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Dynamic performance assessment of bus transit with the multi-activity network structure

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Abstract

This paper proposes a multi-activity dynamic network data envelopment analysis model that combines the multi-activity, network and dynamic DEA models to assess the performance in terms of individual activities, individual processes, individual periods and overall operation. The main advantage of this model is that the linkages among activities and processes, the shared inputs among activities and processes, and the carry-over items among periods are included in a unified model. It can provide more appropriate performance measures. An empirical application of 20 bus transit firms in Taiwan for the period 2004-2012 is provided. Based on the operational characteristics of bus transit firms, both desirable and undesirable outputs are also incorporated into this model. The results show that none of the bus transit firms was effective in terms of the operational effectiveness, and the sources of operational ineffectiveness among bus transit firms were different. Over the period 2004-2012, the period-operational effectiveness scores maintained stable variance, the period efficiencies of highway and urban bus services appeared to have similar patterns, and transit bus firms performed well in the consumption process.

Keyword: multi-activity network data envelopment analysis; dynamic data envelopment analysis; bus transit; efficiency; effectiveness; undesirable output.

1. Introduction

Bus transit systems play an important role in the regional development of a country. Hence, the issue of bus transit performance is of widespread concern. Traditionally, partial indicators are used to measure the operational performance (e.g., average vehicle-miles per vehicle). However, partial indicators only focus on single or parts of operational factors.

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They may lead to misleading results in the bus transit industry, because bus transit operations are characterized by multiple inputs and multi-product capability (Odeck, 2006). Performance measurement based on the conventional data envelopment analysis (DEA) model, which aggregates multiple inputs and multiple outputs, can overcome the weakness of partial indicators. The extant literature on performance measurement for bus transit firms has evaluated efficiency by using the conventional DEA model (e.g., Chang and Kao, 1992; Nolan, 1996; Viton, 1997, 1998; Cowie and Asenova, 1999; Nolan et al., 2001; Odeck, 2006).

In Taiwan, a bus transit firm primarily operates two activities: highway bus (HB) service and urban bus (UB) service. Services provided by bus transit firms are unstorable and must be consumed immediately. If they are not consumed, they will disappear (Tomazinis, 1975). The quantities of consumed service may be a proportion of the quantities of produced service. Hence, the operation of a bus transit firm further involves two processes: production process and consumption process. When bus transit performance is estimated, these unique characteristics of bus transit services should be reflected in the difference between the concepts of efficiency and effectiveness (Hatry, 1980). Efficiency represents "do things right" and is measured by production efficiency (PE), which describes the ratio of actual outputs produced to inputs, while effectiveness represents "do the right things" and is measured by service effectiveness (SEV), which describes the ratio of consumed outputs to produced outputs, and operational effectiveness (OEV), which is the combination of PE and SEV (Yu and Fan, 2009). Since a bus transit firm includes multiple activities and multiple processes, parts of its resources belong to the specific activity or process, while others are shared among different activities and/or processes (e.g., management staff). Furthermore, when bus transit operators plan operationally, they will consider the inter-relationship between consecutive terms, and reserve a proportion of outputs or revenue to the next period (e.g., network length). Hence, in order to understand the operational performance for a bus transit firm, the allocation of shared inputs and the effect of carry-over items between two consecutive terms also need to be taken into account.

Conventional DEA models treat the operational process as a "black box", and use aggregate data to evaluate efficiency, without considering the linking items in parallel and in series, the existence of shared inputs among activities, or carry-over items between two consecutive terms. However, the structure of a bus transit firm is complex, with different activities and processes. In response to these operational characteristics of bus transit firms, a method that combines multiple activities, multiple processes and carry-over items, and that considers the allocation of shared resources, is designed to improve the weaknesses of the conventional DEA model. Early studies have tried to solve these weaknesses. In terms of the problem of "black box", Beasley (1995) and Mar Molinero (1996) proposed a multi-activity DEA model to evaluate performance in each activity, simultaneously. Färe and Grosskopf

