Author's Accepted Manuscript

www.elsevier.com/locate/omega

Managing channel profits of different cooperative Models in closed-loop supply chains

Zu-JunMa , Nian Zhang, Ying Dai, Shu Hu

PII: S0305-0483(15)00141-3
DOI: http://dx.doi.org/10.1016/j.omega.2015.06.013
Reference: OME1565

To appear in: *Omega*

Received date: 23 January 2013
Revised date: 5 June 2015
Accepted date: 27 June 2015

Cite this article as: Zu-JunMa , Nian Zhang, Ying Dai, Shu Hu, Managing channel profits of different cooperative Models in closed-loop supply chains, *Omega,* http://dx.doi.org/10.1016/j.omega.2015.06.013

Managing Channel Profits of Different Cooperative Models in Closed-Loop Supply Chains

Abstract: The importance of closed-loop supply chains has been widely recognized in literature and in practice. The paper investigates interactions among the different parties in a three-echelon closed-loop supply chain consisting of a single manufacturer, a single retailer and two recyclers and focuses on how cooperative strategies affect closed-loop supply chain decision-making. Various cooperative models are considered by observing recent research and current cases, and the optimal decisions and supply chain profits of these models are discussed. By comparing various coalition structures, we discover that cooperative strategies can lead to win-win outcomes and increase an alliance's profit and can be effective ways of achieving greater efficiency from the point of view of the overall supply chain. Finally, the paper presents a detailed comparative analysis of these models and provides insights into the management of closed-loop supply chains.

Keywords: Closed-loop supply chain; Cooperative strategies; Channel profits

1 Introduction

Closed-loop supply chains (CLSCs) focus on taking back products from customers and recovering added value by reusing the entire product and/or certain of its modules, components, and parts. Over the past 20 years, CLSCs have gained considerable attention in industry and academia [1, 2]. To achieve high supply chain efficiency, some channel members in CLSCs may choose to cooperate with other channel members to form an alliance; such cooperation can bring great benefits or competitive advantages [3]. This paper focuses on developing a detailed comprehension of the implications that interactions among the different parties in a CLSC have for optimal decisions and supply chain profits and on how cooperative strategies affect the CLSC decision.

In current practice, we find various coalition structures in CLSCs. In some cases, manufacturers establish strategic alliances with recyclers or invest in their own collection channel for collecting used products. For instance, the "big three" auto manufacturers (i.e., GM, Ford, Chrysler) have made large investments in remanufacturing programs and have established a long-term cooperative partnership with recyclers in the United States [4]. Nike has created a strategic alliance with an eco-non-profit organization, the "National Recycling Coalition", to collect used tennis shoes [5]. Some companies, such as IBM [6] and Dell [7], have designed their own reverse supply chain and formed a department or subsidiary to take part in collecting used products [8], a similar approach to a coalition consisting of a manufacturer and a recycler forming to produce products and recycle used products.

In real life, many manufacturers cooperate with retailers not only in the selling market but also in the collecting market. For example, Haier and Changhong not only set up their own subsidiaries that primarily engage in collecting and handling used products but also established a coalition with large retailers (e.g., Suning, Gome) in China [9, 10]. Xerox and Eastman Kodak Company also established cooperative relationships with retailers, in which the coalition not only produces and sells products but also participates in collecting and handling used products [11, 12]. These alliances function as coalitions including manufacturer, retailer and recycler, all taking part in the operations of a CLSC.

In other cases, independent and non-overlapping recyclers are utilized for collecting and handling used products. For instance, there are two large, independent and non-overlapping Industry Alliances (IA) that manage their own recovery, reuse and recycling of used products in Japan [13]. Hewlett Packard Corporation also built two independent factories to collect and handle its own used computers in the US [14].

Based on observations of current practice and the literature, it is necessary to conduct a deeper study of how cooperative strategies affect the equilibrium profits and optimal decisions of all channel members in CLSCs. In this paper, we consider four cooperative formats in a three-echelon closed-loop supply chain consisting of a single manufacturer (M), a single retailer (R) and two recyclers (C): (1) The manufacturer cooperates with one of the recyclers (M-C coalition structure). (2) The manufacturer builds a coalition with the retailer (M-R coalition structure). (3) The manufacturer builds a coalition with two recyclers (M-C-C coalition structure). (4) The manufacturer, the recycler and the retailer build alliances with one another (M-C-R coalition structure). We analyze the results of the cooperative models by contrasting them with a completely decentralized structure (all channel members enter into an alliance with one another and act as a single entity) and a completely centralized structure (all channel members independently make their own decisions) to illustrate potential sources of efficiencies in CLSCs.

More specifically, we address the following research questions: (1) Should channel members cooperate with one another and, if so, how should they cooperate with one another? (2) How do coalition structures affect the equilibrium profits and optimal decisions of the members in CLSCs?

Some of the key results of this paper demonstrate that the cooperation between the manufacturer and the retailer would increase each's profits and return rates. By approaching the selling market together, they can jointly optimize the final price of the product and efficiently reflect unit net savings from manufacturing and remanufacturing. Additionally, return rates are sensitive to changes in demand. When a manufacturer establishes a coalition with recycler/recyclers, the coalition structure may improve the return rates, the alliance's profit and the retailers' profit. From economies of scale and by being closer to the final demand, they jointly optimize return rates and net savings by remanufacturing directly and efficiently controlling the wholesale price. The manufacturer has a dual role because it produces products by using either new materials or remanufactured materials in CLSCs. Although the manufacturer creates an alliance with both the retailer and the recycler/recyclers, this coalition structure is the most-preferred option because of direct proximity to the selling market and the recycling market, and of jointly optimizing the retail price and the return rate. By comparing various coalition structures, we find that cooperative strategies can lead to win-win outcomes and increase the alliance's profit. Additionally, more members entering into an alliance increase return rates.

On a broader level, this paper contributes to our understanding about interactions among the different parties in a CLSC and the effects of cooperative strategies on the CLSC decision.

The rest of the paper is organized as follows. A comprehensive literature review is exhibited in Section 2. The notations and assumptions of models are described in Section 3. Various cooperative models are considered, and optimal decisions and supply chain profits are analyzed in Section 4. A detailed comparative analysis of these models is made and some interesting propositions are presented about the relationships of various coalition structures in Section 5. Research contributions are summarized, and future research directions are outlined in Section 6.

2 Literature Review

A broader collection and comprehensive review of reverse supply chains and CLSCs can be found in review articles [15, 16]. From a survey of the literature, reverse channel management of CLSCs is one of the most important topics. Savaskan et al. introduced and compared three different reverse channels (i.e., the manufacturer collecting channel, the retailer collecting channel and the third-party collecting channel) and summarized some results from the three channels [1]. Savaskan and Wassenhove studied a two-stage CLSC consisting of a single manufacturer and two retailers and primarily discussed the manufacturer collecting model and the retailer collecting model [17]. Wei and Zhao considered a CLSC with one manufacturer and two competitive retailers and extended the manufacturer collecting model with fuzzy demand [18]. Hong and Yeh proposed a retailer collecting model, in which the retailer collected used products and the manufacturer cooperated with a third-party recycler to handle used products [19]. They demonstrated that the manufacturer might cooperate with a recycler without considering the cooperation of other members in CLSC. Huang et al. considered three decentralized third-party collecting models and represented a CLSC consisting of a recycler, a manufacturer and a retailer, in which the retailer, the recycler and the manufacturer act as the channel leader (Stackelberg leader), respectively [20]. The above studies largely focus on different reverse channel structures. However, due to economies of scale and the fixed investment, the collecting and handling cost paid by the third-party recycler is usually lower than that paid by the manufacturer or the retailer [21, 22], and the third-party collecting model is common in current practical activities. Therefore, in this paper, we focus on the third-party collecting model, in which the manufacturer produces the product, the retailer sells the product and the recycler collects the used product. Moreover, the literature did not study interactions among the different parties in a CLSC. In contrast, we will investigate various cooperative models in a CLSC with a third-party collecting channel. Specifically, we examine the effect of these cooperation models on optimal decisions and supply chain profits.

Cooperative interactions in a supply chain have been comprehensively researched in the past. Cachon investigated several types of supply chain contracts to promote cooperation between a manufacturer and a retailer [23]. Li et al. [9], Huang and Li [24], and Zhang et al. [25] discussed cooperative advertising models in a manufacturer-retailer supply chain and investigated the effect of cooperation on investment effort levels. Gurnani et al. analyzed the effect of supply chain co-opetition on product prices and investment decisions [26, 27]. Leng and Parlar analyzed how the cooperative effect would influence cost savings from a supply chain with a manufacturer, a distributor and a retailer [28]. The above studies aim at the issues of cooperation in forward supply chains. In contrast, in this paper, we specifically investigate cooperative interactions among members in CLSCs.

Next, we present our modeling assumptions and the four cooperative models in CLSCs.

3 Model Assumptions and Notations

We consider a three-echelon CLSC consisting of a single manufacturer, a single retailer and two recyclers. The manufacturer can manufacture a new product directly from raw materials, or remanufacture part or all of a returned unit into a new product. We consider product categories in which there is no distinction between a remanufactured product and a manufactured product [17]. The manufacturer sets the wholesale price paid to the retailer per unit of product and the transfer price paid to the recycler for per unit used product. The retailer sets the selling price and sells the product to consumers. The recyclers collect used products and sell them to the manufacturer, who also determines the return rate affecting the investment in the collection of used products.

The primary goal of this paper is to understand the implications of different cooperative strategies in CLSCs for optimal decisions and supply chain profits. Hence, we extend the models of Savaskan et al. [1] and Jena et al. [29] to a single period model with a three-echelon CLSC consisting of a single manufacturer (M), a single retailer (R) and two recyclers (C). We specifically consider four cooperative models in CLSCs, viz., the M-C cooperative model (Model MC, see Figure 1c), the M-R cooperative model (Model MR, see Figure 1d), the M-C-C cooperative model (Model MCC, see Figure 1e), and the M-C-R cooperative model (Model MCR, see Figure 1f). In addition, the completely centralized model (Model C, see Figure 1a) and the completely decentralized model (Model D, see Figure 1b) are provided as benchmark cases. For each model, we characterize the optimal decision variables and the supply chain profits, respectively. We also examine the sensitivity of the optimal return rate, retail price, demand, and total channel profits to various parameters to reveal the effect of interactions among the different parties on the CLSC decision.

We use the following notations throughout the paper.

c_m : Unit cost of manufacturing a new product.

c_r : Unit cost of remanufacturing a used product into a new product, where $c_r \le c_m$ [17, 30].

Δ : Unit saving cost from remanufacturing, where $\Delta = c_m - c_r$ [31, 32].

ω : Unit wholesale price of a new product.

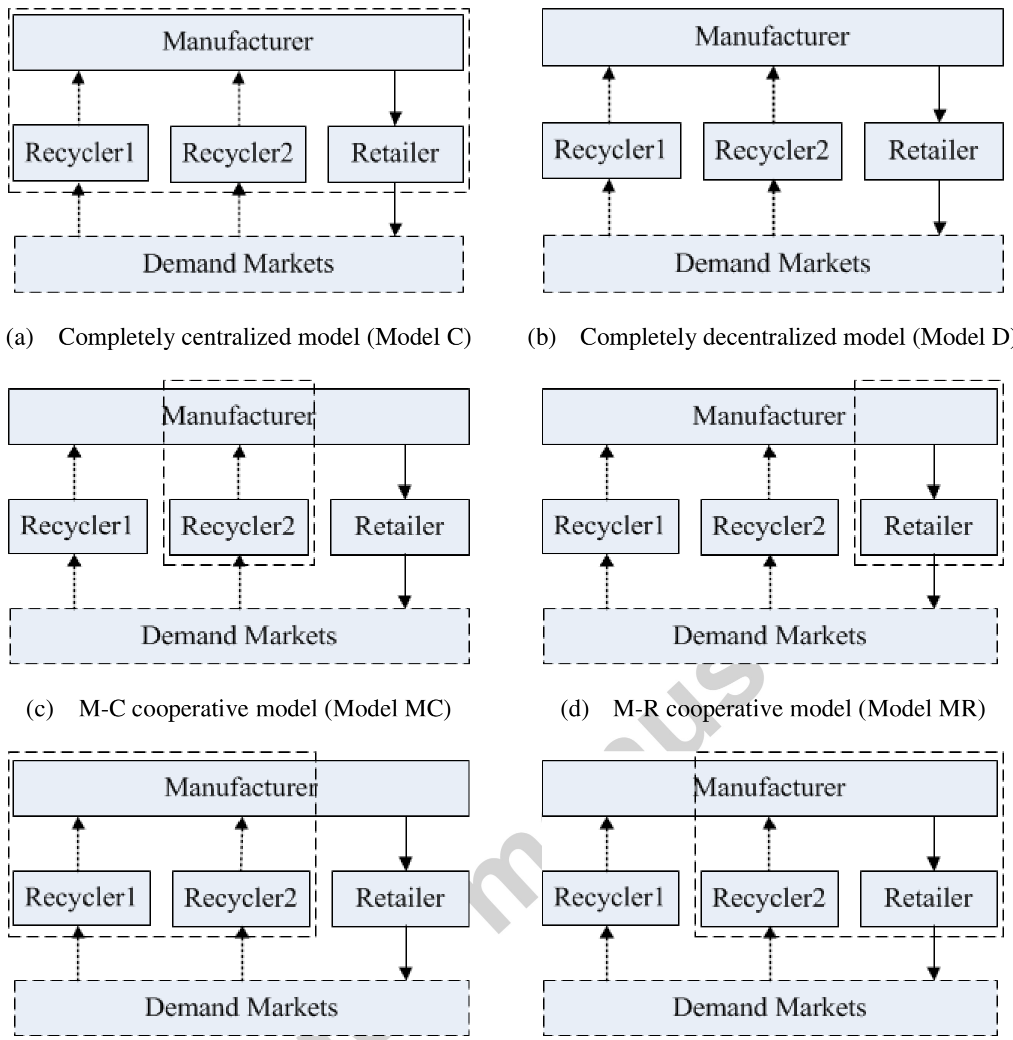
p : Unit retail price of a new product.

