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Waldemar Grauberger, Alf Kimms

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Address of correspondence:
Waldemar Grauberger and Prof. Dr. Alf Kimms
Chair of Logistics and Operations Research
Mercator School of Management
University of Duisburg–Essen
Lotharstr. 65 · 47057 Duisburg · Germany
email: *waldemar.grauberger@uni–due.de*
email: *alf.kimms@uni–due.de*
URL: *http://www.msm.uni–due.de/log/*

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Abstract

We present a model to optimize a competitor's behavior in a network revenue management game within an airline alliance. In particular, we model two forms of competition; horizontal competition with parallel substitutable flights and vertical competition with both competitors operating adjacent connecting flights in a code sharing agreement. We compute pure Nash equilibria with an iterative algorithm presented in an earlier paper. A computational study shows that the algorithm is also suited for computing Nash equilibria taking both types of competition into account and that code sharing increases revenues for both competitors. However, the difference decreases as the network size and mean demand increase.

Keywords: Airline Alliances Network Revenue Management, Code Sharing, Horizontal/ Vertical Competition, Pure Nash Equilibrium

1 Introduction

The deregulation of the airspace in the US, followed by deregulations in other countries in the past two decades, allowed airlines to enlarge their networks (Oum and Park, 1997). In order to avoid many of the efforts connected to the entry into new markets, airlines allied in alliances. See e.g. Oum and Park (1997) and Park (1997) for further reasons for alliance formation as well as its (economic) effects. Chapter 3.2 in Çetiner (2013) provides a detailed treatment of this topic as well as a thorough literature overview.

A key characteristic of airline alliances are code sharing agreements that allow the airlines to sell products which involve utilization of partners' capacities as if they were their own (Oum et al., 2001, p. 57). Code sharing allows the partners to extend their networks, improve customer service, and raise their efficiency through a higher capacity load factor, among other things. With code sharing agreements in use, the problem is not only to allocate capacity to different products, but also to divide the available capacity among the partners within the alliance.

O'Neal et al. (2007) presented a mixed-integer problem to select those flights which should be made available for code sharing. Given these decisions, it must be decided how much of an airline's capacity should be made available to the alliance partners. Graf and Kimms (2011, 2013) have developed procedures based on real options to solve this problem for a two-airline alliance. However, with more than one airline involved in the sale of tickets, the problem of how to divide the profit amongst them arises. Kimms and Çetiner (2012) as well as Çetiner and Kimms (2013) introduced procedures to allocate the alliance's revenue among the partners in fair ways so that none of them has an incentive to leave the alliance. Their procedures are based on the nucleolus concept from cooperative game theory and turned out to be very effective. Topaloglu (2012) proposed a decomposition approach for determining alliance booking limits and transfer prices based on a centralized Deterministic Linear Program (DLP). Belobaba and Jain (2013) described the technical difficulties involved in the information sharing process faced by alliance RM and proposed information sharing mechanisms to overcome these.

All these authors assume a cooperative attitude on the side of the partners. However, despite cooperating in certain aspects, the alliance members often remain competitors in other aspects and strive for revenue maximization. In this paper, we investigate the problem of two airlines that on the one hand cooperate within an alliance but on the other hand continue to compete for customers within a revenue management (RM) setting. In this setting, two types of competition arise which are called horizontal and vertical competition in Netessine and Shumsky (2005) who first took this circumstance into account. We adopt their terminology in this paper.

In *vertical competition* two airlines have to decide on the number of seats to reserve for connecting (code shared) passengers changing planes at a stopover city. Different legs of a multi-leg itinerary are operated by different airlines. Thus, one airline can sell tickets for products which occupy the partner's aircraft. In this setting the airlines must choose how many seats to protect for local and connecting passengers in absence of cooperation or coordination. An airline's booking limit for code-shared tickets is thus affected by the partner's booking limit for them. In *horizontal competition*, on the other hand, the competitors offer identical substitutable products and customers can request a ticket from the competitor if they are denied by their preferred airline.

Wright et al. (2010) also investigated some competitive issues that arise within an alliance, but their approach is different. Namely, they ignored horizontal competition and focused only on the vertical competition the airlines face. Secondly, they did not compute the players' booking limits but rather assumed that the airlines decide for every request whether it should be accepted or not within a Markov game. In this case, the authors provided rules based on bid prices for accepting or denying a request. Altogether, the authors focused on examining which type of revenue sharing agreements, static or dynamic, generated the highest revenues to the alliance and at the same time provided incentives for the airlines to stay in the alliances. Wright (2014) implied incomplete information for the case that the alliance partners were unable or unwilling to share certain information concerning code sharing. He introduced a decomposition rule for a central dynamic program to determine approximate bid prices in an airline alliance for the individual partners.

Our approach is more related to that of Netessine and Shumsky (2005) who were the first ones to use non-cooperative game theory for determining optimal booking limits for code shared products in an alliance.. However, they considered only horizontal or only vertical competition and focused on one-leg and two-leg itineraries only, respectively. Further, the authors considered only two fare classes. Hu et al. (2013) used the same setup, yet they described the alliance formation and operation process as a two-stage game. In the first stage, a cooperative game was used to determine optimal revenue sharing rules for code shared products, while the second stage uses a non-cooperative game to determine optimal booking limits for all products using the airlines' legs. The authors focused on revenue sharing mechanisms that lead to maximal revenues for the complete alliance and modeled the individual partners' decisions so that their models incorporated the central solution. We, on the other hand, consider simultaneous horizontal and vertical competition in $\mathcal{F}$ classes. To the best of our knowledge, we are the first to address both of these types of competition simultaneously within airline alliance networks. Decisions about revenue sharing scheme are not treated here.

Transchel and Shumsky (2012) introduced a closed-loop dynamic pricing game for alliance partners that operate a parallel and substitutable flight. On the one hand the competitors are assumed to compete horizontally on this flight while on the other hand they have to set prices for their local and for the their code-shared products. We do not consider the situation in which both partners operate the same route and at the same time share codes on it. Instead, we assume that products subject to horizontal competition are not subject to vertical competition and vice versa.

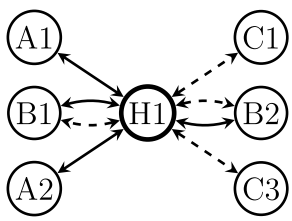
In the next Section our model is formulated. Section 3 provides a computational study followed by a conclusion in Section 4.

2 Model Formulation

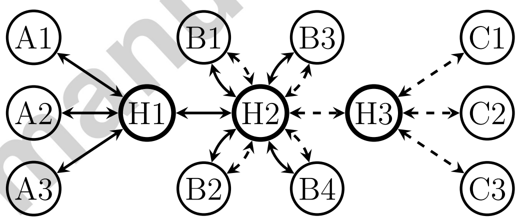
As the previous sections showed, the amount of research considering competition in airline alliances is close to nothing. The mentioned publications are very restricted in use since the competitors operate different legs. To the best of our knowledge, no paper so far considered simultaneous horizontal and vertical competition within alliances on a network with $\mathcal{F}$ classes. We intend to fill this gap in this paper. The model we present here is an extension of the DLP first described by Williamson (1992). The DLP ignores all stochastic information and is thus very simple. Yet, it allows computing an approximation of the optimal solution of the network capacity allocation problem effectively and fast which cannot be solved optimally except for very small instances because the number of states in a dynamic program used for solving the problem grows exponentially (Talluri and van Ryzin, 2004, p. 92).

In the remainder of this paper we will call the involved airlines interchangeably *competitors* or, in a game-theoretic sense *players*. Consider two airlines, denoted as $a \in \{1, -1\}$. In this context, index a refers to the considered player, while $-a$ refers to the competitor. The sets of Origin–Destination (O&D) pairs and fares offered by both players together are P and F, respectively. A combination of an O&D pair in a fare class will be called *product*. The set of flight legs served by both airlines together is L. Player a offers a set $P^a \subseteq P$ of O&D pairs (indexed $1, \ldots, \mathcal{P}^a$) in a set F of fare classes (indexed $1, \ldots, \mathcal{F}$) on a network with a set of $L^a \subseteq L$ legs (indexed $1, \ldots, \mathcal{L}^a$).

We will define the further notions with the help of Figure 1 which shows simplified versions of the networks used in the computational study. The solid and dashed arcs represent the different airlines' networks L^a, respectively. We do not show connecting routes, but these are certainly available in the networks, e.g. a flight from A1 to A2 with a stopover in H1. As the figure shows, in order for code-sharing to be applicable, the networks have to have a common connecting airport at which the code shared customers can switch from one airline's plane to the other's. In our case the connecting hubs are the hubs H1 in Figure 1a) and H2 in Figure 1b).



a) Network with One Hub



b) Network with Two Hubs

Figure 1: Simple Network Structures

The O&D pairs for which the players' demand is affected by horizontal competition are contained in the set $A = \{P^a \cap P^{-a}\}$. These are made up of itineraries offered by both competitors, e.g. itinerary B1–H1 in Figure 1a). Since we have only two competitors, we need no superscript denoting the player for this set. We assume that both players offer the same fare classes and that competition and code sharing thus affects all classes of an O&D pair.