(1996, 2000) proposed a network DEA model for measuring performance with multiple processes. Afterward various models are proposed to measure the efficiencies of individual activities/processes. In terms of the problem of inter-temporal dependence, Färe and Grosskopf (1996) also introduced a dynamic DEA model to study dynamical and historical systems. Nemoto and Goto (1999) used the value-based model to examine dynamic structures. Emrouznejad and Thanassoulis (2005) applied an input process distance measure to overcome the problem of inter-temporal input-output dependence. Tone and Tsutsui (2010) proposed a dynamic slack-based DEA model to explore the effects of carry-over items. Kao and Liu (2014) proposed a relational model to take the operations of individual periods into consideration. In addition, Yu and Lin (2008), Yu and Fan (2009), Chen (2012), Chen et al. (2012) and Wang et al. (2013) provided multi-activity network DEA models which incorporated multiple activities and multiple processes into a unified framework. Bogetoft et al. (2009) used a process distance measure model to study the dynamic network structure. Tone and Tsutsui (2014) developed the dynamic network DEA model within the framework of slacks-based measures to deal with the effects of inter-connected processes and carry-over items. The dynamic network DEA models have been applied in the bank industry (e.g., Fukuyama and Weber, 2014; Avkiran, 2015; Zha et al., 2015) and hospital industry (e.g., Kawaguch et al., 2014).

With regard to studies in the bus transit industry, De Borger et al. (2002) and De Borger and Kerstens (2008) provided the comprehensive survey of the literature on the performance for bus transit operators. From their survey, we can find that most studies neglect the internal structure of bus transit firms and the effects of carry-over items. Although Chu et al. (1992) considered the internal structure of bus transit firms and divided transit performance into efficiency and effectiveness, they applied separate models to evaluate these two performance indices, and ignored the inter-relationship between these two dimensions. Yu and Fan (2009) combined these measures of PE, SEV and OEV into a single multi-activity network model to evaluate bus transit performance, but failed to take effects of carry-over items into consideration. However, the operation of a bus transit firm is not independent among periods. Some variables serve as carry-overs, persisting from one period to another. In consideration of long-term planning and investments, a single-period optimization model is not favorable. Hence, this paper proposes a novel method that combines the multi-activity, network and dynamic DEA models, called the multi-activity dynamic network DEA (MDNDEA) model, to assess performance. This model is designed to evaluate the performance achieved by firms which have several operational activities, processes and carry-over items between two consecutive terms. This framework provides the basis to explore the performance of individual activities, individual processes, individual periods, and overall operation in a unified model. In order to obtain more accurate measures and provide operators and policy makers more information on operational performance in the bus transit industry, the

MDNDEA model is more appropriate.

The contributions of this paper are threefold. First, we propose an MDNDEA model, which accounts for the effects of inter-relationships among activities and processes as well as the impacts of carry-over items between two consecutive terms in a unified DEA framework. Second, undesirable outputs are included in this model to fully evaluate the performance of bus transit firms. Third, we use this model to assess the OEV of bus transit firms in Taiwan, and decompose OEV into the period-production efficiency of the HB service (PHBPE), period-production efficiency of the UB service (PUBPE) and period-service effectiveness (PSEV).

The rest of this paper is organized as follows. Section 2 proposes the methodology for evaluating various performance types in a unified model. Section 3 describes the data and discusses the empirical results and managerial implications. Finally, Section 4 presents the conclusions.