D : The demand of new products in the market, where $D(p) = \alpha - \beta p$, α is the market size, and β is the elasticity of demand [33, 34].

b : Unit transfer price of a used product from the manufacturer to the recyclers.

A : Unit cost of recycling a used product, where $A < \Delta$ [1, 35].

I : The investment in product collection activities, where $I = B\tau^2$, where B is the scaling parameter and τ denotes the return rate of used products [1, 36, 37].



(a) Completely centralized model (Model C) (b) Completely decentralized model (Model D)

(c) M-C cooperative model (Model MC) (d) M-R cooperative model (Model MR)

(e) M-C-C cooperative model (Model MCC) (f) M-C-R cooperative model (Model MCR)

Figure 1 Cooperative models

Π_l^i: The profit function of participant l in cooperative model i. The superscript i will take values C, D, MC, MR, MCC and MCR. The subscript l will take values C, M, C_1, C_2 and R, denoting the cooperative alliance, the manufacturer, recycler 1, recycler 2 and the retailer, respectively.

τ_j^i: The return rate of used products for recycler j ($j=1,2$) in cooperative model i, $0 \le \tau_j^i \le 1$.

τ^i: The total return rates of used products in cooperative model i, $\tau^i = \tau_1^i + \tau_2^i$, $0 \le \tau^i \le 1$.

Without loss of generality, we make the following modeling assumptions.

Assumption 1. Manufacturing a new product by using a used product is less costly than using raw materials, i.e., $c_r \le c_m$. Additionally, c_r is the same for all remanufactured products, while c_m is the same for all new products [1, 29].

Assumption 2. To ensure profitable remanufacturing, the unit cost of collecting and handling a used product is not higher than the unit cost saving from remanufacturing, i.e., $A < \Delta$, and A is an exogenously specified payment [1, 35].

Assumption 3. The market size α and the elasticity of demand β are positive, and $\alpha > \beta c_m$ [1, 34].

Assumption 4. To analyze the different coalition structures of reverse channels, used products are collected by two independent recyclers [13, 38].

This assumption enables us to focus on the game between the manufacturer and the recycler, which demonstrates several characteristics of the recovery market in Japan. Given the restriction on techniques for collecting and handling used products, and the existing large quantity of used products, the competition between recyclers is not intense in the recovery market [39].

Assumption 5. In CLSCs, the manufacturer has absolute channel power over the retailer and the recyclers, acting as a Stackelberg leader [1, 40].

In CLSCs, used products have a positive economic value, and the manufacturer hopes to take back as many used products as possible. However, the return rate is affected by the recyclers' willingness, and that willingness depends on monetary rewards relating to the transfer price. Simultaneously, the wholesale price and the retail price are affected by the transfer price. In the following parts of this paper, we deepen our study of the optimal decisions and supply chain profits of the cooperative models. Based on the notations and the assumptions, we derive the profit functions of the manufacturer, the retailer and the recyclers.

$$
\begin{cases}
\Pi_M = \left(\omega - c_m + \Delta\tau\right) D - b\tau D \\
\Pi_R = \left(p - \omega\right) D \\
\Pi_{C_1} = b\tau_1 D - \left(B\tau_1^{\,2} + A\tau_1 D\right) \\
\Pi_{C_2} = b\tau_2 D - \left(B\tau_2^{\,2} + A\tau_2 D\right)
\end{cases}
$$

4 Cooperative Models in CLSCs

This section primarily analyzes various coalition structures in a three-echelon CLSC, viz., Model MC, Model MR, Model MCC, and Model MCR, and their effect on optimal decisions and supply chain profits. As benchmark cases, Models C and D are analyzed to highlight benefits resulting from cooperative strategies. We solve these models to obtain optimal decisions and compare the optimal decision variables such as the wholesale price, retail price, product return rate, and total chain profits. All the profit functions are shown to be concave in the decision variables (see Appendix A), so we use the first-order conditions throughout to characterize the optimality of the decision variables (see Appendix B).

4.1. Completely centralized model (Model C)

In Model C (as shown in Figure 1a), all members enter into an alliance with one another and act as a single entity. Therefore, there is only a single decision maker, and the wholesale price and the transfer price are irrelevant to the profit function. Hence, the central planner optimizes

$$\underset{p,\tau_1,\tau_2}{Max}\,\Pi^C = \Pi_M + \Pi_R + \Pi_{C_1} + \Pi_{C_2} = (p - c_m)D + (\Delta - A)(\tau_1 + \tau_2)D - B(\tau_1^2 + \tau_2^2) \tag{1}$$

Proposition 1. *In Model C, the optimal values of the retail price and return rates are given by*

$$p^{*C} = \frac{\alpha}{\beta} - \frac{B(\alpha - \beta c_m)}{\beta\left(2B - (\Delta - A)^2 \beta\right)} \quad and \quad \tau_1^{*C} = \tau_2^{*C} = \frac{(\Delta - A)(\alpha - \beta c_m)}{2\left(2B - (\Delta - A)^2 \beta\right)}.$$

Proposition 1 states that there exist the optimal solutions (p^{*C} , τ_1^{*C} , τ_2^{*C}), which maximize the sum of the supply chain members' profits. Model C provides a benchmark scenario to compare with other models. We find that the optimal retail price is smallest among all models (see Table 1 and Table 2). With a lower retail price, which also improves the demand for products, more profits are gained from the demand of new customers. Substituting the values of p^{*C} , τ_1^{*C} , and τ_2^{*C} back in Π^C, the total profit can be obtained.

4.2. Completely decentralized model (Model D)

In Model D (as shown in Figure 1b), the manufacturer decides wholesale price and unit transfer price. Recycler determines return rate. The retailer sets retail price by considering wholesale price. Thus, the problem of Model D can be stated as

$$\begin{aligned} &\underset{\omega,b}{Max}\,\Pi_M^D = (\omega - c_m + \Delta\tau)D - b\tau D \\ &s.t.\begin{cases} \underset{\tau_j}{Max}\,\Pi_{C_j}^D = b\tau_j D - B\tau_j^2 - A\tau_j D \\ \underset{p}{Max}\,\Pi_R^D = (p - \omega)D \end{cases} \end{aligned} \tag{2}$$

***Proposition** 2. In Model D, the optimal values of the retail price, the wholesale price, the unit transfer price, and return rates are as follows:*

$$p^{*D}=\frac{\alpha}{\beta}-\frac{2B\left(\alpha-c_m\beta\right)}{\beta\left(8B-\beta\left(\Delta-A\right)^2\right)},\quad \omega^{*D}=\frac{\alpha}{\beta}-\frac{4B\left(\alpha-c_m\beta\right)}{\beta\left(8B-\beta\left(\Delta-A\right)^2\right)},$$

$$b^{*D}=\frac{A+\Delta}{2},\ and\ \ \tau_1^{*D}=\tau_2^{*D}=\frac{\left(\Delta-A\right)\left(\alpha-\beta c_m\right)}{2\left(8B-\beta\left(\Delta-A\right)^2\right)}.$$

Proposition 2 shows that the optimal solutions ($p^{*D}, \omega^{*D}, b^{*D}, \tau_1^{*D}, \tau_2^{*D}$) of Model D can be obtained. In this model, the optimal return rate is the outcome of the trade-off between the investment in collection effort and the cost saving from remanufacturing. The optimal wholesale price is determined by considering two effects: the direct effect of the wholesale price on demand and the indirect effect on the return rate. In particular, higher demand (i.e., a lower wholesale price) from the consumer market is associated with a higher number of return products for remanufacturing, resulting in obtaining higher marginal benefit from investing in the collection effort.

4.3. M-C cooperative model (Model MC)

In Model MC (as shown in Figure 1c), the manufacturer can ally with either of recyclers. Here we just take the alliance between the manufacturer and recycler2 for example. Thus, the central planner pays transfer price to recycler1, pays wholesale price to the retailer, and determines recycler2's return rate. The retailer decides retail price, and recycler1 sets recycler1's return rate. Therefore, the problem of Model MC is formulated as

$$\begin{aligned}&\underset{\omega,b,\tau_2}{Max}\,\Pi_C^{MC}=\left(\omega-c_m+\Delta\tau\right)D-b\tau_1 D-B\tau_2^{\,2}-A\tau_2 D\\&s.t.\begin{cases}\underset{\tau_1}{Max}\,\Pi_{C_1}^{MC}=bD\tau_1-B\tau_1^{\,2}-A\tau_1 D\\ \underset{p}{Max}\,\Pi_R^{MC}=\left(p-\omega\right)D\end{cases}\end{aligned}\qquad(3)$$

***Proposition** 3. In Model MC, the optimal values of the retail price, the wholesale price, the unit transfer price, and return rates are given by*

$$p^{*MC}=\frac{\alpha}{\beta}-\frac{4B\left(\alpha-c_m\beta\right)}{\beta\left(16B-3\beta\left(\Delta-A\right)^2\right)},\quad \omega^{*MC}=\frac{\alpha}{\beta}-\frac{8B\left(\alpha-c_m\beta\right)}{\beta\left(16B-3\beta\left(\Delta-A\right)^2\right)},$$

$$b^{*MC}=\frac{A+\Delta}{2},\ and\ \ \tau_2^{*MC}=2\tau_1^{*MC}=\frac{2\left(\Delta-A\right)\left(\alpha-\beta c_m\right)}{16B-3\left(\Delta-A\right)^2\beta}.$$

Proposition 3 indicates that we can obtain the optimal solutions ($p^{*MC}, \omega^{*MC}, b^{*MC}, \tau_1^{*MC}, \tau_2^{*MC}$) of Model MC. Surprisingly, we find that return rate τ_2^{*MC} is twice return rate τ_1^{*MC} (i.e., $\tau_2^{*MC}=2\tau_1^{*MC}$).

Additionally, the optimal solutions can then be used to compute the supply chain members' profits (see Table 3). We find that the central planner's profit is greater than the total profit of the manufacturer and recycler2 in Model D (i.e., $\Pi_C^{*MC} \geq \Pi_M^{*D} + \Pi_{C_2}^{*D}$). Recycler1's profit in Model MC is greater than that in Model D (i.e., $\Pi_{C_1}^{*MC} \geq \Pi_{C_1}^{*D}$). Retailer's profit in Model MC is also greater than that in Model D (i.e., $\Pi_R^{*MC} \geq \Pi_R^{*D}$). Therefore, the total profit in Model MC is greater than that in Model D (i.e., $\Pi^{*MC} \geq \Pi^{*D}$). (see Appendix C). It shows that cooperation strategy between the manufacturer and one recycler not only increases the alliance's profit, but also increases the profits of other supply chain members. Therefore, the manufacturer and the recycler would like to cooperate with each other.

