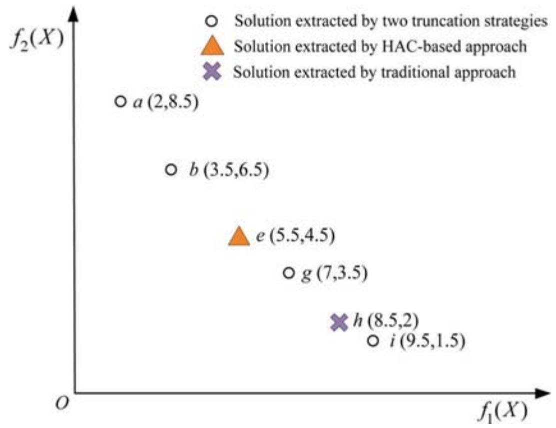

Fig. 4. Results of the proposed and traditional truncation strategies.

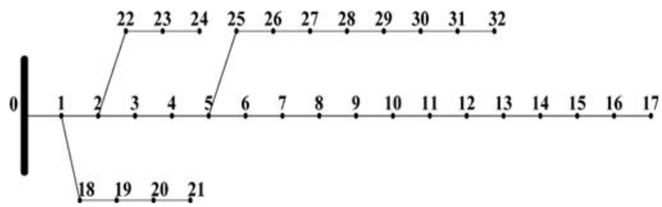

Fig. 5. Diagram of the IEEE 33-bus distribution system.

1) Mutation Operation: For each $x_i (i = 1, 2, \cdots, NP)$, the weighted difference of two randomly chosen population vectors x_{r2} and x_{r3} is added to another randomly selected population member x_{r1}, to form a mutated vector x'_i

$$x'_i = x_{r1} + F \times (x_{r3} - x_{r2}) \tag{22}$$

where x'_i is the new mutated vector, and F is a predefined step size, which is typically chosen from range [0,2]. The mutation operation can improve the local search around the current best solution.

2) Recombination Operation: Assuming that the individual i is composed with chromosome vector $x_i = [u_{i,1}, u_{i,2}, \cdots, u_{i,NC}]^T$, and the mutated individual i is composed with chromosome vector $x'_i = [u'_{i,1}, u'_{i,2}, \cdots, u'_{i,NC}]^T$, then the new individual x''_i is created by (23). The new individual vector is mixed with the original vector x_i and mutated vector x'_i to yield to the new vector x''_i after the recombination operation

$$u''_{i,j} = \begin{cases} u_{i,j}, & \text{if } rand(j) \geq CR \\ u'_{i,j}, & \text{if } rand(j) < CR \end{cases} \quad j = 1, 2, \cdots NC \tag{23}$$

where $rand(j) \in [0, 1]$, and crossover rate $CR \in [0, 1]$. NC is the length of the chromosome. The recombination operation can increase the diversity of the perturbed parameter vector.

F. Choosing the Best Compromise Solution via a Fuzzy Decision

For decision making, it is necessary to select a best compromise solution from the obtained MOO solution sets. Here, using the Fuzzy Set Theory determines the best compromise solution. First, the membership function τ_i^k of the kth solution for the ith objective function F_i^k is defined as

$$\tau_i^k = \frac{F_i^{\max} - F_i^k}{F_i^{\max} - F_i^{\min}} \tag{24}$$

where $F_i^{\max}$ and $F_i^{\min}$ are the maximum and minimum of the ith objective function among all nondominated solutions, respectively. Obviously, τ_i^k gives a measure of the satisfaction degree of the kth solution for the ith objective function. Then, using the fuzzy decision determines the best compromise solution x^{k*} in the Pareto solution set as

$$x^{k*}, \text{ and } \bar{\tau}^{k*} = \max_{k=1,\cdots,M} \left\{ \frac{\sum_{i=1}^{N_{\text{obj}}} \tau_i^k}{\sum_{j=1}^{M} \sum_{i=1}^{N_{\text{obj}}} \tau_i^j} \right\} \tag{25}$$

where M is the number of Pareto solutions.

G. Complete Algorithm of the Proposed Method

The flowchart of the proposed algorithm is shown in following pseudocodes.

Algorithm 1 Procedures of the proposed INSGA-II

Input: The number of objectives, population size NP, maximal iteration $t_{\max}$, etc.