2. Methodology

The operation of a bus transit firm mainly includes two processes: production process and consumption process. The production process can be further divided into two activities: HB service and UB service. Parts of unseparated inputs are shared among activities and/or processes. For example, technicians need to maintain highway and urban buses, simultaneously; management staff members are responsible for the operation of the entire firm. In addition, the carry-over items exist in the bus transit industry, because the operation of a bus transit firm in one period is not independent of that in the next one. A bus transit firm usually continues activities over several periods. Those activities produced in the current period may cause an effect in the next period. For example, the network length generated in current period will drive the levels of production capacity in the next period. If these operational characteristics are ignored, performance measures may be misleading. Thus, the MDNDEA model, which considers the effects of inter-relationships among activities and processes as well as the impacts of carry-over items between two consecutive terms, is more suitable for this industry.

In the case of bus transit firms, outputs of transportation services may involve an undesirable output: the number of accidents. In order to deal with problems where some outputs (desirable outputs) are expected to be maximized and some outputs (undesirable outputs) are expected to be minimized, the directional distance function proposed by Luenberger (1992) will be a more adequate tool. It permits simultaneous expansion of desirable outputs and contraction of undesirable outputs. Hence, we will build the performance measurement model by using the MDNDEA method and the directional distance function.

The operational framework is represented in Fig. 1. Specifically, some inputs are shared in HB and UB production activities (e.g., technicians), and some inputs are shared in HB production activity, UB production activity and consumption process (e.g., management staff). These production capacities of two activities are utilized as the inputs in the consumption process. In addition, some outputs in the production process in the current period will be transferred into the next period.

<Figure 1 insert here>

Suppose that there are J bus transit firms in period t ($t=1,\ldots,T$), and that each firm engages in HB and UB production activities as well as consumption process. Let $X^t_{aj,H}=(x^t_{1j,H},\ldots,x^t_{m_a j,H})$, $X^t_{bj,U}=(x^t_{1j,U},\ldots,x^t_{m_b j,U})$ and $X^t_{ej,C}=(x^t_{1j,C},\ldots,x^t_{m_e j,C})$ denote the dedicated input vectors associated with the HB production activity, UB production activity and consumption process in period *t*, respectively, and let $X^t_{cj,S}=(x^t_{1j,S},\ldots,x^t_{m_c j,S})$ and $X^t_{dj,SC}=(x^t_{1j,SC},\ldots,x^t_{m_d j,SC})$ be input vectors shared by HB production activity and UB production activity as well as by HB production activity, UB production activity and consumption process in period *t*, respectively. It is assumed that, in period *t*, firm *j* allocates some portion, $\mu^t_{cj,H}$, of the shared input quantities $x^t_{cj,S}$ to the HB production activity and the remaining $(1-\mu^t_{cj,H})$ to the UB production activity, while it allocates some portion, $\gamma^t_{dj,H}$, of the shared input quantities $x^t_{dj,SC}$ to the HB production activity, some portion, $\gamma^t_{dj,U}$, to the UB production activity and the remaining $(1-\gamma^t_{dj,H}-\gamma^t_{dj,U})$ to the consumption process. $\mu^t_{cj,H}$, $\gamma^t_{dj,H}$ and $\gamma^t_{dj,U}$ are decision variables determined by firm *j* in period *t*. For the HB production process, in period *t*, each firm produces the intermediate output vector $M^t_{ij,HC}=(m^t_{1j,HC},\ldots,m^t_{n_i j,HC})$ to flow into the consumption process, and carries the discretionary item vector $D^{(t,t+1)}_{pj,H}=(d^{(t,t+1)}_{1j,H},\ldots,d^{(t,t+1)}_{n_p j,H})$ to period *t*+1. For the UB production process, in period *t*, each firm produces the intermediate output vector $M^t_{lj,UC}=(m^t_{1j,UC},\ldots,m^t_{n_l j,UC})$ to flow into the consumption process and carries the discretionary item vector $D^{(t,t+1)}_{qj,U}=(d^{(t,t+1)}_{1j,U},\ldots,d^{(t,t+1)}_{n_q j,U})$ to period *t*+1. For the consumption process, the desirable output vector $Y^t_{fj,C}=(y^t_{1j,C},\ldots,x^t_{s_f j,C})$ and the undesirable output vector $B^t_{gj,C} = (b^t_{1j,C}, \ldots, b^t_{s_g j,C})$ are jointly produced in period *t*. In order to make the development of the model easier to follow, the notations for the model are shown in Table 1.