O&D pairs with demand not affected by horizontal competition make up the set $NA^a = \{P^a \backslash P^{-a}\}$ and are indicated by arcs without overlapping legs by the competitor. These itineraries are available for vertical competition. E.g. O&D pairs A1–H1 in Figure 1a) and A1–H2 in Figure 1b) belong to the set NA^a with the solid arcs belonging to player

a's network. This set includes the set $CS^a \subseteq NA^a$ of itineraries available for code-shared products. One part of a code-shared itinerary is carried out by one airline and another part is carried out by the competitor.

To simplify modeling, we further divide the set CS^a into the sets of inbound itineraries $In^a \subseteq CS^a$ originating in the competitor's network and outbound itineraries $Out^a \subseteq CS^a$ originating in player a's own network. E.g. the itineraries from H1 to A1 or to A2 in Figure 1b) belong to the set In^a of player a if the journey originated in the competitor's network, while the itinerary from A1 or from A2 to H1 belongs to this player's set Out^a if the final destination lies in the competitor's network. A code-shared itinerary is a combination of one player's outbound and the other player's inbound itinerary.

We assume the competitors to interact in a game of complete information, i.e. both have complete knowledge about all relevant information concerning the competitor and both know about this fact. Although this is somewhat unrealistic due to technical or legal aspects concerning alliance cooperation (see e.g. Boyd, 1998, Shumsky, 2006, and Vinod, 2005), it simplifies the analysis and allows focusing on the main issues. This assumption was also made in e.g. Netessine and Shumsky (2005), Li et al. (2008), as well as Grauberger and Kimms (2014b). An airline e.g. can conclude from the type of aircraft employed by the competitor the number of seats available on its plane; prices are public information available through the internet. No buy-ups or buy-downs are considered. After being refused by his preferred airline for a product affected by horizontal competition, a costumer requests the same product from the competitor. If he is refused there too, the customer is lost. No cancellations and no-shows are considered. Every customer who buys a ticket also appears at the time of departure.

The initial demand player a expects to receive for a non-code-shared product during the booking period is d^a_{pf}. The demand for code-shared products is $\hat{d}_{pqf}$ and is not differentiated after players. It is differentiated by the itinerary p served by one player, the itinerary q served by the competitor and the fare class f. $\mathbf{M}^a$ is the incidence matrix for player a's network with $M^a_{lp} = 1$, if leg l is used by O&D pair p and $M^a_{lp} = 0$ otherwise. This means that all products occupy only one seat on the planes they use. Player a's total capacity on leg l is C_l^a. His revenue for a non-code shared product is π_{pf}^a and the revenue for a code-shared-product is $\hat{\pi}_{pqf}^a$. The product's revenue corresponds to the price an airline charges for it because variable costs are negligible in airline RM applications. For horizontal competition, the proportion of customers requesting a product from player a if they are denied a ticket by their preferred airline $-a$ is $\alpha_{pf}^{-a,a} \in [0, 1]$.

The decision variables for airline a are its partitioned booking limits b_{pf}^a for its non-code-shared products and $\hat{b}_{pqf}^a$ for its code shared products. Altogether, airline a solves model $\mathcal{M}^a$ consisting of (1) – (9) in response to the competitor's behavior:

$$\max \quad r^a(b_{pf}^a, \hat{b}_{pqf}^a) = \sum_{p=1}^{\mathcal{P}^a} \sum_{f=1}^{\mathcal{F}} \pi_{pf}^a b_{pf}^a + \sum_{p \in In^a} \sum_{q \in Out^{-a}} \sum_{f=1}^{\mathcal{F}} \hat{\pi}_{pqf}^a \hat{b}_{pqf}^a + \sum_{p \in Out^a} \sum_{q \in In^{-a}} \sum_{f=1}^{\mathcal{F}} \hat{\pi}_{pqf}^a \hat{b}_{pqf}^a \tag{1}$$

$$\text{subject to} \quad b_{pf}^a \le d_{pf}^a \qquad p \in NA^a, f \in F \tag{2}$$

$$b_{pf}^a \le d_{pf}^a + \lfloor \alpha_{pf}^{-a,a} (d_{pf}^{-a} - b_{pf}^{-a})^+ \rfloor \qquad p \in A, f \in F \tag{3}$$

$$\hat{b}_{pqf}^a \le \min\{\hat{d}_{pqf}; \hat{b}_{pqf}^{-a}\} \qquad p \in In^a, q \in Out^{-a}, f \in F \tag{4}$$

$$\hat{b}_{pqf}^a \le \min\{\hat{d}_{pqf}; \hat{b}_{pqf}^{-a}\} \qquad p \in Out^a, q \in In^{-a}, f \in F \tag{5}$$

$$C_l^a \ge \sum_{p=1}^{\mathcal{P}^a} \sum_{f=1}^{\mathcal{F}} M_{lp}^a b_{pf}^a + \sum_{p \in In^a} \sum_{q \in Out^{-a}} \sum_{f=1}^{\mathcal{F}} M_{lp}^a \hat{b}_{pqf}^a + \sum_{p \in Out^a} \sum_{q \in In^{-a}} \sum_{f=1}^{\mathcal{F}} M_{lp}^a \hat{b}_{pqf}^a \qquad l \in L^a \tag{6}$$

$$b_{pf}^a \ge 0 \qquad p \in P^a, f \in F \tag{7}$$

$$\hat{b}_{pqf}^a \ge 0 \qquad p \in In^a, q \in Out^{-a}, f \in F \tag{8}$$

$$\hat{b}_{pqf}^a \ge 0 \qquad p \in Out^a, q \in In^{-a}, f \in F \tag{9}$$

The objective function (1) maximizes the total revenue of airline a summing over the revenues for the products times their booking limits. Constraint (2) limits the booking limit for a product which is not available for code sharing and not affected by horizontal competition to this product's demand. Restriction (3) limits the booking limit for a product affected by horizontal competition to its rounded down total expected demand.

Rounding down the values leads to all parameters on the right hand side being integer-valued which allows for defining the variables continuously and still receiving integer solutions if the entries in the matrix $\mathbf{M}^a$ are only 0 or 1 and the network is acyclic (see also de Boer et al., 2002, p. 77). All these conditions apply in our model. The term $(d_{pf}^{-a} - b_{pf}^{-a})^+$, which is equal to $\max\{0; (d_{pf}^{-a} - b_{pf}^{-a})\}$, on the right hand side of Restriction (3) stands for the non-negative number of booking requests denied by the competing airline $-a$ (spill over demand of $-a$).

Constraints (4) and (5) restrict the booking limits for the code-shared products which are required to not exceed the minimum of the demand for this product and the competitor's booking limit for it. Restriction (6) is the capacity restriction requiring the sum of all booking limits for products utilizing a player's leg not to exceed the airplane's capacity on this leg. Finally, Restrictions (7) – (9) define the booking limits to be non-negative continuous numbers.

In game-theoretic terms, an optimal solution to a player's model $\mathcal{M}^a$ in terms of his vector of booking limits constitutes a *pure strategy*. Moreover, since the respective player responds to his competitor's strategy made up of the booking limits b_{pf}^{-a} and $\hat{b}_{pqf}^{-a}$, an optimal solution for model $\mathcal{M}^a$ constitutes a *best response*. A Nash equilibrium (NE) can be defined as a state in which the players' strategies are mutual best responses (Fudenberg and Tirole, 1991, p. 11). In our context, an NE is a state in a competitive situation in which none of the airlines can increase its revenue by choosing a different vector of booking limits (by deviating from its equilibrium strategy). I.e. no strategy can yield a player a higher payoff than the equilibrium strategy given that the competitor plays his equilibrium strategy. A *pure-strategy NE* is one in which all both players employ only pure strategies.

3 Computational Study

We used the model from the previous section to compute a pure-strategy NE and compared the results with those from the case when a central decision maker controlled the capacity. Computation of pure NE is motivated by the fact that it makes decision making easier because the actions to take (booking limits to set) can be interpreted and implemented directly. An algorithm for computing pure-strategy NE as mutual best responses based on DLPs was recently introduced by Grauberger and Kimms (2014c) and is described in Appendix A of this paper. This algorithm was employed for our computational study. It also came to use in Grauberger and Kimms (2014a) for computing approximate NE in airline RM games under simultaneous price and quantity competition.