Output: Optimal solution x^{k*}

1: Initialize $P_0 = \{x_1, x_2, \ldots, x_{\text{NP}}\}$, and set iteration number $t = 0$;

2: Powerflow computation, and compute objective values f_1, f_2, f_3;

3: **while** $t < t_{\max}$ **do**

4: Make selection, mutation and recombination operations on parent group P_t, and form $NP/2$ new individuals as offspring group Q_t;

5: Power-flow computation, combine current population and offspring group $R_t = P_t \bigcup Q_t$;

6: Use improved nondominated sorting strategy in R_t, and form multiple rank $L = (L_1, L_2, \ldots)$;

7: Truncate the combined population using the HAC-based strategy, and form P_{t+1};

8: $t = t + 1$;

9: **end while**

10: Get the best comprise solution using the Fuzzy Set Theory;

11: **return** x^{k*}.

TABLE I
RESULT WITH THE IEEE 33-BUS TEST SYSTEM

DG planning scheme		Multi-objective values		
Position	Capacity (MW)	Line loss (MW)	Voltage stability index (p.u.)	Voltage deviation
17	0.5180	0.1042	0.0554	3.7282
32	0.4224			

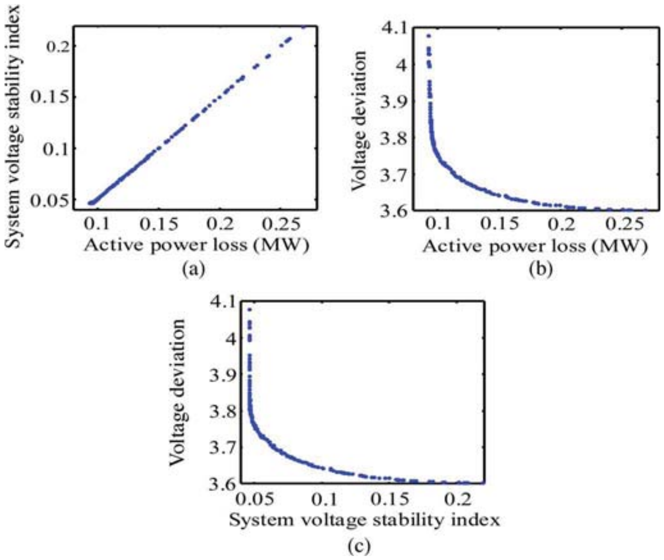

Fig. 6. Relation among different objective functions.

IV. EXPERIMENTS AND RESULTS

A. Experiment Setting and Description

In order to demonstrate the effectiveness of the proposed algorithm, the optimal DG allocation of the IEEE 33-bus, and actual 292-, 588-bus distribution networks with DG are considered and tested. The actual test systems come from real urban DN in North China. The environments of the algorithm implementation and comparison are as follows.

1) The program of the proposed algorithm is developed in MATLAB. In the implementation of the INSGA-II algorithm, the population number $\mathrm{NP} = 200$; the max-iteration number $t_{\max} = 100$; the mutation factor $F = 0.25$; the crossover factor $\mathrm{CR} = 0.9$; and the penalty factor w_1, w_2, and w_3 are all set to 50.
2) The algorithm verification and comparison were made with NSGA-II, strength pareto evolutionary algorithm 2 (SPEA2) [23], and differential evolution multiobjective optimization (DEMO) [24]. The evolution parameters in these algorithms are the same as the ones in INSGA-II.

B. Experiment on the IEEE 33-Bus Case

DG units are considered in the modified IEEE 33-bus system shown in Fig. 5. The algorithm was applied to solve this problem. The peak loading data of IEEE 33-bus test system, seen in Table VII, are utilized as the typical load data. The maximal active power capacities of DG units are set to 1 MW.

In the proposed INSGA-II and the other three evolutionary algorithms for comparison, the initial vector of all independent control variables in each chromosome is set to the random values between their upper and lower limits. The final solution for DG allocation and MOO function values in the IEEE 33-bus system are shown in Table I.