<Table 1 insert here>

Microeconomic theory indicates that one of firms' objectives is to produce the level of outputs where constant returns to scale (CRS) exist. Although firms may operate under variable returns to scale (VRS) in the short run, they will adjust their scale of operations to move toward CRS in the long run (Cummins and Xie, 2013). In addition, Månsson (1996) and Färe et al. (1997) argued that CRS captured the long-run results, while VRS was suitable for the short run. Hence, in a multi-period context, it is reasonable to calculate the efficiency estimates under the assumption of CRS. Accordingly, there are three production technologies, T^t_H, T^t_U and T^t_C in our MDNDEA model, in which the production technology of T^t_H for HB production activity under the assumption of CRS is defined as follows:

$$
\begin{aligned}
T^t_H = \Bigg\{ (x^t, m^t, d^{(t,t+1)}) : & \sum_{j=1}^{J} \lambda^t_{j,H} x^t_{aj,H} \le x^t_{aj,H}, \quad a = 1,\ldots,m_a, \\
& \sum_{j=1}^{J} \mu^t_{cj,H} \lambda^t_{j,H} x^t_{cj,S} \le \mu^t_{cj,H} x^t_{cj,S}, \quad c = 1,\ldots,m_c, \\
& \sum_{j=1}^{J} \gamma^t_{dj,H} \lambda^t_{j,H} x^t_{dj,SC} \le \gamma^t_{dj,H} x^t_{dj,SC}, \quad d = 1,\ldots,m_d, \\
& 0 < \mu^t_{cj,H} < 1, \quad c = 1,\ldots,m_c, \\
& 0 < \gamma^t_{dj,H} < 1, \quad d = 1,\ldots,m_d, \\
& \sum_{j=1}^{J} \lambda^t_{j,H} m^t_{ij,HC} \begin{cases} = m^t_{ij,HC}, \textit{ if the linking item is fixed} \\ \ge m^t_{ij,HC}, \textit{ if the linking item is free} \end{cases}, \quad i = 1,\ldots,n_i, \\
& \sum_{j=1}^{J} \lambda^t_{j,H} d^{(t,t+1)}_{pj,H} \ge d^{(t,t+1)}_{pj,H}, \quad p = 1,\ldots,n_p, \\
& \sum_{j=1}^{J} \lambda^t_{j,H} d^{(t-1,t)}_{pj,H} \le d^{(t-1,t)}_{pj,H}, \quad p = 1,\ldots,n_p, \\
& \lambda^t_{j,H} \ge 0, \quad j = 1,\ldots,J, \quad (t = 1,\ldots,T) \qquad (1)
\end{aligned}
$$

However, if $t=1$, $\sum_{j=1}^{J}\lambda_{j,H}^{1}d_{pj,H}^{(0,1)} \le d_{pj,H}^{(0,1)}$ is substituted for $\sum_{j=1}^{J}\lambda_{j,H}^{1}d_{pj,H}^{(0,1)} = d_{pj,H}^{(0,1)}$; if $t=T$, $\sum_{j=1}^{J}\lambda_{j,H}^{T}d_{pj,H}^{(T,T+1)} \ge d_{pj,H}^{(T,T+1)}$ is removed.