3.1 Test Bed

We assumed two airlines with similar, overlapping networks as in Figure 1 to form an alliance. The number of hubs in the players' networks was identical, i.e. either both had one hub or both had two. Both airlines' networks were assumed to have numbers of 20, 40, 60, 80, and 100 airports to be connected with each hub. These are realistic numbers comparable to the different sized networks of the members of the three major airline alliances OneWorld, SkyTeam, and Star Alliance. There, at least two thirds of the members have one or two hubs and most airlines do not serve more than 200 destinations (see also OneWorld, 2014; SkyTeam, 2014; StarAlliance, 2014). The spoke airports were connected with each other only through the hubs, i.e no direct flights between two spoke airports were allowed. We assumed the percentage of O&D pairs affected by horizontal competition (the competition intensity, CI) to amount to 25%, 50%, and 75% of an airline's total offered itineraries. We set the capacities on half of the flights going in and out of the hubs to 100 seats and 200 each and the capacities on the legs connecting the hubs to 300.

We assumed that products affected by horizontal competition were not subject to code shares and vice versa. The code shared products had to originate in one player's network and had to end in the competitor's network and affected only those itineraries which were not affected by horizontal competition and were available for code sharing. Furthermore, if an itinerary entailed a part served by both players and a part served by only one airline (like e.g. B1–C1 in the above figures), we assumed that the whole itinerary was served by the latter airline. I.e. the mentioned itinerary B1–C1 was not affected by competition but was completely carried out by the airline with the dashed marked network. With two hubs in each of the players' networks, we assumed the networks to be connected only in hub H2.

Out of all possible non-code-shared products, we chose randomly so many that each leg was demanded by at most 60 products. This way we could make sure that no player's demand exceeds his capacity which would make no difference to the non-competitive model. For these products ten pairs of original demand d^a_{pf} for non-code-shared products was generated randomly from the Poisson distribution with means of $\mu \in \{2, 4, 6\}$ for each network setting. The demand $\hat{d}_{pqf}$ for the code shared products was assumed to have a mean of $\mu = 1$. With the different mean demands and capacities, we had different demand-to-capacity ratios, computed as $\frac{60*\mu}{C_l}$. For the non-code-shared products, on the legs with capacities of 100, 200, and 300, these ratios amounted to at most 1.2, 0.6, and 0.4 with a mean demand of $\mu = 2$, at most 2.4, 1.2, and 0.8 with $\mu = 4$, and at most 3.6, 1.8, and 1.2 with $\mu = 6$, respectively. In fact the ratios were lower, though, since the Poisson distribution is positively skewed which means that lower values are more probable than higher ones and which is most noticeable at such low values for the mean demand. Depending on the value of CI, the competition intensity, the code shared products raised the demand-to-capacity ratios by at most $\frac{60*(1-CI)}{C_l}$.

The prices for the non-code-shared products were drawn from the uniform distribution. The intervals for the first (i.e. most expensive, full) class lay between 300 and 400 for single-leg products, between 600 and 800 for two-leg products and between 900 and 1200 for three-leg products, respectively. We assumed four fare classes and the interval boundaries of the higher classes 2 – 4 were 75%, 50%, and 25% of the first class boundaries, respectively. The prices were drawn individually for both players to avoid completely identical prices since in practice competitors seldom charge completely identical prices.

We assumed that a code-shared product's revenue contributed to the operating airlines' total revenue as much as if the airline sold the corresponding single-leg or two-leg flight to local passengers, which was also assumed in Netessine and Shumsky (2005). This allowed for less differentiation of a player's inbound code shared products since they all generated him the same revenue, no matter where they originated in the competitor's network. Hence, we needed only one variable per class of an inbound code shared itinerary which enabled us to replace Constraint (4) with Constraint (10).

$$\hat{b}^a_{pf} \leq \min \left\{ \sum_{q \in Out^{-a}} \hat{d}_{pqf}; \sum_{q \in Out^{-a}} \hat{b}^{-a}_{pqf} \right\} \qquad p \in In^a, f \in F \tag{10}$$

This lead to a reduction of the number of variables needed to model a player's booking limits for his inbound code shared products from the combination $|Out^{-a}| * |In^a| * \mathcal{F}$ to the combination $|In^a| * \mathcal{F}$. This bundling is motivated by the player operating the inbound itineraries merely communicating the number of seats available for code sharing and the competitor having to decide with which of his outbound itineraries he wants to use these seats. The booking limits for the outbound code-shared products could not be bundled in this manner (i.e. irrespective of their final destination) because they had different revenues connected to them and the players had to be able to differentiate between the individual outbound products in case the competitor's inbound booking limit for a product was lower than the total demand for it.

The ratios for the spill-over demand in the horizontal competition were functions of the players' prices to model the customers' willingness to pay: $\alpha^{-a,a}_{pf} = 0.5 - 0.1 \frac{\pi^a_{pf} - \pi^{-a}_{pf}}{\overline{\pi}_{pf} - \underline{\pi}_{pf}}$. Here, $\underline{\pi}_{pf}$ and $\overline{\pi}_{pf}$ are the lower and upper interval boundaries from the price distribution, respectively. The denominator of the fraction is fixed for a product by the interval boundaries $\overline{\pi}_{pf} - \underline{\pi}_{pf}$ from the price distribution. After generating the product prices, the enumerator's value is determined. Since the players' prices lie between the interval boundaries, the fraction's value turns out between -1 and 1. It is negative when $\pi^a_{pf} < \pi^{-a}_{pf}$, positive when $\pi^a_{pf} > \pi^{-a}_{pf}$ or 0 when $\pi^a_{pf} = \pi^{-a}_{pf}$. Multiplying the fraction with -0.1, the value for $\alpha^{-a,a}_{pf}$ is raised above 0.5 when $\pi^a_{pf} < \pi^{-a}_{pf}$, while the value for $\alpha^{-a,a}_{pf}$ is reduced below 0.5 when $\pi^a_{pf} > \pi^{-a}_{pf}$. With $\pi^a_{pf} = \pi^{-a}_{pf}$, we have $\alpha^{-a,a}_{pf} = 0.5$. Hence, if both competitors charged identical prices, half of the customers requested a ticket from the competitor if they were refused by their preferred airline. Charging a higher price than the competitor meant less spill over demand for the respective airline and vice versa. The lowest value $\alpha_{pf}^{-a,a}$ could attain was 0.4 and the highest value was 0.6. Note that the higher a fare class (the cheaper a ticket), the more sensitive the customers reacted to price differences because the intervals for the prices were narrower.

The model presented above might have several optimal solutions which would cause coordination issues concerning the interpretation of the best response computed with it. This, on the other hand, might cause coordination issues about which NE would be found. To avoid these issues, Grauberger and Kimms (2014c) proposed a perturbation of the product prices to receive unique optimal solutions. The perturbing parameter ϵ^a had to be set so that the (unique) optimal solution of the perturbed problem is also an optimal solution in the original one. Hence, we set the value so as to make sure that two (or more) equal prices in the original problem were unequal in the perturbed problem and that unequal prices in the original problem remained unequal. At the same time a (sum of) price(s) which was lower (or higher) then another (sum of) price(s) in the original problem had to remain lower (or higher) in the perturbed problem. Here, we had to make sure that the perturbed summands of the smaller sum did not add up to a larger value than the perturbed summands of the higher sum. Since the price difference only amounted to 1 in the smallest case, it sufficed to make sure that the perturbing values added to the smaller sum's summands did not add up to ≥ 1. Assuming 300 as the highest capacity on a plane and all products only occupying one seat, the highest number of summands in our case was 300. Hence, we had to make sure that (in the worst case) the first 300 values of our perturbing vector did not sum up to ≥ 1. In order to avoid problems of storing the exponentiated perturbing value in sufficient precision, we multiplied all prices by 300 (which led to the smallest positive difference between two prices to amount to 300) and set $\epsilon^a = 0.9999$ for both players. After the optimization we simply had to subtract the perturbing parameters from the prices and divide the prices by 300 to receive the original prices and payoffs. We tried different perturbation orders for the products; "alphabetic by O&D name, then increasing by fare class" and "increasing by fare class, then alphabetic by O&D name".

Altogether, we constructed 1,800 instances (10 types of networks, three values for the competition intensity, two ways of ordering the product prices for the perturbation, and 10 pairs of demand for each of these cases, and 3 values of mean demand). The computation time for each instance was limited to three hours.

3.2 Results

The models and algorithms were implemented in C++ and version 5.6.2 of the Gurobi solver was used to solve the models. The study was done on a computer using an Intel Core i7-620M CPU (2.67 GHz; 2 cores), 8 GB memory and Windows 7 SP1 (64-bit).

The computational results show that the algorithm presented in Grauberger and Kimms (2014c) is suited for real-sized airline networks given in two-partner alliances and can compute NE with simultaneous horizontal and vertical competition. Altogether, an NE was found in 99.33% (1,788 out of 1,800) instances. Next, we only show the results for the instances with the prices perturbed in order "alphabetic by O&D name, then increasing by fare class". The results for perturbation order "increasing by fare class, then alphabetic by O&D name" can be found in Appendix B of this paper. The results for both perturbation orders are similar, though. Figures 2 – 4 show the average computation times in minutes needed for finding a pure NE in the different networks with different competition intensities (CI) as ratios for the products affected by horizontal competition.

