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Supplier selection using ANP and ELECTRE II in interval 2-tuple linguistic environment

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ABSTRACT

In supply chain management, supplier selection can be treated as a type of hierarchical multi-criteria decision-making (MCDM) problems since it involves various criteria and hierarchical structure among criteria often exists. This paper investigates a kind of MCDM problems with two-level criteria and develops a novel hybrid method integrating TL-ANP (2-tuple linguistic analytic network process) and IT-ELECTRE II (interval 2-tuple Elimination and Choice Translating Reality II). Considering interactions among criteria, a TL-ANP approach, in which comparison matrices are consistent 2-tuple linguistic preference relations, is put forward to determine weights of criteria and sub-criteria. To deal with the case of criteria being not compensated, an IT-ELECTRE II approach is proposed. In this approach, ratings of alternatives on sub-criteria are represented as interval 2-tuple linguistic variables. A possible degree and a likelihood-based preference degree are respectively defined, followed by concordance, discordance and indifferent sets. Afterwards, concordance and discordance indices are identified and applied to establish net concordance and net discordance indices. Further, comprehensive dominant values of alternatives are obtained to rank alternatives. Thereby, a novel hybrid method is presented for MCDM with two-level criteria under interval 2-tuple linguistic environment. At length, a real case of supplier selection is examined and comparison analyses are conducted to illustrate the application and superiority of the proposed method.

1. Introduction

With the acceleration of economic globalization process, today's enterprises are exposed to fierce competition. To attract more customers, many enterprises improve the quality and reduce cost (price) of their products. In this process, the raw material supplier plays an important role. Therefore, Enterprises must select appropriate suppliers and retain good relations of cooperation with them. While selecting suppliers, various criteria are involved and some of them are conflict, such as quality and cost. Hence, the supplier selection can be considered as a kind of multi-criteria decision-making (MCDM) problems [8,26,34–36]. Current research on supplier selection mainly focuses on two key issues: evaluation criteria identification and decision-making methods.

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Table 1
Criteria and corresponding sub-criteria for supplier selections.

Criteria	Sub-criteria	Literature
Quality	Quality performance; Quality containment & VDCS feed back	Yang and Tzeng [41]
Price & Terms	Price; Terms; Responsiveness; Lead time, VMI/VOI hub set up cost	
Supply chain support	Purchase order reactiveness; Capacity support & flexibility; Delivery/VMI operation	
Technology	Technical support, Design involvement; ECN/PCN process	
Cost	Product price, Freight cost; Tariff and custom duties	Chen and Yang [2]
Quality	Rejection rate of product; Increased lead time; Quality assessment; Remedy for quality problems	
Service performance	Delivery schedule, Technological and R & D support; Response to changes; Ease of communication	
Supplier Profile	Financial status, Customer base; Performance history; Production facility and capacity	
Risk	Geographical location, Political stability; Economy, Terrorism	
General management capability perspective	Management and strategy; Financial status; Customer relations; Training program; Reputation, History; Language; License; Geographical location	Lee et al. [16]
Manufacturing capability perspective	Production capacity; Product diversity; R & D capability, Safety regulations; Environmental regulations, Quality control; Product price	
Collaboration capability perspective	After-sales service, Delivery reliability	
Agility perspective	Delivery speed; Delivery flexibility; Make flexibility; Source flexibility; Agile customer responsiveness; Collaboration with partners, IT infrastructure	

For evaluation criteria identification, Dickson [6] firstly performed an investigation and proposed 23 different criteria including quality, on-time delivery, price, performance history, warranties policy, technical capability, etc. Among these criteria, the first three criteria are most popular and applied in many supplier selection problems [2,16,27,41]. Subsequently, a lot of new evaluation criteria were introduced, such as finance, management and reputation, service, etc. According to these criteria, different sub-criteria [2,16,41] were presented and listed in Table 1.

In the regard of decision-making methods, earlier studies adopted some classical methods to solve supplier selection problems with crisp numerical assessment information, such as AHP (Analytic Hierarchical Process) [20], ANP (Analytic Network Process) [32] and TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [17]. However, as the complexity of decision-making problems increases, decision information is more and more vague. In this context, Pedrycz [24,25] suggested that linguistic variable [7,18] is suitable to describe quantitative assessment information. For example, when we evaluate the reputation of a supplier, terms like “poor”, “good” and “very good” are usually employed. By converting linguistic variables into triangle fuzzy numbers (TFNs), many fuzzy decision methods [1,11,14,16,21,27,28,33] have been proposed. Roughly speaking, these methods can be divided into two classes: single methods and hybrid methods.

Common single methods are fuzzy AHP (FAHP) [27], fuzzy ANP (FANP) [33] and fuzzy TOPSIS (FTOPSIS) [28]. Hybrid methods are those which fuse at least two single methods. Generally, some single methods are used to determine criteria weights and others are applied to rank suppliers. For example, considering that criteria are independent on each other, Hashemian et al. [11] and Lee et al. [16] derived criteria weights by FAHP, and then ranked suppliers by fuzzy PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) and FTOPSIS, respectively. Considering interactions among criteria, Nguyen et al. [21] used FANP to determine criteria weights and adopted COPRAS-G (Complex Proportional Assessment of alternatives with Grey relations) to rank suppliers; Büyüközkan and Çifçi [1] obtained criteria weights by fuzzy DEMATEL and FANP, and sorted suppliers by FTOPSIS; Karsak and Dursun [14] used QFD (Quality Function Deployment) to derive criteria weights, and then applied DEA (Data Envelopment Analysis) to rank suppliers.

The aforementioned methods demonstrate that most researchers solved supplier selection problems by transforming linguistic variables into TFNs. As a result, computation results usually do not exactly match any of initial linguistic terms and an approximation process must be used to express results in the initial expression domain, which easily leads to loss of information and lack of precision in the final results. To overcome these limitations, Herrera and Martínez [12] introduced 2-tuple linguistic representation model which consists of a linguistic term and a numeric value. The main advantage of this representation is to be continuous in its domain. Therefore, it can express any counting of information in the universe of the discourse. Subsequently, Zhang [44] further extended the 2-tuple linguistic variable into the interval 2-tuple linguistic variable. In 2-tuple linguistic context, Wang [37] proposed Hierarchy Arithmetic Weighted Average approach to rank suppliers; Karsak and Dursun [15] used QFD to give a decision framework for medical supplier selection problems. You et al. [42] addressed an interval 2-tuple linguistic VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje technique) method to tackle anesthetic equipment supplier problems. However, methods [37, 42] assumed that criteria are independent and assigned criteria weights in advance.

Though previous linguistic decision making methods can solve some supplier selection problems, there are some shortcomings: (1) Fuzzy decision methods [1,16,21,28,33] may result in information loss or distortion. (2) Although 2-tuple decision methods [37,42] can overcome the information loss, they did not consider interactions among criteria. The phenomena of interaction among criteria often exist in real-world decision making problems. For instance, while evaluating a supplier, the high quality often brings a good reputation and a good reputation may imply the high quality. (3) Methods [1,16,21,28,33,37,42] regards that values on different criteria can completely compensate for each other. In fact, sometimes, values on some criteria cannot compensate for each other. As an example, if the quality of supplier *A* is much worse than that of supplier *B*, then supplier *A* is not better than supplier *B* even if values of supplier *A* on other criteria are much better than those of supplier *B*.

To make up above shortcomings, this paper proposes a new hybrid method for solving supplier selection problems with two-level criteria. The main motivations of this paper are outlined as follows:

(1) As mentioned before, various criteria and sub-criteria are involved in supplier selection problems. This implies that hierarchical structure often exists in supplier selection. Existing methods [17,28,37,42] only addressed supplier selection with single level criteria and can not handle supplier selection with two-level criteria. Therefore, this paper investigates MCDM with two-level criteria and applies it to supplier selection.
(2) In MCDM, it is very important to reasonably determine criteria weights. However, methods [37,42] did not discuss the determination of criteria weights. Although fuzzy decision methods [11,16,21] derived criteria weights by FAHP or FANP, loss of information happens together with converting linguistic ratings into TFNs. To remedy these disadvantages, this paper develops a 2-tuple linguistic ANP (TL-ANP) approach to deriving criteria and sub-criteria weights.
(3) Due to the fact that methods [1,16,21,28,33,37,42] regards that values on different criteria can completely compensate for each other, these methods are not able to solve MCDM in which values on different criteria are not allowed to compensate for each other. Considering that ELECTRE (Elimination and Choice Translating Reality) family [13,19,23] has a notable merit (namely, alternatives are compared on each criterion and scores on criteria cannot compensate for each other), this paper presents an interval 2-tuple ELECTRE II (IT-ELECTRE II) approach to dealing with the case of criteria being not compensated.

Consequently, this paper formulates supplier selection as a kind of MCDM problems with two-level criteria. Combining TL-ANP and IT-ELECTRE II, a novel hybrid method is proposed for solving such problems. Compared with existing methods, the proposed method has three key features:

(1) While determining weights of criteria and sub-criteria, the proposed TL-ANP approach considers interactions among criteria or within criteria. Thus, decision results may be more consistent with real-world decision situations. However, 2-tuple decision methods [37,42] assumed that criteria are independence on each other and ignored interactions among criteria.
(2) Different from FANP [21,33], the proposed TL-ANP approach employs 2-tuple linguistic variables to express preference relations between two criteria or sub-criteria, by which the loss or distortion of information can be effectively avoided. FANP [21,33] converted linguistic variables into TFNs, which can result in that one linguistic variable may be converted into distinct TFNs or the aggregated results do not belong to the given linguistic term set.
(3) An IT-ELECTRE II approach is proposed to effectively circumvent the issue that criteria cannot compensate for each other. In this approach, ratings of alternatives on sub-criteria are represented by interval 2-tuple linguistic variables which can neatly express uncertain decision information provided by decision makers (DMs). Moreover, this approach inherits the merit of ELECTRE. Hence, decision results may be more reliable compared with VIKOR and TOPSIS.

The rest of this paper is organized as follows. In Section 2, some preliminaries for 2-tuple and interval 2-tuple linguistic variables, 2-tuple linguistic preference relation and ANP are reviewed. Section 3 develops a novel hybrid method for MCDM with two-level criteria in interval 2-tuple linguistic environment. In Section 4, a real case of supplier selection is analyzed to illustrate the application of the proposed method. Section 5 carries out comparison analyses to show the superiority of the proposed method. Finally, some primary conclusions are furnished in Section 6.

2. Preliminaries

This section briefly reviews some basic definitions and properties related to 2-tuple linguistic variables, interval 2-tuple linguistic variables, 2-tuple linguistic preference relation and ANP.

2.1. 2-tuple linguistic variables and interval 2-tuple linguistic variables

Let $S = \{s_0, s_1, s_2, \cdots, s_g\}$ be a predefined linguistic term set with granularity $g + 1$. The 2-tuple linguistic representation model expresses linguistic information by a pair of values called linguistic 2-tuple (s_i, α), where $s_i \in S$ represents the central value of the *i*th linguistic term and $\alpha \in [-0.5, 0.5)$ indicates the deviation to the central value of the *i*th linguistic term.

Example 1. Let $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\}$ be a linguistic term set. The meanings of linguistic terms s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8 respectively are: “extremely poor”, “very poor”, “medium poor”, “poor”, “medium”, “medium good”, “good”, “very good” and “extremely good”. Thus, 2-tuple $(s_7, 0.2)$ means that the real linguistic rating is better than the term “very good (s_7)” and the degree to which the real linguistic rating is preferred to the term “very good (s_7)” is 0.2.

Together with the 2-tuple (s_i, α), there is a numerical value β representing the result of an aggregation of the indices of a set of labels assessed in a linguistic term set *S*. In traditional 2-tuple linguistic model [12], the range of β is between 0 and

g. Hence, there exists a restriction that the range of β varies with the granularity if there exist multi-granularity linguistic term sets [44] in a decision-making problem. To overcome this restriction, Chen and Tai [5] proposed a generalized 2-tuple linguistic model and translation functions.

Definition 1. [5]. Let $S = \{s_0, s_1, s_2, \cdots, s_g\}$ be a linguistic term set and $\beta \in [0, 1]$ be a value representing the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S. To obtain the 2-tuple linguistic variable equivalent to β, the generalized translation function Δ is defined as follows:

$$\Delta(\beta) = (s_i, \alpha) \quad with \quad \begin{cases} s_i, & i = \text{round}(\beta \cdot g) \\ \alpha = \beta - \frac{i}{g}, & \alpha \in [-\frac{1}{2g}, \frac{1}{2g}) \end{cases}. \tag{1}$$

Meanwhile, there is a function Δ^{-1} used to convert a 2-tuple linguistic variable into its equivalent numerical value $\beta \in [0, 1]$. The reverse function Δ^{-1} is defined as

$$\Delta^{-1}: S \times [-\tfrac{1}{2g}, \tfrac{1}{2g}) \to [0, 1], \quad \Delta^{-1}(s_i, \alpha) = \tfrac{i}{g} + \alpha = \beta. \tag{2}$$

Specially, a linguistic term s_i can be converted into a 2-tuple linguistic variable $(s_i, 0)$.

Example 2. Let $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\}$ be a linguistic term set with $g = 8$. From Eq. (2), we have $\alpha \in [-\frac{1}{16}, \frac{1}{16}) = [-0.0625, 0.0625)$. According to Eq. (2), one has

$$\Delta^{-1}(s_1, 0.025) = \tfrac{1}{8} + 0.025 = 0.15, \quad \Delta^{-1}(s_0, 0) = \tfrac{0}{8} + 0 = 0.$$

On the other hand, suppose the symbolic aggregation operation $\beta = 0.35$. In virtue of Eq. (1), it yields that $i = \text{round}(0.35 \times 8) = 3$ and $\alpha = 0.35\text{-}3/8 = -0.025$. Therefore, $\Delta(\beta) = (s_3, -0.025)$.

