

# Wind farm layout optimization using genetic algorithm with different hub height wind turbines



Ying Chen, Hua Li<sup>\*</sup>, Kai Jin, Qing Song

Department of Mechanical and Industrial Engineering, Texas A&M University–Kingsville, MSC 191, 700 University Blvd., Kingsville, TX 78363, USA

## ARTICLE INFO

### Article history:

Received 20 October 2012

Accepted 14 February 2013

Available online 21 March 2013

### Keywords:

Wind farm

Layout

Genetic algorithm

Optimization

## ABSTRACT

Layout optimization is one of the methods to increase wind farm's utilization rate and power output. Previous researches have revealed that different hub height wind turbines may increase wind farm's power output. However, few researches focus on optimizing a wind farm's layout in a two-dimensional area using different hub height wind turbines. In this paper, the authors first investigate the effect of using different hub height wind turbines in a small wind farm on power output. Three different wind conditions are analyzed using nested genetic algorithm, where the results show that power output of the wind farm using different hub height wind turbines will be increased even when the total numbers of wind turbines are same. Different cost models are also taken into account in the analysis, and results show that different hub height wind turbines can also improve cost per unit power of a wind farm. At last, a large wind farm with commercial wind turbines is analyzed to further examine the benefits of using different hub height wind turbines in more realistic conditions.

Published by Elsevier Ltd.

## 1. Introduction

Serious environmental pollution is threatening humans' health, which has raised a lot of public concerns. Fossil fuels as the main energy sources in current society are not sustainable and will be exhausted in the foreseeable future due to limited resources, rapid consumption, climate change, global warming, etc. [1]. In academia, a lot of researchers are investigating how to use renewable energy, such as solar energy, biomass, and wind energy, as substitutes of traditional energy resources. Meanwhile, most countries are trying to use renewable energy to replace fossil fuels so as to keep a better environment. Regarding about wind energy, higher conversion rate, clean and safety are its major advantages compared to other types of renewable energy [1–6]. However, the large variation, uncertainty, and other non-predictable factors and issues impact the wind power estimation and the energy capture efficiency tremendously [1–6]. In US, the cumulative wind power installed capacity is only equal to about 3.3% of the nation's electricity demand at the end of 2011 [7]. In order to increase power output and economic performance of a wind farm, wind speed estimation, wind turbine and gearbox design, and layout optimization are several focal research areas. Liu et al. [8] introduced a quantitative methodology of building an ARMA-GARCH-M model to improve the forecasted rate of wind speeds. Mohammadi and Mostafaeipour [9] demonstrated that Weibull distribution based on the standard deviation method and the

power density method was able to estimate the mean wind power as an alternative method much better. Hall et al. [10] proposed variable ratio gearbox which was integrated into fixed speed wind turbine to adapt to the variable speeds in order to boost efficiency. Kenway and Martin [11] presented a multidisciplinary optimization framework for the design of wind turbine rotors to maximize the power output of a wind turbine by changing the blade geometry and structural sizes without any cost changes and non-compatibility with the rest part of turbine system. Ramos et al. [12] analyzed the factors that affected the wind farm energy output, and indicated that the location selection of a wind farm was very important. In Refs. [13–21], the researchers tried to optimize the layout of a wind farm by the intelligent algorithms with the objective of maximizing its power output or minimizing its cost per unit power.

In this paper, the authors mainly focus on wind farm layout optimization. Most previous research conducted on this topic made use of genetic algorithm (GA) to realize different research objectives. Mosetti et al. [13] first used GA to optimize the layout of a wind farm under three scenarios: constant wind speed and direction, constant wind speed and various wind directions, and various wind speeds and directions. Based on Mosetti et al.'s research, Grady et al. [14] employed more individuals (600) and generations (3000) in GA to achieve better layout for a wind farm. Mittal [15] proposed the micro-sitting method with GA in order to find more accurate positions in a wind farm, and the cell size in his research was  $1\text{ m} \times 1\text{ m}$ , which was different from  $200\text{ m} \times 200\text{ m}$  cell size used in Refs. [13,14]. Compared to Grady et al.'s results, Mittal's results indicated that the cost per unit

<sup>\*</sup> Corresponding author. Tel.: +1 361 593 4057; fax: +1 361 593 4026.

E-mail address: [hua.li@tamuk.edu](mailto:hua.li@tamuk.edu) (H. Li).

power could be reduced in all three scenarios. In order to increase the wind farm power output, Acero et al. [16] investigated the possibility of applying wind turbines with two hub heights in a straight line, and the results demonstrated that using different hub height wind turbines might help generate more power output. Chen and McDonald [17] considered the landowners' decision on wind turbines installation into wind farm layout optimization process. Regarding the tower cost, turbine machine cost and foundation cost, Mora et al. [18] proposed an evolution algorithm based on GA to design a wind farm layout with minimum investment and most efficient use of the wind resource. Later, Gonzalez et al. [19] extended the cost model developed in Ref. [18] to an integral cost model based on a life cycle cost approach. Besides GA, Marmidis et al. [20] tried to use Monte Carlo algorithm to optimize layout of a wind farm under constant wind speed and direction. And under the same wind condition in Ref. [20], Chowdhury et al. [21] employed particle swarm optimization (PSO) algorithm to exhibit that the wind turbines with different diameters could improve the power output of a wind farm.

In this paper, the authors use GA in MATLAB to optimize the layout of a given wind farm with different hub height wind turbines in order to investigate the benefits of using wind turbines with different hub heights. Among all kinds of factors affecting wind farm layout design, the authors only consider the following factors based on the scope of this study: (1) number of turbines installed in a given wind farm, (2) hub heights of wind turbines, (3) wind directions and speeds, and (4) power output and cost per unit power of a given wind farm. As to the factors including local topography, wind farm soil conditions, the construction of roadways, nameplate capacity of a wind farm, and the local vegetation coverage [21], they are not taken into account within the scope of this study. The authors first conduct the layout optimization of a 500 m × 500 m wind farm in three scenarios: (1) constant wind speed and direction, (2) constant wind speed and various wind directions, and (3) various wind speeds and directions. In each case, the power outputs of the optimal layouts using same hub height wind turbines and using different hub height wind turbines are compared, so that the effect of different hub height wind turbines on power output can be investigated. Different cost models are also taken into account in the analysis, and results show that different hub height wind turbines can also improve cost per unit power of the wind farm. At last, a large wind farm with commercial wind turbines is further analyzed to examine the benefits of using different hub height wind turbines in more realistic conditions.

## 2. Methodology

In the first part of this section, the wake model used in this paper is introduced. And the details of modified GA method used in this paper are discussed in the second part of this section.

### 2.1. Wake model

In Ref. [22], it has been declared that a wind turbine's efficiency would be reduced after putting it in a wind farm with other turbines due to the wake effect. When wind flows through a wind turbine, part of kinetic energy is transferred to the turbine blades. As the wind speed is decreased by the blades, it will produce a volumetric expansion regarding the mass accumulation before the blades. To simplify the wake model without considering the near turbulence intensity, this effect is assumed to propagate continuously and linearly as shown in Fig. 1. The wake effect will increase when multiple wakes apply to the same wind turbine. The analytical wake model used in this paper was first developed by Jensen [23] and later improved by Katic et al. [24] and Frandsen [25]. In

this model, the momentum is assumed to be constant inside the wake. And it is possible to treat the resulting wake caused by a wind turbine as a turbulent wake if the near field behind the turbine is not taken into account [13–25]. In Ref. [22], it has been validated that the traditional Jensen's wake model is more precise than other ones at predicting the wake loss. Most of the parameters involved in the wake model are listed in Table 1.