4.4. M-R cooperative model (Model MR)

In this model (as shown in Figure 1d), the manufacturer and the retailer build a coalition with one another and are viewed as a central planner. Wholesale price is irrelevant in this model. The central planner decides retail price and transfer price. Recyclers set their respective return rates. Hence, the problem of Model MR is given by

$$\begin{aligned} &\underset{b,p}{Max}\,\Pi_C^{MR} = \left(p - c_m\right) D + \left(\Delta - b\right)\tau D \\ &s.t. \begin{cases} \underset{\tau_1}{Max}\,\Pi_{C_1}^{MR} = bD\tau_1 - B\tau_1^{\,2} - A\tau_1 D \\ \underset{\tau_2}{Max}\,\Pi_{C_2}^{MR} = bD\tau_2 - B\tau_2^{\,2} - A\tau_2 D \end{cases} \end{aligned} \tag{4}$$

***Proposition** 4. In Model MR, the optimal vales of the retail price, the unit transfer price, and return rates are as follows:*

$$p^{*MR} = \frac{\alpha}{\beta} - \frac{2B\left(\alpha - c_m\beta\right)}{\beta\left(4B - \left(\Delta - A\right)^2 \beta\right)}, \quad b^{*MR} = \frac{A + \Delta}{2}, \ and \ \ \tau_1^{*MR} = \tau_2^{*MR} = \frac{\left(\Delta - A\right)\left(\alpha - c_m\beta\right)}{2\left(4B - \left(\Delta - A\right)^2 \beta\right)}.$$

Proposition 4 implies that there exist the optimal solutions ($p^{*MR}, b^{*MR}, \tau_1^{*MR}, \tau_2^{*MR}$), which maximize the profit functions in Model MR. Comparing equilibrium channel profits in Table 3, we find that the central planner's profit in Model MR is greater than the total profit of the manufacturer and the retailer in Model D (i.e., $\Pi_C^{*MR} \geq \Pi_M^{*D} + \Pi_R^{*D}$). Recycler1's profit in Model MR is greater than that in Model D (i.e., $\Pi_{C_1}^{*MR} \geq \Pi_{C_1}^{*D}$). Additionally, recycler2's profit in Model MR is also greater than that in Model D (i.e., $\Pi_{C_2}^{*MR} \geq \Pi_{C_2}^{*D}$) (see Appendix C). It indicates that cooperation strategy between the manufacturer and the retailer also not only increases the alliance's profit, but also increases the profits of other supply chain members. Therefore, the manufacturer and the retailer are willing to cooperate with each other.

4.5. M-C-C cooperative model (Model MCC)

In this model (as shown in Figure 1e), the manufacturer and recyclers establish the coalition structure and are viewed as a central planner. Transfer price is irrelevant in this model. The central planner decides wholesale price and return rates. The retailer sets retail price. Thus, the problem of Model MCC can be written as

$$\begin{aligned} &\underset{\omega,\tau_1,\tau_2}{Max}\,\Pi_C^{MCC} = (\omega - c_m + \Delta\tau)D - B\tau_1^{\,2} - B\tau_2^{\,2} - A\tau_1 D - A\tau_2 D \\ &s.t.\quad \underset{p}{Max}\,\Pi_R^{MCC} = (p-\omega)D \end{aligned} \tag{5}$$

***Proposition** 5. In Model MCC, the optimal values of the retail price, the wholesale price, and return rates are given by*

$$p^{*MCC} = \frac{\alpha}{\beta} - \frac{B(\alpha - \beta c_m)}{\beta\left(4B - \beta(\Delta - A)^2\right)},\quad \omega^{*MCC} = \frac{\alpha}{\beta} - \frac{2B(\alpha - \beta c_m)}{\beta\left(4B - \beta(\Delta - A)^2\right)},\ and$$

$$\tau_1^{*MCC} = \tau_2^{*MCC} = \frac{(\Delta - A)(\alpha - \beta c_m)}{2\left(4B - \beta(\Delta - A)^2\right)}.$$

Proposition 5 states that we can find the optimal solutions (p^{*MCC} , ω^{*MCC} , τ_1^{*MCC} , τ_2^{*MCC}) of Model MCC. Comparing equilibrium channel profits in Table 3, we see that the central planner's profit in Model MCC is greater than the total profit of the central planner and recycler1 in Model MC (i.e., $\Pi_C^{*MCC} \geq \Pi_C^{*MC} + \Pi_{C_1}^{*MC}$). Additionally, the retailer's profit in Model MCC is also greater than that in Model MC (i.e., $\Pi_R^{*MCC} \geq \Pi_R^{*MC}$). Therefore, the total profit in Model MCC is greater than that in Model MC (i.e., $\Pi^{*MCC} \geq \Pi^{*MC}$). Moreover, the central planner's profit is also greater than the total profit of the manufacturer, recycler1 and recycler2 in Model D (i.e., $\Pi_C^{*MCC} \geq \Pi_M^{*D} + \Pi_{C_1}^{*D} + \Pi_{C_2}^{*D}$). The retailer's profit is also greater than that in Model D (i.e., $\Pi_R^{*MCC} \geq \Pi_R^{*D}$) (see Appendix C). It shows that cooperation strategy among the manufacturer and recyclers can increase both the alliance's profit and the retailer's profit. Hence, the manufacturer and the recyclers are willing to cooperate with one another.

4.6. M-C-R cooperative model (Model MCR)

In Model MCR (as shown in Figure 1f), the manufacturer and the retailer can ally with either of recyclers. Here we also just take the alliance among the manufacturer, the retailer and recycler2 for example. Wholesale price is irrelevant in this model. The central planner decides transfer price, retail price and recycler2's return rate. Recycler 1 determines recycler1's return rate. Hence, the problem of Model MCR is given by

$$\begin{aligned} &\underset{b,p,\tau_2}{Max}\,\Pi^{MCR} = \Pi_M + \Pi_R + \Pi_{C_2} = \left(p - c_m\right)D + \left(\Delta - A\right)\tau_2 D + \left(\Delta - b\right)\tau_1 D - B\tau_2^{\,2} \\ &s.t.\quad \underset{\tau_1}{Max}\,\Pi_{C_1}^{MCR} = b\tau_1 D - B\tau_1^{\,2} - A\tau_1 D \end{aligned} \tag{6}$$

Proposition 6. *In Model MCR, the optimal values of the retail price, the unit transfer price, and return rates are as follows:*

$$p^{*MCR} = \frac{\alpha}{\beta} - \frac{4B\left(\alpha - \beta c_m\right)}{\beta\left(8B - 3\left(\Delta - A\right)^2 \beta\right)},\quad b^{*MCR} = \frac{\Delta + A}{2},\quad \tau_1^{*MCR} = \frac{\left(\Delta - A\right)\left(\alpha - \beta c_m\right)}{8B - 3\left(\Delta - A\right)^2 \beta},\ \textit{and}$$

$$\tau_2^{*MCR} = \frac{2\left(\Delta - A\right)\left(\alpha - \beta c_m\right)}{8B - 3\left(\Delta - A\right)^2 \beta}.$$

Proposition 6 states that there exist the optimal solutions ($p^{*MCR}, b^{*MCR}, \tau_1^{*MCR}, \tau_2^{*MCR}$), which maximize the profit functions in Model MCR. Comparing equilibrium channel profits in Table 3, we can find that the central planner's profit in Model MCR is greater than the total profit of the central planner and the retailer in Model MC (i.e., $\Pi_C^{*MCR} \geq \Pi_C^{*MC} + \Pi_R^{*MC}$). Additionally, recycler1's profit is also greater than that in Model MC (i.e., $\Pi_{C_1}^{*MCR} \geq \Pi_{C_1}^{*MC}$). Thus, the total profit in Model MCR is greater than that in Model MC (i.e., $\Pi^{*MCR} \geq \Pi^{*MC}$). Moreover, the central planner's profit is greater than the total profit of the central planner's profit and recycler2's profit in Model MR (i.e., $\Pi_C^{*MCR} \geq \Pi_C^{*MR} + \Pi_{C_2}^{*MR}$). Recycler1's profit is also greater than in Model MR (i.e., $\Pi_{C_1}^{*MCR} \geq \Pi_{C_1}^{*MR}$). Therefore, the total profit in Model MCR is greater than that in Model MR (i.e., $\Pi^{*MCR} \geq \Pi^{*MR}$) (see Appendix C). It indicates that cooperation strategy among the manufacturer, the retailer and one recycler can increase both the alliance's profits and the other recycler's profit. Therefore, the manufacturer, the retailer and the recycler would like to cooperate with one another.

In the following section, we perform a detailed comparative analysis of these models.

5 Comparison of Various Cooperative Models

In this section, we make a detailed analysis and careful comparison of the results in various cooperative models. The main objective is to develop a general understanding of each coalition structure. The results are summarized in Table 1, Table 2 and Table 3. The optimal decisions in Model C and Model D are displayed in Table 1, which provides benchmark scenarios for comparing with other models. Table 2 shows optimal decisions in various cooperative models such as Model MC, Model MR, Model MCC, and Model MCR. Additionally, the channel members' profits in various models for different coalition structures are shown in Table 3.

5.1. Analytical Results

By comparative analysis of the results in Table 1, Table 2 and Table 3, we make some interesting observations to reveal the relationships of different coalition structures in CLSCs. All proofs are provided in the Appendix D.

Proposition 7. *When* $\Delta \to A$*, total channel profits in various models are related as*

$$\lim_{\Delta \to A} \Pi^{*D} = \lim_{\Delta \to A} \Pi^{*MC} = \lim_{\Delta \to A} \Pi^{*MCC}, \quad \lim_{\Delta \to A} \Pi^{*C} = \lim_{\Delta \to A} \Pi^{*MR} = \lim_{\Delta \to A} \Pi^{*MCR}, \textit{ and}$$

$$\lim_{\Delta \to A} \frac{\Pi^{*D}}{\Pi^{*C}} = \frac{\Pi^{*MC}}{\Pi^{*MR}} = \frac{\Pi^{*MCC}}{\Pi^{*MCR}} = \frac{3}{4}.$$

Note that if $\Delta \to A$, then $\tau^* \to 0$, because the recyclers have no profit and may not be willing to collect any used products. Thus the reverse channel would no longer exist. When $\Delta \to A$, we compare the two groups of models and determine that the profit ratios equal $3/4$. In this situation, there exists only the forward supply chain in CLSCs. Model MC and Model MCC have a structure similar to that of Model D, and Model MR and Model MCR have a structure similar to that of Model C. In particular, when the manufacturer and the retailer establish cooperation with one another, the increased profit equals a quarter of the total channel profits.

Proposition 7 states that the total channel profits in CLSCs mainly come from the forward supply chain. Hence, the manufacturer and the retailer ought to establish a good cooperative relationship. Many observations in the industry empirically corroborate the findings of the proposition, as in the case of the partnership between P&G and Wal-Mart [41].

Table 1 Comparison of optimal decisions in Model C and Model D

Channel decision and profits	Model C	Model D
Π^*	$\frac{B(\alpha-\beta c_m)^2}{2\beta\left(2B-(\Delta-A)^2\beta\right)}$	$\frac{\left(24B-(\Delta-A)^2\beta\right)B(\alpha-\beta c_m)^2}{2\beta\left(8B-(\Delta-A)^2\beta\right)^2}$
p^*	$\frac{\alpha}{\beta}-\frac{B(\alpha-\beta c_m)}{\beta\left(2B-(\Delta-A)^2\beta\right)}$	$\frac{\alpha}{\beta}-\frac{2B(\alpha-c_m\beta)}{\beta\left(8B-(\Delta-A)^2\beta\right)}$
D	$\frac{B(\alpha-\beta c_m)}{\left(2B-(\Delta-A)^2\beta\right)}$	$\frac{2B(\alpha-c_m\beta)}{\left(8B-(\Delta-A)^2\beta\right)}$
τ_1^*	$\frac{(\Delta-A)(\alpha-\beta c_m)}{2\left(2B-(\Delta-A)^2\beta\right)}$	$\frac{(\Delta-A)(\alpha-\beta c_m)}{2\left(8B-(\Delta-A)^2\beta\right)}$
τ_2^*	$\frac{(\Delta-A)(\alpha-\beta c_m)}{2\left(2B-(\Delta-A)^2\beta\right)}$	$\frac{(\Delta-A)(\alpha-\beta c_m)}{2\left(8B-(\Delta-A)^2\beta\right)}$
ω^*	N/A	$\frac{\alpha}{\beta}-\frac{4B(\alpha-c_m\beta)}{\beta\left(8B-(\Delta-A)^2\beta\right)}$
b^*	N/A	$\frac{A+\Delta}{2}$