Definition 2. A matrix $\tilde{\boldsymbol{Y}} = (\tilde{y}_{ij})_{m\times n}$ is called a 2-tuple linguistic matrix if $\tilde{y}_{ij} = (s_{ij}, \alpha_{ij})(i = 1, 2, \cdots, m; j = 1, 2, \cdots, n)$ are 2-tuples defined in Definition 1.

Definition 3. Given a 2-tuple linguistic matrix $\tilde{\boldsymbol{Y}} = (\tilde{y}_{ij})_{m\times n}$, if $\beta_{ij} = \Delta^{-1}(\tilde{y}_{ij})$, matrix $\boldsymbol{\Delta^{-1}}(\tilde{\boldsymbol{Y}}) = (\beta_{ij})_{m\times n}$ is called a 2-tuple linguistic transformed matrix of matrix $\tilde{\boldsymbol{Y}}$. Meanwhile, matrix $\tilde{\boldsymbol{Y}}$ is called the corresponding 2-tuple linguistic matrix of matrix $\boldsymbol{\Delta^{-1}}(\tilde{\boldsymbol{Y}})$.

Example 3. Let the linguistic term set S come from Example 1. Matrix

$$\tilde{\boldsymbol{Y}} = \begin{pmatrix} (s_3, 0.03) & (s_7, 0.00) & (s_1, 0.02) \\ (s_5, -0.01) & (s_2, 0.04) & (s_6, 0.01) \\ (s_1, 0.05) & (s_6, 0.03) & (s_4, -0.02) \end{pmatrix}$$

is a 2-tuple linguistic matrix. Thus, the 2-tuple linguistic transformed matrix $\boldsymbol{\Delta^{-1}}(\tilde{\boldsymbol{Y}})$ is obtained as

$$\boldsymbol{\Delta}^{-1}(\tilde{\boldsymbol{Y}}) = (\beta_{ij})_{3\times 3} = \begin{pmatrix} 0.405 & 0.875 & 0.145 \\ 0.615 & 0.290 & 0.760 \\ 0.175 & 0.780 & 0.480 \end{pmatrix}.$$

For example, $\beta_{11} = \Delta^{-1}(\tilde{y}_{11}) = \Delta^{-1}(s_3, 0.03) = 0.405$. Similarly, other elements can be acquired.

Definition 4. [40]. Let $S = \{s_0, s_1, s_2, \cdots, s_g\}$ be a linguistic term set. An uncertain linguistic variable is defined as $\tilde{s} = [s_k, s_l]$, where s_k, $s_l \in S$ and $k \le l$.

Definition 5. [44]. An interval 2-tuple linguistic variable is composed of two 2-tuples, denoted by $[(s_i, \alpha_1), (s_j, \alpha_2)]$ with $(s_i, \alpha_1) \le (s_j, \alpha_2)$(see [5]). An interval 2-tuple expressing the equivalent information to an interval value $[\beta_1, \beta_2](\beta_1, \beta_2 \in [0, 1])$ is obtained by the following function:

$$\Delta[\beta_1, \beta_2] = [(s_i, \alpha_1), (s_j, \alpha_2)] \quad \text{with} \quad \begin{cases} s_i, & i = \text{round}(\beta_1 \cdot g) \\ s_j, & j = \text{round}(\beta_2 \cdot g) \\ \alpha_1 = \beta_1 - \frac{i}{g}, & \alpha_1 \in [-\frac{1}{2g}, \frac{1}{2g}) \\ \alpha_2 = \beta_2 - \frac{j}{g}, & \alpha_2 \in [-\frac{1}{2g}, \frac{1}{2g}) \end{cases} \tag{3}$$

Correspondingly, there is always a function Δ^{-1} such that an interval 2-tuple can be transformed into an interval value $[\beta_1, \beta_2](\beta_1, \beta_2 \in [0, 1])$ as follows:

$$\Delta^{-1}[(s_i, \alpha_1), (s_j, \alpha_2)] = [\tfrac{i}{g} + \alpha_1, \tfrac{j}{g} + \alpha_2] = [\beta_1, \beta_2]. \tag{4}$$

Particularly, an uncertain linguistic variable $\tilde{s} = [s_k, s_l]$ can be transformed into an interval 2-tuple linguistic variable $[(s_k, 0), (s_l, 0)]$.

Definition 6. Let $\tilde{a} = [(s_i, \alpha_1), (s_j, \alpha_2)]$ and $\tilde{b} = [(t_i, \gamma_1), (t_j, \gamma_2)]$ $(s_i, s_j, t_i, t_j \in S)$ be two interval 2-tuples. Minkowski distance between two interval 2-tuples is defined as

$$d_q(\tilde{a}, \tilde{b}) = [\tfrac{1}{2}(|\Delta^{-1}(s_i, \alpha_1) - \Delta^{-1}(t_i, \gamma_1)|^q + |\Delta^{-1}(s_j, \alpha_2) - \Delta^{-1}(t_j, \gamma_2)|^q)]^{1/q}, \tag{5}$$

where $q > 0$ is a distance parameter. Specially, distances $d_1(\tilde{a}, \tilde{b})$, $d_2(\tilde{a}, \tilde{b})$ and $d_{+\infty}(\tilde{a}, \tilde{b})$ are called Hamming distance, Euclidean distance and Chebyshev distance, respectively.

Definition 7. Let $\tilde{y}_1 = [(s_i, \alpha_1), (s_j, \alpha_2)]$ and $\tilde{y}_2 = [(t_i, \gamma_1), (t_j, \gamma_2)]$ be two interval 2-tuples. A possibility degree to which $\tilde{y}_1$ is preferred to $\tilde{y}_2$ (denoted by $\tilde{y}_1 \geq \tilde{y}_2$) is defined as

$$\phi(\tilde{y}_1 \geq \tilde{y}_2) = \frac{1}{2}\left(1 + \frac{(\Delta^{-1}(s_j,\alpha_2) - \Delta^{-1}(t_j,\gamma_2)) + (\Delta^{-1}(s_i,\alpha_1) - \Delta^{-1}(t_i,\gamma_1))}{|\Delta^{-1}(s_j,\alpha_2) - \Delta^{-1}(t_j,\gamma_2)| + |\Delta^{-1}(s_i,\alpha_1) - \Delta^{-1}(t_i,\gamma_1)| + l_{\tilde{y}_1\tilde{y}_2}}\right), \tag{6}$$

where $l_{\tilde{y}_1\tilde{y}_2}$ indicates the length of intersection of interval values $\Delta^{-1}(\tilde{y}_1)$ and $\Delta^{-1}(\tilde{y}_2)$.

Example 4. Let the linguistic term set S still come from Example 1. Consider two interval 2-tuples $\tilde{y}_1 = [(s_2, 0), (s_3, -0.025)]$ and $\tilde{y}_2 = [(s_2, 0.05), (s_3, 0.025)]$. The possibility degree $\phi(\tilde{y}_1 \geq \tilde{y}_2)$ is computed as follows:

From Eq. (4), it derives $\Delta^{-1}(\tilde{y}_1) = [\frac{2}{8} + 0,\ \frac{3}{8} - 0.025] = [0.25, 0.35]$. Similarly, $\Delta^{-1}(\tilde{y}_2) = [0.30, 0.40]$. Then, the intersection of $\Delta^{-1}(\tilde{y}_1)$ and $\Delta^{-1}(\tilde{y}_2)$ is obtained as [0.30, 0.35]. Thereby, it follows that $l_{\tilde{y}_1\tilde{y}_2} = 0.35 - 0.30 = 0.05$. On the other hand, by Eq. (2), we have $\Delta^{-1}(s_2, 0) = \frac{2}{8} + 0 = 0.25$. Analogously, it is obtained that $\Delta^{-1}(s_3, -0.025) = 0.35$, $\Delta^{-1}(s_2, 0.05) = 0.30$, $\Delta^{-1}(s_3, 0.025) = 0.40$. Thus, by Eq. (6), one has

$$\phi(\tilde{y}_1 \geq \tilde{y}_2) = \frac{1}{2}\left(1 + \frac{(0.35-0.40)+(0.25-0.30)}{|0.35-0.40|+|0.25-0.30|+0.05}\right) = 0.1667.$$

Property 1. The possible degree of Definition 7 satisfies the following desirable properties:

(i) $0 \leq \phi(\tilde{y}_1 \geq \tilde{y}_2) \leq 1$;
(ii) $\phi(\tilde{y}_1 \geq \tilde{y}_2) + \phi(\tilde{y}_2 \geq \tilde{y}_1) = 1$;
(iii) $\phi(\tilde{y}_1 \geq \tilde{y}_2) = 1$ if $(s_i, \alpha_1) > (t_j, \gamma_2)$;
(iv) $\phi(\tilde{y}_1 \geq \tilde{y}_2) = \phi(\tilde{y}_2 \geq \tilde{y}_1) = 0.5$ if $\phi(\tilde{y}_1 \geq \tilde{y}_2) = \phi(\tilde{y}_2 \geq \tilde{y}_1)$.

Proof. See Appendix A.

2.2. 2-tuple linguistic preference relations

In a decision-making problem, let $S = \{s_0, s_1, s_2, \cdots, s_g\}$ be a given linguistic term set and $U = \{U_1, U_2, \cdots, U_n\}$ be the set of criteria. DMs compare each pair of criteria in U and acquire a pair-wise comparison matrix, denoted by $\boldsymbol{P} = (p_{ij})_{n \times n}$, where $p_{ij} = (s_{ij}, \alpha_{ij})(i, j = 1, 2, \cdots, n)$ are 2-tuples and indicate the importance degree of criterion U_i over U_j. If $p_{ij} = (s_{g/2}, 0)$, then criterion U_i is as important as criterion U_j; If $p_{ij} > (s_{g/2}, 0)$, then criterion U_i is more important than criterion U_j; If $p_{ij} < (s_{g/2}, 0)$, then criterion U_j is more important than criterion U_i.

Definition 8. [10]. Let $S = \{s_0, s_1, s_2, \cdots, s_g\}$ be a linguistic term set. A linguistic matrix $\boldsymbol{P} = (p_{ij})_{n \times n}$ is called a 2-tuple linguistic preference relation (TLPR) if it satisfies $p_{ii} = (s_{g/2}, 0)$ and $\Delta(\Delta^{-1}(p_{ij}) + \Delta^{-1}(p_{ji})) = (s_g, 0)$, where $p_{ij} = (s_{ij}, \alpha_{ij})$, $s_{ij} \in S, \alpha_{ij} \in [-\frac{1}{2g}, \frac{1}{2g})$.

Definition 9. [10]. A TLPR $\boldsymbol{P} = (p_{ij})_{n \times n}$ is additive consistent if and only if elements in matrix $\boldsymbol{P}$ satisfy $p_{ij} = \Delta(\Delta^{-1}(p_{ik}) + \Delta^{-1}(p_{kj}) - \Delta^{-1}(s_{g/2}, 0))$ *for* $\forall i, j, k = 1, 2, \cdots, n$.

Theorem 1. *Let $\boldsymbol{P} = (p_{ij})_{n \times n}$ be an incomplete TLPR and only elements of the first row p_{1j} $(j = 1, 2, \cdots, n)$ are given a prior. Assume rest elements are completed as follows:*

$$p_{ij} = \Delta(\Delta^{-1}(p_{1j}) - \Delta^{-1}(p_{1i}) + \Delta^{-1}(s_{g/2}, 0)) \quad (i, j = 1, 2, \cdots, n), \tag{7}$$

then $\boldsymbol{P} = (p_{ij})_{n \times n}$ is additive consistent.

Proof. See Appendix B.

Remark 1. To derive an additive consistent TLPR $\boldsymbol{P} = (p_{ij})_{n \times n}$, DMs only need to furnish elements of its first row and rest elements can be determined via Eq. (7).

Example 5. The linguistic term set S is described in Example 1. In a decision making problem with four evaluation criteria, DMs respectively compare the first criterion with other criteria and provide preference information as $p_{11} = (s_4, 0)$, $p_{12} = (s_5, 0.05)$, $p_{13} = (s_6, -0.01)$ and $p_{14} = (s_3, 0)$. By Eq. (7), rest elements can be generated below:

$$p_{23} = \Delta(\Delta^{-1}(p_{13}) - \Delta^{-1}(p_{12}) + \Delta^{-1}(s_{g/2}, 0)) = \Delta(0.74 - 0.725 + 0.5) = \Delta(0.515) = (s_4, 0.015).$$

From Definition 8, one has $p_{32} = \Delta(\Delta^{-1}(s_8, 0) - \Delta^{-1}(p_{23})) = \Delta(0.485) = (s_4, -0.015)$.
Similarly, it yields $p_{24} = (s_1, 0.025)$, $p_{42} = (s_7, -0.025)$, $p_{34} = (s_1, 0.01)$, $p_{43} = (s_7, -0.01)$.

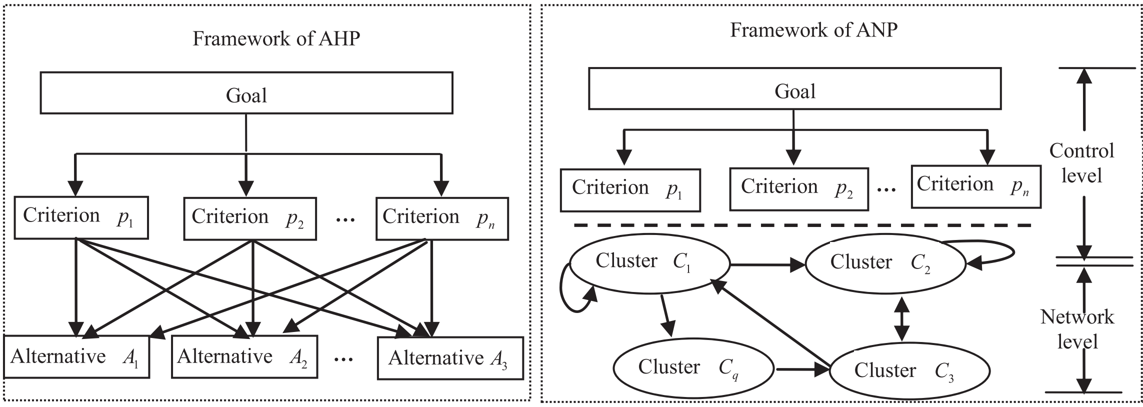

Fig. 1. Frameworks of AHP and ANP.