The power generated by a wind turbine is computed through the following equations. The Eq. (1) is to calculate power output of  $i$ th wind turbine with the wind speed  $U_i$ , where  $U_i$  can be decided based on Eqs. (2) and (3).

$$p_i = \frac{1}{2} \rho A U_i^3 C_p \quad (1)$$

$$U_i = U_0 (1 - U_{\text{def}} \times (A_{\text{overlap}}/A)) \quad (2)$$

$$U_0^j(z_j) = \frac{u^*}{k} \ln \left( \frac{z_j}{z_0} \right) + \psi \quad (3)$$

where  $U_0^j(z_j)$  is the free stream velocity before wind turbine  $j$ ,  $u^*$  is the friction velocity corresponding to the turbine's hub height,  $k$  is the von Karman constant that is set to 0.4 as usual [2], and  $\psi$  is stability term. The value of  $\psi$  will be zero in neutral conditions, positive in stable conditions, and negative in unstable conditions. The condition is assumed to be neutral in this study so that the value of  $\psi$  is zero. Based on empirical values [13], the surface roughness length  $z_0$  is assumed to be 0.3 in this paper. Velocity loss  $U_{\text{def}}$  is expressed in following equation:

$$U_{\text{def}} = \frac{2a}{\left(1 + \alpha \frac{x}{r_1}\right)^2} \quad (4)$$

where  $\alpha$  is entrainment constant, and  $x$  is the downstream distance from the wind turbine that generates the wake. And  $\alpha$  can be calculated using Eq. (5). Since we use wind turbines with different hub heights, the value of  $\alpha$  will change when hub height  $z_j$  changes. Wake radius  $r_1$  is related to entrainment constant  $\alpha$  and distance  $x$ , which can be determined by Eq. (6) [25]. Eq. (8) introduces the relationship between thrust coefficient and axial induction factor, which is used to calculate downstream rotor radius as Eq. (7). When several wakes merge together, the resultant velocity  $u$  is calculated by equating the kinetic energy deficit of the mixed wake to the sum of kinetic energy deficits of each individual wake at that point, which is shown in Eq. (9). The objective function used in GA is to maximize the total power output of a wind farm as shown in Eq. (10).

$$\alpha = \frac{0.5}{\ln(z_j/z_0)} \quad (5)$$

$$r_1 = \alpha x + r_r \quad (6)$$

$$r_r = r \sqrt{(1-a)/(1-2a)} \quad (7)$$

$$C_T = 4a \times (1-a) \quad (8)$$

$$(1 - u/U_0)^2 = \sum_{i=1}^N (1 - U_i/U_0)^2 \quad (9)$$

$$\text{Max } P = \sum_{i=1}^N p_i \quad (10)$$

When a turbine is covered partially in a wake, the forces on the turbine blades are uneven and thus, the entire turbine will face unsteady operation and high turbulence [27]. As same as previous research [13–21], we assume that the unsteady operation due to

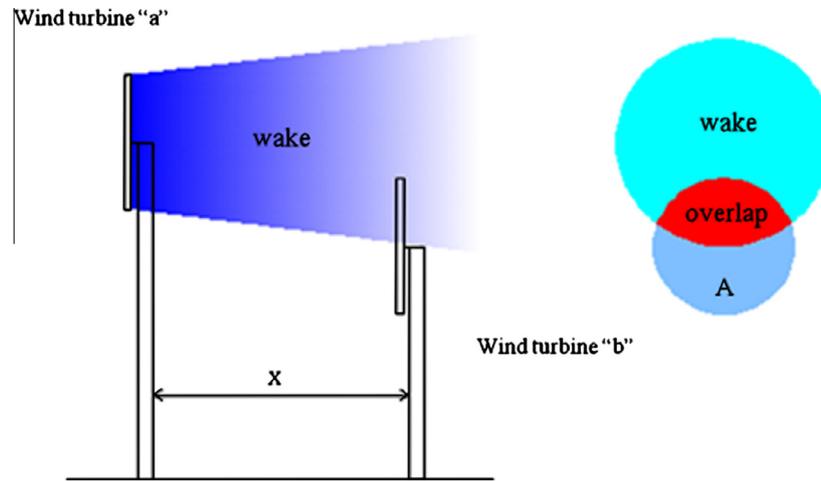


Fig. 1. Schematic of a wake model involving wind turbines with different hub heights.

Table 1

Major nomenclature used.

$U_0$	Velocity of free stream (m/s)	$U_{def,tot}$	Total velocity deficit (m/s)
$U_{def}$	Velocity loss in the wake (m/s)	$a$	Axial induction factor
$A$	Swept area of wind blades ( $m^2$ )	$C_T$	Turbine thrust coefficient
$z_0$	Surface roughness length (m)	$U_i$	Downstream wind speed in one wake (m/s)
$z_j$	Hub height of wind turbine $j$ (m)	$N$	Number of wind turbines in a wind farm
$r_r$	Downstream rotor radius (m)	$p_i$	Power output of a wind turbine (MW)
$r_1$	Wake radius (m)	$P$	Total power output of a wind farm (MW)
$r$	Turbine radius (m)	$u$	Downstream wind speed in several wakes (m/s)
$\rho$	Air density ( $1.2254 \text{ kg/m}^3$ )	$C_p$	Power coefficient of a wind turbine
$\alpha$	Entrainment constant	$A_{overlap}$	Overlapping area between wake and a turbine ( $m^2$ )

partial coverage will not have effect on the power output of a wind farm. Based on this assumption, the effects of using different hub height wind turbines into a wind farm are investigated in this study. Meanwhile, according to Eq. (3), the velocity of free stream at lower turbines is smaller than that at higher turbines, which may decrease power generation. For example, in Fig. 1, assuming that the hub height of wind turbine “a” is 78 m, the hub height of wind turbine “b” is 50 m, and the free stream velocity at 78 m is 12 m/s, we can calculate the free stream velocity at 50 m height as 11.0404 m/s when  $\psi = 0$  and  $z_0 = 0.3$  m. Even though Acero et al. [16] showed that wind turbines with different hub heights could increase the power generation in a straight line layout under condition of single wind speed and two opposite wind directions, the lower wind turbine used in his study is 50 m hub height with 77 m diameter which is irrational in reality. Thus, it is still necessary to investigate whether using different hub height wind turbines in a two dimensional wind farm would help generate more power under complicated wind conditions.

## 2.2. Genetic algorithm

In this paper, the authors use genetic algorithm in MATLAB to search the optimal layout of a given wind farm. As a global search tool, GA may avoid the local optimal solutions by generating solutions randomly [26]. First of all, GA will generate binary chromosomal strings randomly. Every chromosomal string is an individual, which represents a layout of the given wind farm in this study. Selection, crossover and mutation are another three major steps in GA. Selection is to select and retain a certain proportion of individuals that can generate better results according to a given selection probability. After selection, GA will conduct crossover and mutation based on corresponding given probabilities. Different probabilities will generate different results. And the probabilities

need to be optimized according to the problem itself. Crossover proceeds randomly among the selected individuals in order to find the better individuals. Mutation proceeds on the individuals after crossover, which is used to increase the diversity of individuals so as to avoid the premature convergence. After selection, crossover and mutation, individuals with better results will be carried over to the next generation, and the rest individuals will be eliminated simultaneously. The algorithm then will reinsert some new random individuals to replace the deleted ones in order to maintain the same number of individuals in each generation. GA will be continuously carried on until it reaches the given maximum number of generation.