TABLE II
ALGORITHM COMPARISONS IN THE C INDEX

Algorithm	INSGA-II	NSGA-II	DEMO	SPEA2
INSGA-II	-	26.17%	12.21%	26.40%
NSGA-II	5.99%	-	9.05%	19.63%
DEMO	8.44%	24.02%	-	24.78%
SPEA2	5.96%	14.41%	8.04%	-

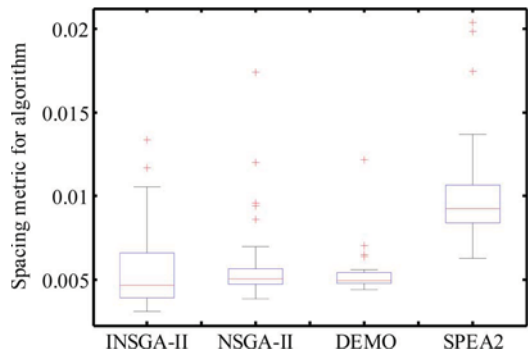

Fig. 7. Box-plots of spacing metric values from different multiobjective optimization algorithms.

TABLE III
SIZES OF THE DIFFERENT DISTRIBUTION POWER SYSTEMS

Test System	N node number	N_{DG} number of DG	N_C number of constraints	N_B branch number
IEEE 33-bus	33	2	104	32
Actual 292-bus	292	6	902	296
Actual 588-bus	588	10	1806	592

Distribution relations of the optimal solution set among different objective functions are shown in Fig. 6. As shown in Fig. 6(a), the optimal solution set is a linear distribution for line loss objective and system voltage stability objective. The relation between line loss objective and voltage deviation objective is shown in Fig. 6(b). The solution result in Fig. 6(b) shows a reciprocal distribution for two objectives. It also shows that two objectives have a contrasting relation. Fig. 6(c) shows that there is a contrasting relation between the system voltage stability objective and voltage deviation objective.

C. Performance Analysis of the INSGA-II Algorithm

The comparisons of the proposed INSGA-II with NSGA-II, SPEA2 and DEMO were made. For each algorithm, 30 runs with different random seeds have been carried out. In the case of MOO, the comparison of searching performance is substantially more complex than for the single-objective optimization problem. The following comparisons are based on the C index [25] and spacing metric [26].

TABLE IV
OPTIMIZATION RESULT OF INSGA-II ON DIFFERENT DISTRIBUTION POWER SYSTEMS

ID	Test System	Active load (MW)	Time (s)	Optimal objectives			
				Optimization	Line loss (MW)	Voltage stability index	Voltage deviation
1	IEEE 33-bus	3.715	33.49	Before /	0.2026	0.0746	11.7053
				After	0.0950	0.0462	3.7420
2	292-bus	11.2557	42.48	Before /	0.8627	0.0177	73.4851
				After	0.4321	0.0131	31.5791
3	588-bus	15.8306	48.56	Before /	1.6193	0.0116	250.0983
				After	0.7116	0.0078	90.2717

1) C Index:

Definition 4: Let X', and $X'' \subseteq X$ be two sets of decision vectors, the function C maps the ordered pair (X', X'') to be the interval [0, 1]

$$C(X', X'') = \frac{|\{a'' \in X''; \exists a' \in X' : a' \prec a''\}|}{|X''|}. \tag{26}$$

The value $C(X', X'') = 1$ means that all solutions in X'' are dominated by solutions in X'. The opposite $C(X', X'') = 0$ represents the situation that none of the solutions in X'' are covered by the set X'.

As shown in Table II, 26.17%, 12.21%, and 26.40% solutions of NSGA-II, DEMO, and SPEA2 are dominated by solutions of INSGA-II, respectively, while there are 5.99%, 8.44%, and 5.96% solutions of NSGA-II, DEMO, and SPEA2 dominating solutions of INSGA-II. It shows that the INSGA-II has more excellent searching performance and better Pareto-optimal front.

2) Spacing Metric: In order to judge the evenly distributed performance of the Pareto solution set, the spacing metric is defined as the distance variance of each solution to its closest neighbor, that is

$$S = \sqrt{\frac{1}{\mathrm{NP}-1}\sum_{i=1}^{\mathrm{NP}}(\bar{d}-d_i)^2} \tag{27}$$

where d_i is the distance of the ith individual to its closest neighbor, and $\bar{d}$ is the mean of d_i among individuals. The d_i and $\bar{d}$ can be calculated as follows:

$$d_i = \min\left\{\sum_{m=1}^{N_{\mathrm{obj}}}\frac{|f_m(x_i)-f_m(x_j)|}{f_{m\max}-f_{m\min}}\right\} \tag{28}$$

$$\bar{d} = \frac{\left(\sum_{i=1}^{\mathrm{NP}} d_i\right)}{\mathrm{NP}}. \tag{29}$$

A smaller value of spacing metric means that the solutions in the Pareto solution set are more evenly distributed. And the value of zero for the spacing metric means that all solutions in the Pareto solution set are equally spaced. The box plots of spacing metric values from different MOO algorithms are shown in Fig. 7.