Thus, a completed TLPR is derived as

$$\boldsymbol{P}=\begin{pmatrix} (s_4,0) & (s_5,0.050) & (s_6,-0.010) & (s_3,0.00) \\ (s_3,-0.05) & (s_4,0) & (s_4,0.015) & (s_1,0.025) \\ (s_2,0.01) & (s_4,-0.015) & (s_4,0) & (s_1,0.01) \\ (s_5,0.00) & (s_7,-0.025) & (s_7,-0.01) & (s_4,0) \end{pmatrix}.$$

According to Definition 9, it is easily proved that matrix $\boldsymbol{P}$ is additive consistent.

Suppose a TLPR $\boldsymbol{P}=(p_{ij})_{n\times n}$ is additive consistent and $\boldsymbol{w}=(w_1, w_2, \cdots, w_n)^T$ is an underlying priority vector of TLPR $\boldsymbol{P}$. Then, the vector $\boldsymbol{w}$ can be determined as follows:

$$w_i=\Delta\left(\sum_{j=1}^{n}\Delta^{-1}(p_{ij})/\sum_{i=1}^{n}\sum_{j=1}^{n}\Delta^{-1}(p_{ij})\right)\quad (i=1,2,\cdots,n). \tag{8}$$

2.3. Analysis net process

Analytic net process (ANP) [30] is a generalization of the famous AHP [29]. AHP is a hierarchical structure with independent criteria, whereas ANP is a network structure that can deal with interactions among criteria. Frameworks of AHP and ANP are depicted in Fig. 1.

As shown in Fig. 1, a network structure of ANP includes control level and network level. Control level consists of goal and independent criteria whose weights can be obtained by AHP. There is at least one goal in control level. In network level, a network spreads out in all directions and involves arrows between clusters or loops within the same cluster. These arrows and loops indicate the relations among clusters or within cluster. For example, a single arrow from cluster C_1 to cluster C_2 means that cluster C_1 impacts on cluster C_2. A two-way arrow between cluster C_2 and C_3 indicates that cluster C_2 impacts on C_3, meanwhile, cluster C_3 also impacts on C_2. A loop in cluster C_2 implies that there are interactions among elements within cluster C_2.

In this paper, we only consider that there is one goal and criteria are omitted in control level. Suppose there are n clusters, denoted by $C_1, C_2, \cdots, C_n$, and each cluster C_i has n_i elements, $e_{i1}, e_{i2}, \cdots, e_{in_i}(i=1, 2, \cdots, n)$. To determine weights of all elements in clusters $C_i(i = 1, 2, \cdots, n)$ by ANP, the following procedure needs to be performed:

(i) Determine the weighting matrix $\boldsymbol{A}$

Considering interactions and feedback among clusters or within clusters, judgment matrices $\boldsymbol{A}^i=(a^i_{kj})_{n\times n}(i=1, 2, \cdots, n)$ are constructed by pair-wise comparisons with the 1–9 scale, where a^i_{kj} indicates that an influence degree of cluster C_k on cluster C_i is a^i_{kj} times as important as that of cluster C_j on cluster C_i. If $\boldsymbol{A}^i$ is completely consistent or of acceptable consistency, the priority vector of matrix $\boldsymbol{A}^i$, denoted by $\boldsymbol{w}_i=(w_{i1}, w_{i2}, \cdots w_{in})^T$, can be computed by the eigen-value method. Otherwise, $\boldsymbol{A}^i$ should be modified. If cluster C_i is independent of C_j, then $w_{ij}=0$. Thus, the weighting matrix $\boldsymbol{A}=(\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots, \boldsymbol{w}_n)$ can be determined and simply denoted by $\boldsymbol{A}=(a_{ij})_{n\times n}$.

(ii) Construct the supermatrix $\boldsymbol{W}$

The supermatrix $\boldsymbol{W}$ is composed of many submatrices $\boldsymbol{W}_{ij}(i, j = 1, 2, \cdots, n)$ as follows:

$$\boldsymbol{W} = \begin{array}{c} \\ C_1 e_{11}, \ldots e_{1n_1} \\ C_2 e_{21}, \ldots e_{1n_2} \\ \ldots \\ C_n e_{n1}, \ldots e_{nn_n} \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \\ {\scriptstyle e_{11},\ldots e_{1n_1}} & {\scriptstyle e_{21},\ldots e_{1n_2}} & & {\scriptstyle e_{n1},\ldots e_{nn_n}} \end{array} \\ \begin{pmatrix} \boldsymbol{W}_{11} & \boldsymbol{W}_{12} & \cdots & \boldsymbol{W}_{1n} \\ \boldsymbol{W}_{21} & \boldsymbol{W}_{22} & \cdots & \boldsymbol{W}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \boldsymbol{W}_{n1} & \boldsymbol{W}_{n2} & \cdots & \boldsymbol{W}_{nn} \end{pmatrix} \end{array}. \tag{9}$$

In submatrix $\boldsymbol{W}_{ij}$, the kth column vector is a local priority vector representing influence degrees of all elements in cluster C_i on the element e_{jk} in cluster C_j. Therefore, the sum of elements in kth column of $\boldsymbol{W}_{ij}$ should be equal to 1. This property also holds for other columns of $\boldsymbol{W}_{ij}$. The process of determining matrix $\boldsymbol{W}_{ij}$ is similar to that of determining matrix $\boldsymbol{A}$. Additionally, if the ith cluster has no influence on the jth cluster, then $\boldsymbol{W}_{ij} = \boldsymbol{0}$. For instance, it can be seen from Fig. 1 that $\boldsymbol{W}_{21} = \boldsymbol{0}$, but $\boldsymbol{W}_{12} \neq \boldsymbol{0}$.

(iii) Compute the weighted supermatrix $\bar{\boldsymbol{W}}$

By multiplying supermatrix $\boldsymbol{W}$ with matrix $\boldsymbol{A}$, the weighted supermatrix can be derived as

$$\bar{\boldsymbol{W}} = (\bar{\boldsymbol{W}}_{ij})_{n\times n} = A \times \boldsymbol{W}, \tag{10}$$

where $\bar{\boldsymbol{W}}_{ij} = a_{ij} \times \boldsymbol{W}_{ij}$ $(i, j = 1, 2, \cdots, n)$.

(iv) Determine the limit matrix $\boldsymbol{W}^{\infty}$

The limit matrix can be calculated as $\boldsymbol{W}^{\infty} = \lim_{k\to\infty} \bar{\boldsymbol{W}}^k$. In this limit matrix, all components in each row are the same.

(v) Determine overall weights of elements with respect to the goal

Since each cluster C_i has n_i elements, there are $\sum_{i=1}^{n} n_i$ elements in the ANP model. The limit matrix $\boldsymbol{W}^{\infty}$ is a $t \times t$ matrix, where $t = \sum_{i=1}^{n} n_i$. The overall weight vector of elements with respect to the goal, denoted by $\boldsymbol{\omega} = (\omega_1, \omega_2, \cdots, \omega_t)^{\mathrm{T}}$, can be determined from the limit matrix $\boldsymbol{W}^{\infty}$, where ω_k is the element of the kth row of matrix $\boldsymbol{W}^{\infty}$.

3. A novel hybrid method for solving MCDM problems with two-level criteria

In this section, a type of MCDM problems with two-level criteria is described and a new hybrid method is proposed to solve such problems.

3.1. Description of problems

Let $A = \{A_1, A_2, \cdots, A_m\}$ be the set of alternatives, $U = \{U_1, U_2, \cdots, U_n\}$ be the set of the first level criteria and $T = \{u_1, u_2, \cdots, u_t\}$ be the set of the sub-criteria. Assume each criteria U_j has n_j sub-criteria, denoted by $u_{j1}, u_{j2}, \cdots, u_{jn_j}$, then $t = \sum_{j=1}^{n} n_j$. The weight vector of sub-criteria, $\boldsymbol{\omega} = (\omega_1, \omega_2, ..., \omega_t)^T$, is unknown and needs to be determined. Suppose that the rating of alternative A_i with respect to sub-criterion u_j can be represented as an uncertain linguistic variable $\tilde{x}_{ij} = [s_{ij}, t_{ij}]$, where s_{ij} and t_{ij} come from the predefined linguistic term set $S = \{s_0, s_1, s_2, \cdots, s_g\}$. Thus, a linguistic decision matrix $\tilde{\boldsymbol{X}} = (\tilde{x}_{ij})_{m\times n}$ is elicited and then converted into an interval 2-tuple linguistic matrix $\tilde{\boldsymbol{R}} = (\tilde{r}_{ij})_{m\times n}$, where $\tilde{r}_{ij} = ([s_{ij}, 0], [t_{ij}, 0])$.

This paper focuses on how to select the best alternative (s) considering interactions among criteria. The proposed hybrid method, integrating TL-ANP and IT-ELECTRE II, concentrates on solving two key issues: (1) Determine sub-criteria weights; (2) Rank alternatives.

3.2. Determine sub-criteria weights using TL-ANP

In classical ANP, elements in judgment matrices are characterized by 1–9 scale. However, due to the inherent complexity and uncertainty in the real-world decision problems, it is difficult for DMs to express their preferences by numerical values with full confidence. In this case, linguistic variables are more appropriate to capture uncertainties occurring in pair-wise comparison judgments. This paper utilizes linguistic variables to represent ratings on pair-wise comparisons of criteria and sub-criteria and proposes TL-ANP to determine sub-criteria weights.

The criterion U_j and sub-criterion u_{jk} are respectively corresponding to the cluster C_j and its element e_{jk} in ANP stated in Section 2.3. By using linguistic variables instead of numerical values in1-9 scale, the classical ANP is extended to propose TL-ANP described as follows:

(1) Construct the network structure based on interactions among criteria and sub-criteria
(2) Determine the weighting matrix $\boldsymbol{A}$.

To determine the weighting matrix, 2-tuple linguistic judgment matrices,$\boldsymbol{A}^i = (a^i_{kj})_{n\times n}$ $(i = 1, 2, \cdots, n)$, are first built based on Remark 1. (See Section 4.3 in detail).

Let $\boldsymbol{w}_i = (w_{i1}, w_{i2}, \cdots w_{in})^T$ be the priority vector of matrices $\boldsymbol{A}^i$ $(i = 1, 2, \cdots, n)$. By Eq. (9), components of $\boldsymbol{w}_i$ can be obtained as

$$w_{ik} = \Delta\left(\sum_{j=1}^{n}\Delta^{-1}(a^i_{kj})/\sum_{k=1}^{n}\sum_{j=1}^{n}\Delta^{-1}(a^i_{kj})\right) \quad (i = 1, 2, \cdots, n;\ k = 1, 2, \cdots, t). \tag{11}$$

Thus, the weighting matrix $\boldsymbol{A} = (\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots, \boldsymbol{w}_n)$ can be determined.

(3) Determine the supermatrix $\boldsymbol{W}$.

By comparing the influences of the sub-criteria in criterion U_i on sub-criteria in criterion U_j, submatrix $\boldsymbol{W}_{ij}$ can be obtained in a similar way as the weighting matrix $\boldsymbol{A}$ is obtained. All matrices $\boldsymbol{W}_{ij}(i, j = 1, 2, \cdots, n)$ compose of the supermatrix $\boldsymbol{W}$, i.e.,

$$\boldsymbol{W} = \begin{array}{l} \\ U_1\ u_{11}, \ldots u_{1n_1} \\ U_2\ u_{21}, \ldots u_{1n_2} \\ \ldots \\ U_n\ u_{n1}, \ldots u_{nn_n} \end{array} \begin{array}{c} \begin{array}{cccc} \underset{u_{11},\ldots u_{1n_1}}{U_1} & \underset{u_{21},\ldots u_{1n_2}}{U_2} & \cdots & \underset{u_{n1},\ldots u_{nn_n}}{U_n} \end{array} \\ \begin{pmatrix} \boldsymbol{W}_{11} & \boldsymbol{W}_{12} & \cdots & \boldsymbol{W}_{1n} \\ \boldsymbol{W}_{21} & \boldsymbol{W}_{22} & \cdots & \boldsymbol{W}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \boldsymbol{W}_{n1} & \boldsymbol{W}_{n2} & \cdots & \boldsymbol{W}_{nn} \end{pmatrix} \end{array} \tag{12}$$

(4) Calculate the weighted supermatrix $\bar{\boldsymbol{W}}$

Motivated by Eq. (10), we calculate the weighted supermatrix as

$$\bar{\boldsymbol{W}} = \boldsymbol{\Delta}(\boldsymbol{\Delta}^{-1}(\boldsymbol{A}) \times \boldsymbol{\Delta}^{-1}(\boldsymbol{W})), \tag{13}$$

where $\boldsymbol{\Delta}^{-1}(\boldsymbol{A})$ and $\boldsymbol{\Delta}^{-1}(\boldsymbol{W})$ are respectively the 2-tuple linguistic transformed matrices of matrices $\boldsymbol{A}$ and $\boldsymbol{W}$.

(5) Compute the limit matrix $\boldsymbol{W}^\infty$ as follows:

$$\boldsymbol{W}^\infty = \boldsymbol{\Delta}(\lim_{k\to\infty}(\boldsymbol{\Delta}^{-1}(\bar{\boldsymbol{W}}))^k). \tag{14}$$

It is worth mentioning that components of each row in matrix $\boldsymbol{W}^\infty$ are the same.

(6) Identify the weight vector of sub-criteria with respect to overall goal.

From matrix $\boldsymbol{W}^\infty$, the weight vector of sub-criteria $\boldsymbol{\omega}= (\omega_1, \omega_2, \cdots, \omega_t)^{\mathrm{T}}$ can be determined, where ω_k $(k = 1, 2, \cdots, t)$ is a component of kth row of matrix $\boldsymbol{W}^\infty$.