Previous researches [13–15] used single hub height wind turbines, and they only needed to consider the position of each wind turbine in the given wind farm, so that one binary string in GA is enough to represent the layout of a wind farm. When the variable is not only the position of each wind turbine, Chowdhury et al. [21] introduced PSO algorithm to optimize wind farm layout by determining the number of turbines first with the given nameplate capacity of a wind farm and then optimizing each turbine’s diameter and position. In this study, the second variable in the layout optimization is hub height. With traditional GA method used in Refs. [13–15], one binary string is not enough to represent two variables. Instead of using PSO with predetermined wind farm nameplate capacity, the authors develop a nested GA with randomized initial number of wind turbines, which means one binary string representing the turbine positions and the other one representing the turbine hub heights. In this way, the nested GA can also examine the best number of wind turbines suitable to a given wind farm.

First, GA will generate an  $m \times n$  binary matrix including different individuals representing wind farm layouts, where  $m$  is the total number of individuals in one generation and  $n$  is the length of

**Table 2**  
Major GA parameters used.

	Selection rate	Crossover rate	Mutation rate	# of Generations	# of Individuals	Length of individual
GA1	0.9	1	0.01	200	$m = 200$	$n = 361$
GA2	0.9	1	0.01	50	$g = 50$	$h$ (variable)

**Table 3**  
Parameter used in Grady et al.'s [14] research.

Hub height (m)	60	Surface roughness length (m)	0.3
Diameter (m)	40	Velocity of free stream (m/s)	12
Wind farm size	2 km $\times$ 2 km	Thrust coefficient	0.88
Cell size	200 m $\times$ 200 m	Air density (kg/m <sup>3</sup> )	1.2254

each individual representing potential positions of wind turbines. The values of  $m$  and  $n$  are decided at the beginning of optimization. This is position optimization, which is named as GA1. In each individual of GA1, it is defined that “0” represents no wind turbines in that position and “1” represents one wind turbine in that position. Considering the collision and the interference in the case of a wind turbine falling down or rotating, the closest distance between any two turbines in this research is set to be 100 m based on the turbines’ hub heights and diameters. With micro-sitting method, similar to Mittal’s research [15], the unsuitable “1”s in GA1 will be deleted first to meet the minimum distance constraint between any two turbines. Then, the hub height of each wind turbine will be specified in order to compute the power output of each potential wind farm layout. Thus, another GA is nested to GA1 to optimize turbines’ hub heights in each potential layout. This is hub height optimization, which is named as GA2 in this study. Based on the determined number of turbines in each individual of GA1, a  $g \times h$  binary matrix will be created in GA2, where  $g$  is the number of individuals representing hub heights combination. The value of  $h$  is decided by the number of “1” left in each individual after eliminating the unsuitable turbines in GA1, which values are always changing. The lower wind turbine is represented by “0” and the higher one is represented by “1”. After selecting the best combination of hub heights in GA2 for the corresponding individual in GA1, the power output of potential wind farm layouts could be determined. Then, GA2 will continue to optimize the hub heights of all potential layouts generated by GA1. After obtaining the power output of all potential layouts, GA1 will select the layouts that reach the better results according to objective function, and then proceed to crossover and mutation on the selected layouts to complete the simulation for one generation. Basically, GA1 handles the position selection of wind turbines while GA2 deals with the hub heights selection. In this study, we first conduct preliminary tests to decide the values of all the GA parameters. The major GA parameters used in the three case studies are shown in Table 2.

### 3. Pretest and GA parameters validation

Before conducting the three case studies mentioned in Section 1, the authors conduct a pretest to validate the selected GA parameters shown in Table 2 in order to make sure the GA parameters are suitable since generations and individuals used in this research are

much less than Grady et al.’s [14]. The parameters of wind turbines and wind farm used in Ref. [14] are summarized in Table 3. Although there were three case studies conducted in Ref. [14] as mentioned before, only the first two case studies are compared in the pretest since the parameters of the wind distribution in Grady et al.’s third case study was not given.

Meanwhile, the rated power in Ref. [14] and Mosetti’s study [13] was not clear. But it can be found out that the rated wind speed is more than 12 m/s and the rated power is more than 600 kW from the power curve in Ref. [13]. The GA parameters in the pretest are shown in Table 4. Pretest 1 and pretest 2 correspond to the case study 1 and 2 in Grady et al.’s research, respectively.

The pretest results are shown Table 5, and Fig. 2 shows the fitness value variation chart for the pretest. It is clear that same or even better results can be reached with the selected GA parameters in this study even though Grady et al. used 600 individuals and 3000 generations in his case study 1 and 2. In order to improve the diversity of individuals, the number of individuals are increased from 50 in pretest to 200 in all three case studies in this paper since the chromosomal string is 361 not 10 or 100 in the three case studies.

### 4. Case studies with results and discussion

Due to the limitation of computing capacity, we decide the given wind farm’s size as 500 m  $\times$  500 m, which is further divided into 400 cells with a cell size of 25 m  $\times$  25 m as shown in Fig. 3. Although this cell size is not as small as 1 m  $\times$  1 m cell size used in Refs. [15,21], it should be good enough to provide sufficient potential locations for wind turbines. Instead of locating wind turbines into the center of each cell, we decide to place all wind turbines at the intersection points in the grid (shown in Fig. 3) within the area of the given wind farm. Considering the size of tower foundation, the available positions will not include those intersection points at four edges. For example, the intersection point (25, 25) is a potential position, but (0, 25) is not a potential position. Therefore, there are 361 potential positions available in this small wind farm, which means  $n$  is equal to 361 in Table 2. However, according to the closest distance constraint, the actual number of wind turbines that can be placed in this wind farm will be less than 361. To allow larger turbine spacing, the closest distance between any two turbines is set as 100 m considering the turbine hub height and rotor radius used in this paper. However, a higher value may be used in practice, such as 4 or 5 times of rotor diameter.

Rated wind speed of a wind turbine is another very important factor that requires consideration. If actual wind speed exceeds the wind turbine’s rated wind speed, power generation will not increase. When calculating power output, two conditions are

**Table 4**  
Major GA parameters used in pretests.

	Selection rate	Crossover rate	Mutation rate	# of Generations	# of Individuals	Length of individual
Pretest 1	0.9	1	0.01	200	50	10
Pretest 2	0.9	1	0.01	200	50	100

**Table 5**  
Pretest results versus Grady et al.'s [14] research results.

	Power output (MW)	Fitness value	No. of turbines
Pretest 1	14.31	0.001543	30
Case study 1 in Ref. [14]	14.31	0.001543	30
Pretest 2	18.183	0.001544	41
Case study 2 in Ref. [14]	17.22	0.001566	39

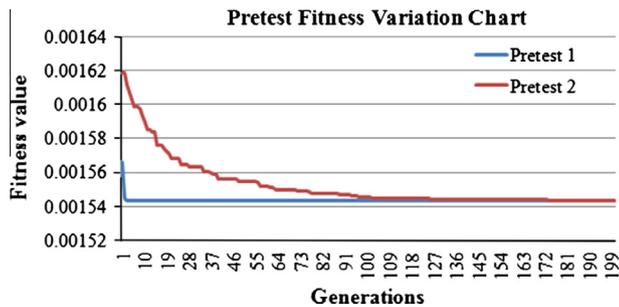


Fig. 2. Fitness value variation chart.