The computation time needed to find an NE rose with the network size, the mean demand, and the competition intensity. While an NE was found in seconds in the smallest networks with a mean demand of $\mu = 2$, in the largest networks with a mean demand of $\mu = 6$, and CI $= 0.75$ about 39 minutes were needed on average to find an NE. The most time needed by the algorithm to terminate with an NE was 161 minutes. Such "outliers" are responsible for the graphs being not monotone since they raise the average. Altogether, the computation times are acceptable even for large networks and are similar to the times without vertical competition.

We compared the summed equilibrium payoffs to the cooperative (central) payoff and to the summed non-competitive payoffs, i.e. when both airlines ignored competition and

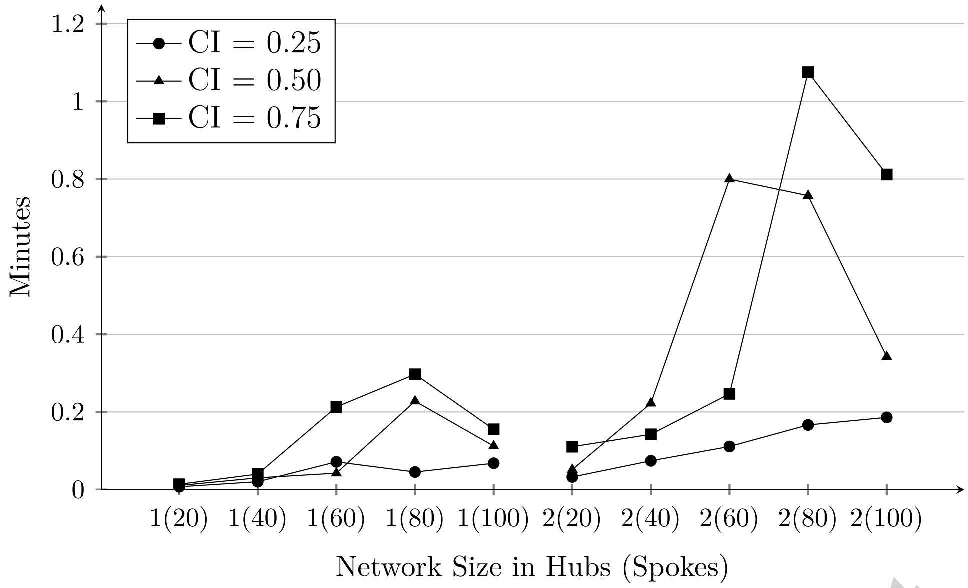


Figure 2: Average Computation Times in Minutes with $\mu = 2$

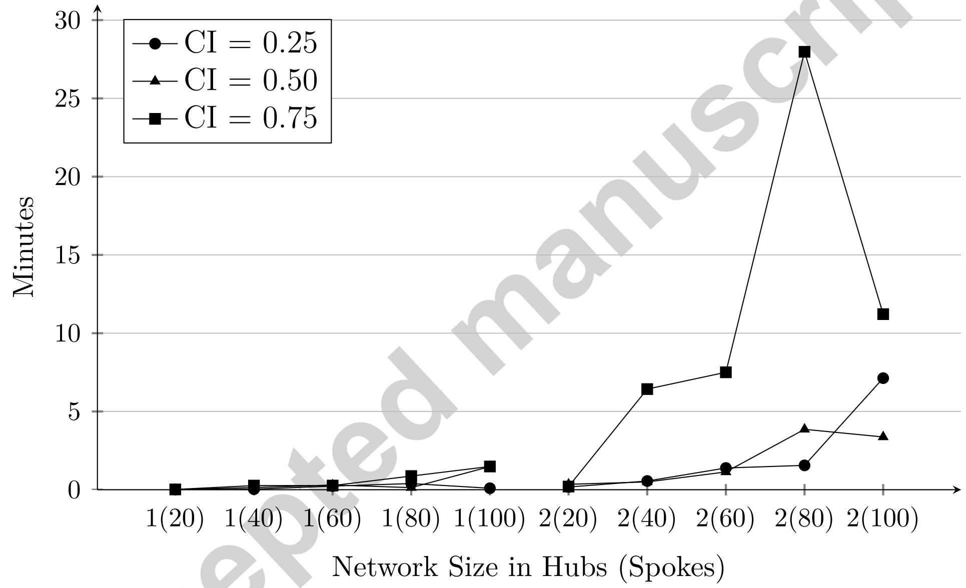


Figure 3: Average Computation Times in Minutes with $\mu = 4$

did not take into account neither spill over nor code shared demand. For the cooperative case the prices for the products affected by competition were set as average prices charged by the players. The other products' prices were taken directly as the player's respective price. The demand for the products and the capacities on the legs used by these products were taken as the sum of the players' values. The results are shown in Tables 1 – 3. Here, C, NE, and NC stand for central, equilibrium summed, and non-competitive summed payoffs, respectively. The entries in the first row of columns 3 – 12 stand for the networks

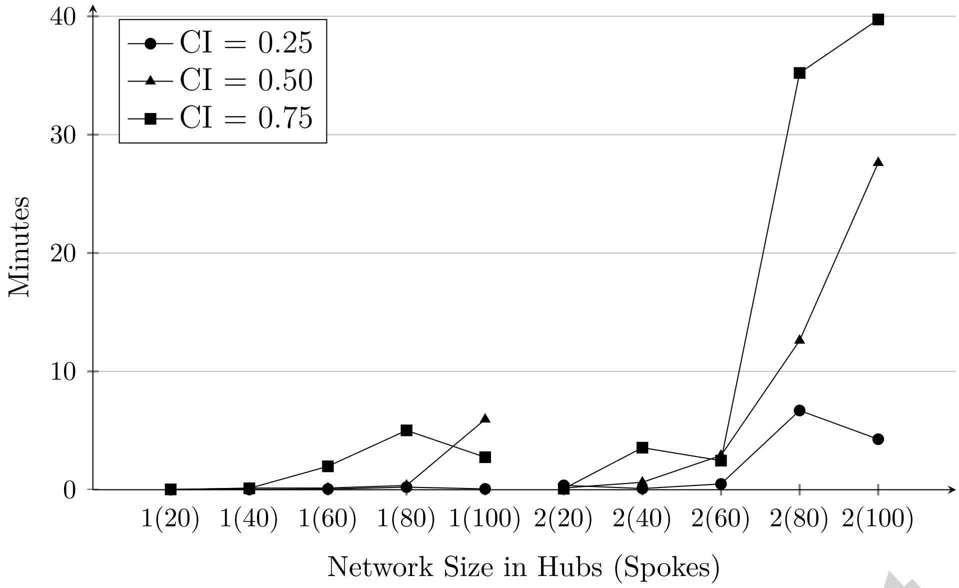


Figure 4: Average Computation Times in Minutes with $\mu = 6$

as "Hubs(Spokes)". The revenues were computed without the perturbing parameters ϵ. These were not needed for the cooperative case and for comparing the values with each other, we used the non-perturbed prices for the other settings as well.

CI	Ratio	1(20)	1(40)	1(60)	1(80)	1(100)	2(20)	2(40)	2(60)	2(80)	2(100)
0.25	NE/C	91.44%	91.66%	95.39%	95.07%	97.10%	97.46%	97.95%	99.33%	99.37%	98.92%
	NC/C	81.27%	91.04%	95.13%	94.16%	96.75%	87.27%	92.23%	96.22%	96.08%	96.91%
	NC/NE	88.87%	99.33%	99.72%	99.04%	99.64%	89.55%	94.17%	96.87%	96.69%	97.96%
0.50	NE/C	94.56%	92.51%	96.02%	95.56%	97.54%	97.97%	98.32%	99.58%	99.47%	99.00%
	NC/C	83.57%	91.91%	95.75%	94.59%	97.12%	88.32%	92.24%	96.22%	96.00%	96.91%
	NC/NE	88.38%	99.35%	99.72%	98.99%	99.57%	90.15%	93.82%	96.63%	96.52%	97.89%
0.75	NE/C	96.37%	95.73%	98.95%	97.94%	99.21%	99.19%	99.66%	99.69%	99.73%	99.59%
	NC/C	91.00%	95.31%	98.83%	97.32%	99.04%	93.78%	97.44%	97.67%	97.87%	98.54%
	NC/NE	94.42%	99.56%	99.88%	99.36%	99.83%	94.55%	97.77%	97.98%	98.13%	98.95%