As shown in Fig. 7, each box plot represents distributions of spacing metric values. The top and bottom horizontal lines in each box plot present the boundary values except the outliers. The exceptional value has been plotted as outliers using "+". The rectangular box contains half of the spacing metric values, and the red line within the rectangular box shows the median for spacing metric values. Compared with the other three algorithms, the INSGA-II has the minimal median (see red line) and minimum value (see bottom solid line). Therefore, INSGA-II has advantages in finding evenly distributed solutions, and the Pareto set derived by INSGA-II has better diversity in selecting the best compromise solution.

TABLE V
INSGA-II RESULT WITH A DIFFERENT POPULATION SCALE AND ITERATIONS

Experiment	Test system	Active load(MW)	Population size(*NP*)	Iterations ($t_{\max}$)	Time (s)
1	588-bus	15.8306	200	100	48.56
2	588-bus	15.8306	300	200	198.30

D. Cases Analysis on Different Distribution Systems

The proposed INSGA-II is tested on different distribution power systems. Table III shows the size of each set of various test systems.

Table IV shows the optimization result of INSGA-II in three cases. The computation time will slowly increase when the scale of distribution grids increases. Most of the increased time is spent in power-flow computation for large-scale distribution power systems. For the optimization objectives, the integration of DG units significantly reduces the line loss and voltage deviation and improves the system voltage stability.

Additional experiments were made on the actual 588-bus test systems with different NP and $t_{\max}$. As shown in Table V, two experiments can almost get the same optimization objectives value. Compared with experiment 1, the population size in experiment 2 increases 50%, and the iteration number increases accordingly; however, the computation time increases 400%. It can be concluded that the population size and maximal iteration number are two major influencing parameters in INSGA-II.

E. Method Validation on Average and Light Loading

The optimal planning result for two DG units in Table I is based on the peak loading of the IEEE 33-bus system. To validate the proposed method suitable for average and light loading, the optimal result for two DG units is put in the cases of average and light loading, respectively. The three objectives function values in the two cases are computed based on the optimal allocation, that is, DG 1 with 0.5180 MW is integrated in node

TABLE VI
THREE-OBJECTIVE FUNCTION VALUES IN AVERAGE AND LIGHT LOADING

Load type	Optimization	Line loss (MW)	Voltage stability index (p. u.)	Voltage deviation
Peak loading	Before /	0.2027	0.0746	11.7102
	After	0.1042	0.0554	3.7282
Average loading	Before /	0.1502	0.064	8.6641
	After	0.0647	0.0363	2.2318
Light loading	Before /	0.1201	0.0571	6.9216
	After	0.0486	0.0299	1.4699

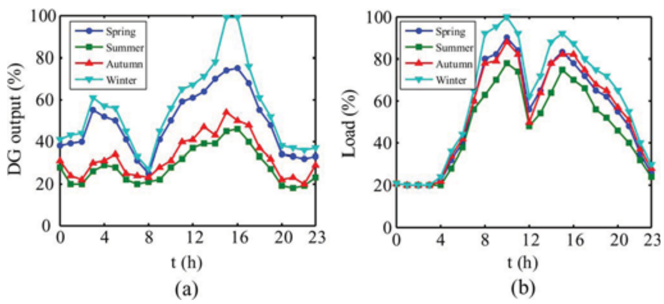

Fig. 8. Forecasted time-series data of wind generation output and industrial load.

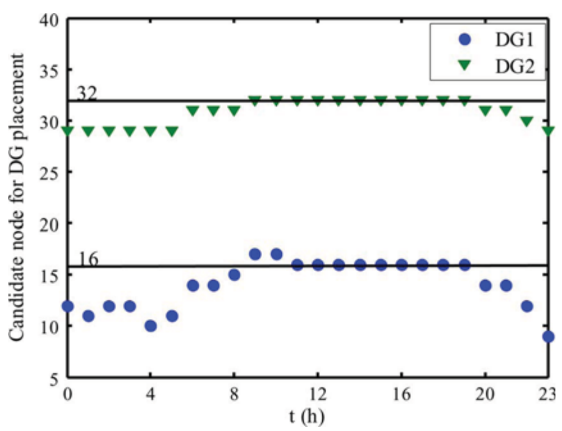

Fig. 9. Independent optimal placement for DG units in 24 h.