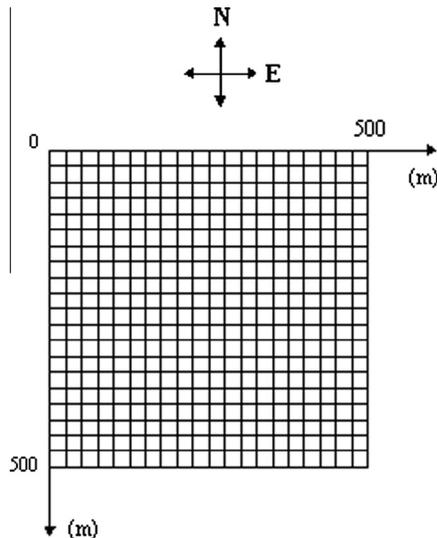


Fig. 3. Grid of the given wind farm used in three case studies.

**Table 6**  
Parameter of wind turbines used.

Hub height	Diameter	$C_T$	$C_P$	Installed capacity	Rated wind speed
78 m/50 m	40 m	0.8888	0.4	680 kW	13.0158 m/s

considered: (1) actual wind speed is equal to or greater than rated wind speed, where power output is equal to installed capacity; and (2) actual wind speed is less than rated wind speed, where power output is calculated using Eq. (1). Meanwhile, most commercial wind turbines have a cut-off wind speed, which means the rotor will be disconnected with gearbox when wind speed exceeds the cut-off speed. The cut-off wind speed is normally about 20–25 m/s, which is higher than the reference wind speeds used in this paper, so that the scenario of rotor being disconnected with gearbox is not considered in this study.

In current commercial wind turbine manufacturing market, one type wind turbine usually has several different hub height towers.

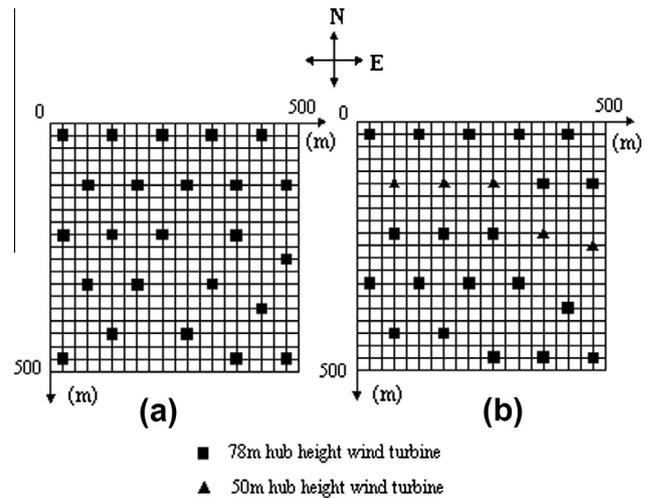


Fig. 4. Optimal layouts generated by GA in first case study: (a) using 78 m hub height wind turbines, (b) using 78 m and 50 m hub heights wind turbines.

In Sections 4.1–4.3, the hub heights of wind turbines used are based on enercon\_e40–600 [28–30] that range from 46 m to 78 m. And the following parameters (shown in Table 6) of wind turbines are selected based in Refs. [13–15,17] for simplification purpose. In Section 4.4, commercial wind turbines are introduced to make the case study more close to realistic conditions.

Since two free stream velocities exist due to two different hub heights considered, free stream velocity at 78 m above the ground is chosen as the reference wind speed, and free stream velocity at 50 m is calculated based on the reference wind speed. In first case study, the reference wind speed is set to 12 m/s and the wind direction is assumed as from north to south. In second case study, the reference wind speed is 12 m/s, while wind directions are separated into 36 directions with 10° increment between two adjacent directions. The probability of occurrence of each direction is assumed to be same. In third case study, it is based on various wind speeds and directions, and wind distribution chart (shown in Fig. 7) used is similar to previous research [13–15,17], where the sum of the occurrence probability for each wind speed at each direction is uniform and 8 m/s, 12 m/s, 17 m/s at the height of 78 m above the ground are used as the three variable reference wind speeds.

In each case study, the two best layout results within at least five GA runs are presented, and the power outputs of the given wind farm between using 78 m hub height wind turbines and using 78 m and 50 m hub height wind turbines are compared. In the third case study, two cost models are taken into account to investigate the effect on cost per unit power. At last, a large wind farm with commercial wind turbines is analyzed to further test the benefits of using different hub height wind turbines in realistic conditions.

#### 4.1. First case study: constant wind speed and direction

The first case study is based on the condition of constant wind speed and direction. The optimal layouts of the wind farms using

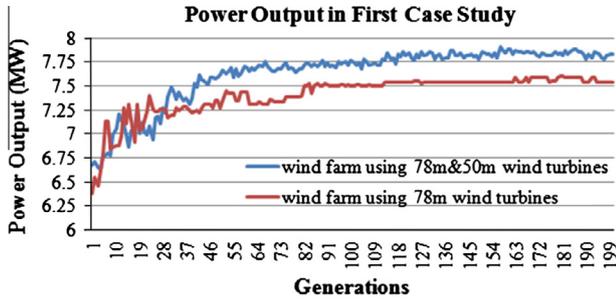


Fig. 5. Power output chart of first case study.

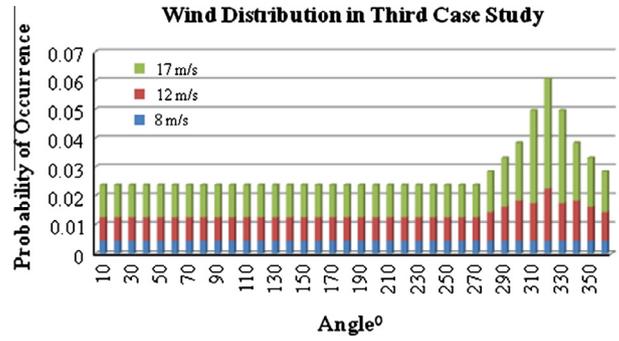


Fig. 8. Wind distribution chart in third case study.

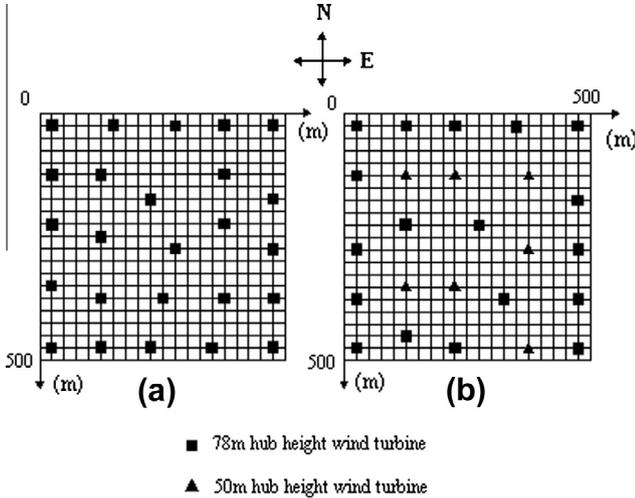


Fig. 6. Optimal layouts generated by GA in second case study: (a) using 78 m hub height wind turbines, (b) using 78 m and 50 m hub heights wind turbines.