The production technology of T_U^t for UB production activity under the assumption of CRS is expressed as follows:

$$
\begin{aligned}
T_U^t = \Bigg\{ (x^t, m^t, d^{(t,t+1)}) : & \sum_{j=1}^{J}\lambda_{j,U}^{t}x_{bj,U}^{t} \le x_{bj,U}^{t}, \quad b=1,\ldots,m_b, \\
& \sum_{j=1}^{J}(1-\mu_{cj,H}^{t})\lambda_{j,U}^{t}x_{cj,S}^{t} \le (1-\mu_{cj,H}^{t})x_{cj,S}^{t}, \quad c=1,\ldots,m_c, \\
& \sum_{j=1}^{J}\gamma_{dj,U}^{t}\lambda_{j,U}^{t}x_{dj,SC}^{t} \le \gamma_{dj,U}^{t}x_{dj,SC}^{t}, \quad d=1,\ldots,m_d, \\
& 0<\mu_{cj,H}^{t}<1, \quad c=1,\ldots,m_c, \\
& 0<\gamma_{dj,U}^{t}<1, \quad d=1,\ldots,m_d, \\
& \sum_{j=1}^{J}\lambda_{j,U}^{t}m_{lj,UC}^{t}\begin{cases} =m_{lj,UC}^{t}, \text{if the linking item is fixed} \\ \ge m_{lj,UC}^{t}, \text{if the linking item is free} \end{cases}, \quad l=1,\ldots,n_l, \\
& \sum_{j=1}^{J}\lambda_{j,U}^{t}d_{qj,U}^{(t,t+1)} \ge d_{qj,U}^{(t,t+1)}, \quad q=1,\ldots,n_q, \\
& \sum_{j=1}^{J}\lambda_{j,U}^{t}d_{qj,U}^{(t-1,t)} \le d_{qj,U}^{(t-1,t)}, \quad q=1,\ldots,n_q, \\
& \sum_{j=1}^{J}\lambda_{j,U}^{t}=1 \Bigg\} \quad (t=1,\ldots,T)
\end{aligned} \tag{2}
$$

Similarly, if $t=1$, $\sum_{j=1}^{J}\lambda_{j,U}^{1}d_{qj,U}^{(0,1)} \le d_{qj,U}^{(0,1)}$ is substituted for $\sum_{j=1}^{J}\lambda_{j,U}^{1}d_{qj,U}^{(0,1)} = d_{qj,U}^{(0,1)}$; if $t=T$, $\sum_{j=1}^{J}\lambda_{j,U}^{T}d_{qj,U}^{(T,T+1)} \ge d_{qj,U}^{(T,T+1)}$ is removed.

Further, the production technology of T_C^t for consumption service under the assumption of CRS is written as follows:

$$
\begin{aligned}
T_C^t = \Bigg\{ (x^t, m^t, y^t) : & \sum_{j=1}^{J} \lambda_{j,C}^t x_{ej,C}^t \le x_{ej,C}^t, \quad e = 1,\ldots,m_e, \\
& \sum_{j=1}^{J} (1-\gamma_{dj,H}^t - \gamma_{dj,U}^t)\lambda_{j,C}^t x_{dj,SC}^t \le (1-\gamma_{dj,H}^t - \gamma_{dj,U}^t) x_{dj,SC}^t, \quad d = 1,\ldots,m_d, \\
& 0 < \gamma_{dj,H}^t < 1, \quad d = 1,\ldots,m_d, \\
& 0 < \gamma_{dj,U}^t < 1, \quad d = 1,\ldots,m_d, \\
& \sum_{j=1}^{J} \lambda_{j,C}^t m_{ij,HC}^t \begin{cases} = m_{ij,HC}^t, \text{if the linking item is fixed} \\ \le m_{ij,HC}^t, \text{if the linking item is free} \end{cases}, \quad i = 1,\ldots,n_i, \\
& \sum_{j=1}^{J} \lambda_{j,C}^t m_{lj,UC}^t \begin{cases} = m_{lj,UC}^t, \text{if the linking item is fixed} \\ \le m_{lj,UC}^t, \text{if the linking item is free} \end{cases}, \quad l = 1,\ldots,n_l, \\
& \sum_{j=1}^{J} \lambda_{j,C}^t y_{fj,C}^t \ge y_{fj,C}^t, \quad f = 1,\ldots,s_f, \\
& \sum_{j=1}^{J} \lambda_{j,C}^t b_{gj,C}^t = b_{gj,C}^t, \quad g = 1,\ldots,s_g, \\
& \sum_{j=1}^{J} \lambda_{j,C}^t = 1 \Bigg\} \quad (t = 1,\ldots,T)
\end{aligned}
\tag{3}
$$