3.3. Rank alternatives by IT-ELECTE II approach

In this subsection, ELECTE II is generalized to suit interval 2-tuple linguistic environment and thereby an IT-ELECTE II approach is developed to rank alternatives. This approach is completed by following three phases.

(I) First phase: likelihood-based preference degree

Definition 10. Let $\tilde{r}_{ij} = ([s_{ij}, 0], [t_{ij}, 0])$ and $\tilde{r}_{lj} = ([s_{lj}, 0], [t_{lj}, 0])$ be two ratings of alternatives A_i and A_l on sub-criterion u_j, the possible degree of $\tilde{r}_{ij}$ preferred to $\tilde{r}_{lj}$ $(\tilde{r}_{ij} \geq \tilde{r}_{lj})$ is defined as

$$\phi^j_{il} = \phi(\tilde{r}_{ij} \geq \tilde{r}_{lj}) = \frac{1}{2}\left(1 + \frac{(\Delta^{-1}(t_{ij}, 0) - \Delta^{-1}(t_{lj}, 0)) + (\Delta^{-1}(s_{ij}, 0) - \Delta^{-1}(s_{lj}, 0))}{|\Delta^{-1}(t_{ij}, 0) - \Delta^{-1}(t_{lj}, 0)| + |\Delta^{-1}(s_{ij}, 0) - \Delta^{-1}(s_{lj}, 0)| + l_{\tilde{r}_{ij}\tilde{r}_{lj}}}\right). \tag{15}$$

Similar to Property 1, the possible degree $\phi(\tilde{r}_{ij} \geq \tilde{r}_{lj})$ has following features.

Property 2. The possible degree $\phi(\tilde{r}_{ij} \geq \tilde{r}_{lj})$ satisfies:

(i) $0 \leq \phi(\tilde{r}_{ij} \geq \tilde{r}_{lj}) \leq 1$;
(ii) $\phi(\tilde{r}_{ij} \geq \tilde{r}_{lj}) + \phi(\tilde{r}_{lj} \geq \tilde{r}_{ij}) = 1$;
(iii) $\phi(\tilde{r}_{ij} \geq \tilde{r}_{lj}) = 1$ if $(s_{ij}, 0) > (t_{lj}, 0)$;
(iv) $\phi(\tilde{r}_{ij} \geq \tilde{r}_{lj}) = \phi(\tilde{r}_{lj} \geq \tilde{r}_{ij}) = 0.5$ if $\phi(\tilde{r}_{ij} \geq \tilde{r}_{lj}) = \phi(\tilde{r}_{lj} \geq \tilde{r}_{ij})$.

Matrix $\boldsymbol{\Phi}^j = (\phi^j_{il})_{m\times m}(j = 1, 2, \cdots, t)$ are called possible degree matrices. According to Property 2, matrices $\boldsymbol{\Phi}^j$ $(j = 1, 2, \cdots, t)$ are fuzzy complementary preference relations [31]. Based on method [42], dominant indices of alternatives are defined below.

Definition 11. A dominant index of an alternative A_i on sub-criterion u_j is defined as

$$DI^j_i = \tfrac{1}{m(m-1)}\left(\sum_{l=1}^{m}\phi^j_{il} + \tfrac{m}{2} - 1\right). \tag{16}$$

For sub-criterion u_j, if $DI_i^j > DI_l^j$, then alternative A_i is preferred to A_l, denoted by $A_i \succ A_l$; If $DI_i^j = DI_l^j$, then alternatives A_i and A_l are indifferent, denoted by $A_i \sim A_l$; If $DI_i^j < DI_l^j$, then alternative A_l is preferred to A_i, denoted by $A_l \succ A_i$.

According to Eq. (16), Theorem 2 can be easily proved.

Theorem 2. *The dominant index DI_i^j satisfies following properties:*

(1) $0 \le DI_i^j \le 1$ $(i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, t)$;

(2) $\sum\limits_{i=1}^{m} DI_i^j = 1$ $(j = 1, 2, \cdots, t)$.

Definition 12. *A likelihood-based preference degree of $A_i \succ A_l$ on sub-criterion u_j is defined as*

$$L_{il}^j = L^j(A_i \succ A_l) = \begin{cases} 1, & \text{if } DI_i^j - DI_l^j \ge q \\ (DI_i^j - DI_l^j)/q, & \text{if } 0 < DI_i^j - DI_l^j < q \\ 0, & \text{if } DI_i^j - DI_l^j \le 0 \end{cases} \tag{17}$$

where $q > 0$ is a threshold value of strict preference and may be determined by DMs.

The likelihood-based preference degree $L^j(A_i \succ A_l)$ indicates the degree to which alternative A_i is preferred to A_l with respect to sub-criterion u_j. If $L^j(A_i \succ A_l) = 1$, then alternative A_i is absolutely preferred to A_l; If $0 < L^j(A_i \succ A_l) < 1$, then alternative A_i is weakly preferred to A_l; If $L^j(A_i \succ A_l) = 0$, then alternative A_i is not preferred to A_l. Matrix $\boldsymbol{L}^j = (L_{il}^j)_{m\times m}$ is called a likelihood-based preference degree matrix. Based on the likelihood-based preference degree, concordance and discordance indices are defined in the second phase to respectively describe one alternative is preferred or inferior to the other.

(II) Second phase: concordance and discordance indices

Employing the likelihood-based preference degree, concordance sets, including the strong concordance set J_{il}^{+s} and the weak concordance set J_{il}^{+w}, are respectively defined as follows:

$$J_{il}^{+s} = \{ j \,|\, L^j(A_i \succ A_l) = 1 \}, \tag{18}$$

$$J_{il}^{+w} = \{ j \,|\, 0 < L^j(A_i \succ A_l) < 1 \}. \tag{19}$$

Analogously, discordance sets are composed of the subscripts of those sub-criteria where A_i is inferior to A_l. Discordance sets also can be divided into two categories: a strong discordance set J_{il}^{-s} and a weak discordance set J_{il}^{-w} which are respectively defined as

$$J_{il}^{-s} = \{ j \,|\, L^j(A_l \succ A_i) = 1 \}, \tag{20}$$

$$J_{il}^{-w} = \{ j \,|\, 0 < L^j(A_l \succ A_i) < 1 \}. \tag{21}$$

In addition, it can be seen from Eq. (17) that $L^j(A_i \succ A_l) = L^j(A_l \succ A_i) = 0$ if $DI_i^j = DI_l^j$. In this case, alternatives A_i and A_l are considered to be indifferent on sub-criterion u_j. Thereby, an indifferent set is defined as

$$J_{il}^{=} = \{ j \,|\, L^j(A_i \succ A_l) = 0 \text{ and} DI_i^j = DI_l^j \}. \tag{22}$$

Based on concordance, discordance and indifferent sets, a concordance index is defined as the ratio of the sum of the weighted likelihood-based preference degree in concordance sets and indifference set to the sum of all weighted likelihood-based preference degrees, i.e.,

$$C(i, l) = \frac{\sum_{j \in J_{il}^{+s} \cup J_{il}^{+w} \cup J_{il}^{=}} (\Delta^{-1}(\omega_j) L^j(A_i \succ A_l))}{\sum_{j=1}^{t} ((\Delta^{-1}(\omega_j)(L^j(A_i \succ A_l) + L^j(A_l \succ A_i))}, \tag{23}$$

where $\boldsymbol{\omega} = (\omega_1, \omega_2, \cdots, \omega_n)^{\mathrm{T}}$ is the 2-tuple weight vector of sub-criteria obtained in Section 3.2.

The concordance index $C(i, l)$ shows the comprehensive degree to which alternative A_i is preferred to A_l. The larger the $C(i, l)$, the bigger the degree of alternative A_i is preferred to A_l.

Remark 2. In methods [3,4,39,43], the concordance index is defined as

$$C^G(i, l) = \sum_{j \in J_{il}^{+s} \cup J_{il}^{+w} \cup J_{il}^{=}} \omega'_j \Big/ \sum_{j=1}^{n} \omega'_j, \tag{24}$$

where $\boldsymbol{\omega'} = (\omega'_1, \omega'_2, \cdots, \omega'_n)^{\mathrm{T}}$ is a numerical weight vector of criteria.

Obviously, the concordance index $C^G(i, l)$ did not consider likelihood-based preference degree $L^j(A_i \succ A_l)$, whereas the proposed concordance index $C(i, l)$ does, which may improve the distinguishing power of $C(i, l)$ and increase the sensitivity to the ratings of alternatives on criteria.

Example 6. In a MCDM problem, let $\{A_1, A_2, A_3\}$ be an alternative set and $\{U_1, U_2, U_3, U_4\}$ be a set of criteria whose weight vector is $\boldsymbol{\omega}'= (0.4, 0.3, 0.1, 0.2)^T$. The linguistic term set S comes from Example 1. Suppose the decision matrix is provided as

$$\tilde{\boldsymbol{R}} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} U_1 & U_2 & U_3 & U_4 \end{array} \\ \begin{pmatrix} [(s_7, 0.025), (s_8, 0.000)] & [(s_4, 0.000), (s_5, -0.025)] & [(s_1, 0.025), (s_2, -0.025)] & [(s_4, -0.050), (s_5, -0.025)] \\ [(s_1, 0.025), (s_2, -0.025)] & [(s_4, -0.050), (s_5, -0.025)] & [(s_7, 0.025), (s_8, 0.000)] & [(s_4, 0.000), (s_5, -0.025)] \\ [(s_1, 0.025), (s_2, -0.025)] & [(s_1, 0.025), (s_2, -0.025)] & [(s_7, 0.025), (s_8, 0.000)] & [(s_4, 0.000), (s_5, -0.025)] \end{pmatrix} \end{array}$$

Take $C(1, 2)$ as an example to illustrate how to compute the concordance index $C(i, l)$.

Similar to Example 4, the possible degree matrix on criterion U_1 is obtained as $\boldsymbol{\Phi}^1 = \begin{pmatrix} 0.5 & 1 & 1 \\ 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \end{pmatrix}$. By Eq. (16), the dominant index vector on criterion U_1 is derived as $\boldsymbol{DI}^1 = (DI_1^1, DI_2^1, DI_3^1) = (0.50, 0.25, 0.25)$. Let $q = 0.1$, then the likelihood-based preference degree matrix on criterion U_1 is computed as $\boldsymbol{L}^1 = (L_{il}^1)_{3\times 3} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Likewise, dominant index vectors on other criteria can also be derived, i.e.,

$$\boldsymbol{L}^2 = \begin{pmatrix} 0 & 0.555 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad \boldsymbol{L}^3 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \boldsymbol{L}^4 = \begin{pmatrix} 0 & 0 & 0 \\ 0.8333 & 0 & 0 \\ 0.8333 & 0 & 0 \end{pmatrix}.$$

Further, since $L_{12}^1 = 1$ and $L_{12}^2 = 0.555 < 1$, the strong concordance set $J_{12}^{+s} = \{1\}$ and the weak concordance set $J_{12}^{+w} = \{2\}$ can be respectively acquired by Eqs. (18) and (19). Analogously, one gets the strong discordance set $J_{12}^{-s} = \{3\}$, the weak discordance set $J_{12}^{-w} = \{4\}$ and the indifferent set $J_{12}^{=} = \Phi$, where Φ is an empty set. Thereby, the likelihood-based preference degree $C(1, 2)$ can be obtained by Eq. (23) as $C(1, 2) = \frac{0.4+0.3\times 0.555}{0.4+0.3\times 0.555+0.1\times 1+0.2\times 0.8333} = 0.6799$.

To verify advantages of the proposed concordance index Eq. (23) compared with that described in Eq. (24), some meaningful analyses are conducted as follows.

Observing matrix $\tilde{\boldsymbol{R}}$, one has $\tilde{r}_{11} = \tilde{r}_{23}$, $\tilde{r}_{12} = \tilde{r}_{24}$, $\tilde{r}_{13} = \tilde{r}_{21}$ and $\tilde{r}_{14} = \tilde{r}_{22}$. On the other hand, it is noticed that the sum of ω'_1 and ω'_2 (i.e., 0.7) is greater than that of ω'_3 and ω'_4 (i.e., 0.3). Therefore, alternative A_1 should be preferred to A_2(i.e., $A_1 \succ A_2$). For alternatives A_2 and A_3, their ratings are all the same except that the rating of A_2 on criterion U_2 is greater than that of A_3. Hence, it is concluded that $A_2 \succ A_3$. Thus, the relation $A_1 \succ A_2 \succ A_3$ should hold intuitively. Thereby, it is deduced that the concordance index of alternative A_1 with respect to A_2 should be less than that of alternative A_1 with respect to A_3.

Indeed, by the proposed concordance index$C(i, l)$ (i.e., Eq. (23)), one gets $C(1, 2) = 0.6799 < C(1, 3) = 0.7241$. This result is accordance with the above analysis.

On the other hand, using Eq. (24) yields that $C^G(1, 2) = C^G(1, 3) = 0.7$ which is not consistent with the above analysis. In other words, $C^G(i, l)$ in Eq. (24) has no ability to distinguish $C^G(1, 2)$ and $C^G(1, 3)$. Therefore, the concordance index $C(i, l)$ has stronger distinguishing power than index $C^G(i, l)$. Moreover, when $\tilde{r}_{14}$ and $\tilde{r}_{22}$ simultaneously increase to $[(s_4, -0.02), (s_5, -0.025)]$ while other elements are fixed, it obtains $C(1, 2) = 0.7253 < C(1, 3) = 0.7924$. However, indices $C^G(1, 2) = C^G(1, 3) = 0.7$ remain unchanged. This observation implies that index $C(i, l)$ is sensitive to variations of criterion ratings, whereas index $C^G(i, l)$ is not sensitive.