Table 2 Comparison of optimal decisions in cooperative models

Channel decision and profits	Model MC	Model MCC	Model MR	Model MCR
Π^*	$\frac{\left(48B-5\beta(\Delta-A)^2\right)B(\alpha-\beta c_m)^2}{\beta\left(16B-3(\Delta-A)^2\beta\right)^2}$	$\frac{\left(6B-\beta(\Delta-A)^2\right)B(\alpha-\beta c_m)^2}{2\beta\left(4B-(\Delta-A)^2\beta\right)^2}$	$\frac{\left(8B-(\Delta-A)^2\beta\right)B(\alpha-c_m\beta)^2}{2\beta\left(4B-(\Delta-A)^2\beta\right)^2}$	$\frac{B\left(16B-5(\Delta-A)^2\beta\right)(\alpha-\beta c_m)^2}{\beta\left(8B-3(\Delta-A)^2\beta\right)^2}$
p^*	$\frac{\alpha}{\beta}-\frac{4B(\alpha-c_m\beta)}{\beta\left(16B-3(\Delta-A)^2\beta\right)}$	$\frac{\alpha}{\beta}-\frac{B(\alpha-c_m\beta)}{\beta\left(4B-(\Delta-A)^2\beta\right)}$	$\frac{\alpha}{\beta}-\frac{2B(\alpha-c_m\beta)}{\beta\left(4B-(\Delta-A)^2\beta\right)}$	$\frac{\alpha}{\beta}-\frac{4B(\alpha-\beta c_m)}{\beta\left(8B-3(\Delta-A)^2\beta\right)}$
τ_1^*	$\frac{(\Delta-A)(\alpha-\beta c_m)}{16B-3(\Delta-A)^2\beta}$	$\frac{(\Delta-A)(\alpha-c_m\beta)}{2\left(4B-(\Delta-A)^2\beta\right)}$	$\frac{(\Delta-A)(\alpha-c_m\beta)}{2\left(4B-(\Delta-A)^2\beta\right)}$	$\frac{(\Delta-A)(\alpha-\beta c_m)}{8B-3(\Delta-A)^2\beta}$
τ_2^*	$\frac{2(\Delta-A)(\alpha-\beta c_m)}{16B-3(\Delta-A)^2\beta}$	$\frac{(\Delta-A)(\alpha-c_m\beta)}{2\left(4B-(\Delta-A)^2\beta\right)}$	$\frac{(\Delta-A)(\alpha-c_m\beta)}{2\left(4B-(\Delta-A)^2\beta\right)}$	$\frac{2(\Delta-A)(\alpha-\beta c_m)}{8B-3(\Delta-A)^2\beta}$
$\tau_1^*+\tau_2^*$	$\frac{3(\Delta-A)(\alpha-\beta c_m)}{16B-3(\Delta-A)^2\beta}$	$\frac{(\Delta-A)(\alpha-c_m\beta)}{4B-(\Delta-A)^2\beta}$	$\frac{(\Delta-A)(\alpha-c_m\beta)}{4B-(\Delta-A)^2\beta}$	$\frac{3(\Delta-A)(\alpha-\beta c_m)}{8B-3(\Delta-A)^2\beta}$
ω^*	$\frac{\alpha}{\beta}-\frac{8B(\alpha-c_m\beta)}{\beta\left(16B-3(\Delta-A)^2\beta\right)}$	$\frac{\alpha}{\beta}-\frac{2B(\alpha-c_m\beta)}{\beta\left(4B-(\Delta-A)^2\beta\right)}$	N/A	N/A
b^*	$\frac{A+\Delta}{2}$	N/A	$\frac{A+\Delta}{2}$	$\frac{\Delta+A}{2}$

Table 3 Comparison of channel members' profits in the decentralized model and cooperative models

<table>
<tr><th>Profits</th><th>Model MC</th><th>Model MCC</th><th>Model D</th><th>Model MR</th><th>Model MCR</th></tr>
<tr><td>Π_R^*</td><td>$\frac{16B^2(\alpha-\beta c_m)^2}{\beta\left(16B-3(\Delta-A)^2\beta\right)^2}$</td><td>$\frac{B^2(\alpha-\beta c_m)^2}{\beta\left(4B-(\Delta-A)^2\beta\right)^2}$</td><td>$\frac{4B^2(\alpha-\beta c_m)^2}{\beta\left(8B-(\Delta-A)^2\beta\right)^2}$</td><td rowspan="2">$\frac{B(\alpha-c_m\beta)^2}{\beta\left(4B-(\Delta-A)^2\beta\right)}$</td><td rowspan="3">$\frac{2B(\alpha-\beta c_m)^2}{\beta\left(8B-3(\Delta-A)^2\beta\right)}$</td></tr>
<tr><td>Π_M^*</td><td rowspan="2">$\frac{B(\alpha-\beta c_m)^2}{\beta\left(8B-\frac{3}{2}(\Delta-A)^2\beta\right)}$</td><td rowspan="3">$\frac{B(\alpha-\beta c_m)^2}{\beta\left(8B-2(\Delta-A)^2\beta\right)}$</td><td>$\frac{B(\alpha-\beta c_m)^2}{\beta\left(8B-(\Delta-A)^2\beta\right)}$</td></tr>
<tr><td>$\Pi_{C_2}^*$</td><td>$\frac{B(\Delta-A)^2(\alpha-\beta c_m)^2}{\left(16B-2(\Delta-A)^2\beta\right)^2}$</td><td>$\frac{B(\Delta-A)^2(\alpha-c_m\beta)^2}{\left(8B-2(\Delta-A)^2\beta\right)^2}$</td></tr>
<tr><td>$\Pi_{C_1}^*$</td><td>$\frac{B(\Delta-A)^2(\alpha-\beta c_m)^2}{\left(16B-3(\Delta-A)^2\beta\right)^2}$</td><td>$\frac{B(\Delta-A)^2(\alpha-\beta c_m)^2}{\left(16B-2(\Delta-A)^2\beta\right)^2}$</td><td>$\frac{B(\Delta-A)^2(\alpha-c_m\beta)^2}{\left(8B-2(\Delta-A)^2\beta\right)^2}$</td><td>$\frac{B(\Delta-A)^2(\alpha-\beta c_m)^2}{\left(8B-3(\Delta-A)^2\beta\right)^2}$</td></tr>
</table>

Proposition 8. *In Model D, Model MC, Model MR and Model MCR, the manufacturer sets all* $b^* = (\Delta + A)/2$.

The reverse supply chain consists of the manufacturer and the recyclers. The incentive for remanufacturing is directly driven by $\Delta - b$, and the incentive of the recyclers to recycle is directly driven by $b - A$. The transfer price affects the game between the manufacturer and recyclers. Namely, the transfer price varies from the minimum of quantity A to the highest Δ. If the manufacturer chooses a large transfer price, the recyclers would increase return rate. However, the manufacturer's net savings from remanufacturing products would diminish (i.e., $\Delta - b$ decreases), and the manufacturer's profits would decrease as the transfer price approaches the saving cost from remanufacturing. We obtain equilibrium between the manufacturer's profit and the recycler's profit, and the balance point is $b^* = (\Delta + A)/2$.

Proposition 8 immediately implies that for different coalition structures, the transfer price paid by the manufacturer to recyclers is stable. The proposition will help recyclers to choose the proper investment level in the collection of used products.

Proposition 9. *The optimal return rates are related as* $\tau^{*C} \geq \tau^{*MCR} \geq \tau^{*MCC} = \tau^{*MR} \geq \tau^{*MC} \geq \tau^{*D}$.

Note that the recycler j's total profit is $(b - A)\tau_j D(p(w)) - B\tau_j^2$, in which retail price is determined by the retailer, transfer price is determined by the manufacturer, and return rate is determined by recycler. Because $\Delta \geq b \geq A$, we know that the marginal benefit is $2(b^{*D} - A) \leq (\Delta - A) + (b^{*MC} - A) \leq 2(\Delta - A)$ in Model D, Model MC, Model MCC and Model C. To increase profits, these cooperative structures would moderately increase the investment in the collection of used products and improve the return rate ($\tau^{*C} \geq \tau^{*MCC} \geq \tau^{*MC} \geq \tau^{*D}$). Moreover, we also find that $\tau^{*C} \geq \tau^{*MCR} \geq \tau^{*MR} \geq \tau^{*D}$. The alliance of manufacturer and retailer can increase the demand of the market, which may indirectly influence the amount of used products in the reverse channel. Retail price can be strategically determined to make used-product collection more profitable, a result of a second-degree effect on return rate.

Proposition 9 contains some potentially interesting results. In CLSCs, more members in an alliance are associated with higher return rates. Specifically, the cooperation between the manufacturer and the retailer could increase return rates. Hence, the government should encourage the manufacturer and retailer to participate in collecting used products.

Proposition 10. *Wholesale prices and retail prices in various models are related as* $\omega^{*D} \geq \omega^{*MC} \geq \omega^{*MCC}$, *and* $p^{*D} \geq p^{*MC} \geq p^{*MCC} \geq p^{*MR} \geq p^{*MCR} \geq p^{*C}$. *Consequently,* $D^{*D} \leq D^{*MC} \leq D^{*MCC} \leq D^{*MR} \leq D^{*MCR} \leq D^{*C}$, *where* $D = \alpha - \beta p$.

From the above proposition, the wholesale price and the retail price would be influenced by various coalition structures. Moreover, wholesale price is determined by the manufacturer and retail price is determined by the retailer. From Proposition 9, we know that the alliance of manufacturer and recycler would increase the investment in the collection of used product ($\tau^{*D} \leq \tau^{*MC} \leq \tau^{*MCC}$) and collect more used products from the market. The manufacturer setting a lower wholesale price to increase the demand and increasing the cost savings from remanufacturing may cause the wholesale price to decrease ($\omega^{*D} \geq \omega^{*MC} \geq \omega^{*MCC}$). When the manufacturer establishes an alliance with the retailer, wholesale price would be irrelevant in these models (Model MR, Model MCR, and Model C). The retail price in the completely centralized model is lower than that of the others because the lower price can increase both demand and profits.

The implication of Proposition 10 is that the more the members enter into an alliance among the manufacturer and recyclers, the lower the wholesale prices are. In addition, the cooperation between the manufacturer and the retailer might decrease retail prices and be beneficial to consumers.

Proposition 11. *The retailer's profits and total channel profits in various models are related as*

$$\prod_R^{*MCC} > \prod_R^{*MC} > \prod_R^{*D} \quad and \quad \prod^{*C} \geq \prod^{*MCR} \geq \prod^{*MR} \geq \prod^{*MCC} \geq \prod^{*MC} \geq \prod^{*D}.$$

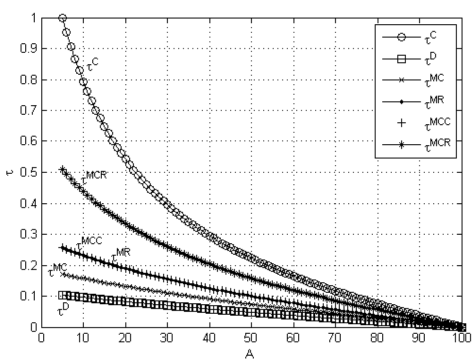
The results reveal the varying of profits among various cooperative models. When the manufacturer allies with the retailer, the retail price is reduced and more products are sold into the market. When the manufacturer builds alliances with recyclers, more used products are collected from the market. In short, cooperative strategies can increase the yield of products in CLSCs and promote the selling of products to obtain more profit at the best price.

The implication of Proposition 11 is that the alliance among the manufacturer and recyclers would improve the retailers' profit, and cooperation strategy can improve supply chain efficiency. The manufacturer may be apt to cooperate with the retailer rather than the recycler.

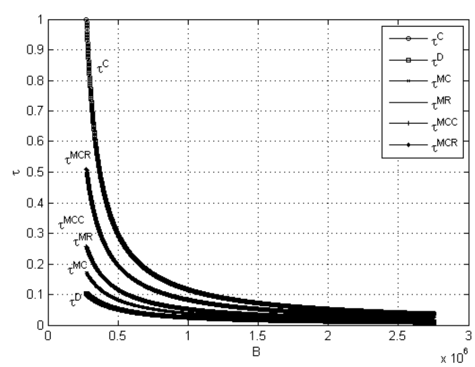
5.2. Numerical Study

Because recycling fee and scaling parameter cannot be solved analytically, a numerical example is provided to examine the effect of the autocorrelation coefficients A and B on return rates, retail prices, demand and total channel profits in various models. We assume that $\alpha = 10000$, $\beta = 40$, $c_m = 200$, and $\Delta = 100$ [1]. And Matlab software is utilized to calculate optimal channel decisions and profits. The results are shown in the Figure 2, Figure 3, Figure 4, and Figure 5.