Table 1: Average Payoff Ratios with $\mu = 2$

CI	Ratio	1(20)	1(40)	1(60)	1(80)	1(100)	2(20)	2(40)	2(60)	2(80)	2(100)
0.25	NE/C	94.67%	97.88%	98.94%	98.50%	99.33%	95.93%	97.88%	98.95%	98.83%	99.26%
	NC/C	94.58%	97.82%	98.87%	98.42%	99.25%	95.82%	97.80%	98.85%	98.68%	99.20%
	NC/NE	99.90%	99.93%	99.92%	99.92%	99.92%	99.89%	99.92%	99.90%	99.85%	99.94%
0.50	NE/C	95.44%	98.36%	99.29%	98.85%	99.61%	96.67%	97.99%	99.08%	98.92%	99.37%
	NC/C	95.27%	98.21%	99.13%	98.69%	99.46%	96.49%	97.86%	98.91%	98.73%	99.25%
	NC/NE	99.82%	99.85%	99.84%	99.84%	99.84%	99.81%	99.87%	99.83%	99.81%	99.87%
0.75	NE/C	97.93%	99.35%	100.15%	99.74%	100.21%	98.55%	99.60%	99.58%	99.53%	99.79%
	NC/C	97.71%	99.16%	99.94%	99.52%	99.98%	98.35%	99.43%	99.39%	99.35%	99.63%
	NC/NE	99.77%	99.81%	99.79%	99.78%	99.77%	99.80%	99.82%	99.81%	99.82%	99.84%

Table 2: Average Payoff Ratios with $\mu = 4$

As the tables show, the summed competitive and non-competitive payoffs were usually lower that the central payoffs. The payoffs in the NE were slightly higher than those in the

CI	Ratio	1(20)	1(40)	1(60)	1(80)	1(100)	2(20)	2(40)	2(60)	2(80)	2(100)
0.25	NE/C	97.48%	99.20%	99.74%	99.39%	99.88%	98.17%	99.17%	99.67%	99.54%	99.74%
	NC/C	97.40%	99.08%	99.61%	99.28%	99.76%	98.10%	99.10%	99.58%	99.44%	99.66%
	NC/NE	99.91%	99.87%	99.87%	99.89%	99.88%	99.93%	99.93%	99.91%	99.90%	99.92%
0.50	NE/C	98.06%	99.63%	100.11%	99.77%	100.24%	98.68%	99.37%	99.82%	99.70%	99.90%
	NC/C	97.91%	99.38%	99.85%	99.53%	99.99%	98.51%	99.23%	99.66%	99.54%	99.74%
	NC/NE	99.84%	99.75%	99.74%	99.76%	99.75%	99.83%	99.86%	99.84%	99.83%	99.84%
0.75	NE/C	99.35%	100.35%	100.73%	100.40%	100.73%	99.69%	100.20%	100.12%	100.09%	100.18%
	NC/C	99.16%	99.95%	100.33%	100.03%	100.34%	99.47%	99.99%	99.90%	99.86%	99.96%
	NC/NE	99.81%	99.60%	99.61%	99.63%	99.61%	99.78%	99.79%	99.78%	99.77%	99.78%

Table 3: Average Payoff Ratios with $\mu = 6$

non-competitive situations, though. This is due to Constraints (3) – (5) which take into account a player's spill over and code shared demand and increase a player's demand and thus payoff. This effect increases with a higher competition intensity because the demand of more products is potentially raised through spill-over and code shared demand. The higher the competition intensity (the more products are affected by spillover demand), the higher an airline's NE payoff and the lower the ratio between the NE payoffs and the non-competitive payoffs. However, the difference between the central payoff compared to the competitive and non-competitive ones decreases with a higher demand and CI indicating that with a high demand and/or competition intensity, the revenues in the decentralized decisions are almost as high as those in the centralized case.

Note, however, that we only used the cooperative payoffs for relative comparison with the other values and that the cooperative payoffs cannot be compared to the other values directly because of several reasons. First, the cooperative prices were taken as averages from the competitors' prices (which falsifies the cooperative payoff) while the cooperative demand is the sum of the individual players' demand. This is also the reason why some of the ratios turn out higher than 100% (especially with a high CI) indicating a higher revenue under competition than in the central case, which would make no sense. Secondly, in case of competition not all denied customers turn to the competitor but the spill-over demand is decreased by the parameter α^a_{pf}. Under cooperation this issue does not exist and all demand is considered. This also leads to those products not affected by competition being able to access the extended capacities in the cooperative case and raising the overall load factor by using the partner's capacities. With both competitors charging identical prices for all products, CI $= 1$, and $\alpha^a_{pf} = 1$ for all products, the ratios would turn out closer to 1.

The latter case was considered in Grauberger and Kimms (2014b). There, in 105 of 120 considered instances, the summed payoffs in the NE were exactly as high as the cooperative payoff. If a difference did exist, it amounted to less than 1%.

We also compared the central and NE payoffs including code shared products and horizontal competition with the corresponding payoffs without code shared products and only horizontal competition. Tables 4 – 6 show these results. The entries therein were computed as $\frac{\text{Payoff with code sharing}}{\text{Payoff without code sharing}}$. As before, C and NE stand for "central" and "Nash equilibrium". One can see that code sharing tendentially leads to higher payoffs. Including code shared products never lead to lower payoffs than ignoring it, which is obvious since the Constraints (4) and (5) can only affect the revenue positively.

CI	Ratio	1(20)	1(40)	1(60)	1(80)	1(100)	2(20)	2(40)	2(60)	2(80)	2(100)
0.25	C	122.90%	109.75%	105.04%	106.12%	103.30%	114.52%	108.41%	103.90%	104.00%	103.10%
	NE	112.48%	100.62%	100.24%	100.92%	100.32%	111.60%	106.14%	103.19%	103.38%	102.04%
0.50	C	119.48%	108.71%	104.39%	105.68%	102.95%	113.29%	108.48%	103.93%	104.12%	103.12%
	NE	113.06%	100.57%	100.21%	100.94%	100.35%	110.77%	106.49%	103.40%	103.53%	102.08%
0.75	C	109.72%	104.84%	101.11%	102.72%	100.97%	106.77%	102.78%	102.44%	102.21%	101.45%
	NE	105.82%	100.34%	100.02%	100.54%	100.07%	105.58%	102.15%	101.97%	101.82%	100.98%

Table 4: Average Payoff Ratios with and without Code Sharing with $\mu = 2$

CI	Ratio	1(20)	1(40)	1(60)	1(80)	1(100)	2(20)	2(40)	2(60)	2(80)	2(100)
0.25	C	105.75%	102.30%	101.19%	101.62%	100.80%	104.43%	102.23%	101.11%	101.25%	100.74%
	NE	100.00%	100.00%	100.00%	100.00%	100.00%	100.03%	100.02%	100.03%	100.09%	100.01%
0.50	C	105.07%	102.03%	101.04%	101.48%	100.70%	103.91%	102.25%	101.13%	101.26%	100.75%
	NE	100.00%	100.00%	100.00%	100.00%	100.00%	100.01%	100.01%	100.06%	100.08%	100.02%
0.75	C	102.43%	101.11%	100.25%	100.70%	100.23%	102.02%	100.68%	100.67%	100.66%	100.37%
	NE	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.03%	100.03%	100.00%

Table 5: Average Payoff Ratios with and without Code Sharing with $\mu = 4$

CI	Ratio	1(20)	1(40)	1(60)	1(80)	1(100)	2(20)	2(40)	2(60)	2(80)	2(100)
0.25	C	102.81%	101.01%	100.49%	100.78%	100.34%	101.99%	100.94%	100.44%	100.55%	100.33%
	NE	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
0.50	C	102.44%	100.91%	100.43%	100.71%	100.30%	101.75%	100.95%	100.45%	100.54%	100.33%
	NE	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
0.75	C	101.19%	100.48%	100.08%	100.33%	100.09%	100.88%	100.26%	100.25%	100.28%	100.16%
	NE	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.01%	100.00%

Table 6: Average Payoff Ratios with and without Code Sharing with $\mu = 6$

It becomes clear once more, that a centralized decision leads to better (i.e. more profitable) capacity control decisions than a decentralized competitive one. This is due to code sharing improving the revenues in the centralized cases more than the summed NE revenues. Results of the experiment shown in tables also reveal that with a higher mean demand and a higher competition intensity the gain from code sharing decreases since there are less routes to share codes on. Instead, the horizontal competition is more dominant here. Also, with a higher mean demand the capacity load factor is already fairly high without code shared products which leaves less space for them, as the demand-to-capacity ratios showed in the previous section.

4 Conclusion and Outlook

We have presented a model to optimize a competitor's booking limits when it faces competition from an alliance partner. Both, horizontal and vertical competition was considered. In horizontal competition the competitors offer parallel and substitutable products and thus compete directly for customers, while in vertical competition the partners operate connecting, adjacent flights and one airline might sell tickets for products occupying the partner's aircraft. Here, a capacity competition arises because customers for the code shared products and local customers must be seated in the same plane. We distinguished the code shared products after their origin and destination into inbound and outbound products. Assuming that the code shared products generated the same revenue like the corresponding local products, we were able to reduce the number of decision variables for the inbound code shared products.