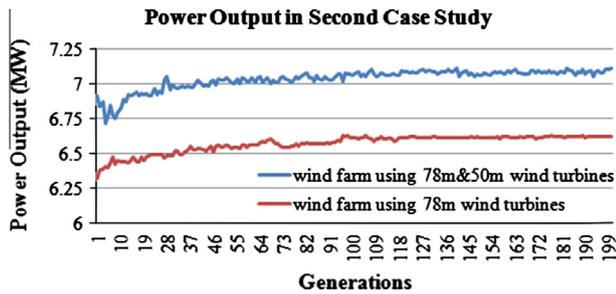


Fig. 7. Power output chart of second case study.

same hub height wind turbines and two hub height wind turbines are shown in Fig. 4a and b. Basically, wind turbines in both layouts are sort of uniformly located facing the wind direction. In Fig. 4a, there are 24 wind turbines used and the maximum total power output is 7.5985 MW. Its power variation curve with generation increasing is shown as the red line in Fig. 5. In Fig. 4b, the number of 78 m wind turbines and 50 m wind turbines are 20 and 5, respectively. By placing one more wind turbine, the layout shown in Fig. 4b generates 7.9058 MW power output. The blue line in Fig. 5 represents its power variation trend. The difference between two lines clearly demonstrates a wind farm using wind turbines with two hub heights can generate more power and it increases 4.04% of total power output when using wind turbines with different hub heights in this wind farm. From Fig. 4a and b, we can find out that both first rows are placed maximum allowable number of wind turbines because wind turbines in first row have no wake affecting them and normally generate more power output than

other turbines. We can also find out that turbines are trying to avoid full wake coverage area (location right behind its frontal turbine) when using same hub height wind turbines. However, using lower wind turbines decreases the wake effect, thus in the 9th row of Fig. 4b, higher ones choose to locate directly behind the lower ones in the 5th row.

#### 4.2. Second case study: constant wind speed and various wind directions

In second case study, the wind condition is constant wind speed and various wind directions. It is assumed occurrence probabilities of all directions are same, so that the layout will not be direction oriented. Meanwhile, it will also cause multiple optimal layouts that generate same or very close power output due to no direction orientation. A wind turbine may be in others' wakes when wind direction changes, so the optimal layouts will be compromised results. The layout shown in Fig. 6a has 25 wind turbines with only 78 m hub height, and will generate maximum power output of 6.6271 MW. In Fig. 6b, it has same number of wind turbines as Fig. 6a including eighteen 78 m wind turbine and seven 50 m wind turbines, but it can generate maximum power output of 7.1112 MW that increases 7.3% than the layout in Fig. 6a. The power variation chart (Fig. 7) also clearly shows that using two hub height wind turbines will increase wind farm power output without increasing the total number of wind turbines compared to using same height wind turbine. Meanwhile, we can notice that wind turbines in the two layouts shown in Fig. 6 seem to prefer to locate around the outline of the wind farm so that wind turbines can face the wind in all directions with less wake effect to generate more power output.

#### 4.3. Third case study: various wind speeds and directions

Since wind speed and direction are critical factors for wind energy conversion [31], it is more important and practical to consider the condition of various wind speeds and directions that is more close to reality compared to case 1 and 2. Although there are some methods to predicate wind speed and direction trend [32], the probability of occurrence for each wind speed in each direction we used in case 3 is simplified and shown in Fig. 8.

From Fig. 8, we can see that the occurrence frequencies of 12 m/s and 17 m/s between 280° and 360° are greater than other angles, which means 280° to 360° directions have more effects on the wind farm layout than other directions. Among the three reference wind speeds, 17 m/s is greater than the rated wind speed. It may be possible that 17 m/s is high enough to make a wind turbine reach its rated power output even though it is in several wakes. The maximum power outputs of the layouts shown in Fig. 9a and b are 9.2825 MW and 9.9081 MW, respectively. The power variation

chart shown in Fig. 10 clearly demonstrate that the layout with different hub height wind turbines is able to generate more power output when the total number of wind turbines remains same in more complex wind conditions.

After we analyze the optimization results in detail, we find out that the power output of a wind farm will not increase as much as expected by placing additional wind turbines in it after the existing number of wind turbines reaches certain level. However, it may increase the payback period of a wind farm project. Therefore, we decide to include cost in the original objective function of third case study to avoid this situation by searching for the minimum cost per unit power generated by the wind farm. Two cost models are used to further investigate whether using different hub height wind turbines will actually increase power output of a wind farm. The first cost model is a simplified cost model proposed by Mosetti et al. [13], and the second one is a comprehensive model developed by Mora et al. [18].

4.3.1. Third case study with simplified cost model

$$\text{Cost}_{\text{simplified}} = N \left( \frac{2}{3} + \frac{1}{3} e^{-0.00174 \times N^2} \right) \quad (11)$$

$$\text{Obj}_{\text{modified}} = \text{Min Cost}_{\text{simplified}}/P \quad (12)$$

Proposed by Mosetti et al. [13], the total investment cost of a wind park can be calculated using Eq. (11), which assumes that the non-dimensional cost of a wind turbine is one and the maximum reduction is 1/3 for each additional wind turbine [13]. And the modified objective function shown in Eq. (12) is to minimize the cost per unit power as in Refs. [13–15,17,20]. We assume that 50 m hub height wind turbine costs as same as 78 m hub height wind turbine in this case. By using the modified objective function, the optimal layouts generated by GA are shown in Fig. 11. In Fig. 11a, the layout has 17 wind turbines with 78 m hub height, and its cost per unit power is 1.9682/MW with total power output of 7.4995 MW. After introducing 50 m hub height wind turbines, the layout shown in Fig. 11b has 22 wind turbines: 15 wind turbines with 78 m hub height and 7 wind turbines with 50 m hub height, and its cost per unit power is 1.9351/MW with total power output of 9.2117 MW. Even though the cost per unit power only decrease 1.68% (see Fig. 12), the total power output generated by

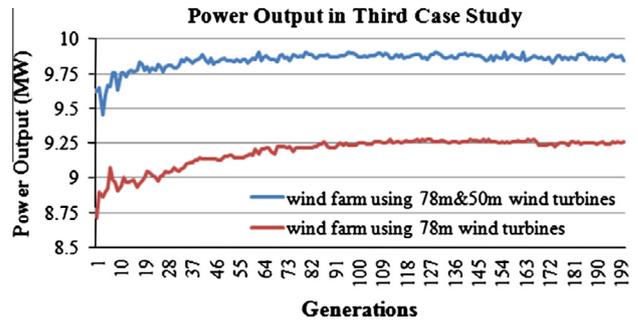


Fig. 10. Power output chart of third case study.

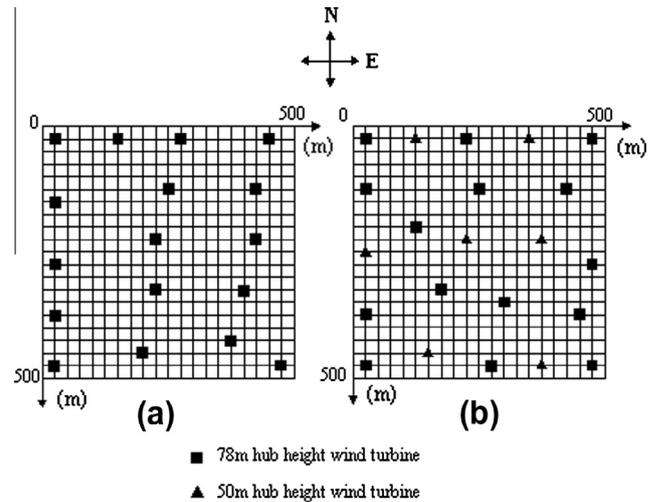


Fig. 11. Optimal layouts generated by GA in third case study with simplified cost model: (a) using 78 m hub height wind turbines, (b) using 78 m and 50 m hub heights wind turbines.

the wind farm using different height wind turbines increases 22.83%, which demonstrates that using multiple hub height wind turbines in a limited wind farm will increase the farm efficiency.