Concordance index $C(i, l)$ shows the comprehensive degree to which one alternative is preferred to the other. Conversely, to describe the comprehensive degree to which one alternative is inferior to the other, the discordance index is defined as

$$D(i, l) = \max_{j \in J_{il}^{-s} \cup J_{il}^{-w}} \{d_1(\tilde{y}_{ij}, \tilde{y}_{lj})\} / \max_{j \in K} \{d_1(\tilde{y}_{ij}, \tilde{y}_{lj})\}, \tag{25}$$

where

$$\tilde{y}_{ij} = \Delta(\Delta^{-1}(w_j)\Delta^{-1}(\tilde{r}_{ij})) = [\Delta((\Delta^{-1}(w_j)\Delta^{-1}(s_{ij}, 0)), \Delta((\Delta^{-1}(w_j)\Delta^{-1}(t_{ij}, 0))] \tag{26}$$

and $d_1(\tilde{y}_{ij}, \tilde{y}_{lj})$ represents the Hamming distance between $\tilde{y}_{ij}$ and $\tilde{y}_{lj}$ (see Eq. (5)).

Matrices $\boldsymbol{C}= (c_{il})_{m\times m}$ and $\boldsymbol{D}= (d_{il})_{m\times m}$ are respectively called concordance matrix and discordance matrix, where $c_{il} = C(i, l)$ and $d_{il} = D(i, l)$.

According to concordance and discordance indices, alternatives are ranked in the third phase.

(III) **Third phase: ranking alternatives**

Using the concordance index, the net concordance index is defined as

$$NC_i = \sum_{l \neq i} C(i,l) / \sum_{l \neq i} C(l,i). \tag{27}$$

Similarly, the net discordance index is defined as

$$ND_i = \sum_{l \neq i} D(i,l) / \sum_{l \neq i} D(l,i) \tag{28}$$

It is clearly that the larger NC_i and the smaller ND_i, the better the alternative A_i. Hence, comprehensive dominant values of alternatives can be defined as

$$CD_i = NC_i / (NC_i + ND_i). \tag{29}$$

The greater comprehensive dominant value CD_i, the better the alternative A_i. Alternatives can be ranked based on descending orders of comprehensive dominant value CD_i $(i = 1, 2, \cdots, m)$.

3.4. Proposed algorithm for MCDM with two-level criteria

According to the aforementioned analyses, a new algorithm for MCDM with two-level criteria in interval 2-tuple environment is designed below.

Step 1. Identify the criteria set, sub-criteria set and alternative set.
Step 2. Construct the network structure based on relations among criteria.
Step 3. Determine the 2-tuple linguistic weight vector of sub-criteria by TL-ANP in Section 3.2.
Step 4. Establish the uncertain linguistic decision matrix $\tilde{\boldsymbol{X}} = (\tilde{x}_{ij})_{m\times t}$ with $\tilde{x}_{ij} = [s_{ij}, t_{ij}]$ and convert it into an interval 2-tuple linguistic matrix $\tilde{\boldsymbol{R}} = (\tilde{r}_{ij})_{m\times t}$, where $\tilde{r}_{ij} = [(s_{ij}, 0), (t_{ij}, 0)]$.
Step 5. Calculate the possible degree ϕ_{il}^{j} $(i, l = 1, 2, \cdots, m;\ j = 1, 2, \cdots, t)$ using Eq. (15).
Step 6. Derive likelihood-based preference degrees $L_{il}^{j}(i, l = 1, 2, \cdots, m;\ j = 1, 2, \cdots, t)$ by Eqs. (16) and (17).
Step 7. Construct concordance, discordance and indifference sets according to Eqs. (18)–(22).
Step 8. Determine concordance and discordance indices via Eqs. (23), (25) and (26).
Step 9. Compute net concordance and discordance indices by Eqs. (27) and (28).
Step 10. Identify comprehensive dominant value of alternatives by Eq. (29).
Step 11. Rank alternatives according to their comprehensive dominant values.

4. An application to a real-life supplier selection case

In this section, a real case of Yutong Bus supplier selection with two-level criteria is examined to demonstrate the application of the proposed method.

4.1. Case description of Yutong Bus supplier selection

Yutong Bus Co., Ltd. (YBC for short), an auto manufacture company focusing on passenger car, was funded in 1963 and became the first listed company among the bus industry in China in 1997. The primary area of YBC is located in Yutong industrial Park in Zhengzhou. In 2014, bus sales in YBC reached 61,398 units and the turnover exceeded 25.7 billion yuan. With the increasing commercial competitions, YBC has to improve the quality and reduce the cost of products to enhance its core competitiveness. To gain this goal, it is very important to select an appropriate supplier for automotive upholstery. As a corporate consultant employed by YBC from 2013, the first author is responsible for the decision making and consultation on supply chain management. According to purchasing records of automotive upholstery from the purchasing department, the first author and his team screen suppliers and five suppliers remain for further evaluation, including Wuxi Huaguang Auto Parts Group Co.,Ltd (A_1), Shanghai Edscha Machinery Co., Ltd (A_2), Anhui Qingsong Tools Co., Ltd (A_3), Shandong Sanling Automotive upholstery Co., Ltd (A_4) and Henan Kiekert Xingguang Co., Ltd (A_5). Next subsections show how the first author and his team utilize our proposed method to help YBC select a best supplier from these five suppliers. In addition, the discussion on validation of obtained results is performed.

4.2. Determination of criteria and sub-criteria and network relationships

Employing the general criteria stated in Introduction and consulting a DM committee consisting of five managers from different functional departments of YBC, including purchasing manager, quality manager, production manager and technical director, the first author and his team determine the evaluation criteria and their sub-criteria which are depicted in Table 2. By the multi-session brainstorming, network relationships among criteria are determined. Fig. 2 shows interdependences among criteria and the feedback within a criterion.

Table 2
Criteria and sub-criteria for supplier selection.

Criteria	Sub-criteria
Quality (U_1)	Quality performance (u_1); Quality containment & VDCS feed back (u_2)
Cost (U_2)	Product cost (u_3); Logistics cost(u_4)
Technology (U_3)	R & D ability (u_5);Design and manufacture ability (u_6); Information technology (u_7)
Agility (U_4)	Lead time (u_8); Delivery time (u_9)
General management capability (U_5)	Management and strategy (u_{10}); Financial status (u_{11}); Reputation (u_{12})

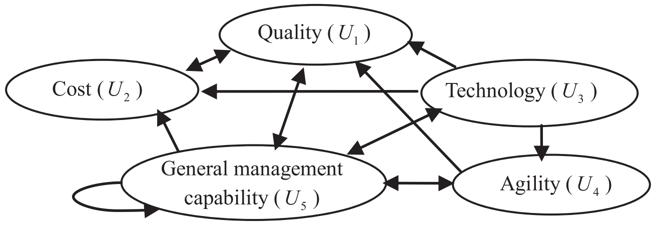

Fig. 2. Network relationship map of interactions among or within criteria.

Table 3
Linguistic terms corresponding to linguistic variables for pair-wise comparisons.

Linguistic variable	Linguistic terms	Linguistic variable	Linguistic terms
Extreme weak	s_0	Moderately strong	s_5
Very weak	s_1	Strong	s_6
Weak	s_2	Very strong	s_7
Moderately weak	s_3	Extremely strong	s_8
Equally strong	s_4		

Table 4
Elements in the first row of matrix $\boldsymbol{A}^1$.

U_1	U_2	U_3	U_4	U_5
U_2	s_4	s_1	s_3	s_1

Table 5
Matrix $\boldsymbol{A}^1$.

U_1	U_2	U_3	U_4	U_5	$\boldsymbol{w_1}$
U_2	$(s_4, 0)$	$(s_1, 0)$	$(s_3, 0)$	$(s_1, 0)$	$(s_1, 0.0156)$
U_3	$(s_7, 0)$	$(s_4, 0)$	$(s_6, 0)$	$(s_4, 0)$	$(s_3, -0.0469)$
U_4	$(s_5, 0)$	$(s_2, 0)$	$(s_4, 0)$	$(s_2, 0)$	$(s_2, -0.0469)$
U_5	$(s_7, 0)$	$(s_4, 0)$	$(s_6, 0)$	$(s_4, 0)$	$(s_3, -0.0469)$

4.3. Determine weights of sub-criteria

(1) **Determine the weighting matrix**

For determining the weighting matrix, it is necessary to construct 2-tuple linguistic judgment matrices $\boldsymbol{A}^j = (a^j_{ki})_{5\times 5}(j = 1, 2, \cdots, 5)$. As a preparation, the first author provides a linguistic term set as shown in Table 3 to help DMs compare criteria and sub-criteria. In what follows, take matrix $\boldsymbol{A}^1$ as an example to illustrate the construction of matrices $\boldsymbol{A}^j$ $(j = 1, 2, 3, 4, 5)$.

From Remark 1, the first author only needs to obtain the elements in the first row of $\boldsymbol{A}^1$ by consulting DM committee. For instance, to obtain a^1_{12}, the first author asks DM committee such a question: "Compared with the influence of technology (U_3) on quality (U_1), how much is the influence degree of cost (U_2) on quality (U_1)?". DM committee answers "very weak" according to the given linguistic term set. Thus, element $a^1_{12} = s_1$ is identified. Likewise, other elements in the first row of $\boldsymbol{A}^1$ can be obtained and represented in Table 4. Converting elements in Table 4 into 2-tuples and then employing Eq. (7), rest elements of $\boldsymbol{A}^1$ can be yielded in a similar way to Example 5. Thus, a 2-tuple linguistic matrix $\boldsymbol{A}^1$ is constructed and listed in Table 5. Finally, the priority vector $\boldsymbol{w}_1$ of matrix $\boldsymbol{A}^1$ is computed by Eq. (11) and shown in the last column in Table 5.

Similarly, other matrices $\boldsymbol{A}^j$ and their corresponding priority vectors $\boldsymbol{w}_j (j = 2, 3, 4, 5)$ can be derived. These priority vectors $\boldsymbol{w}_j$ compose of the weighting matrix $\boldsymbol{A}$. Please see Table 6.

Table 6
Weighting matrix $\boldsymbol{A}$.

	U_1	U_2	U_3	U_4	U_5
U_1	$(s_0, 0)$	$(s_4, -0.0278)$	$(s_0, 0)$	$(s_0, 0)$	$(s_2, -0.04)$
U_2	$(s_1, 0.0156)$	$(s_0, 0)$	$(s_0, 0)$	$(s_0, 0)$	$(s_1, -0.015)$
U_3	$(s_3, -0.0469)$	$(s_2, 0.0556)$	$(s_0, 0)$	$(s_5, -0.0625)$	$(s_2, 0.06)$
U_4	$(s_2, -0.0469)$	$(s_0, 0)$	$(s_0, 0)$	$(s_0, 0)$	$(s_2, 0.01)$
U_5	$(s_3, -0.0469)$	$(s_2, -0.0278)$	$(s_8, 0)$	$(s_5, -0.0625)$	$(s_1, -0.015)$

Table 7
Supermatrix.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	e_{12}
u_1	$(s_0,0)$	$(s_0,0)$	$(s_6,-0.0625)$	$(s_5,-0.0625)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_5,-0.0625)$	$(s_5,0)$	$(s_4,0)$
u_2	$(s_0,0)$	$(s_0,0)$	$(s_3,-0.0625)$	$(s_4,-0.0625)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_4,-0.0625)$	$(s_3,0)$	$(s_4,0)$
u_3	$(s_6,0)$	$(s_4,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_4,0)$	$(s_6,-0.0625)$	$(s_6,-0.0625)$
u_4	$(s_2,0)$	$(s_4,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_4,0)$	$(s_3,-0.0625)$	$(s_3,-0.0625)$
u_5	$(s_4,-0.0278)$	$(s_2,-0.0556)$	$(s_2,0.0556)$	$(s_2,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_1,0.0139)$	$(s_2,-0.0278)$	$(s_4,-0.0278)$	$(s_3,0.0139)$	$(s_4,-0.0278)$
u_6	$(s_2, 0.0556)$	$(s_3,0.0417)$	$(s_4,-0.0278)$	$(s_2,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_4,-0.0278)$	$(s_2,-0.0278)$	$(s_2,0.0556)$	$(s_4,-0.0278)$	$(s_2,0.0556)$
u_7	$(s_2,-0.0278)$	$(s_3,0.0139)$	$(s_2,-0.0278)$	$(s_4,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_3,0.0139)$	$(s_4,0.0556)$	$(s_2,-0.0278)$	$(s_1,0.0139)$	$(s_2,-0.0278)$
u_8	$(s_4,0)$	$(s_3,-0.0625)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_5,0)$	$(s_5,-0.0625)$	$(s_4,0)$
u_9	$(s_4,0)$	$(s_6,-0.0625)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_0,0)$	$(s_3,0)$	$(s_4,-0.0625)$	$(s_4,0)$
u_{10}	$(s_2,0.0278)$	$(s_4,-0.0278)$	$(s_4,-0.0278)$	$(s_2,-0.0278)$	$(s_2,0.0278)$	$(s_3,-0.0139)$	$(s_3,-0.0139)$	$(s_3,-0.0139)$	$(s_3,-0.0139)$	$(s_3,-0.0139)$	$(s_3,0.0417)$	$(s_4,-0.0556)$
u_{11}	$(s_4,0.0278)$	$(s_2,-0.0278)$	$(s_2,0.0565)$	$(s_2,-0.0278)$	$(s_4,0.0278)$	$(s_4,-0.0556)$	$(s_4,-0.0556)$	$(s_2,0.0278)$	$(s_2,0.0278)$	$(s_2,0.0278)$	$(s_2,0)$	$(s_3,-0.0139)$
u_{12}	$(s_2,-0.0556)$	$(s_2,0.0556)$	$(s_2,-0.0278)$	$(s_4,0.0556)$	$(s_2,-0.0556)$	$(s_2,-0.0556)$	$(s_2,-0.0556)$	$(s_3,-0.0139)$	$(s_3,-0.0139)$	$(s_3,-0.0139)$	$(s_3,-0.0417)$	$(s_2,-0.0556)$