The proposed model was used in an iterative algorithm introduced in an earlier paper to compute pure-strategy Nash equilibria based on mutual best responses. We compared the NE revenues with the case when a centralized decision maker controlled the capacity. We found that the central revenues were higher than in the NE, but that this difference decreased with the network size, the competition intensity, and mean demand. After all, with a high competition intensity, NE revenues were almost as high as in the centralized case indicating almost (Pareto) optimal NE. We also compared the situations with simultaneous horizontal and vertical competition to those without code sharing and only horizontal competition. These results revealed that code sharing benefits the partners because new demand is generated and raises the revenues. The revenue of the central decision maker increased most. Yet, again the benefits decreased with larger networks, higher competition intensities and higher mean demand. The computation times were all together acceptable and an NE could be found in 1,788 out of 1,800 cases within three hours. The most time needed to find an NE was 161 minutes.

Future studies could include more than two partners in the alliance. There, attention will have to be paid about who competes with whom horizontally and vertically, since probably not all partners' networks will interfere with one another. Here, the algorithm by Daskalakis and Papadimitriou (2006) for computing pure NE in graphical games might be applicable. In such games there exist ≥ 2 players and the players' payoffs are only affected by the behavior of their "neighbors" and not by all players. The recently introduced concept of targeted competition for more than two players by Dubovik and Parakhonyak (2014) might be of help as well. Here, each player must choose the intensity with which he wants to compete with his rivals.

Also, we did not differentiate by which airline received a request for a code-shared product. In reality this might matter and would intensify competition for capacity on the code-shared flights, especially when the prices an airline receives for a code-shared product depend on who sells the ticket or on the partners' market strength. The operating carrier might accept less code-shared flights sold by the partner than if he received the requests. If the airlines sell code shared products at different prices (or in case of preferences for certain airlines on the side of the customers) a different type of horizontal competition might arise since the customers could request the same code shared product from the competitor if they are denied once by their preferred airline.

Further, while we assumed the players to have equal market power (or decision power within the alliance), it might be different in certain cases. Here, a Stackelberg game could be used to model the players' different market power or importance within the alliance in future research. Connected to this, an interesting point to be considered is when only the total demand for a product is given, not each competitor's individual demand. Then, the total demand for a product might be split based on the players' booking limits. Li et al. (2007) as well as Li et al. (2008) have proposed such demand splitting rules for single-leg, two-class problems. Their work could serve as a ground to develop such rules for the network case with $\mathcal{F}$ classes. Finally, the alliance problem could be modeled and solved via a multi-objective optimization problem with the different objectives standing for the partners' revenue interests.

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References

Belobaba, P. P. and Jain, H., (2013), Alliance Revenue Management in Practice: Impacts of Bid Price Sharing and Dynamic Valuation, *Journal of Revenue and Pricing Management*, **12** (6), pp. 475–488.

Boyd, E. A., (1998), Airline Alliances, *OR/MS Today*, **25** (5), pp. 28–31.

Çetiner, D., (2013), *Fair Revenue Sharing Mechanisms for Strategic Passenger Airline Alliances*, Springer, Berlin.

Çetiner, D. and Kimms, A., (2013), Assessing Fairness of Selfish Revenue Sharing Mechanisms for Airline Alliances, *Omega*, **41** (4), pp. 641–652.

Daskalakis, C. and Papadimitriou, C. H., (2006), Computing Pure Nash Equilibria in Graphical Games via Markov Random Fields, in: Feigenbaum, J., ed., *Proceedings of the 7th ACM Conference on Electronic Commerce*, ACM Press, New York, pp. 91–99.

de Boer, S. V., Freling, R., and Piersma, N., (2002), Mathematical Programming for Network Revenue Management Revisited, *European Journal of Operational Research*, **137** (1), pp. 72–92.

Dubovik, A. and Parakhonyak, A., (2014), Drugs, Guns, and Targeted Competition, *Games and Economic Behavior*, **87**, pp. 497–507.

Fudenberg, D. and Tirole, J., (1991), *Game Theory*, MIT Press, Cambridge.

Graf, M. and Kimms, A., (2011), An Option-Based Revenue Management Procedure for Strategic Airline Alliances, *European Journal of Operational Research*, **215** (2), pp. 459–469.

Graf, M. and Kimms, A., (2013), Transfer Price Optimization for Option-Based Airline Alliance Revenue Management, *International Journal of Production Economics*, **145** (1), pp. 281–293.

Grauberger, W. and Kimms, A., (2014a), Airline Revenue Management Games under Simultaneous Price and Quantity Competition, Working Paper, University of Duisburg-Essen, Duisburg.

Grauberger, W. and Kimms, A., (2014b), Computing Approximate Nash Equilibria in General Network Revenue Management Games, *European Journal of Operational Research*, **237** (3), pp. 1008–1020.

Grauberger, W. and Kimms, A., (2014c), Computing Pure Nash Equilibria in Network Revenue Management Games, Working Paper, University of Duisburg-Essen, Duisburg.

Hu, X., Caldentey, R., and Vulcano, G., (2013), Revenue Sharing in Airline Alliances, *Management Science*, **59** (5), pp. 1177–1195.

Kimms, A. and Çetiner, D., (2012), Approximate Nucleolus-Based Revenue Sharing in Airline Alliances, *European Journal of Operational Research*, **220** (2), pp. 510–521.

Li, M. Z. F., Oum, T. H., and Anderson, C. K., (2007), An Airline Seat Allocation Game, *Journal of Revenue and Pricing Management*, **6** (4), pp. 321–330.

Li, M. Z. F., Zhang, A., and Zhang, Y., (2008), Airline Seat Allocation Competition, *International Transactions in Operational Research*, **15** (4), pp. 439–459.

Netessine, S. and Shumsky, R. A., (2005), Revenue Management Games: Horizontal and Vertical Competition, *Management Science*, **51** (5), pp. 813–831.

O'Neal, J. W., Jacob, M. S., Farmer, A. K., and Martin, K. G., (2007), Development of a Codeshare Flight-Profitability System at Delta Air Lines, *Interfaces*, **37** (5), pp. 436–444.

OneWorld, (2014), Member Airlines, URL: http://www.oneworld.com/member-airlines/overview.

Oum, T. H. and Park, J.-H., (1997), Airline Alliances: Current Status, Policy Issues, and Future Directions, *Journal of Air Transport Management*, **3** (3), pp. 133–144.

Oum, T. H., Yu, C., and Zhang, A., (2001), Global Airline Alliances: International Regulatory Issues, *Journal of Air Transport Management*, **7** (1), pp. 57–62.

Park, J.-H., (1997), The Effects of Airline Alliances on Markets and Economic Welfare, *Transportation Research Part E: Logistics and Transportation Review*, **33** (3), pp. 181–195.

Shumsky, R., (2006), The Southwest Effect, Airline Alliances and Revenue Management, *Journal of Revenue and Pricing Management*, **5** (1), pp. 83–89.

SkyTeam, (2014), SkyTeam Members, URL: http://www.skyteam.com/en/About-us/Our-members/.

StarAlliance, (2014), Member Airlines, URL: http://www.staralliance.com/en/about/member_airlines/.

Talluri, K. T. and van Ryzin, G. J., (2004), *The Theory and Practice of Revenue Management*, Kluwer Academic Publishers, Boston.

Topaloglu, H., (2012), A Duality Based Approach for Network Revenue Management in Airline Alliances, *Journal of Revenue and Pricing Management*, **11** (5), pp. 500–517.

Transchel, S. and Shumsky, R. A., (2012), Frenemies: Price Competition between Codesharing Airlines, *International Annual Conference of the German Operations Research Society (GOR)*, Hannover (Germany).

Vinod, B., (2005), Alliance Revenue Management, *Journal of Revenue and Pricing Management*, **4** (1), pp. 66–82.

Williamson, E. L., (1992), Airline Network Seat Inventory Control: Methodologies and Revenue Impacts, Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge.

Wright, C. P., (2014), Decomposing Airline Alliances: A Bid-Price Approach to Revenue Management with Incomplete Information Sharing, *Journal of Revenue and Pricing Management*, **13** (3), pp. 164–182.

Wright, C. P., Groenevelt, H., and Shumsky, R. A., (2010), Dynamic Revenue Management in Airline Alliances, *Transportation Science*, **44** (1), pp. 15–37.

Appendix A: Computing a Pure-Strategy Nash Equilibrium

Algorithms to compute a pure-strategy NE based on the DLP were just recently proposed in Grauberger and Kimms (2014b,c). While the former is a heuristic not guaranteeing to find an exact NE, the latter is an exact algorithm that finds a pure NE with certainty if one exists in the game or terminates with the information that none exists. We applied the latter algorithm for our purposes. It was also employed in Grauberger and Kimms (2014a) to compute NE in RM settings under simultaneous price and quantity competition. To be self-contained we briefly reflect here the working principle of the algorithm and refer to the paper for further technical details.