Furthermore, we calculate the cost per unit power for the layout shown in Fig. 9b as 1.966/MW using the cost model shown in Eq. (11), which is also less than the cost per unit power of layout shown in Fig. 11a. So it means that a wind farm using different hub height wind turbines will be able to generate more power output with short payback period compared to the one using same height wind turbines. However, in practice, the lower hub height wind turbine does not cost as same as higher one. Therefore, the further study is preceded in Section 4.3.2.

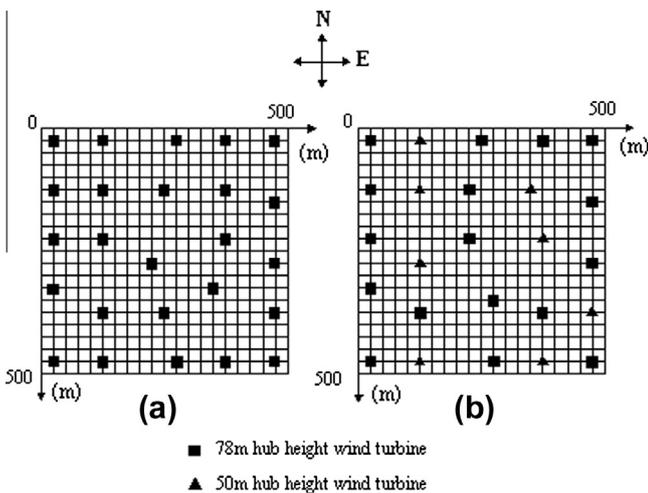


Fig. 9. Optimal layouts generated by GA in third case study: (a) using 78 m hub height wind turbines, (b) using 78 m and 50 m hub heights wind turbines.

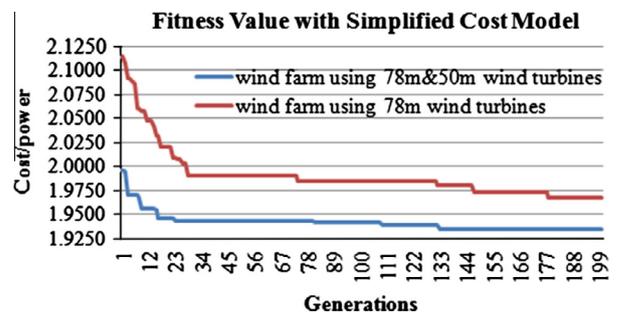


Fig. 12. Cost per unit power chart with simplified cost model.

**Table 7**  
Wind turbines characteristics.

Wind turbine model	78 m Hub height with variable-pitch	50 m Hub height with variable-pitch
Power (kW)	680	680
Hub height (m)	78	50
Turbine cost (€)	593,867	593,867
Tower cost (€/m)	1500	1500

**Table 8**  
Summary of results related to Figs. 9 and 13.

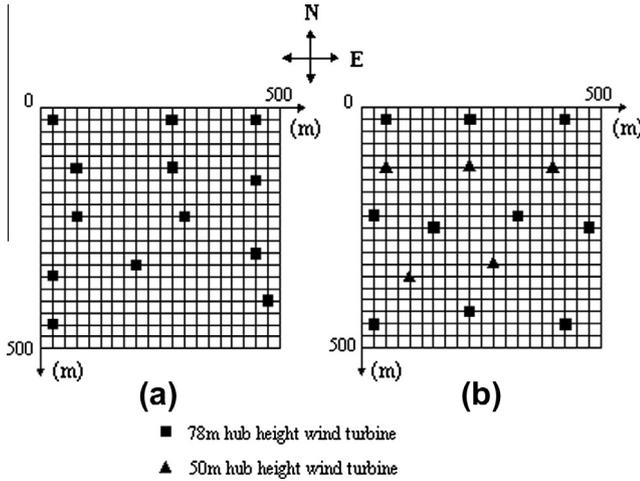
	Fig. 9a	Fig. 9b	Fig. 13a	Fig. 13b
Initial investment cost (M€)	17.7717	17.4357	9.2413	10.453
Power output (MW)	9.2825	9.9081	5.9743	6.7828
Cost per unit power (M€/MW)	1.9145	1.7597	1.5468	1.5411

**Table 9**  
Parameters of GE 1.6 MW wind turbines.

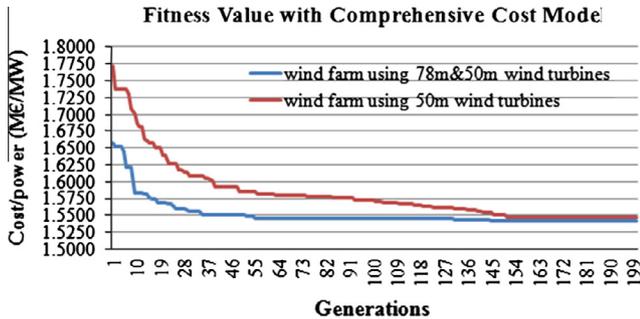
Turbine type	Nameplate capacity	Hub height	Rotor diameter	Installed capital cost
GE 1.62-100	1.62 MW	80 m	100 m	1850 \$/kW
GE 1.62-100	1.62 MW	100 m	100 m	2025 \$/kW

**Table 10**  
Multi-objective optimization results with given wind farm nameplate capacity.

Objective	No. of wind turbines		Power output	Cost per unit power
	100 m	80 m		
Max power	15	3	26.2099 MW	2.1931 M\$/MW
Min cost per unit power	2	16	26.021 MW	2.0691 M\$/MW



**Fig. 13.** Optimal layouts generated by GA in third case study with comprehensive cost model: (a) using 78 m hub height wind turbines, (b) using 78 m and 50 m hub heights wind turbines.



**Fig. 14.** Cost per unit power chart with comprehensive cost model.

4.3.2. Third case study with comprehensive cost model

To further validate the conclusion obtained in Section 4.3.1, we use another comprehensive cost model presented in Ref. [18] since it considered the tower cost. According to the wind turbine characteristics and corresponding cost for each part in Ref. [18], we assume the cost according to the turbine and its tower. In Ref. [18], the rated power of the turbine is 600 kW with a total cost of 524,000 €. Assuming the price per unit kilowatts is same, the cost for the turbine (680 kW) used in this study will be 593,867 € and the cost for tower per meter is the same as [18]. The detail information of wind turbine and the corresponding cost is shown in Table 7.

The comprehensive cost model can be expressed as below:

$$\text{Cost\_comprehensive} = \sum_{n=1}^T \sum_{i=1}^N (\text{Cost}_{ni} + \text{Tower\_cost}_{ni} \times \text{Hub\_height}_{ni}) \quad (13)$$

where  $T$  represents the total turbine types and  $N$  represents the total number of  $i$ th type wind turbines. By substituting the simplified cost model with the comprehensive cost model in Eq. (12), the layouts with the comprehensive cost model generated by GA are shown in Fig. 13. And the fitness value that varies with generations increasing is shown in Fig. 14.