In Figure 2, we focus on how recycling fee and scaling parameter affect return rates in various cooperative models. In Figure 2a, we first plot changes in return rates when A is increased from 5 to 100 with an increment of 1. Figure 2a validates Proposition 9. We find that these return rates are negatively related to the recycling fee, which increases at the same rate. Additionally, the relationship between the recycling fee and the return rates are almost linearly dependence. In Figure 2b, we plot these return rates when scaling parameter B is increased from 275,500 to 2,800,000 in increments of 1000. We find that return rates are also negatively related to the scaling parameter and that the growth inflection is 500,000. In other words, value B also has a great influence on the return rates when value B is increased from 275,500 to 500,000. When B increases from 500,000 to 2,800,000, value B has a slight effect on the return rates in these models.



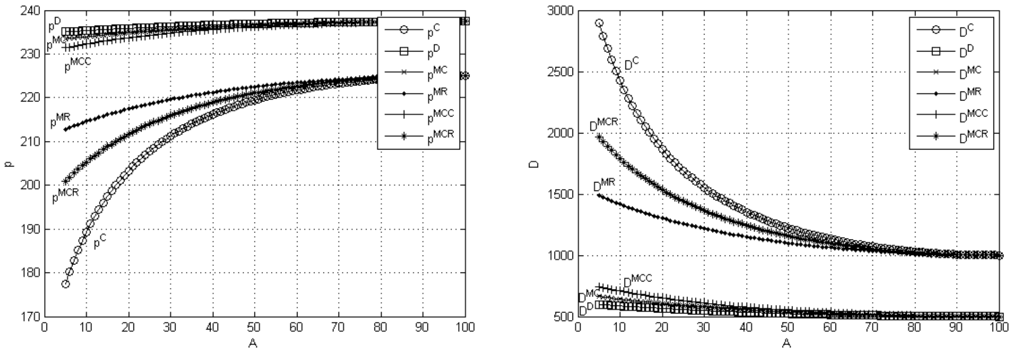
(a) Effect of A on return rates



(b) Effect of B on return rates

Figure 2 Effect of some factors on the return rate in the supply chain

Figure 2 shows that the more the members enter into an alliance, the higher the return rates are in various cooperative models. Hence, cooperative strategies have a major impact on the return rate in a CLSC. Specifically, when the manufacturer and the retailer establish cooperation with one another, the return rate will increase prominently.

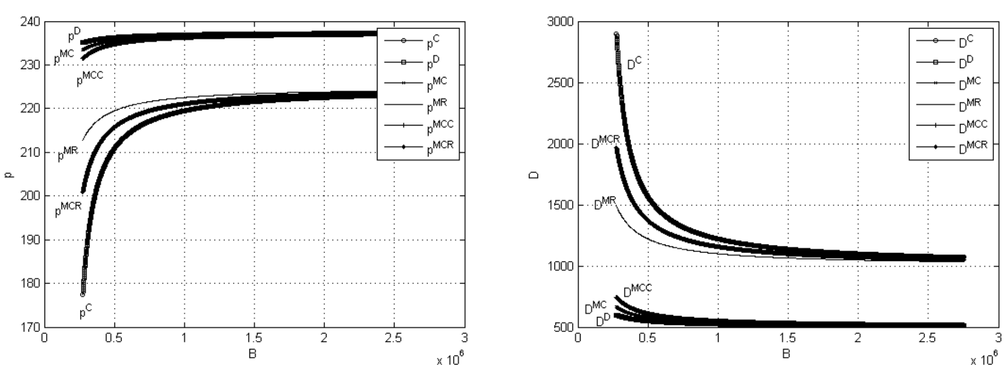


(a) Effect of A on retail prices (b) Effect of A on demand

Figure 3 Effect of the recycling fee on retail prices and demand

In Figure 3, we perform sensitivity analysis of retail prices and demand with respect to recycling fee. We plot changes in retail prices and demand when recycling fee A changes from 5 to 100 with an increment of 1. Figure 3 validates Proposition 10. We find that wholesale price and retail price are positively related to recycling fee, and demand is negatively related to recycling fee, which increases at the same rate.

In Figure 4, we investigate the effect of scaling parameter on retail prices and demand. We plot changes in retail price and demand when B is increased from 275,500 to 2,800,000 in increments of 1000. Figure 4 confirms Proposition 10. In Figure 4, we find that retail price is positively related to scaling parameter, and demand is negatively related to scaling parameter. These curves have the same growth inflection, which is 500,000. When B increases from 275,500 to 500,000, value B has a great influence on retail prices and demand. When B increases from 500,000 to 2,800,000, value B has a slight effect on retail prices and demand.

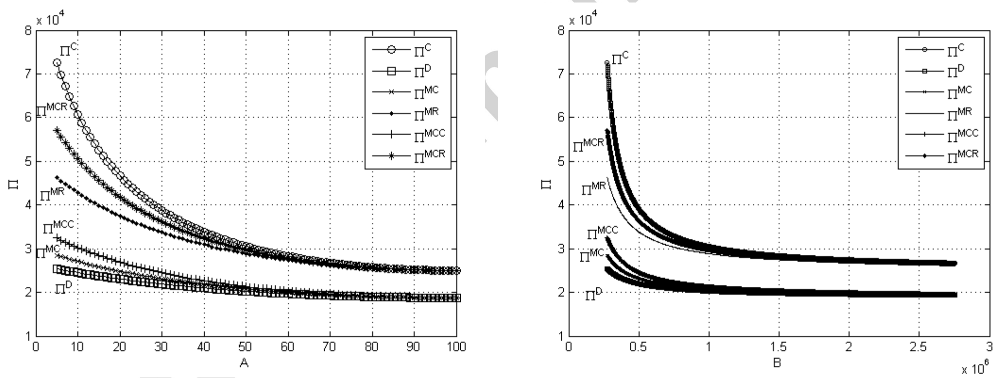


(a) Effect of B on retail prices (b) Effect of B on demand

Figure 4 Effect of the scaling parameter on retail prices and demand

Both in Figure 3 and Figure 4, the retail price in the completely centralized model is smaller than that in other models. In addition, we see that the manufacturer and the retailer are more likely to choose low-price strategy when the value of scaling parameter is smaller. However, when the scaling parameter is big enough, the manufacturer and retailer may choose stable prices, i.e., their cooperative strategies are no longer affected by scaling parameter.

To further study the effects of recycling fee and scaling parameter on total channel profits, we plot the varying of profits when A increases from 5 to 100 with an increment of 1 in Figure 5a, which indicates that profits are negatively related to value A, which increases at the same rate. In Figure 5b, we plot the varying of profits when B increases from 275,500 to 2,800,000 in increments of 1000. In Figure 5b, we find that profits are negatively related to value B, which increases at the same rate as does value B. When B increases from 275,500 to 500,000, value B changes slightly, whereas profit $\prod^*$ varies significantly. When B increases from 500,000 to 2,800,000, value B has a slight effect on profits. These curves have the same growth inflection of 500,000.



(a) Effect of A on total channel profits

(b) Effect of B on total channel profits

Figure 5 Effect of some factors on profits

Figure 5 depicts the intuitive effect of recycling fee and scaling parameter on total channel profits in various cooperative models. We see that total channel profits fall off rapidly with the increasing in scaling parameter. In addition, the total channel profit in the completely centralized model dominates total channel profits in other models, because forward and reverse supply chain decisions are integral coordinated in a CLSC. Specifically, cooperation structures in the forward supply chain are better than that in the reverse supply chain. Hence, the manufacturer should pay more attention to the relationship with the retailer.

6 Summary

In this paper, we consider a three-echelon supply chain consisting of a single manufacturer, a single retailer and two recyclers. We investigate interactions between the different parties in CLSCs and present four cooperative models: Model MC, Model MR, Model MCC and Model MCR. And we solve these cooperative models and make a detailed analysis and careful comparison of the results in various cooperative models. In particular, the following results are of managerial relevance. First, cooperative strategies have an impact on return rates of used products, and return rates increase with the number of alliance members. Specially, the cooperation between the manufacturer and the retailer would prominently increase return rates. Second, wholesale prices would decrease with the number of members in an alliance among the manufacturer and recyclers. The cooperation between the manufacturer and the retailer would decrease retail prices and be beneficial to consumers. Third, cooperative strategies can lead to win-win outcomes and increase the total channel profit. Total channel profits in the forward supply chain are larger than that in the reverse supply chain.

In future research, we may remove some assumptions to develop more comprehensive supply chain systems, such as a case in which recyclers compete against one another in the same market. Both consumer utility and government subsidy mechanisms affecting the supply chain should be considered. The modeling framework in this paper can be easily extended to consider such questions.

Acknowledgments

We are grateful to the associate editor and the two anonymous reviewers for their valuable comments and useful suggestions, which are all valuable and very helpful for revising and improving our paper.

Appendix A

Proof of the profit functions of supply chain members are concave in the decision variables.

Proof. Consider *Model C.*

To have an interior point solution for τ_j $(0 \le \tau_1 + \tau_2 \le 1)$ [1, 29, 42], Π^C should satisfy $\partial \Pi^C / \partial \tau_j \big|_{\tau_1+\tau_2=1} \le 0$ and $\partial^2 \Pi^C / \partial (\tau_j)^2 \le 0$. That is, $2B \ge (\alpha - \beta c_m)(\Delta - A) + \beta (\Delta - A)^2$. Then, we calculate the Hessian matrix as follows:

$$H=\begin{bmatrix}\partial^2\Pi^C/\partial p^2 & \partial^2\Pi^C/\partial p\partial\tau_1 & \partial^2\Pi^C/\partial p\partial\tau_2\\ \partial^2\Pi^C/\partial p\partial\tau_1 & \partial^2\Pi^C/\partial\tau_1{}^2 & \partial^2\Pi^C/\partial\tau_2\partial\tau_1\\ \partial^2\Pi^C/\partial p\partial\tau_2 & \partial^2\Pi^C/\partial\tau_2\partial\tau_1 & \partial^2\Pi^C/\partial\tau_2{}^2\end{bmatrix}=\begin{bmatrix}-2\beta & \beta(A-\Delta) & \beta(A-\Delta)\\ \beta(A-\Delta) & -2B & 0\\ \beta(A-\Delta) & 0 & -2B\end{bmatrix}$$

Since $\Delta\ge A$, $\beta\ge 0$, and $B\ge 0$, we can obtain

$$|H_1|=-2\beta<0,\ |H_2|=\beta\left(4B-\beta(\Delta-A)^2\right)\ge 0 \text{ and } |H_3|=-4\beta B\left(2B-\beta(\Delta-A)^2\right)\le 0.$$

Hence, Π^C is jointly concave in p, τ_1 and τ_2.

Consider *Model D.*

To have an interior point solution for τ_j ($0\le\tau_1+\tau_2\le 1$), $\Pi^D_{C_j}$ must satisfy $\partial\Pi^D_{C_j}/\partial\tau_j\big|_{\tau_1+\tau_2=1}\le 0$ and $\partial^2\Pi^D_{C_j}\big/\partial(\tau_j)^2\le 0$. That is, $B\ge(\alpha-\beta c_m)(\Delta-A)$. Π^D_R is concave in p since $\partial^2\Pi^D_R/\partial p^2=-2\beta\le 0$. $\Pi^D_{C_j}$ is concave in τ_j since $\partial^2\Pi^D_{C_j}/\partial\tau_j{}^2=-2B\le 0$. For $A\le b\le\Delta$, Π^D_M is jointly concave in ω and b since

$$\frac{\partial^2\Pi^D_M}{\partial\omega^2}=-\frac{\left(\beta\left(2B-\beta(b-A)(\Delta-b)\right)\right)}{(2B)}\le-\frac{\left(8B-\beta(\Delta-A)^2\right)}{4}\le 0 \text{ and}$$

$$\left(\frac{\partial^2\Pi^D_M}{\partial\omega^2}\right)\left(\frac{\partial^2\Pi^D_M}{\partial b^2}\right)-\left(\frac{\partial^2\Pi^D_M}{\partial b\partial\omega}\right)\left(\frac{\partial^2\Pi^D_M}{\partial b\partial\omega}\right)=\frac{\beta(\alpha-\beta w)^2}{(2B)^2}\begin{pmatrix}2B-\beta(\Delta-A)^2+\\ 3\beta(b-A)(\Delta-b)\end{pmatrix}\ge 0.$$