The algorithm is initiated by providing a starting player and a starting strategy for the competitor. The players then search for best responses (booking limit strategies) iteratively until a player's best response is chosen a second time. This is called *forward search*.

When a best response is chosen a second time in two consecutive iterations, a pure NE is found. If a best response was chosen more than one iteration ago for the first time, a constraint is added to the model to forbid the current strategy using binary variables and an alternative best response in form of an alternative optimal solution for the current player's model is seeked. This way the algorithm is able to handle degenerated and non-degenerated games. In the former there might be more than one best response to a player's strategy while in the latter all best responses are unique. In our case, the degeneracy of a game depends on the number of optimal solutions to the employed models. If the models always have unique optimal solutions, the game is non-degenerated and vice versa.

If an alternative best response can be found after forbidding a strategy, the forward search continues until another best response is chosen repeatedly. It is then again checked when this best response has been chosen before and it is forbidden, if necessary. If an alternative best response cannot be found (the model becomes infeasible) for this player after forbidding the current strategy, a *backtracking procedure* is initiated which searches for an alternative for the competitor's latest best response. If an alternative best response cannot be found for him either, the algorithm goes back one more step and seeks an alternative best response for the player whose best response has been chosen before. In this manner, the procedure continues until either an alternative best response can be found and the forward search can continue or until the first iteration is reached without an alternative best response for the starting player. In the latter case the algorithm would terminate with the message that no pure NE exists in the game.

If the models employed by the players have several optimal solutions, coordination issues about which best response will be chosen and which NE will be reached may arise. In order to solve this problem and to receive unique best responses Grauberger and Kimms (2014c) suggested adding a matrix $\boldsymbol{\epsilon^a}$ (which has the same dimensions as the player's price matrix) of an exponentiated factor ϵ^a to a player's matrix of prices. For this, the prices must be sorted in a fixed order which must not be changed once it is determined. Depending on the order of perturbation, the elements in the matrix $\boldsymbol{\epsilon^a}$ are exponentiated differently. Equation (11) shows such a matrix with the perturbation order "first alphabetic by O&D name, then by fare class". The rows of the matrix stand for the O&D pairs and the columns stand for the fare classes. The second superscripts in the matrix stand for exponents which in this case are computed as "$\mathcal{F}*$ (row number $-$ 1) + column number". For further technical details we refer to Grauberger and Kimms (2014c).

$$\boldsymbol{\epsilon^a} = \left\{ \begin{array}{ccccc} \epsilon^{a,1} & \epsilon^{a,2} & \epsilon^{a,3} & \cdots & \epsilon^{a,\mathcal{F}} \\ \epsilon^{a,\mathcal{F}+1} & \epsilon^{a,\mathcal{F}+2} & \epsilon^{a,\mathcal{F}+3} & \cdots & \epsilon^{a,\mathcal{F}+\mathcal{F}} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \epsilon^{a,\mathcal{F}*(\mathcal{P}^a-2)+1} & \epsilon^{a,\mathcal{F}*(\mathcal{P}^a-2)+2} & \epsilon^{a,\mathcal{F}*(\mathcal{P}^a-2)+3} & \cdots & \epsilon^{a,\mathcal{F}*(\mathcal{P}^a-2)+\mathcal{F}} \\ \epsilon^{a,\mathcal{F}*(\mathcal{P}^a-1)+1} & \epsilon^{a,\mathcal{F}*(\mathcal{P}^a-1)+2} & \epsilon^{a,\mathcal{F}*(\mathcal{P}^a-1)+3} & \cdots & \epsilon^{a,\mathcal{F}*(\mathcal{P}^a-1)+\mathcal{F}} \end{array} \right\} \quad (11)$$

Appendix B: Computational Results for Perturbation Order "Increasing by Fare Class, then Alphabetic by O&D Name"

In the following we show the computational results for the instances with perturbation order "increasing by fare class, then alphabetic by O&D name". Figures 5 – 7 show the average computation times. These increase with increasing network size and mean demand. Still, the average time needed to compute an NE in the longest case is acceptable with about 27 minutes. Tables 7 – 9 show the average payoff ratios between the central (C), competitive (NE) and non-competitive (NC) single-airline capacity control decisions. The entries in the first row of columns 3 – 12 stand for the networks as "Hubs(Spokes)". Here one can see that as the networks, mean demand, and competition intensity (CI) increase, the difference between the central case and the competitive and non-competitive cases decreases meaning that with a high demand and/or competition intensity, the revenues in the decentralized decisions are almost as high as those in the centralized case. Tables 10 – 12 show the average payoff ratios with simultaneous horizontal and vertical competition versus those without code sharing and only horizontal competition. While code sharing tendentially increases the revenue, it does so more in the central case than in the decentralized. The benefit decreases with a higher competition intensity and mean demand, though.

CI	Ratio	1(20)	1(40)	1(60)	1(80)	1(100)	2(20)	2(40)	2(60)	2(80)	2(100)
0.25	NE/C	91.44%	91.65%	95.39%	95.07%	97.10%	97.46%	97.95%	99.33%	99.37%	98.92%
	NC/C	81.27%	91.04%	95.13%	94.16%	96.75%	87.27%	92.23%	96.22%	96.08%	96.91%
	NC/NE	88.87%	99.33%	99.72%	99.04%	99.64%	89.55%	94.17%	96.87%	96.69%	97.96%
0.50	NE/C	94.56%	92.51%	96.02%	95.56%	97.54%	97.97%	98.32%	99.58%	99.47%	99.00%
	NC/C	83.57%	91.91%	95.75%	94.59%	97.12%	88.32%	92.24%	96.22%	96.00%	96.91%
	NC/NE	88.38%	99.35%	99.72%	98.99%	99.57%	90.15%	93.82%	96.63%	96.52%	97.89%
0.75	NE/C	96.38%	95.73%	98.96%	97.94%	99.21%	99.19%	99.66%	99.69%	99.73%	99.59%
	NC/C	91.00%	95.31%	98.83%	97.32%	99.04%	93.78%	97.44%	97.67%	97.87%	98.54%
	NC/NE	94.42%	99.56%	99.88%	99.36%	99.83%	94.55%	97.77%	97.98%	98.13%	98.95%