Table 8
Limit matrix.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	e_{12}
u_1	$(s_1,-0.0552)$	$(s_1,-0.0552)$	$(s_1,-0.0552)$	$(s_1,-0.0552)$	$(s_1,-0.0552)$	$(s_1,-0.0552)$	$(s_1,-0.0552)$	$(s_1,-0.0552)$	$(s_1,-0.0552)$	$(s_1,-0.0552)$	$(s_1,-0.0552)$	$(s_1,-0.0552)$
u_2	$(s_0,0.0484)$	$(s_0,0.0484)$	$(s_0,0.0484)$	$(s_0,0.0484)$	$(s_0,0.0484)$	$(s_0,0.0484)$	$(s_0,0.0484)$	$(s_0,0.0484)$	$(s_0,0.0484)$	$(s_0,0.0484)$	$(s_0,0.0484)$	$(s_0,0.0484)$
u_3	$(s_0,0.0396)$	$(s_0,0.0396)$	$(s_0,0.0396)$	$(s_0,0.0396)$	$(s_0,0.0396)$	$(s_0,0.0396)$	$(s_0,0.0396)$	$(s_0,0.0396)$	$(s_0,0.0396)$	$(s_0,0.0396)$	$(s_0,0.0396)$	$(s_0,0.0396)$
u_4	$(s_0,0.0234)$	$(s_0,0.0234)$	$(s_0,0.0234)$	$(s_0,0.0234)$	$(s_0,0.0234)$	$(s_0,0.0234)$	$(s_0,0.0234)$	$(s_0,0.0234)$	$(s_0,0.0234)$	$(s_0,0.0234)$	$(s_0,0.0234)$	$(s_0,0.0234)$
u_5	$(s_1,-0.0351)$	$(s_1,-0.0351)$	$(s_1,-0.0351)$	$(s_1,-0.0351)$	$(s_1,-0.0351)$	$(s_1,-0.0351)$	$(s_1,-0.0351)$	$(s_1,-0.0351)$	$(s_1,-0.0351)$	$(s_1,-0.0351)$	$(s_1,-0.0351)$	$(s_1,-0.0351)$
u_6	$(s_1,-0.0281)$	$(s_1,-0.0281)$	$(s_1,-0.0281)$	$(s_1,-0.0281)$	$(s_1,-0.0281)$	$(s_1,-0.0281)$	$(s_1,-0.0281)$	$(s_1,-0.0281)$	$(s_1,-0.0281)$	$(s_1,-0.0281)$	$((s_1,-0.0281)$	$(s_1,-0.0281)$
u_7	$(s_1,-0.0480)$	$(s_1,-0.0480)$	$(s_1,-0.0480)$	$(s_1,-0.0480)$	$(s_1,-0.0480)$	$(s_1,-0.0480)$	$(s_1,-0.0480)$	$(s_1,-0.0480)$	$(s_1,-0.0480)$	$(s_1,-0.0480)$	$(s_1,-0.0480)$	$(s_1,-0.0480)$
u_8	$(s_1,-0.0525)$	$(s_1,-0.0525)$	$(s_1,-0.0525)$	$(s_1,-0.0525)$	$(s_1,-0.0525)$	$(s_1,-0.0525)$	$(s_1,-0.0525)$	$(s_1,-0.0525)$	$(s_1,-0.0525)$	$(s_1,-0.0525)$	$(s_1,-0.0525)$	$(s_1,-0.0525)$
u_9	$(s_0,0.0611)$	$(s_0,0.0611)$	$(s_0,0.0611)$	$(s_0,0.0611)$	$(s_0,0.0611)$	$(s_0,0.0611)$	$(s_0,0.0611)$	$(s_0,0.0611)$	$(s_0,0.0611)$	$(s_0,0.0611)$	$(s_0,0.0611)$	$(s_0,0.0611)$
u_{10}	$(s_1,0.0218)$	$(s_1,0.0218)$	$(s_1,0.0218)$	$(s_1,0.0218)$	$(s_1,0.0218)$	$(s_1,0.0218)$	$(s_1,0.0218)$	$(s_1,0.0218)$	$(s_1,0.0218)$	$(s_1,0.0218)$	$(s_1,0.0218)$	$(s_1,0.0218)$
u_{11}	$(s_1,0.0488)$	$(s_1,0.0488)$	$(s_1,0.0488)$	$(s_1,0.0488)$	$(s_1,0.0488)$	$(s_1,0.0488)$	$(s_1,0.0488)$	$(s_1,0.0488)$	$(s_1,0.0488)$	$(s_1,0.0488)$	$(s_1,0.0488)$	$(s_1,0.0488)$
u_{12}	$(s_1,-0.0241)$	$(s_1,-0.0241)$	$(s_1,-0.0241)$	$(s_1,-0.0241)$	$(s_1,-0.0241)$	$(s_1,-0.0241)$	$(s_1,-0.0241)$	$(s_1,-0.0241)$	$(s_1,-0.0241)$	$(s_1,-0.0241)$	$(s_1,-0.0241)$	$(s_1,-0.0241)$

(2) Determine the supermatrix

By pairwise comparisons on sub-criteria and performing the similar process for deriving matrix $\boldsymbol{A}$, the supermatrix $\boldsymbol{W}$can be obtained by Eq. (12) and shown in Table 7 (see Appendix C).

(3) Compute the weighted supermatrix

As shown in Table 7, each block indicates a block matrix of the supermatrix and denoted by $\boldsymbol{W}_{ij}$. Thus, each component of the weighted supermatrix $\bar{\boldsymbol{W}} = (\bar{\boldsymbol{W}}_{ij})_{5\times5}$ determined by Eq. (13) is also a block matrix, where $\bar{\boldsymbol{W}}_{ij} = \boldsymbol{\Delta}(\boldsymbol{\Delta}^{-1}(a_{ij}) \times \boldsymbol{\Delta}^{-1}(\boldsymbol{W}_{ij}))$. For example, from Tables 6 and 7, one has $a_{12} = (s_4, -0.0278)$ and $\boldsymbol{W}_{12} = \begin{pmatrix} (s_6, -0.0625) & (s_5, -0.0625) \\ (s_3, -0.0625) & (s_4, -0.0625) \end{pmatrix}$. By Eq. (2) and Definition 3, it yields $\Delta^{-1}(a_{12}) = 0.4722$ and $\Delta^{-1}(\boldsymbol{W}_{12}) = \begin{pmatrix} \Delta^{-1}(s_6, -0.0625) & \Delta^{-1}(s_5, -0.0625) \\ \Delta^{-1}(s_3, -0.0625) & \Delta^{-1}(s_4, -0.0625) \end{pmatrix} = \begin{pmatrix} 0.6875 & 0.5625 \\ 0.3125 & 0.4375 \end{pmatrix}$. Hence, it follows from Eq. (13) that $\bar{\boldsymbol{W}}_{12} = \boldsymbol{\Delta}(\boldsymbol{\Delta}^{-1}(a_{12}) \times \boldsymbol{\Delta}^{-1}(\boldsymbol{W}_{12}))$=$\begin{pmatrix} \Delta(0.3246) & \Delta(0.2656) \\ \Delta(0.1476) & \Delta(0.2066) \end{pmatrix}$. In virtue of Eq. (1), one gets $\bar{\boldsymbol{W}}_{12} = \begin{pmatrix} (s_3, -0.0504) & (s_2, 0.0156) \\ (s_1, 0.0226) & (s_2, -0.0434) \end{pmatrix}$.

(4) Determine the limit matrix and sub-criteria weights

By Eq. (14), the convergence of $(\Delta^{-1}(\bar{\boldsymbol{W}}))^k$ becomes stable at $k = 12$. Therefore, matrix $(\Delta^{-1}(\bar{\boldsymbol{W}}))^{12}$ can be considered as the limit matrix which is shown in Table 8 (see Appendix C). From Table 8, each sub-criterion weight, which is corresponding to each row of the limit matrix, is respectively obtained as $\omega_1 = (s_1, -0.0552)$, $\omega_2 = (s_0, 0.0484)$, $\omega_3 = (s_0, 0.0396)$, $\omega_4 = (s_0, 0.0234)$, $\omega_5 = (s_1, -0.0351)$, $\omega_6 = (s_1, -0.0281)$, $\omega_7 = (s_1, -0.0480)$, $\omega_8 = (s_1, -0.0525)$, $\omega_9 = (s_0, 0.0611)$, $\omega_{10} = (s_1, 0.0218)$, $\omega_{11} = (s_1, -0.0488)$, $\omega_{12} = (s_1, -0.0241)$.

Table 9
Assessment ratings of suppliers with respect to each sub-criterion.

	A_1	A_2	A_3	A_4	A_5
u_1	[M, MG]	[MG, G]	[P, M]	[MP, MG]	[M, MG]
u_2	[MG, MG]	[M, G]	[VP, MP]	[P, P]	[MG, G]
u_3	[VG, EG]	[G, G]	[P, MP]	[EP, VP]	[MG, MG]
u_4	[EP, P]	[VP, VP]	[G, VG]	[MG, VG]	[VP, P]
u_5	[G, EG]	[G, G]	[M, MG]	[M, MG]	[MG, VG]
u_6	[G, VG]	[VG, EG]	[VG, EG]	[G, G]	[M, MG]
u_7	[MG, VG]	[MG, VG]	[G, G]	[M, G]	[MG, G]
u_8	[G, EG]	[G, EG]	[VG, VG]	[P, MP]	[M, M]
u_9	[M, G]	[G, G]	[VG, EG]	[VG, VG]	[MG, VG]
u_{10}	[MG, G]	[VG, EG]	[G, G]	[MG, G]	[MP, M]
u_{11}	[MG, VG]	[G, EG]	[MP, MP]	[P, M]	[VP, MP]
u_{12}	[M, G]	[MG, G]	[M, M]	[G, EG]	[MG, VG]

Table 10
Interval 2-tuple linguistic decision matrix.

	A_1	A_2	A_3	A_4	A_5
u_1	$[(b_4,0), (b_5,0)]$	$[(b_5,0), (b_6,0)]$	$[(b_2,0), (b_4,0)]$	$[(b_3,0), (b_5,0)]$	$[(b_4,0), (b_5,0)]$
u_2	$[(b_5,0), (b_5,0)]$	$[(b_4,0), (b_5,0)]$	$[(b_1,0), (b_3,0)]$	$[(b_2,0), (b_3,0)]$	$[(b_5,0), (b_6,0)]$
u_3	$[(b_7,0), (b_8,0)]$	$[(b_6,0), (b_6,0)]$	$[(b_2,0), (b_3,0)]$	$[(b_0,0), (b_1,0)]$	$[(b_5,0), (b_5,0)]$
u_4	$[(b_0,0), (b_2,0)]$	$[(b_1,0), (b_1,0)]$	$[(b_6,0), (b_7,0)]$	$[(b_5,0), (b_7,0)]$	$[(b_1,0), (b_2,0)]$
u_5	$[(b_6,0), (b_8,0)]$	$[(b_6,0), (b_8,0)]$	$[(b_4,0), (b_5,0)]$	$[(b_4,0), (b_5,0)]$	$[(b_5,0), (b_7,0)]$
u_6	$[(b_6,0), (b_7,0)]$	$[(b_7,0), (b_8,0)]$	$[(b_7,0), (b_8,0)]$	$[(b_6,0), (b_6,0)]$	$[(b_4,0), (b_5,0)]$
u_7	$[(b_5,0), (b_7,0)]$	$[(b_5,0), (b_7,0)]$	$[(b_6,0), (b_6,0)]$	$[(b_4,0), (b_6,0)]$	$[(b_5,0), (b_6,0)]$
u_8	$[(b_6,0), (b_8,0)]$	$[(b_6,0), (b_8,0)]$	$[(b_2,0), (b_3,0)]$	$[(b_7,0), (b_7,0)]$	$[(b_4,0), (b_4,0)]$
u_9	$[(b_5,0), (b_6,0)]$	$[(b_6,0), (b_6,0)]$	$[(b_7,0), (b_8,0)]$	$[(b_7,0), (b_7,0)]$	$[(b_5,0), (b_7,0)]$
u_{10}	$[(b_6,0), (b_7,0)]$	$[(b_7,0), (b_8,0)]$	$[(b_6,0), (b_6,0)]$	$[(b_5,0), (b_6,0)]$	$[(b_3,0), (b_4,0)]$
u_{11}	$[(b_5,0), (b_7,0)]$	$[(b_6,0), (b_8,0)]$	$[(b_3,0), (b_3,0)]$	$[(b_2,0), (b_4,0)]$	$[(b_1,0), (b_3,0)]$
u_{12}	$[(b_6,0), (b_7,0)]$	$[(b_5,0), (b_6,0)]$	$[(b_4,0), (b_4,0)]$	$[(b_6,0), (b_8,0)]$	$[(b_5,0), (b_7,0)]$

4.4. Evaluate candidate suppliers and select the best one

(1) Determine the decision matrix

According to the following linguistic terms set, DM committee evaluates candidate suppliers on each sub-criterion and establishes an uncertain linguistic decision matrix shown in Table 9. Further, this matrix is converted into an interval 2-tuple linguistic matrix represented in Table 10.

S = {b_0 = Extreme poor (EP), b_1 = Very poor (VP), b_2 = Poor (P), b_3 = Medium Poor (MP), b_4 = Medium (M), b_5 = Medium good (MG), b_6 = Good (G), b_7 = Very good (VG), b_8 = Extreme good (EG)}.