17, and DG 2 with 0.4224 MW is integrated in node 32, respectively. The computation result is shown in Table VI.

As shown in Table VI, three objective function values in the light loading case are smaller than the average and peak loading cases. It means that the line loss, system stability, and voltage deviation become poor with the loading increase. But after optimization, three objective function values in three cases are all obviously decreased. It shows that the optimal DG planning result based on peak loading can create an active effect on the same distribution system with light and average loading. The aforementioned experiment result provides evidence that the optimal DG allocation scheme in peak loading can be feasible in various loading levels.

TABLE VII
DATA OF THE MODIFIED IEEE 33-BUS DISTRIBUTION TEST SYSTEM

Line	bus i	bus j	Peak loading P_{Dj} (kW)	Peak loading Q_{Dj} (kVAr)	Light loading P_{Dj} (kW)	Light loading Q_{Dj} (kVAr)
1	0	1	100	60	78	47
2	1	2	90	40	70	31
3	2	3	120	80	94	63
4	3	4	60	30	47	23
5	4	5	60	20	47	16
6	5	6	200	100	157	78
7	6	7	200	100	157	78
8	7	8	60	20	47	16
9	8	9	60	20	47	16
10	9	10	45	30	35	23
11	10	11	60	35	47	27
12	11	12	60	35	47	27
13	12	13	120	80	94	63
14	13	14	60	10	47	8
15	14	15	60	20	47	16
16	15	16	60	20	47	16
17	16	17	90	40	70	31
18	1	18	90	40	70	31
19	18	19	90	40	70	31
20	19	20	90	40	70	31
21	20	21	90	40	70	31
22	2	22	90	50	70	39
23	22	23	420	200	329	157
24	23	24	420	200	329	157
25	5	25	60	25	47	20
26	25	26	60	25	47	20
27	26	27	60	20	47	16
28	27	28	120	70	94	55
29	28	29	200	600	157	470
30	29	30	150	70	117	55
31	30	31	210	100	164	78
32	31	32	60	40	47	31

* Note that R and X are the same as those in the original IEEE 33-bus test system.

F. Experiment on Time Series Data of the DG and Load With Seasonal Changes

For the proposed method in the time-series data case, the optimal placement and sizing problem is analyzed with wind generation and industrial load in the modified IEEE 33-bus system. The forecasted time-series data for the wind generator output and industrial load in four seasons based on the real urban distribution power system in Northern China are shown in Fig. 8(a) and (b). Here, assume that the variation of 24 h in each season is kept the same for wind generation output and industrial load. Taking the data in the summer as one example, the proposed multiobjective planning problem is optimized at each time period, and then multiperiod solutions are obtained.

The optimal DG placement at each time period is shown in Fig. 9. There is one independent allocation solution for each hour with time-series data. Based on the density of candidate solutions, nodes 16 and 32 are selected as the best DG integration positions.

As shown in Fig. 9, the most appropriate sizing of DG units can be obtained from the corresponding time period whose DG positions are nodes 16 and 32. Besides, when DG units are integrated into the system using the optimal allocation result at a certain time period, the solution should not violate the penetration rate constraint at other time periods. After individually putting optimal DG allocation results at 11∼19th time period and calculating the wind generation output at the 24-h time period, the optimal allocation result at the 16th time period is most appropriate to the constraint at all 24 time periods. Thus, the sizing values of DG1 and DG2 in the 16th time period can be selected as the optimal sizing of DG units.

V. Conclusion

To summarize the modeling, optimization algorithm improvement, and comparison study for optimal planning of multiple DG units, the following conclusions can be derived as:

1) three objectives to consider minimum line loss, minimum voltage deviation, and maximal voltage stability margin can correctly formulate optimal planning of multiple DG units;
2) by improving the mutation and crossover procedure, strengthening the nondominated sorting and truncation strategies, and determining the Pareto solution set using the fuzzy membership function method, the proposed INSGA-II can obtain the best compromise solution for all objectives. Taking IEEE 33-, actual 292-, and 588-bus systems as test cases, the comparisons of the proposed INSGA-II with the traditional multiobjective optimization algorithms, such as NSGA-II, DEMO and SPEA2, indicate that the proposed method can achieve better precision and diversity.

In practice, the choice of the best site may not always be feasible due to many reality constraints. But the optimization and analysis here suggest that considering multiobjectives helps to decide placement and sizing of DG units for the decision-maker.

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