Consider *Model MC.*

Π^{MC}_R is concave in p since $\partial^2\Pi^{MC}_R/\partial p^2=-2\beta\le 0$. $\Pi^{MC}_{C_1}$ is concave in τ_1 since $\partial^2\Pi^{MC}_{C_1}/\partial\tau_1{}^2=-2B\le 0$. In addition, we calculate the Hessian matrix as follows:

$$H=\begin{bmatrix}\frac{\partial^2\Pi^{MC}_C}{\partial b^2} & \frac{\partial^2\Pi^{MC}_C}{\partial b\partial\omega} & \frac{\partial^2\Pi^{MC}_C}{\partial b\partial\tau_2}\\ \frac{\partial^2\Pi^{MC}_C}{\partial b\partial\omega} & \frac{\partial^2\Pi^{MC}_C}{\partial\omega^2} & \frac{\partial^2\Pi^{MC}_C}{\partial\tau_2\partial\omega}\\ \frac{\partial^2\Pi^{MC}_C}{\partial b\partial\tau_2} & \frac{\partial^2\Pi^{MC}_C}{\partial\omega\partial\tau_2} & \frac{\partial^2\Pi^{MC}_C}{\partial\tau_2{}^2}\end{bmatrix}=\begin{bmatrix}\frac{\beta\left(\beta(b-A)(\Delta-b)-4B\right)}{4B} & \frac{\beta(w\beta-\alpha)(A-2b+\Delta)}{4B} & \frac{1}{2}\beta(A-\Delta)\\ \frac{\beta(w\beta-\alpha)(A-2b+\Delta)}{4B} & -\frac{(\alpha-w\beta)^2}{4B} & 0\\ \frac{1}{2}\beta(A-\Delta) & 0 & -2B\end{bmatrix}$$

Since $\Delta\ge b\ge A$, $\beta\ge 0$, and $B\ge 0$, we can obtain

$$|H_1|=\frac{\beta}{4B}\left(-4B+\beta(b-A)(\Delta-b)\right)\le\frac{\beta}{4B}\left(-16B+\beta(A-\Delta)^2\right)\le 0,$$

$$|H_2|=\beta(\alpha-\beta w)^2\left(4B-\beta(\Delta-A)^2+3\beta(b-A)(\Delta-b)\right)\Big/(4B)^2\ge 0, \text{ and}$$

$$\left|H_3\right| = \frac{\beta(\alpha - w\beta)^2}{16B}\left(3\beta(\Delta - A)^2 - 8B - 6\beta(\Delta - b)(b - A)\right) \le 0.$$

Hence, Π_C^{MC} is jointly concave in ω, b and τ_2.

Consider *Model MR.*

$\Pi_{C_j}^{MR}$ is concave in τ_j since $\partial^2\Pi_{C_j}^{MR} \big/ \partial \tau_j^{\,2} = -2B \le 0$. For $A \le b \le \Delta$, Π_C^{MR} is concave in ω and b since $\frac{\partial^2\Pi_C^{MR}}{\partial b^2} = -\frac{2(\alpha - p\beta)^2}{B} \le 0$ and

$$\left(\frac{\partial^2\Pi_C^{MR}}{\partial b^2}\right)\left(\frac{\partial^2\Pi_C^{MR}}{\partial p^2}\right) - \left(\frac{\partial^2\Pi_C^{MR}}{\partial b \partial p}\right)\left(\frac{\partial^2\Pi_C^{MR}}{\partial b \partial p}\right) = \frac{4\beta(\alpha - p\beta)^2}{B^2}\begin{pmatrix} B - \beta(A - \Delta)^2 + \\ 3\beta(b - A)(\Delta - b) \end{pmatrix} \ge 0.$$

Consider *Model MCC.*

Π_R^{MCC} is concave in p since $\partial^2\Pi_R^{MCC} \big/ \partial p^2 = -2\beta \le 0$. In addition, we calculate the Hessian matrix as follows:

$$H = \begin{bmatrix} \frac{\partial^2\Pi_C^{MCC}}{\partial \omega^2} & \frac{\partial^2\Pi_C^{MCC}}{\partial \omega \partial \tau_1} & \frac{\partial^2\Pi_C^{MCC}}{\partial \omega \partial \tau_2} \\ \frac{\partial^2\Pi_C^{MCC}}{\partial \tau_1 \partial \omega} & \frac{\partial^2\Pi_C^{MCC}}{\partial \tau_1^{\,2}} & \frac{\partial^2\Pi_C^{MCC}}{\partial \tau_1 \partial \tau_2} \\ \frac{\partial^2\Pi_C^{MCC}}{\partial \tau_2 \partial \omega} & \frac{\partial^2\Pi_C^{MCC}}{\partial \tau_1 \partial \tau_2} & \frac{\partial^2\Pi_C^{MCC}}{\partial \tau_2^{\,2}} \end{bmatrix} = \begin{bmatrix} -\beta & \frac{1}{2}\beta(A - \Delta) & \frac{1}{2}\beta(A - \Delta) \\ \frac{1}{2}\beta(A - \Delta) & -2B & 0 \\ \frac{1}{2}\beta(A - \Delta) & 0 & -2B \end{bmatrix}$$

Since $\Delta \ge b \ge A$, $\beta \ge 0$, and $B \ge 0$, we can obtain

$$\left|H_1\right| = -\beta \le 0, \quad \left|H_2\right| = \frac{1}{4}\beta\left(8B - \beta(A - \Delta)^2\right) \ge 0 \quad \text{and} \quad \left|H_3\right| = \beta B\left(-4B + \beta(A - \Delta)^2\right) \le 0.$$

Hence, Π_C^{MCC} is jointly concave in ω, b and τ_2.

Consider *Model MCR.*

$\Pi_{C_1}^{MCR}$ is concave in τ_1 since $\partial^2\Pi_{C_1}^{MCR} \big/ \partial \tau_1^{\,2} = -2B \le 0$. In addition, we calculate the Hessian matrix as follows:

$$H = \begin{bmatrix} \frac{\partial^2\Pi_C^{MCR}}{\partial p^2} & \frac{\partial^2\Pi_C^{MCR}}{\partial b \partial p} & \frac{\partial^2\Pi_C^{MCR}}{\partial p \partial \tau_2} \\ \frac{\partial^2\Pi_C^{MCR}}{\partial b \partial p} & \frac{\partial^2\Pi_C^{MCR}}{\partial b^2} & \frac{\partial^2\Pi_C^{MCR}}{\partial \tau_2 \partial b} \\ \frac{\partial^2\Pi_C^{MCR}}{\partial p \partial \tau_2} & \frac{\partial^2\Pi_C^{MCR}}{\partial b \partial \tau_2} & \frac{\partial^2\Pi_C^{MCR}}{\partial \tau_2^{\,2}} \end{bmatrix} = \begin{bmatrix} \frac{\beta\left(\beta(b - A)(\Delta - b) - 2B\right)}{B} & \frac{\beta(p\beta - \alpha)(A - 2b + \Delta)}{B} & \beta(A - \Delta) \\ \frac{\beta(p\beta - \alpha)(A - 2b + \Delta)}{B} & -\frac{(\alpha - p\beta)^2}{B} & 0 \\ \beta(A - \Delta) & 0 & -2B \end{bmatrix}$$

Since $\Delta \ge b \ge A$, $\beta \ge 0$, and $B \ge 0$, we can obtain

$$\left|H_1\right| = \frac{1}{B}\beta\left(-2B + \beta(b-A)(\Delta-b)\right) \le 0,$$

$$\left|H_2\right| = \frac{\beta(\alpha-p\beta)^2}{B^2}\left(2B - \beta(\Delta-A)^2 + 3\beta(b-A)(\Delta-b)\right) \ge 0, \text{ and}$$

$$\left|H_3\right| = \frac{\beta(\alpha-p\beta)^2}{B}\left(3\beta(\Delta-A)^2 - 4B - 6\beta(\Delta-b)(b-A)\right) \le 0.$$

Hence, Π_C^{MCR} is jointly concave in p, b and τ_2.

Appendix B

Proof of Proposition 1. In Model C, the central planner maximizes $\underset{p,\tau_1,\tau_2}{Max}\Pi^C = \Pi_M + \Pi_R + \Pi_{C_1} + \Pi_{C_2} = (p-c_m)D + (\Delta-A)(\tau_1+\tau_2)D - B\left({\tau_1}^2+{\tau_2}^2\right)$. Because the objective function is jointly concave in p, τ_1 and τ_2 (see Appendix A), the first-order condition characterizes the unique best response, $p^{*C} = \alpha/\beta - B(\alpha-\beta c_m)\Big/\left(\beta\left(2B-\beta(\Delta-A)^2\right)\right)$ and $\tau_1^{*C} = \tau_2^{*C} = (\Delta-A)(\alpha-\beta c_m)\Big/\left(2\left(2B-(\Delta-A)^2\beta\right)\right)$.

Proof of Proposition 2. In Model D, for a given wholesale price ω, the retailer's problem is $\underset{p}{Max}\Pi_R^D = (p-\omega)D$. Because the objective function is concave in p (see Appendix A), it follows that $p^{*D} = (\alpha+\beta\omega)/(2\beta)$. Given p^{*D}, recyclers maximize $\underset{\tau_j}{Max}\Pi_{C_j}^D = b\tau_j D - B{\tau_j}^2 - A\tau_j D$. From the concavity of the objective function in τ_j, it follows that $\tau_j^{*D} = \left((b-A)(\alpha-\beta\omega)\right)/(4B)$. Given p^{*D} and τ_j^{*D}, the manufacturer maximizes $\underset{\omega,b}{Max}\Pi_M^D = (\omega-c_m+\Delta\tau)D - b\tau D$. Because the objective function is jointly concave in ω and b, the first-order condition characterizes the unique best response, $b^{*D} = (A+\Delta)/2$ and $\omega^{*D} = \alpha/\beta - \left(4B(\alpha-c_m\beta)\right)\Big/\left(\beta\left(8B-(\Delta-A)^2\beta\right)\right)$.

Proof of Proposition 3. In Model MC, for a given wholesale price ω, the retailer maximizes $\underset{p}{Max}\Pi_R^{MC} = (p-\omega)D$. From the concavity of the objective function in p (see Appendix A), it follows that $p^{*MC} = (\alpha+\beta\omega)/(2\beta)$. Given p^{*MC}, recycler1 maximizes $\underset{\tau_1}{Max}\Pi_{C_1}^{MC} = (b-A)D\tau_1 - B{\tau_1}^2$. From the concavity of the objective function in τ_1, it follows that $\tau_1^{*MC} = \left((b-A)(\alpha-\beta\omega)\right)/(4B)$. Given p^{*MC} and τ_1^{*MC}, the central planner maximizes $\underset{\omega,b,\tau_2}{Max}\Pi_C^{MC} = \left(\omega-c_m+\Delta\left(\tau_1+\tau_2\right)\right)D - b\tau_1 D - B{\tau_2}^2 - A\tau_2 D$. Because the objective function is jointly concave in ω, b and τ_2, the first-order condition characterizes the unique best response, $\omega^{*MC} = \alpha/\beta - \left(8B\left(\alpha - \beta c_m\right)\right)/\left(\beta\left(16B - 3\beta\left(\Delta - A\right)^2\right)\right)$, $b^{*MC} = \left(A + \Delta\right)/2$, and $\tau_2^{*MC} = \left(2\left(\Delta - A\right)\left(\alpha - \beta c_m\right)\right)/\left(16B - 3\beta\left(\Delta - A\right)^2\right)$.

Proof of Proposition 4. In Model MR, for a given retail price p, recyclers maximize $\underset{\tau_j}{Max}\,\Pi_{C_j}^{MR} = b\tau_j D - B\tau_j^{\,2} - A\tau_j D$. Because the objective function $\Pi_{C_j}^{MR}$ is concave in τ_j (see Appendix A), it follows that $\tau_1^{*MR} = \tau_2^{*MR} = \left(\left(\Delta - A\right)\left(\alpha - c_m\beta\right)\right)/\left(2\left(4B - \left(\Delta - A\right)^2\beta\right)\right)$. Give τ_1^{*MR} and τ_2^{*MR}, the central planner maximizes $\underset{b,p}{Max}\,\Pi_C^{MR} = \left(p - c_m\right)D + \left(\Delta - b\right)\tau D$. Because the objective function is jointly concave in p and b, the first-order condition characterizes the unique best response, $p^{*MR} = \alpha/\beta - \left(2B\left(\alpha - c_m\beta\right)\right)/\left(\beta\left(4B - \left(\Delta - A\right)^2\beta\right)\right)$ and $b^{*MR} = \left(A + \Delta\right)/2$.