where λ_H^t, λ_U^t and λ_C^t are intensity variables associated with the HB production activity, UB production activity and consumption process, respectively. In addition, the production technologies satisfy the assumptions of strong disposability of desirable outputs and inputs, and weak disposability of undesirable outputs suggested by Färe et al. (1989).

Then the operational ineffectiveness for bus transit firm *k* can be estimated by solving the following MDNDEA model based on a directional distance function:

$$
\vec{D}(x_k, m_k, d_k, y_k) = \max\ \beta_k = \sum_{t=1}^{T} W^t \left[w^P (w^H \cdot \beta_{k,H}^t + w^U \cdot \beta_{k,U}^t) + w^C \cdot \beta_{k,C}^t \right] \tag{4}
$$

Subject to

a. HB production activity:

$$\sum_{j=1}^{J} \lambda_{j,H}^{t} x_{aj,H}^{t} \leq (1-\beta_{k,H}^{t}) x_{ak,H}^{t}, \quad a=1,\ldots,m_a,\, t=1,\ldots,T \tag{4.1}$$

$$\sum_{j=1}^{J} \lambda_{j,H}^{t} m_{ij,HC}^{t} = m_{ik,HC}^{t}, \quad i=1,\ldots,n_i,\, t=1,\ldots,T \tag{4.2}$$

$$\sum_{j=1}^{J} \lambda_{j,H}^{t} d_{pj,H}^{(t,t+1)} = \sum_{j=1}^{J} \lambda_{j,H}^{t+1} d_{pj,H}^{(t,t+1)}, \quad p=1,\ldots,n_p,\, t=1,\ldots,T-1 \tag{4.3}$$

$$\sum_{j=1}^{J} \lambda_{j,H}^{t} d_{pj,H}^{(t,t+1)} = d_{pk,H}^{(t,t+1)} - S_{pk,H}^{(t,t+1),\, free}, \quad p=1,\ldots,n_p,\, t=1,\ldots,T-1 \tag{4.4}$$

b. UB production activity:

$$\sum_{j=1}^{J} \lambda_{j,U}^{t} x_{bj,U}^{t} \leq (1-\beta_{k,U}^{t}) x_{bk,U}^{t}, \quad b=1,\ldots,m_b,\, t=1,\ldots,T \tag{4.5}$$

$$\sum_{j=1}^{J} \lambda_{j,U}^{t} m_{lj,UC}^{t} = m_{lk,UC}^{t}, \quad l=1,\ldots,n_l,\, t=1,\ldots,T \tag{4.6}$$

$$\sum_{j=1}^{J} \lambda_{j,U}^{t} d_{qj,U}^{(t,t+1)} = \sum_{j=1}^{J} \lambda_{j,U}^{t+1} d_{qj,U}^{(t,t+1)}, \quad q=1,\ldots,n_q,\, t=1,\ldots,T-1 \tag{4.7}$$

$$\sum_{j=1}^{J} \lambda_{j,U}^{t} d_{qj,U}^{(t,t+1)} = d_{qk,U}^{(t,t+1)} - S_{qk,U}^{(t,t+1),\, free}, \quad q=1,\ldots,n_q,\, t=1,\ldots,T-1 \tag{4.8}$$

c. Consumption process:

$$\sum_{j=1}^{J} \lambda_{j,C}^{t} x_{ej,C}^{t} \leq (1-\beta_{k,C}^{t}) x_{ek,C}^{t}, \quad e=1,\ldots,m_e,\, t=1,\ldots,T \tag{4.9}$$