Table 7: Average Payoff Ratios with $\mu = 2$

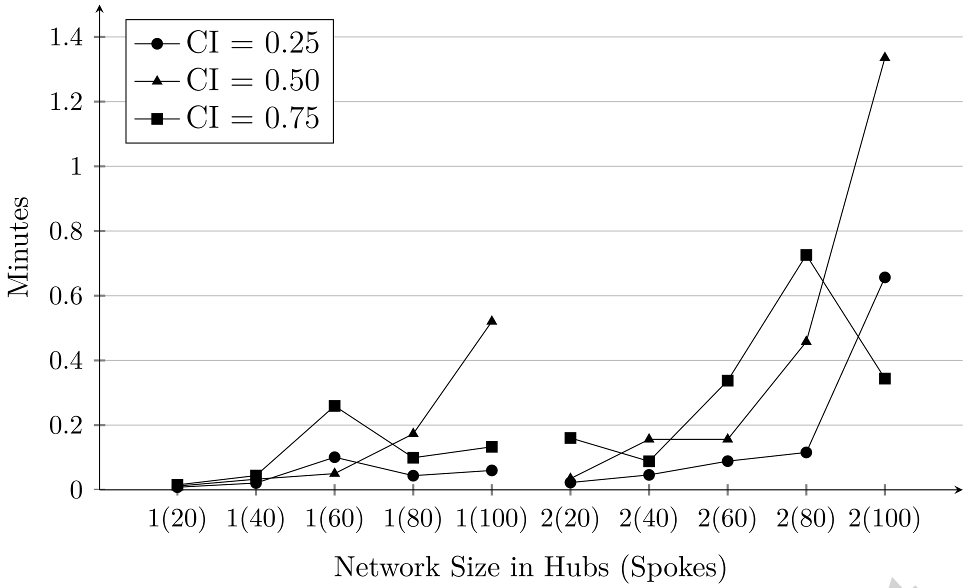

Figure 5: Average Computation Times in Minutes with $\mu = 2$

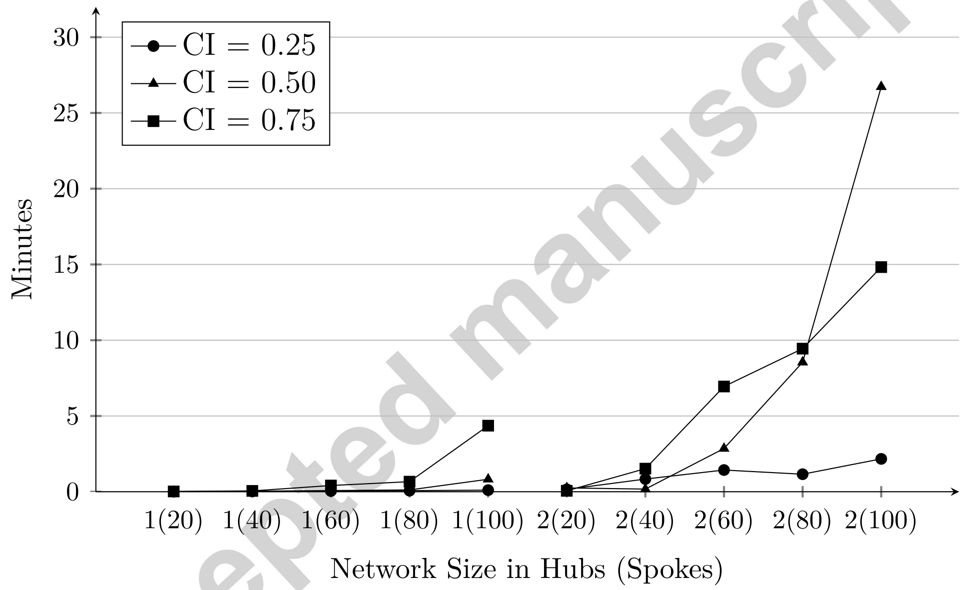

Figure 6: Average Computation Times in Minutes with $\mu = 4$

CI	Ratio	1(20)	1(40)	1(60)	1(80)	1(100)	2(20)	2(40)	2(60)	2(80)	2(100)
0.25	NE/C	94.67%	97.88%	98.94%	98.50%	99.33%	95.94%	97.88%	98.95%	98.83%	99.27%
	NC/C	94.58%	97.82%	98.87%	98.42%	99.25%	95.82%	97.80%	98.85%	98.68%	99.20%
	NC/NE	99.90%	99.93%	99.92%	99.92%	99.92%	99.88%	99.92%	99.90%	99.85%	99.93%
0.50	NE/C	95.44%	98.36%	99.29%	98.85%	99.61%	96.66%	97.99%	99.08%	98.92%	99.37%
	NC/C	95.27%	98.21%	99.13%	98.69%	99.46%	96.48%	97.86%	98.91%	98.73%	99.25%
	NC/NE	99.82%	99.85%	99.84%	99.84%	99.84%	99.81%	99.87%	99.83%	99.81%	99.88%
0.75	NE/C	97.94%	99.35%	100.15%	99.74%	100.21%	98.55%	99.61%	99.58%	99.53%	99.78%
	NC/C	97.71%	99.16%	99.94%	99.52%	99.98%	98.34%	99.43%	99.39%	99.35%	99.63%
	NC/NE	99.77%	99.81%	99.79%	99.78%	99.78%	99.79%	99.82%	99.81%	99.81%	99.85%

Table 8: Average Payoff Ratios with $\mu = 4$

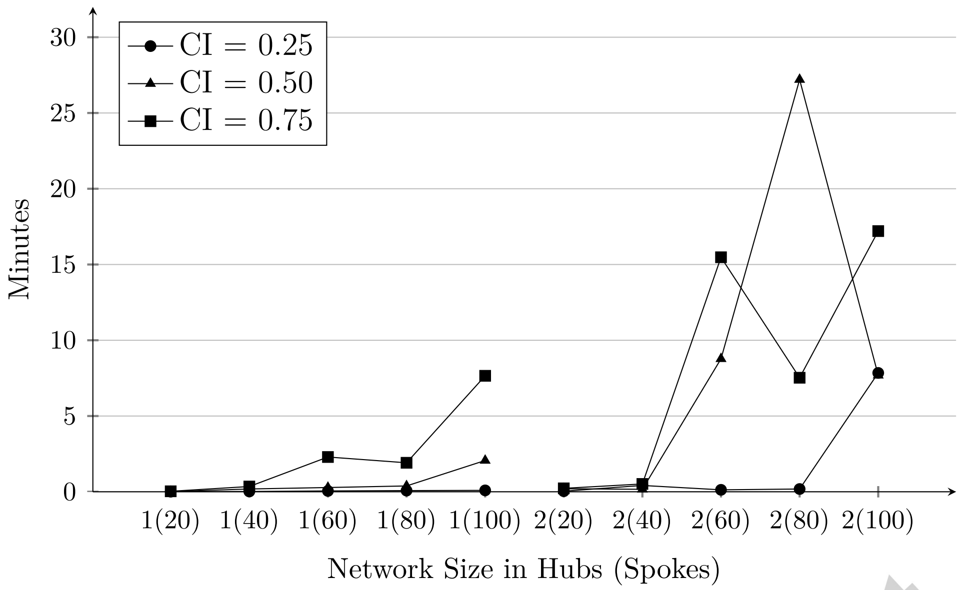

Figure 7: Average Computation Times in Minutes with $\mu = 6$

CI	Ratio	1(20)	1(40)	1(60)	1(80)	1(100)	2(20)	2(40)	2(60)	2(80)	2(100)
0.25	NE/C	97.48%	99.20%	99.74%	99.39%	99.88%	98.17%	99.16%	99.67%	99.54%	99.73%
	NC/C	97.40%	99.08%	99.61%	99.28%	99.76%	98.10%	99.10%	99.58%	99.44%	99.66%
	NC/NE	99.91%	99.87%	99.87%	99.89%	99.88%	99.93%	99.93%	99.91%	99.91%	99.92%
0.50	NE/C	98.06%	99.63%	100.11%	99.77%	100.24%	98.68%	99.38%	99.82%	99.70%	99.90%
	NC/C	97.91%	99.38%	99.85%	99.53%	99.99%	98.52%	99.23%	99.66%	99.54%	99.74%
	NC/NE	99.84%	99.75%	99.74%	99.76%	99.75%	99.84%	99.85%	99.85%	99.83%	99.84%
0.75	NE/C	99.35%	100.35%	100.73%	100.40%	100.73%	99.70%	100.20%	100.12%	100.08%	100.18%
	NC/C	99.16%	99.95%	100.33%	100.03%	100.34%	99.47%	99.99%	99.89%	99.86%	99.96%
	NC/NE	99.81%	99.60%	99.61%	99.63%	99.61%	99.78%	99.79%	99.78%	99.77%	99.78%

Table 9: Average Payoff Ratios with $\mu = 6$

CI	Ratio	1(20)	1(40)	1(60)	1(80)	1(100)	2(20)	2(40)	2(60)	2(80)	2(100)
0.25	C	122.90%	109.75%	105.04%	106.12%	103.30%	114.52%	108.41%	103.90%	104.00%	103.10%
	NE	112.48%	100.62%	100.24%	100.92%	100.32%	111.60%	106.14%	103.19%	103.38%	102.04%
0.50	C	119.48%	108.71%	104.39%	105.68%	102.95%	113.29%	108.48%	103.93%	104.12%	103.12%
	NE	113.06%	100.57%	100.21%	100.94%	100.35%	110.77%	106.49%	103.40%	103.53%	102.08%
0.75	C	109.72%	104.84%	101.11%	102.72%	100.97%	106.77%	102.78%	102.44%	102.21%	101.45%
	NE	105.82%	100.34%	100.02%	100.54%	100.07%	105.58%	102.15%	101.97%	101.82%	100.98%

Table 10: Payoff Rations with and without Code Sharing with $\mu = 2$

CI	Ratio	1(20)	1(40)	1(60)	1(80)	1(100)	2(20)	2(40)	2(60)	2(80)	2(100)
0.25	C	105.75%	102.30%	101.19%	101.62%	100.80%	104.43%	102.23%	101.11%	101.25%	100.74%
	NE	100.00%	100.00%	100.00%	100.00%	100.00%	100.03%	100.01%	100.03%	100.09%	100.01%
0.50	C	105.07%	102.03%	101.04%	101.48%	100.70%	103.91%	102.25%	101.13%	101.26%	100.75%
	NE	100.00%	100.00%	100.00%	100.00%	100.00%	100.01%	100.01%	100.05%	100.08%	100.02%
0.75	C	102.43%	101.11%	100.25%	100.70%	100.23%	102.02%	100.68%	100.67%	100.66%	100.37%
	NE	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.03%	100.03%	100.00%

Table 11: Payoff Rations with and without Code Sharing with $\mu = 4$

CI	Ratio	1(20)	1(40)	1(60)	1(80)	1(100)	2(20)	2(40)	2(60)	2(80)	2(100)
0.25	C	102.81%	101.01%	100.49%	100.78%	100.34%	101.99%	100.94%	100.44%	100.55%	100.33%
	NE	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
0.50	C	102.44%	100.91%	100.43%	100.71%	100.30%	101.75%	100.95%	100.45%	100.54%	100.33%
	NE	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
0.75	C	101.19%	100.48%	100.08%	100.33%	100.09%	100.88%	100.26%	100.25%	100.28%	100.16%
	NE	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

Table 12: Payoff Rations with and without Code Sharing with $\mu = 6$