Proof of Proposition 5. In Model MCC, for a given wholesale price ω, the retailer maximizes $\underset{p}{Max}\,\Pi_R^{MCC} = \left(p - \omega\right)D$. From the concavity of the objective function in p (see Appendix A), it follows that $p^{*MCC} = \left(\alpha + \beta\omega\right)/\left(2\beta\right)$. Given p^{*MCC}, the central planner maximizes $\underset{\omega,\tau_1,\tau_2}{Max}\,\Pi_C^{MCC} = \left(\omega - c_m + \Delta\left(\tau_1 + \tau_2\right)\right)D - B\tau_1^{\,2} - B\tau_2^{\,2} - \left(\tau_1 + \tau_2\right)AD$. Because the objective function is jointly concave in τ_1, τ_2, and ω, the first-order condition characterizes the unique best response, $\tau_1^{*MCC} = \tau_2^{*MCC} = \left(\left(\Delta - A\right)\left(\alpha - \beta c_m\right)\right)/\left(8B - 2\beta\left(\Delta - A\right)^2\right)$ and $\omega^{*MCC} = \alpha/\beta - \left(2B\left(\alpha - \beta c_m\right)\right)/\left(\beta\left(4B - \beta\left(\Delta - A\right)^2\right)\right)$.

Proof of Proposition 6. In Model MCR, the recycler1's problem can be stated as $\underset{\tau_1}{Max}\,\Pi_{C_1}^{MCR} = b\tau_1 D - B\tau_1^{\,2} - A\tau_1 D$. From the concavity of the objective function in τ_1 (see Appendix A), it follows that $\tau_1^{*MCR} = \left(\left(b - A\right)\left(\alpha - \beta p\right)\right)/\left(2B\right)$. Given τ_1^{*MCR}, the central planner maximizes $\underset{b,p,\tau_2}{Max}\,\Pi^{MCR} = \Pi_M + \Pi_R + \Pi_{C_2} = \left(p - c_m\right)D + \left(\Delta - A\right)\tau_2 D + \left(\Delta - b\right)\tau_1 D - B\tau_2^{\,2}$. Because the objective function is jointly concave in b, p and τ_2, it follows that $b^{*MCR} = \left(A + \Delta\right)/2$, $p^{*MCR} = \alpha/\beta - \left(4B\left(\alpha - \beta c_m\right)\right)/\left(\beta\left(8B - 3\beta\left(\Delta - A\right)^2\right)\right)$, and $\tau_2^{*MCR} = \left(2\left(\Delta - A\right)\left(\alpha - \beta c_m\right)\right)/\left(\left(8B - 3\beta\left(\Delta - A\right)^2\right)\right)$.

Appendix C

Because the demand is nonnegative, i.e., $D \ge 0$, we obtain an equivalent $2B - \beta(\Delta - A)^2 \ge 0$.

Comparison of the supply chain profit between Model MC and Model D.

Note the following:

$$\Pi_C^{*\,MC} - \left(\Pi_M^{*\,D} + \Pi_{C_2}^{*\,D}\right) = \frac{B(\alpha - \beta c_m)^2 \left(16B - \beta(\Delta - A)^2\right)(\Delta - A)^2}{4\left(16B - 3\beta(\Delta - A)^2\right)\left(8B - (\Delta - A)^2 \beta\right)^2}$$

$$\Pi_{C_1}^{*\,MC} - \Pi_{C_1}^{*\,D} = B(\Delta - A)^2 (\alpha - \beta c_m)^2 \left(\frac{1}{\left(16B - 3(\Delta - A)^2 \beta\right)^2} - \frac{1}{\left(16B - 2(\Delta - A)^2 \beta\right)^2} \right)$$

$$\Pi_R^{*\,MC} - \Pi_R^{*\,D} = \frac{4B^2 (\alpha - \beta c_m)^2}{\beta} \left(\frac{1}{\left(8B - \frac{3}{2}\beta(\Delta - A)^2\right)^2} - \frac{1}{\left(8B - (\Delta - A)^2 \beta\right)^2} \right)$$

Then, we can obtain

$$\Pi_C^{*\,MC} - \left(\Pi_M^{*\,D} + \Pi_{C_2}^{*\,D}\right) \ge 0, \quad \Pi_{C_1}^{*\,MC} - \Pi_{C_1}^{*\,D} \ge 0, \quad \Pi_R^{*\,MC} - \Pi_R^{*\,D} \ge 0, \text{ and } \Pi^{*MC} \ge \Pi^{*D}.$$

Comparison of the supply chain profit between Model MR and Model D.

Note the following:

$$\Pi_C^{*\,MR} - \left(\Pi_M^{*\,D} + \Pi_R^{*\,D}\right) = \frac{16B^3 (\alpha - \beta c_m)^2}{\beta\left(8B - (\Delta - A)^2 \beta\right)^2 \left(4B - \beta(\Delta - A)^2\right)}$$

$$\Pi_{C_1}^{*\,MR} - \Pi_{C_1}^{*\,D} = B(\Delta - A)^2 (\alpha - c_m \beta)^2 \left(\frac{1}{\left(8B - 2(\Delta - A)^2 \beta\right)^2} - \frac{1}{\left(16B - 2(\Delta - A)^2 \beta\right)^2} \right)$$

$$\Pi_{C_2}^{*\,MR} - \Pi_{C_2}^{*\,D} = B(\Delta - A)^2 (\alpha - c_m \beta)^2 \left(\frac{1}{\left(8B - 2(\Delta - A)^2 \beta\right)^2} - \frac{1}{\left(16B - 2(\Delta - A)^2 \beta\right)^2} \right)$$

Then, we can obtain

$$\Pi_C^{*\,MR} - \left(\Pi_M^{*\,D} + \Pi_R^{*\,D}\right) \ge 0, \quad \Pi_{C_1}^{*\,MR} - \Pi_{C_1}^{*\,D} \ge 0, \text{ and } \Pi_{C_2}^{*\,MR} - \Pi_{C_2}^{*\,D} \ge 0.$$

Comparison of the supply chain profit between Model MCC and Model D/ Model MC.

Note the following:

$$\Pi_C^{*\,MCC} - \left(\Pi_C^{*\,MC} + \Pi_{C_1}^{*\,\,MC}\right) = \frac{B\left(\alpha-\beta c_m\right)^2\left(8B-(\Delta-A)^2\beta\right)(\Delta-A)^2}{2\left(16B-3(\Delta-A)^2\beta\right)^2\left(4B-(\Delta-A)^2\beta\right)}$$

$$\Pi_R^{*\,MCC} - \Pi_R^{*\,MC} = \frac{B^2\left(\alpha-\beta c_m\right)^2}{\beta}\left(\frac{1}{\left(4B-(\Delta-A)^2\beta\right)^2} - \frac{1}{\left(4B-\frac{3}{4}(\Delta-A)^2\beta\right)^2}\right)$$

$$\Pi_C^{*\,MCC} - \left(\Pi_M^{*\,D} + \Pi_{C_1}^{*\,D} + \Pi_{C_2}^{*\,D}\right) = \frac{2B^2\left(\alpha-\beta c_m\right)^2(\Delta-A)^2}{\left(4B-\beta(\Delta-A)^2\right)\left(8B-(\Delta-A)^2\beta\right)^2}$$

$$\Pi_R^{*\,MCC} - \Pi_R^{*\,D} = \frac{B^2\left(\alpha-\beta c_m\right)^2}{\beta}\left(\frac{1}{\left(4B-(\Delta-A)^2\beta\right)^2} - \frac{1}{\left(4B-\frac{1}{2}(\Delta-A)^2\beta\right)^2}\right)$$

Then, we can obtain

$$\Pi_C^{*\,MCC} - \left(\Pi_C^{*\,MC} + \Pi_{C_1}^{*\,MC}\right) \geq 0,\;\; \Pi_R^{*\,MCC} - \Pi_R^{*\,MC} \geq 0,\;\; \Pi^{*MCC} \geq \Pi^{*MC},$$

$$\Pi_C^{*\,MCC} - \left(\Pi_M^{*\,D} + \Pi_{C_1}^{*\,D} + \Pi_{C_2}^{*\,D}\right) \geq 0, \text{ and }\; \Pi_R^{*\,MCC} - \Pi_R^{*\,D} \geq 0,$$

Comparison of the supply chain profit between Model MCR and Model D/ Model MC/ Model MR.

Note the following:

$$\Pi_C^{*\,MCR} - \left(\Pi_C^{*\,MC} + \Pi_R^{*\,MC}\right) = \frac{128B^3\left(\alpha-\beta c_m\right)^2}{\beta\left(16B-3(\Delta-A)^2\beta\right)^2\left(8B-3(\Delta-A)^2\beta\right)}$$

$$\Pi_{C_1}^{*\,\,MCR} - \Pi_{C_1}^{*\,\,MC} = B(\Delta-A)^2\left(\alpha-\beta c_m\right)^2\left(\frac{1}{\left(8B-3(\Delta-A)^2\beta\right)^2} - \frac{1}{\left(16B-3(\Delta-A)^2\beta\right)^2}\right)$$

$$\Pi_C^{*\,MCR} - \left(\Pi_C^{*\,MR} + \Pi_{C_2}^{*\,\,MR}\right) = \frac{B\left(\alpha-\beta c_m\right)^2\left(8B-(\Delta-A)^2\beta\right)(\Delta-A)^2}{\left(8B-3(\Delta-A)^2\beta\right)\left(4B-(\Delta-A)^2\beta\right)}$$

$$\Pi_{C_1}^{*\,\,MCR} - \Pi_{C_1}^{*\,\,MR} = B(\Delta-A)^2\left(\alpha-\beta c_m\right)^2\left(\frac{1}{\left(8B-3(\Delta-A)^2\beta\right)^2} - \frac{1}{\left(8B-2(\Delta-A)^2\beta\right)^2}\right)$$

Then, we can obtain

$$\Pi_C^{*\,MCR} - \left(\Pi_C^{*\,MC} + \Pi_R^{*\,MC}\right) \geq 0,\;\; \Pi_{C_1}^{*\,\,MCR} - \Pi_{C_1}^{*\,\,MC} \geq 0,\;\; \Pi^{*MCR} \geq \Pi^{*MC},$$

$$\Pi_C^{*\,MCR} - \left(\Pi_C^{*\,MR} + \Pi_{C_2}^{*\,\,MR}\right) \geq 0,\;\; \Pi_{C_1}^{*\,\,MCR} - \Pi_{C_1}^{*\,\,MR} \geq 0, \text{ and }\; \Pi^{*MCR} \geq \Pi^{*MR}.$$

Appendix D

Proof of Proposition 9. We can obtain from the optimal values of $\tau_j^*\left(j=1,2\right)$ in Table 1 and Table 2. Note the following:

$$2B-\beta\left(\Delta-A\right)^2 \le 4B-\frac{3}{2}\beta\left(\Delta-A\right)^2 \le 4B-\beta\left(\Delta-A\right)^2 \le 8B-\frac{3}{2}\beta\left(\Delta-A\right)^2 \le 8B-\beta\left(\Delta-A\right)^2$$

$$2B-\beta\left(\Delta-A\right)^2 \le 2B-\frac{3}{4}\beta\left(\Delta-A\right)^2 \le 4B-\beta\left(\Delta-A\right)^2 \le 4B-\frac{3}{4}\beta\left(\Delta-A\right)^2 \le 8B-\beta\left(\Delta-A\right)^2$$

Then, we can obtain

$$\frac{\left(\Delta-A\right)\left(\alpha-\beta c_m\right)}{2B-\beta\left(\Delta-A\right)^2} \ge \frac{\left(\Delta-A\right)\left(\alpha-\beta c_m\right)}{4B-\frac{3}{2}\beta\left(\Delta-A\right)^2} \ge \frac{\left(\Delta-A\right)\left(\alpha-\beta c_m\right)}{4B-\beta\left(\Delta-A\right)^2} \ge \frac{\left(\Delta-A\right)\left(\alpha-\beta c_m\right)}{8B-\frac{3}{2}\beta\left(\Delta-A\right)^2} \ge \frac{\left(\Delta-A\right)\left(\alpha-\beta c_m\right)}{8B-\beta\left(\Delta-A\right)^2}$$

$$\frac{\left(\Delta-A\right)\left(\alpha-\beta c_m\right)}{2B-\beta\left(\Delta-A\right)^2} \ge \frac{\left(\Delta-A\right)\left(\alpha-\beta c_m\right)}{2B-\frac{3}{4}\beta\left(\Delta-A\right)^2} \ge \frac{\left(\Delta-A\right)\left(\alpha-\beta c_m\right)}{4B-\beta\left(\Delta-A\right)^2} \ge \frac{\left(\Delta-A\right)\left(\alpha-\beta c_m\right)}{4B-\frac{3}{4}\beta\left(\Delta-A\right)^2} \ge \frac{\left(\Delta-A\right)\left(\alpha-\beta c_m\right)}{8B-\beta\left(\Delta-A\right)^2}$$

Hence, $\tau_1^{*C} \ge \tau_1^{*MCR} \ge \tau_1^{*MCC} = \tau_1^{*MR} \ge \tau_1^{*MC} \ge \tau_1^{*D}$ and $\tau_2^{*C} \ge \tau_2^{*MCR} \ge \tau_2^{*MCC} = \tau_2^{*MR} \ge \tau_2^{*MC} \ge \tau_2^{*D}$, then, we obtain $\tau^{*C} \ge \tau^{*MCR} \ge \tau^{*MCC} = \tau^{*MR} \ge \tau^{*MC} \ge \tau^{*D}$, where $\tau = \tau_1 + \tau_2$.