(2) Calculate the possible degree matrix

According to Table 10, the possible degrees on sub-criterion u_1 can be obtained in a way similar to Example 4. Thus, the possible degree matrix is generated as

$$\mathbf{\Phi}^1 = (\phi^1_{il})_{5\times5} = \begin{pmatrix} 0.5 & 0 & 1 & 0.75 & 0.5 \\ 1 & 0.5 & 1 & 1 & 1 \\ 0 & 0 & 0.5 & 0.1667 & 0 \\ 0.25 & 0 & 0.8333 & 0.5 & 0.25 \\ 0.5 & 0 & 1 & 0.75 & 0.5 \end{pmatrix}$$

(3) Compute the likelihood-based preference degree matrix on each sub-criterion

Using Eq. (16), the dominant index vector on sub-criterion u_1 is obtained as $\boldsymbol{DI}^1$ = (0.2125, 0.3000, 0.1083, 0.1667, 0.2125). Taking $q = 0.1$ in Eq. (17), the likelihood-based preference degree matrix on sub-criterion u_1 is obtained as:

$$\boldsymbol{L}^1 = (L^1_{il})_{5\times5} = \begin{pmatrix} 0 & 0 & 1 & 0.458 & 0 \\ 0.875 & 0 & 1 & 1 & 0.875 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.583 & 0 & 0 \\ 0 & 0 & 1 & 0.458 & 0 \end{pmatrix}.$$

For example, as $DI^1_1 = 0.2125$ and $DI^1_2 = 0.3$, it follows that $0 < DI^1_2 - DI^1_1 = 0.0875 < q$. Thus, by Eq. (17), one has $L^1_{21} = 0.0875/0.1 = 0.875$. Meanwhile, $L^1_{12} = 0$ as $DI^1_1 - DI^1_2 < 0$.

Table 11
Concordance, discordance and indifferent sets.

Strong concordance sets		Weak concordance sets		Indifferent sets		Strong discordance sets		Weak discordance sets	
J_{12}^{+s}	∅	J_{12}^{+w}	{3,4,5,12}	$J_{12}^{=}$	{7,8}	J_{12}^{-s}	∅	J_{12}^{-w}	{1,2,6, 9,10,11}
J_{13}^{+s}	{1,2,3,5,8,12}	J_{13}^{+w}	{7,10,11}	$J_{13}^{=}$	∅	J_{13}^{-s}	{4,9}	J_{13}^{-w}	{6}
J_{14}^{+s}	{2,3,5,7,10}	J_{14}^{+w}	{1, 6, 11}	$J_{14}^{=}$	{8}	J_{14}^{-s}	{4,9}	J_{14}^{-w}	{12}
J_{15}^{+s}	{3,6,8,11,10}	J_{15}^{+w}	{5,7,12}	$J_{15}^{=}$	{1}	J_{15}^{-s}	∅	J_{15}^{-w}	{2,4,9}
J_{21}^{+s}	∅	J_{21}^{+w}	{1,2,6,9,10,11}	$J_{21}^{=}$	{7,8}	J_{21}^{-s}	∅	J_{21}^{-w}	{3,4,5,12}
J_{23}^{+s}	{1,2,3,8,10,11}	J_{23}^{+w}	{5,7,12}	$J_{23}^{=}$	{6}	J_{23}^{-s}	{4,9}	J_{23}^{-w}	∅
J_{24}^{+s}	{1,2,3,6,7,10,11}	J_{24}^{+w}	{5}	$J_{24}^{=}$	{8}	J_{24}^{-s}	{4,12}	J_{24}^{-w}	{9}
J_{25}^{+s}	{6, 8,10,11}	J_{25}^{+w}	{1,3,7}	$J_{25}^{=}$	∅	J_{25}^{-s}	∅	J_{25}^{-w}	{2,4,5,9,12}
J_{31}^{+s}	{4, 9}	J_{31}^{+w}	{6}	$J_{31}^{=}$	∅	J_{31}^{-s}	{1,2,3 8,5,12}	J_{31}^{-w}	{7,10,11}
J_{32}^{+s}	{4,9}	J_{32}^{+w}	∅	$J_{32}^{=}$	{6}	J_{32}^{-s}	{1,2,3, 8,10,11}	J_{32}^{-w}	{5,7,12}
J_{34}^{+s}	{6,7}	J_{34}^{+w}	{3,4, 9,10,11}	$J_{34}^{=}$	{2,5}	J_{34}^{-s}	{8,12}	J_{34}^{-w}	{1}
J_{35}^{+s}	{6,9,10}	J_{35}^{+w}	{4,7,11}	$J_{35}^{=}$	∅	J_{35}^{-s}	{1,2,5,7 10}	J_{35}^{-w}	{3}
J_{41}^{+s}	{4,9}	J_{41}^{+w}	{12}	$J_{41}^{=}$	{8}	J_{41}^{-s}	{2,3,5710}	J_{41}^{-w}	{1,6,11}
J_{42}^{+s}	{4,12}	J_{42}^{+w}	{9}	$J_{42}^{=}$	{8}	J_{42}^{-s}	{1,2,3 ,6,7, 10,11}	J_{42}^{-w}	{5}
J_{43}^{+s}	{8, 12}	J_{43}^{+w}	{1}	$J_{43}^{=}$	{2,5}	J_{43}^{-s}	{6,7}	J_{43}^{-w}	{3,4,9, 10,11}
J_{45}^{+s}	{8}	J_{45}^{+w}	{4,6,9,10,11,12}	$J_{45}^{=}$	∅	J_{45}^{-s}	{2,3,5}	J_{45}^{-w}	{1,7}
J_{51}^{+s}	∅	J_{51}^{+w}	{2,4,9}	$J_{51}^{=}$	{1}	J_{51}^{-s}	{3,6,8, 10,11}	J_{51}^{-w}	{5,7,12}
J_{52}^{+s}	∅	J_{52}^{+w}	{2,4,5,9, 12}	$J_{52}^{=}$	∅	J_{52}^{-s}	{6,8, 10,11}	J_{52}^{-w}	{1,3,7}
J_{53}^{+s}	{1,2,5,12}	J_{53}^{+w}	{3,8}	$J_{53}^{=}$	∅	J_{53}^{-s}	{6, 9,10}	J_{53}^{-w}	{4,7, 11}
J_{54}^{+s}	{2,3,5}	J_{54}^{+w}	{1,7}	$J_{54}^{=}$	∅	J_{54}^{-s}	{8}	J_{54}^{-w}	{4,6,9, 10,11,12}

Note: ∅ indicates an empty set.

(4) Determine concordance, discordance and indifference sets

Similar to Example 6, concordance, discordance and indifference sets are obtained by Eqs. (18)–(22) and listed in Table 11.

(5) Compute concordance matrix and discordance matrix

In virtue of Eq. (23), the concordance matrix is identified as

$$\boldsymbol{C} = \begin{pmatrix} - & 0.3958 & 0.8422 & 0.8523 & 0.8901 \\ 0.6042 & - & 0.8921 & 0.8090 & 0.8420 \\ 0.1578 & 0.1079 & - & 0.5965 & 0.5790 \\ 0.1477 & 0.1910 & 0.4035 & - & 0.6347 \\ 0.1099 & 0.1580 & 0.4210 & 0.3653 & - \end{pmatrix}.$$

Using Eqs. (5), (25) and (26), the discordance matrix is obtained as

$$\boldsymbol{D} = \begin{pmatrix} - & 1.0000 & 0.2317 & 0.2187 & 0.0581 \\ 0.6087 & - & 0.1853 & 0.1887 & 0.0829 \\ 1.0000 & 1.0000 & - & 1.0000 & 0.6035 \\ 1.0000 & 1.0000 & 0.5319 & - & 0.5520 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & - \end{pmatrix}.$$

(6) Compute net concordance and discordance indices

By Eqs. (27) and (28), net concordance and discordance vectors are respectively derived as

$$\boldsymbol{NC} = (2.9229, 3.6907, 0.5632, 0.5250, 0.3579), \quad \boldsymbol{ND} = (0.4180, 0.2664, 1.8491, 1.2810, 3.0853).$$

(7) Compute comprehensive dominant values of suppliers

Comprehensive dominant values of suppliers are determined via Eq. (29), i.e.,

$$CD_1 = 0.8749, \quad CD_2 = 0.9327, \quad CD_3 = 0.2335, \quad CD_4 = 0.2907, \quad CD_5 = 0.1039.$$

Since $CD_2 > CD_1 > CD_4 > CD_3 > CD_5$, the ranking is $A_2 \succ A_1 \succ A_4 \succ A_3 \succ A_5$. Hence, Shanghai Edscha Machinery Co., Ltd (namely A_2) is the best supplier, followed by Wuxi Huaguang (A_1) and then by Sanling (A_4), Henan Kiekert (A_3) and Anhui Qingsong (A_5) are ranked at the bottom.

4.5. Discussion on validation of the obtained results

It is often that different decision making methods may yield different ranking when they are fed with exactly the same assessment data. Wang and Triantaphyllou [38] established three testing criteria which are usually used to evaluate the relative performance of various MCDM methods:

Test criterion 1: An effective MCDM method should not change the indication of the best alternative on replacing a non-optimal alternative by another worse alternative without changing the relative importance of each decision criteria.
Test criterion 2: An effective MCDM method should following transitive property.
Test criterion 3: When a MCDM problem is decomposed into smaller problems and the same MADM method is applied on smaller problems to rank alternatives, combined ranking of alternatives should be identical to the original ranking of un-decomposed problem.

The validity of the results obtained by the method of this paper is tested using these criteria.

4.5.1. Validity test of the results obtained by the proposed method using criterion 1

In order to test the validity of the results obtained by the proposed method under test criterion 1, alternative A_4, a non-optimal alternative, is replaced by the following worse alternative:

$$A'_4 = ([\mathrm{P}, \mathrm{M}], [\mathrm{VP}, \mathrm{P}], [\mathrm{EP}, \mathrm{EP}], [\mathrm{M}, \mathrm{G}], [\mathrm{MP}, \mathrm{M}], [\mathrm{MG}, \mathrm{MG}], [\mathrm{MP}, \mathrm{M}], [\mathrm{MG}, \mathrm{G}], [\mathrm{MG}, \mathrm{MG}], [\mathrm{M}, \mathrm{MG}], [\mathrm{EP}, \mathrm{P}], [\mathrm{MG}, \mathrm{G}]).$$

Since the relative importance of the criteria remains unchanged in this modified MCDM problem, employing the same steps of the proposed method, corresponding results are obtained as

$$CD_1 = 0.9015, \quad CD_2 = 0.9505, \quad CD_3 = 0.3369, \quad CD_4 = 0.0508, \quad CD_5 = 0.2168.$$

According to the descending order of comprehensive values, alternatives are ranked as $A_2 \succ A_1 \succ A_3 \succ A_5 \succ A'_4$. Thus, the indication of the best alternative for the modified MCDM problem is still A_2 which is same as that for the original MCDM problem, Therefore, it is confirmed that the proposed method does not change the indication of the best alternative when a non-optimal alternative is replaced by another worse alternative. Hence, the proposed MCDM method is valid under test criterion 1. For other non-optimal alternatives, such as A_1, A_3 and A_5, the same conclusion holds.

4.5.2. Validity test of the results obtained by the proposed method under criteria 2 and 3

In order to test validity of the obtained results by the proposed method using criteria 2 and 3, original MCDM problem is decomposed into two sets of smaller MCDM problems $\{A_1, A_2, A_3\}$ and $\{A_1, A_3, A_4, A_5\}$, respectively. Following the steps of the proposed method, corresponding rankings are respectively generated as $A_2 \succ A_1 \succ A_3$ and $A_1 \succ A_4 \succ A_3 \succ A_5$for these two sub-problems.

Combining the rankings of above sub-problems together, the final ranking is obtained as $A_2 \succ A_1 \succ A_4 \succ A_3 \succ A_5$ which is identical to the ranking of un-decomposed MCDM problem and exhibits transitive property. Hence, the obtained results are valid under test criteria 2 and 3.

5. Thorough comparative analyses with existing MCDM methods

To demonstrate the superiority of the proposed method, this section conducts thorough comparative analyses with existing MCDM methods, including interval-valued 2-tuple weighted average (IVTWA) operator aggregated method [44], interval 2-tuple linguistic VIKOR (ITL-VIKOR) method [42] and a fuzzy integrated method [9].

5.1. Ranking results obtained by existing methods and Spearman's rank-correlation test

To compare with methods [9,42,44], we first use these methods to solve the above real case.

(1) According to the aggregation method [44] and ITL-VIKOR method [42], sub-criteria weights are assigned in advance. To make this comparison more validly, suppose sub-criteria weights are the numerical values corresponding to sub-criteria weights obtained by the proposed method, i.e.,

$$\boldsymbol{\omega}'_1 = (0.0698, 0.0484, 0.0396, 0.0234, 0.0899, 0.0969, 0.0770, 0.0725, 0.0611, 0.1468, 0.1738, 0.1009).$$

In virtue of method [44] and method [42], the ranking orders of alternatives are generated as $A_2 \succ A_1 \succ A_4 \succ A_3 \succ A_5$ and $A_2 \sim A_1 \succ A_4 \succ A_3 \succ A_5$, respectively.

(2) Method [9] is an integrated method combining FAHP and FTOPSIS, where FAHP is used to determine sub-criteria weights and FTOPSIS is applied to rank alternatives. Since method [11] assumed that criteria and sub-criteria are independent on each other, whereas the proposed method considers the interactions among some criteria, sub-criteria weights cannot be determined by FAHP. To make comparison validly, sub-criteria weights are still assigned as $\boldsymbol{\omega}'_1$. Employing FTOPSIS, the ranking order of alternatives is obtained as $A_2 \succ A_3 \succ A_1 \succ A_4 \succ A_5$.

To measure the ranking differences between methods [9,42,44] and the proposed method, Spearman's rank-correlation test, a technique allowing for ascertaining whether there is statistically significant rank-correlation between two sets of values, is applied to the decision. In this test, ranking values of alternatives are computed by their corresponding ranking orders. For example, given a ranking order $A_2 \succ A_1 \succ A_4 \succ A_3 \succ A_5$, ranking values of alternatives A_1, A_2, A_3, A_4 and A_5 are 2, 1,

Table 12
Rankings by four methods and their differences.