The initial investment cost, power output and cost per unit power for Figs. 9 and 13 are summarized in Table 8. From this table, by comparing the results of Fig. 13a and b, it is clear to show that the power output increases 13.53% by using different hub height wind turbines when the cost per unit power decreases 0.37%, which means that using different hub height wind turbines may not only reduce cost per unit power, but also improve the power output of a wind farm. And the results of Fig. 9a and b in Table 8 also reveal this conclusion.

Apparently, the application of different hub height wind turbines will increase the partial wake coverage so that the maintenance cost will be increased due to the partial load. Since partial wake coverage also exists when the wind direction changes in a commercial wind farm with single hub height wind turbines, it is hard to determine within current research scope how much the partial wake coverage will affect the wind turbines system.

4.4. Large wind farm with commercial wind turbines

From the above theoretical analysis, we find out that the application of wind turbines with different hub heights can improve power output and cost per unit power of a wind farm. However, the size of commercial wind farm is usually much bigger than 500 m × 500 m and the safety distance between any two turbines is four or five times of wind turbine's rotor diameter. Therefore, in order to test above results in realistic conditions, we decide to apply the same concept on a 2800 m × 1200 m wind farm using GE 1.62 MW wind turbines with four times of rotor diameter safety distance. The cell size is still 25 m × 25 m and the wind condition is same as shown in Fig. 8. Table 9 shows parameters of GE 1.62 MW wind turbine [33].

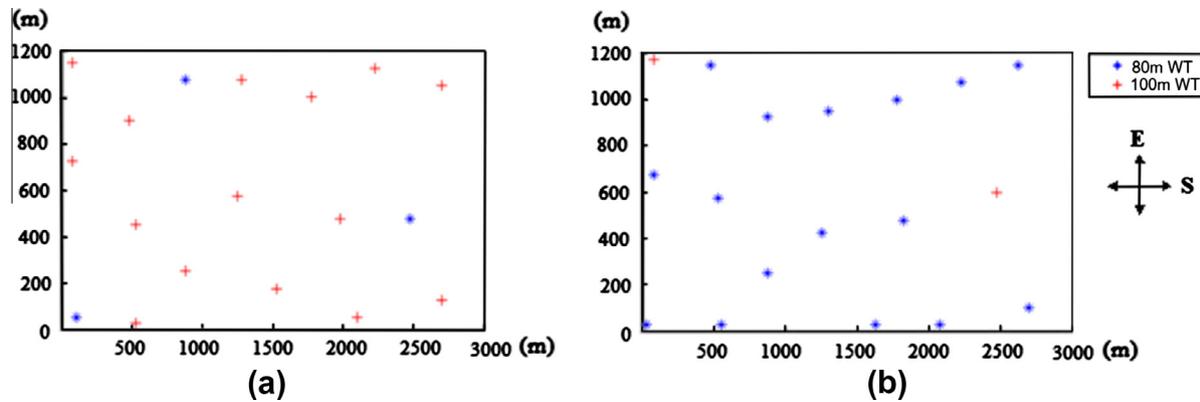


Fig. 15. Typical optimal layouts generated by parallel multi-objective GA optimization with given nameplate capacity: (a) wind farm layout with objective of maximizing power output, (b) wind farm layout with objective of minimizing cost per unit power.

In Ref. [33], the installed capital cost has already included the cost of tower. Meanwhile, in most real commercial cases, the number of turbines is fixed in the farm because the power capacity of the farm is generally planned at the beginning of the investment [21,34,35]. So the number of wind turbines is decided at beginning in this case study based on given wind farm nameplate capacity. From [36], it is found out that the average area requirement is  $34.5 \pm 22.4$  (hectare/MW) for US wind farms. Assuming that there is no road crossing the  $2800 \text{ m} \times 1200 \text{ m}$  area and 12.1 hectare/MW is selected, the nameplate capacity of the wind farm will be 28 MW and the number of wind turbines will be 18. Instead of single objective optimization, parallel multi-objective (Eqs. (10) and (12)) GA optimization is used to examine the benefits of applying different hub height wind turbines in commercial wind farms since multi-objective optimization will be more close to real commercial considerations. Due to the natural differences between single objective optimization and multi-objective optimization [37], real code GA is used in the following case study where the codes are used to represent wind turbine coordinates and heights, and numbers of individuals and generations are increased to 400 and 1000, respectively. With the parameters shown in Table 9 and wind conditions shown in Fig. 8, the results are shown in Table 10 with two typical layouts shown in Fig. 15.

From Table 10 and Fig. 15, it is clearly shown that different hub height wind turbines are selected in both layouts. Furthermore, the wind condition used is simplified with more than 80% of wind speed is larger than 11 m/s, which is the rated wind speed of GE 1.62-100. Therefore, it makes both high and low GE wind turbines mostly operate with their rated power. When more realistic wind condition applies, differences of power output and cost per unit power caused by different hub height wind turbines will be magnified, and the advantages of using different hub height wind turbines may be increased.

## 5. Conclusion

In this paper, the authors first investigate the effects of using different hub height wind turbines in a small onshore wind farm with nested GA. The GA parameters are first validated through pre-test and comparison with previous research results. Three case studies are conducted by comparing wind farms using different hub height wind turbines and using same hub height wind turbines. The results demonstrate that the power output of wind farm with different hub height wind turbines will be better even when the total numbers of wind turbines are same. Different cost models are also taken into account in the analysis, and results indicate that

different hub height wind turbines can also improve cost per unit power generated by the wind farm. At last, a large wind farm with commercial wind turbines is analyzed to further test the benefits of using different hub height wind turbines in realistic conditions, where the results demonstrate the conclusion obtained from first three case studies. The method presented in this paper is not only limited to two hub height wind turbines. Real coded GA can be employed with proposed nested GA by using the codes to represent different hub heights choices in GA2. For example, if there are five hub height wind turbines as candidates, in GA2, we can use “0–4” to represent the corresponding hub height with same possibility for GA to choose each hub height. The proposed nested GA can be also used to solve problems involving both wind turbine types and hub height selection. For example, if GA1 obtains the number of wind turbines placed in the wind farm as 20, the string length in GA2 should be 40, where the first 20 bits represent the turbine types and the rest 20 bits represent turbine hub heights. However, real coded method may have some drawbacks when the hub height choice increases to three or more in one position since the possibility of choosing a wind turbine to locate there will be decreased. For example, if there are five potential hub heights, the possibility that the position has no wind turbine will be 0.2 while the position can be placed a wind turbine will be 0.8. This may also increase the computational time. The same problem will not exist in the nested GA method since position selection is handled by GA1 with uniform possibility. In addition, real codes can represent coordinates of wind turbines' positions and hub heights in one chromosomal string in parallel multi-objective GA as used in Section 4.4. Meanwhile, the method can be also applied into other types of wind farms as long as the wake model stays the same. It is noted that the application of different hub height wind turbines in a wind farm actually takes advantages of decreasing wake effects, thus, this application will not be very helpful if the difference of two or more hub heights is too small.