Proof of Proposition 10. Note the following:

$$2B-\frac{1}{4}\left(\Delta-A\right)^2\beta \ge 2B-\frac{3}{8}\left(\Delta-A\right)^2\beta \ge 2B-\frac{1}{2}\left(\Delta-A\right)^2\beta$$

$$\begin{Bmatrix} 4B-\frac{1}{2}\left(\Delta-A\right)^2\beta \ge 4B-\frac{3}{4}\left(\Delta-A\right)^2\beta \ge 4B-\left(\Delta-A\right)^2\beta \ge \\ 2B-\frac{1}{2}\left(\Delta-A\right)^2\beta \ge 2B-\frac{3}{4}\left(\Delta-A\right)^2\beta \ge 2B-\left(\Delta-A\right)^2\beta \end{Bmatrix}$$

Then, we can obtain

$$\frac{\alpha}{\beta}-\frac{B\left(\alpha-c_m\beta\right)}{\beta\left(2B-\frac{1}{4}\left(\Delta-A\right)^2\beta\right)} \ge \frac{\alpha}{\beta}-\frac{B\left(\alpha-c_m\beta\right)}{\beta\left(2B-\frac{3}{8}\left(\Delta-A\right)^2\beta\right)} \ge \frac{\alpha}{\beta}-\frac{B\left(\alpha-c_m\beta\right)}{\beta\left(2B-\frac{1}{2}\left(\Delta-A\right)^2\beta\right)}$$

$$\begin{Bmatrix} \frac{\alpha}{\beta}-\frac{B\left(\alpha-c_m\beta\right)}{\beta\left(4B-\frac{1}{2}\left(\Delta-A\right)^2\beta\right)} \ge \frac{\alpha}{\beta}-\frac{B\left(\alpha-c_m\beta\right)}{\beta\left(4B-\frac{3}{4}\left(\Delta-A\right)^2\beta\right)} \ge \frac{\alpha}{\beta}-\frac{B\left(\alpha-c_m\beta\right)}{\beta\left(4B-\left(\Delta-A\right)^2\beta\right)} \ge \\ \frac{\alpha}{\beta}-\frac{B\left(\alpha-c_m\beta\right)}{\beta\left(2B-\frac{1}{2}\left(\Delta-A\right)^2\beta\right)} \ge \frac{\alpha}{\beta}-\frac{B\left(\alpha-\beta c_m\right)}{\beta\left(2B-\frac{3}{4}\left(\Delta-A\right)^2\beta\right)} \ge \frac{\alpha}{\beta}-\frac{B\left(\alpha-\beta c_m\right)}{\beta\left(2B-\left(\Delta-A\right)^2\beta\right)} \end{Bmatrix}$$

Hence, $\omega^{*D} \ge \omega^{*MC} \ge \omega^{*MCC}$ and $p^{*D} \ge p^{*MC} \ge p^{*MCC} \ge p^{*MR} \ge p^{*MCR} \ge p^{*C}$. We also obtain $D^{*D} \le D^{*MC} \le D^{*MCC} \le D^{*MR} \le D^{*MCR} \le D^{*C}$, where $D = \alpha - \beta p$.

Proof of Proposition 11. From Appendix C, we obtain

$$\prod^{*MC} \geq \prod^{*D},\ \prod^{*MCC} \geq \prod^{*MC}, \text{ and } \prod^{*MCR} \geq \prod^{*MR}.$$

Note the following:

$$\Pi_R^{*MCC} - \Pi_R^{*MC} = \frac{B^2(\alpha-\beta c_m)^2}{\beta}\left(\frac{1}{\left(4B-\beta(\Delta-A)^2\right)^2} - \frac{1}{\left(4B-\frac{3}{4}\beta(\Delta-A)^2\right)^2}\right)$$

$$\Pi_R^{*MC} - \Pi_R^{*D} = \frac{4B^2(\alpha-\beta c_m)^2}{\beta}\left(\frac{1}{\left(8B-\frac{3}{2}\beta(\Delta-A)^2\right)^2} - \frac{1}{\left(8B-\beta(\Delta-A)^2\right)^2}\right)$$

$$\Pi^{*C} - \Pi^{*MCR} = \frac{B(\alpha-\beta c_m)^2\left(4B-(\Delta-A)^2\beta\right)(\Delta-A)^2}{2\left(8B-3\beta(\Delta-A)^2\right)^2\left(2B-\beta(\Delta-A)^2\right)}$$

$$\Pi^{*MR} - \Pi^{*MCC} = \frac{B^2(\alpha-\beta c_m)^2}{\beta\left(-4B+\beta(A-\Delta)^2\right)^2}$$

Then, we can obtain

$$\Pi_R^{*MCC} \geq \Pi_R^{*MC}, \text{ and } \Pi_R^{*MC} \geq \Pi_R^{*D}.$$

$$\prod^{*C} \geq \prod^{*MCR},\ \prod^{*MCR} \geq \prod^{*MR},\ \prod^{*MR} \geq \prod^{*MCC},\ \prod^{*MCC} \geq \prod^{*MC}, \text{ and } \prod^{*MC} \geq \prod^{*D}.$$

Hence, $\Pi_R^{*MCC} \geq \Pi_R^{*MC} \geq \Pi_R^{*D}$ and $\prod^{*C} \geq \prod^{*MCR} \geq \prod^{*MR} \geq \prod^{*MCC} \geq \prod^{*MC} \geq \prod^{*D}$.

References:

[1] Savaskan RC, Bhattacharya S, Van Wassenhove LN. Closed-loop supply chain models with product remanufacturing. Manage Sci. 2004;50:239-52.

[2] Abbey JD, Meloy MG, Guide VDR, Atalay S. Remanufactured Products in Closed‐Loop Supply Chains for Consumer Goods. Prod Oper Manag. 2014;24:488–503.

[3] Chen J-M, Chang C-I. The co-opetitive strategy of a closed-loop supply chain with remanufacturing. Transportation Research Part E: Logistics and Transportation Review. 2012;48:387-400.

[4] Bylinsky G. Manufacturing for reuse. Fortune. 1995;131:102-7.

[5] Kumar S, Malegeant P. Strategic alliance in a closed-loop supply chain, a case of manufacturer and eco-non-profit organization. Technovation. 2006;26:1127-35.

[6] Germans R. Reuse and IBM. Proceedings of the first international working seminar on reuse Eindhoven: Netherlands; 1996. p. 119-31.

[7] Cottrill K. Dell and the reverse computer boom. Traffic World. 2003;267:23-4.

[8] Karakayali I, Emir-Farinas H, Akcali E. An analysis of decentralized collection and processing of end-of-life products. J Oper Manag. 2007;25:1161-83.

[9] Li SX, Huang Z, Zhu J, Chau PY. Cooperative advertising, game theory and manufacturer–retailer supply chains. Omega. 2002;30:347-57.

[10] Swami S, Shah J. Channel Coordination in Green Supply Chain Management: The Case of

Package Size and Shelf-Space Allocation. Technology Operation Management. 2011;2:50-9.
[11] Fiona E. Xerox: design for the environment. Harvard Business School Case. 1993;7:94-122.
[12] Ginsburg J. Once is not enough. Bus Week. 2001:682-4.
[13] Aizawa H, Yoshida H, Sakai S-i. Current results and future perspectives for Japanese recycling of home electrical appliances. Resources, Conservation and Recycling. 2008;52:1399-410.
[14] Hammer M. Reengineering work: don't automate, obliterate. Harvard Bus Rev. 1990;68:104-12.
[15] Guide VDR, Van Wassenhove LN. OR FORUM---The Evolution of Closed-Loop Supply Chain Research. Oper Res. 2009;57:10-8.
[16] Govindan K, Soleimani H, Kannan D. Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future. Eur J Oper Res. 2015;240:603-26.
[17] Savaskan RC, Van Wassenhove LN. Reverse channel design: the case of competing retailers. Manage Sci. 2006;52:1-14.
[18] Wei J, Zhao J. Pricing decisions with retail competition in a fuzzy closed-loop supply chain. Expert Syst Appl. 2011;38:11209-16.
[19] Hong I, Yeh J-S. Modeling closed-loop supply chains in the electronics industry: A retailer collection application. Transportation Research Part E: Logistics and Transportation Review. 2012;48:817-29.
[20] Huang M, Song M, Lee LH, Ching WK. Analysis for strategy of closed-loop supply chain with dual recycling channel. Int J Product Econ. 2013;144:510-20.
[21] Arruñada B, Vázquez XH. When your contract manufacturer becomes your competitor. Harvard Bus Rev. 2006;84:135-44.
[22] Aguezzoul A. Third-party logistics selection problem: A literature review on criteria and methods. Omega. 2014;49:69-78.
[23] Cachon GP. Supply chain coordination with contracts. Handbooks in operations research and management science. 2003;11:227-339.
[24] Huang Z, Li SX. Co-op advertising models in manufacturer–retailer supply chains: A game theory approach. Eur J Oper Res. 2001;135:527-44.
[25] Zhang J, Gou Q, Liang L, Huang Z. Supply chain coordination through cooperative advertising with reference price effect. Omega. 2013;41:345-53.
[26] Gurnani H, Erkoc M, Luo Y. Impact of product pricing and timing of investment decisions on supply chain co-opetition. Eur J Oper Res. 2007;180:228-48.
[27] Gurnani H, Erkoc M. Supply contracts in manufacturer‐retailer interactions with manufacturer‐quality and retailer effort‐induced demand. Naval Research Logistics (NRL). 2008;55:200-17.
[28] Leng M, Parlar M. Allocation of cost savings in a three-level supply chain with demand information sharing: A cooperative-game approach. Oper Res. 2009;57:200-13.
[29] Jena SK, Sarmah S. Price competition and co-operation in a duopoly closed-loop supply chain. Int J Product Econ. 2014;156:346-60.
[30] Giutini R, Gaudette K. Remanufacturing: The next great opportunity for boosting US productivity. Bus Horiz. 2003;46:41-8.
[31] Gupta S, Loulou R. Process innovation, product differentiation, and channel structure: Strategic incentives in a duopoly. Marketing Sci. 1998;17:301-16.
[32] Gilbert SM, Cvsa V. Strategic commitment to price to stimulate downstream innovation in a supply chain. Eur J Oper Res. 2003;150:617-39.
[33] Bulow JI. Durable-goods monopolists. The Journal of Political Economy. 1982;90:314-32.

[34] Lee E, Staelin R. Vertical strategic interaction: Implications for channel pricing strategy. Marketing Sci. 1997;16:185-207.

[35] Swan PL. Optimum durability, second-hand markets, and planned obsolescence. The Journal of Political Economy. 1972;80:575-85.

[36] Lilien GL, Kotler P, Moorthy KS. Marketing models: Prentice Hall; 1992.

[37] Zhao H. Raising awareness and signaling quality to uninformed consumers: A price-advertising model. Marketing Sci. 2000;19:390-6.

[38] Nnorom I, Osibanjo O. Overview of electronic waste (e-waste) management practices and legislations, and their poor applications in the developing countries. Resources, conservation and recycling. 2008;52:843-58.

[39] Gungor A, Gupta SM. Issues in environmentally conscious manufacturing and product recovery: a survey. Comput Ind Eng. 1999;36:811-53.

[40] Choi S, Fredj K. Price Competition and Store Competition: Store Brands vs. National Brand. Eur J Oper Res. 2013;225:166-78.

[41] Prahalad CK, Ramaswamy V. Co-opting customer competence. Harvard Bus Rev. 2000;78:79-90.

[42] Zhang J, Liu G, Zhang Q, Bai Z. Coordinating a supply chain for deteriorating items with a revenue sharing and cooperative investment contract. Omega. 2015;56:37-49.

Highlights

1. The interactions among the different parties in a three-echelon closed-loop supply chain are investigated.
2. The optimal decisions and the supply chain profits in various cooperative models are discussed.
3. The more the members enter into an alliance, the higher the return rates are.
4. Cooperative strategies can lead to win-win outcomes and increase the total channel profit.