Suppliers	Ranking				Ranking differences		
	The proposed method (A)	Method [44](B)	Method [42](C)	Method [9](D)	A-B	A-C	A-D
A_1	2	2	1.5	3	0	0.5	1
A_2	1	1	1.5	1	0	0.5	0
A_3	4	4	4	2	0	0	2
A_4	3	3	3	4	0	0	1
A_5	5	5	5	5	0	0	0
Spearman's test result				r_s	1	0.925	0.7
				Z	2	1.85	1.4

Table 13
Comparison with existing MCDM methods.

	Method [44]	Method [42]			Method [9]	The proposed method
Hierarchical structure	Single-level criteria	Single-level criteria			Two-level criteria	Two-level criteria
Determination of criteria weights	Given in advance	Given in advance			Fuzzy AHP	TL-ANP
Decision making approach	Aggregation operator	ITL-VIKOR			Fuzzy TOPSIS	IT-ELECTREE II
		(S_i, α_i)	(R_i, α_i)	(O_i, α_i)		
Vector of ranking values	(0.7618, 0.8034, 0.5745, 0.6127, 0.5420)	$(\Delta(0.3374), \Delta(0.2688), \Delta(0.5968), \Delta(0.5429), \Delta(0.6531))$	$(\Delta(0.0509), \Delta(0.0631), \Delta(0.1241), \Delta(0.1241), \Delta(0.1490))$	$(\Delta(0.0892), \Delta(0.0619), \Delta(0.8002), \Delta(0.7300), \Delta(1.0000))$	(0.0040, 0.0062, 0.0042, 0.0030, 0.0023)	(0.8749, 0.9327, 0.2335, 0.2907, 0.1039)
Ranking order	$A_2 \succ A_1 \succ A_4 \succ A_3 \succ A_5$	$A_2 \succ A_1 \succ A_4 \succ A_3 \succ A_5$	$A_2 \succ A_3 \succ A_1 \succ A_4 \succ A_5$	$A_2 \succ A_1 \succ A_4 \succ A_3 \succ A_5$		

4, 3 and 5, respectively. Then, to measure the ranking differences between two ranking orders, two test statistics r_s and Z are needed to be calculated by the following expressions [22]:

$$r_s = 1 - \frac{6}{n(n^2-1)} \sum_{i=1}^{n} (d_i)^2 \tag{30}$$

$$Z = r_s\sqrt{n-1} \tag{31}$$

where d_i is the difference between two ranking values of alternative A_i in two ranking orders, n is the number of alternatives. According to [22], one has $-1 \le r_s \le 1$. Furthermore, the closer the value of r_s is to 1or -1, the stronger the correlation between two ranking orders. If $Z \ge 1.645$, then it can be concluded that there is evidence of a positive relation between two ranking orders. Otherwise, it is considered that two ranking orders are dissimilar. Employing Eqs. (30) and (31), differences between ranking orders of alternatives obtained by methods [9,42,44] and those generated by the proposed method are given in Table 12.

It is observed from Table 12 that the two sets of rankings obtained by method [44] and the proposed method have the highest rank-correlation of 1 and the largest test value of $Z = 2$. Therefore, it is concluded that these two sets of rankings have completely positive correlation. The test value between the rankings obtained by method [42] and those obtained by the proposed method is 1.85, which exceeds the critical 1.645. Thus, we affirm that the rankings obtained by method [42] are strongly positively correlated with those obtained by the proposed method. However, as for the rankings obtained by method [9] and those obtained by the proposed method, the test value is 1.4, which is lower than 1.645. Therefore, these two sets of rankings are considered to be dissimilar. In summary, the rankings obtained by the proposed method are statistically positively correlated with those obtained by methods [42,44], which demonstrates the validity of the proposed method.

5.2. Further comparative analysis

To show advantages of the proposed method, this section further compares the proposed method with existing methods [11,42,44] from hierarchical structure, determination of criteria weights and decision making approaches. The detailed comparison results are described in Table 13. In addition, to intuitively compare the ranking results of alternatives obtained by different methods, we depict these results in Fig. 3.

(1) Compared with methods [42,44], the proposed method is able to solve more complex MCDM problems because the former only can handle MCDM problems with single level criteria, while the latter can handle MCDM problems with

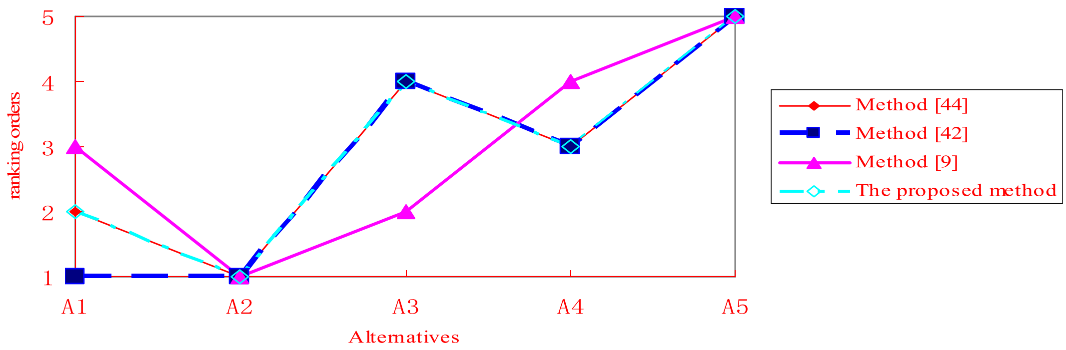

Fig. 3. Ranking orders of alternatives obtained by different methods.

two-level criteria. Thus, the latter has wider scope of applications than the former. Although method [9] also can tackle the MCDM problems with two-level criteria, it transformed linguistic variables into TFNs, which may cause loss or distortion of information. The proposed method deals with MCDM problems by interval 2-tuple linguistic variables which can effectively overcome this shortcoming.

(2) The proposed method determines criteria and sub-criteria weights objectively by TL-ANP, which can avoid the subjective randomness. However, methods [42,44] gave criteria weights in advance and did not consider the determination of criteria weights. Although method [9] employed fuzzy AHP to derive criteria weights, there are two limitations: 1) it is supposed that criteria are independent on each other; 2) it did not discuss how to repair the consistency of preference relations when preference relations are unacceptable consistent. On the other hand, the proposed method not only considers interactions among criteria, but also constructs consistent preference relations.

(3) As for the decision making approach, the proposed method utilizes IT-ELECTRE II to rank alternatives. Compared with decision making approaches used in other methods [9,42,44], the conditions of IT-ELECTRE II (i.e., alternatives are compared on each criterion and the comparison scores on criteria cannot compensate for each other) are stricter. Therefore, the results obtained by IT-ELECTRE II are more cautious and more reliable.

6. Conclusions

In today's fierce competitive market, it is necessary for enterprises to select a suitable supplier to win a space in their business. Therefore, supplier selection is a critical issue for enterprises. This paper investigated a type of supplier selection problems with two-level criteria and proposed a hybrid method by combining ANP with ELECRE II in interval 2-tuple linguistic environment. Primary contributions are summarized as follows:

(1) Interactions among criteria or within criteria are considered while determining weights of criteria and sub-criteria, which is more consistent with real-world decision situations.

(2) A TL-ANP approach is proposed to determine weights of criteria and sub-criteria. There are two prominent characteristics of this approach: 1) 2-tuple linguistic variables are used in comparison matrices, which not only can help DMs supply their information more flexible and easier, but can avoid the loss and distortion of evaluation information; 2) The proposed technique for constructing comparison matrix of a TLPR can guarantee the consistency of TLPR. Moreover, DMs are only required to supply the elements in the first row of comparison matrix. Thus, the workload of DMs and the cost of enterprise may be remarkably reduced.

(3) An IT-ELECTRE II approach is developed to rank alternatives. In this approach, ratings of alternatives on sub-criteria are in the form of interval 2-tuples, which can suitably model the quantitative and qualitative criteria involved in supplier selection. Furthermore, IT-ELECTRE II is able to neatly compare alternative suppliers and has a strong distinguishing power.

Future study will extend the proposed hybrid method to group decision-making problems and other decision environments, such as hesitant fuzzy set and linguistic hesitant fuzzy set.

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Appendix A

Proof of Property 1. Denote $\Delta^{-1}(s_i,\alpha_1)=\beta_1^-$, $\Delta^{-1}(t_i,\alpha_2)=\beta_1^+$, $\Delta^{-1}(s_j,\gamma_1)=\beta_2^-$, $\Delta^{-1}(t_j,\gamma_2)=\beta_2^+$.

(i) Since $-(|\beta_1^+-\beta_2^+|+|\beta_1^--\beta_2^-|)\le(\beta_1^+-\beta_2^+)+(\beta_1^--\beta_2^-)\le|\beta_1^+-\beta_2^+|+|\beta_1^--\beta_2^-|$, it obtains that $-1\le\frac{(\beta_1^+-\beta_2^+)+(\beta_1^--\beta_2^-)}{|\beta_1^+-\beta_2^+|+|\beta_1^--\beta_2^-|}\le 1$. In the following, the proof is completed in two cases.

Case 1. $(\beta_1^+-\beta_2^+)+(\beta_1^--\beta_2^-)\ge 0$. Thus, it yields that

$$0\le\frac{(\beta_1^+-\beta_2^+)+(\beta_1^--\beta_2^-)}{|\beta_1^+-\beta_2^+|+|\beta_1^--\beta_2^-|+l_{\tilde{y}_1\tilde{y}_2}}\le\frac{(\beta_1^+-\beta_2^+)+(\beta_1^--\beta_2^-)}{|\beta_1^+-\beta_2^+|+|\beta_1^--\beta_2^-|}\le 1.$$

Consequently,$0\le\frac{1}{2}(1+\frac{(\beta_1^+-\beta_2^+)+(\beta_1^--\beta_2^-)}{|\beta_1^+-\beta_2^+|+|\beta_1^--\beta_2^-|+l_{\tilde{y}_1\tilde{y}_2}})=\frac{1}{2}(1+\frac{(\Delta^{-1}(t_i,\alpha_2)-\Delta^{-1}(t_j,\gamma_2))+(\Delta^{-1}(s_i,\alpha_1)-\Delta^{-1}(s_j,\gamma_1))}{|\Delta^{-1}(t_i,\alpha_2)-\Delta^{-1}(t_j,\gamma_2)|+|\Delta^{-1}(s_i,\alpha_1)-\Delta^{-1}(s_j,\gamma_1)|+l_{\tilde{y}_1\tilde{y}_2}})\le 1$.

Namely, $0\le\phi(\tilde{y}_1\ge\tilde{y}_2)\le 1$.

Case 2. $(\beta_1^+-\beta_2^+)+(\beta_1^--\beta_2^-)<0$. The proof process is similar to Case 1.

(ii) To prove this conclusion, it is only needed to perform the operation $\phi(\tilde{y}_1\ge\tilde{y}_2)+\phi(\tilde{y}_2\ge\tilde{y}_1)$ based on the expression of $\phi(\tilde{y}_1\ge\tilde{y}_2)$. Here, the proof process is omitted.

(iii) If $(s_i^1,\alpha_1^1)\succ(s_j^2,\alpha_2^2)$, then $l_{\tilde{y}_1\tilde{y}_2}=0$ and $\beta_2^-<\beta_2^+<\beta_1^-<\beta_1^+$.
Thereby, $(\beta_1^+-\beta_2^+)+(\beta_1^--\beta_2^-)=|\beta_1^+-\beta_2^+|+|\beta_1^--\beta_2^-|$.
Similar to the proof process of Case 1, the conclusion $\phi(\tilde{y}_1\ge\tilde{y}_2)=1$ can be easily derived.

(iv) It is easily obtained from conclusion (ii).
The proof of Property 1 is completed.

Appendix B

Proof of Theorem 1. From $p_{ij}=\Delta(\Delta^{-1}(p_{1j})-\Delta^{-1}(p_{1i})+\Delta^{-1}(s_{g/2},0))\,(i,j=1,2,\cdots,n)$, we have

$$p_{ik}=\Delta(\Delta^{-1}(p_{1k})-\Delta^{-1}(p_{1i})+\Delta^{-1}(s_{g/2},0)),\quad p_{kj}=\Delta(\Delta^{-1}(p_{1j})-\Delta^{-1}(p_{1k})+\Delta^{-1}(s_{g/2},0)).$$

Thereby,

$$\Delta^{-1}(p_{ik})+\Delta^{-1}(p_{kj})=(\Delta^{-1}(p_{1k})-\Delta^{-1}(p_{1i})+\Delta^{-1}(s_{g/2},0))+(\Delta^{-1}(p_{1j})-\Delta^{-1}(p_{1k})+\Delta^{-1}(s_{g/2},0)).\tag{B.1}$$

Simplifying Eq. (B.1), one has

$$\Delta^{-1}(p_{ik})+\Delta^{-1}(p_{kj})=\Delta^{-1}(p_{1j})-\Delta^{-1}(p_{1i})+\Delta^{-1}(s_{g/2},0)+\Delta^{-1}(s_{g/2},0).\tag{B.2}$$

Subtracting $\Delta^{-1}(s_{g/2},0)$ on both sides of Eq. (B.2), it follows that

$$\Delta^{-1}(p_{ik})+\Delta^{-1}(p_{kj})-\Delta^{-1}(s_{g/2},0)=\Delta^{-1}(p_{1j})-\Delta^{-1}(p_{1i})+\Delta^{-1}(s_{g/2},0).\tag{B.3}$$

Consequently, $\Delta(\Delta^{-1}(p_{ik})+\Delta^{-1}(p_{kj})-\Delta^{-1}(s_{g/2},0))=\Delta(\Delta^{-1}(p_{1j})-\Delta^{-1}(p_{1i})+\Delta^{-1}(s_{g/2},0))=p_{ij}$. Namely, $p_{ij}=\Delta(\Delta^{-1}(p_{ik})+\Delta^{-1}(p_{kj})-\Delta^{-1}(s_{g/2},0))$. From Definition 9, matrix $\boldsymbol{P}=(p_{ij})_{n\times n}$ is additively consistent. This completes the proof.

Appendix C

(See Tables 7 and 8).

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