Because of nested GA may require more computational efforts, the number of individuals and generations may not be set too high, and the size of selected wind farm is small due to the same reason. Parallel computing method in MATLAB will be used later to increase our computing capacity in the future research, so that more individuals and generations with large wind farm can be tested using proposed nested GA method. In the future research, we will compare the computational efforts needed of using nested GA and real coded GA. Meanwhile, although the analytical wake model used in this paper is commonly used in other similar research works, it is actually a very simplified model that may not be suitable for complex terrain conditions. And the surface roughness length in this paper is also an assumed value. Different wake model

with real surface roughness length values generated by geographic information system should be tested in the future research to make the proposed method more applicable. Finally, we need to pay attention to how much the maintenance cost will increase due to increased partial load after introducing different hub height wind turbines.

### Acknowledgements

This work is supported by Frank H. Dotterweich College of Engineering at Texas A&M University–Kingsville. The authors are also thankful to HP Catalyst Initiative for providing computational equipment for this project.

### References

- [1] Mostafaeipour A, Sedaghat A, Dehghan-Niri AA, Kalantar V. Wind energy feasibility study for city of Shahrbabak in Iran. *Renew Sustain Energy Rev* 2011;15:2545–56.
- [2] Burton T, Sharpe D, Jenkins N, Bossanyi E. *Wind energy handbook*. John Wiley & Sons; 2001.
- [3] Joselin Herbert GM, Iniyan S, Sreevalsan E, Rajapandian S. A review of wind energy technologies. *Renew Sustain Energy Rev* 2007;11:1117–45.
- [4] Akpinar EK, Akpinar S. An assessment on seasonal analysis of wind energy characteristics and wind turbine characteristics. *Energy Convers Manage* 2005;46(11–12):1848–67.
- [5] Carta JA, Velázquez S, Matías JM. Use of Bayesian networks classifiers for long-term mean wind turbine energy output estimation at a potential wind energy conversion site. *Energy Convers Manage* 2011;52(2):1137–49.
- [6] Adaramola MS, Paul SS, Oyedepo SQ. Assessment of electricity generation and energy cost of wind energy conversion systems in north-central Nigeria. *Energy Convers Manage* 2011;52:3363–8.
- [7] Wiser R, Bolinger M. 2011 Wind Technologies Market Report. Lawrence Berkeley National Laboratory; August 2012.
- [8] Liu H, Shi J, Qu X. Empirical investigation on using wind speed volatility to estimate the operation probability and power output of wind turbines. *Energy Convers Manage* 2013;67:8–17.
- [9] Mohammadi K, Mostafaeipour A. Using different methods for comprehensive study of wind turbine utilization in Zarrineh, Iran. *Energy Convers Manage* 2013;65:463–70.
- [10] Hall JF, Mecklenborg CA, Chen D, Pratap SB. Wind energy conversion with a variable-ratio gearbox: design and analysis. *Renewable Energy* 2011;36(3):1075–80.
- [11] Kenway G, Martin J. Aerostructural shape optimization of wind turbine blades considering site-specific winds. In: 12th AIAA/ISSMO multidisciplinary analysis and optimization conference. Victoria, British Columbia, Canada; 2008.
- [12] Ramos CA, Ramirez D, Jin K, Li H. Factorial analysis of selected factors affecting the electricity production of onshore wind farms. *Int J Energy Clean Environ* 2011;12(1):67–77.
- [13] Mosetti G, Poloni C, Diviacco B. Optimization of wind turbine positioning in a large windfarms by means of a genetic algorithm. *J Wind Eng Ind Aerodyn* 1994;51(1):105–16.
- [14] Grady SA, Hussaini MY, Abdullah MM. Placement of wind turbines using genetic algorithms. *Renewable Energy* 2005;30:259–70.
- [15] A. Mittal, Optimization of the layout of large wind farms using genetic algorithm. M.S. Thesis. Case Western Reserve University; 2010.
- [16] Acero J, Acevedo J, Rendon M, Oleszewski O. Linear wind farm layout optimization through computational intelligence. In: Proceedings of 8th Mexican international conference on artificial intelligence; 2009. p. 692–703.
- [17] Chen L, McDonald E. A new model for wind farm layout optimization with landowner decisions. In: ASME 2011 international design engineering technical conference & computers and information in engineering conference. Washington (DC), United States; 2011.
- [18] Mora JC, Baron JC, Santos JR, Payan MB. An evaluative algorithm for wind farm optimal design. *Neurocomputing* 2007;70:2651–8.
- [19] Gonzalez JS, Rodriguez AG, Mora JC, Santos JR, Payan MB. Optimization of wind farm turbines layout using an evaluative algorithm. *Renewable Energy* 2010;35:1671–81.
- [20] Marmidis G, Lazarou S, Pyrgioti E. Optimal placement of wind turbines in a wind park using Monte Carlo simulation. *Renewable Energy* 2008;33:1455–60.
- [21] Chowdhury S, Zhang J, Messac A, Castillo L. Unrestricted wind farm layout optimization (UWFLO): investigating key factors influencing the maximum power generation. *Renewable Energy* 2012;38(1):16–30.
- [22] Sorensen P, Nielsen T. Recalibrating wind turbine wake model parameters—validating the wake model performance for large offshore wind farms, European wind energy conference and exhibition. Athens: Greece; 2006.
- [23] Jensen NO. A note on wind generator interaction. Roskilde (Denmark): Risø National Laboratory; 1983.
- [24] Katic I, Højstrup J, Jensen NO. A simple model for cluster efficiency, European wind energy association conference and exhibition. Rome, Italy; 198. p. 407–10.
- [25] Frandsen S. On the wind speed reduction in the center of large clusters of wind turbines. *J Wind Eng Ind Aerodyn* 1992;39(1–3):251–65.
- [26] Pohlheim H. GEATbx: genetic and evolutionary algorithm toolbox for use with MATLAB; 1999.
- [27] Ciprian V, Iulian M, Bratcu AI, Ceangă E. Output power maximization of low-power wind energy conversion systems revisited: possible control solutions. *Energy Convers Manage* 2010;51(2):305–10.
- [28] <<http://www.buildings.gr/enercon/E40/E40.htm>> [accessed 01.12.12].
- [29] <[http://www.en.wind-turbine-models.com/turbine/68/enercon-gmbh/e40-44\\_600](http://www.en.wind-turbine-models.com/turbine/68/enercon-gmbh/e40-44_600)> [accessed 01.12.12].
- [30] <[http://www.thewindpower.net/turbine\\_en\\_173\\_enercon\\_e40-600.php](http://www.thewindpower.net/turbine_en_173_enercon_e40-600.php)> [accessed 01.12.12].
- [31] Akpinar S, Akpinar EK. Estimation of wind energy potential using finite mixture distribution models. *Energy Convers Manage* 2009;50(4):877–84.
- [32] Carta JA, Ramirez P, Bueno C. A joint probability density function of wind speed and direction for wind energy analysis. *Energy Convers Manage* 2008;49(6):1309–20.
- [33] Wiser R, Lantz E, Bolinger M, Hand M. Recent developments in the levelized cost of Energy from US wind power projects. National Renewable Energy Laboratory; 2012.
- [34] Kusiak A, Song Z. Design of wind farm layout for maximum wind energy capture. *Renewable Energy* 2010;35:685–94.
- [35] Eroğlu Y, Seçkiner SU. Design of wind farm layout using ant colony algorithm. *Renewable Energy* 2012;44:53–62.
- [36] Denholm P, Hand M, Jackson M, Ong S. Land-use requirements of modern wind power plants in the United States. Technical, Report NREL/TP-6A2 45834; 2009.
- [37] Murata T, Ishibuchi H, Tanaka H. Multi-objective genetic algorithm and its applications to flowshop scheduling. *Comput Ind Eng* 1996;30(4):957–68.