$$\sum_{j=1}^{J} \lambda_{j,C}^{t} m_{ij,HC}^{t} = m_{ik,HC}^{t}, \quad i=1,\ldots,n_i,\, t=1,\ldots,T \tag{4.10}$$

$$\sum_{j=1}^{J} \lambda_{j,C}^{t} m_{lj,UC}^{t} = m_{lk,UC}^{t}, \quad l=1,\ldots,n_l,\, t=1,\ldots,T \tag{4.11}$$

$$\sum_{j=1}^{J} \lambda_{j,C}^{t} y_{fj,C}^{t} \geq (1+\beta_{k,C}^{t}) y_{fk,C}^{t}, \quad f=1,\ldots,s_f,\, t=1,\ldots,T \tag{4.12}$$

$$\sum_{j=1}^{J} \lambda_{j,C}^{t} b_{gj,C}^{t} = (1-\beta_{k,C}^{t}) b_{gk,C}^{t}, \quad g=1,\ldots,s_g,\, t=1,\ldots,T \tag{4.13}$$

d. Shared inputs:

$$\sum_{j=1}^{J} \mu_{cj,H}^{t} \lambda_{j,H}^{t} x_{cj,S}^{t} \leq (1-\beta_{k,H}^{t}) \mu_{ck,H}^{t} x_{ck,S}^{t}, \quad c=1,\ldots,m_c,\, t=1,\ldots,T \tag{4.14}$$

$$\sum_{j=1}^{J} (1-\mu_{cj,H}^{t}) \lambda_{j,U}^{t} x_{cj,S}^{t} \leq (1-\beta_{k,U}^{t})(1-\mu_{ck,H}^{t}) x_{ck,S}^{t}, \quad c=1,\ldots,m_c,\, t=1,\ldots,T \tag{4.15}$$

$$\sum_{j=1}^{J}\gamma^t_{dj,H}\lambda^t_{j,H}x^t_{dj,SC} \le (1-\beta^t_{k,H})\gamma^t_{dk,H}x^t_{dk,SC}, \quad d=1,\ldots,m_d, t=1,\ldots,T \tag{4.16}$$

$$\sum_{j=1}^{J}\gamma^t_{dj,U}\lambda^t_{j,U}x^t_{dj,SC} \le (1-\beta^t_{k,U})\gamma^t_{dk,U}x^t_{dk,SC}, \quad d=1,\ldots,m_d, t=1,\ldots,T \tag{4.17}$$

$$\sum_{j=1}^{J}(1-\gamma^t_{dj,H}-\gamma^t_{dj,U})\lambda^t_{j,C}x^t_{dj,SC} \le (1-\beta^t_{k,C})(1-\gamma^t_{dk,H}-\gamma^t_{dk,U})x^t_{dk,SC}, \quad d=1,\ldots,m_d, t=1,\ldots,T \tag{4.18}$$

$$L^t_{c,H} < \mu^t_{c,H} < U^t_{c,H}, \quad c=1,\ldots,m_c, t=1,\ldots,T \tag{4.19}$$

$$L^t_{d,H} < \gamma^t_{d,H} < U^t_{d,H}, \quad d=1,\ldots,m_d, t=1,\ldots,T \tag{4.20}$$

$$L^t_{d,U} < \gamma^t_{d,U} < U^t_{d,U}, \quad d=1,\ldots,m_d, t=1,\ldots,T \tag{4.21}$$

e. Initial conditions:

$$\sum_{j=1}^{J}\lambda^1_{j,H}d^{(0,1)}_{pj,H} = d^{(0,1)}_{pk,H}, \quad p=1,\ldots,n_p, \tag{4.22}$$

$$\sum_{j=1}^{J}\lambda^1_{j,U}d^{(0,1)}_{qj,U} = d^{(0,1)}_{qk,U}, \quad q=1,\ldots,n_q, \tag{4.23}$$

$$\sum_{t=1}^{T}W^t = 1 \tag{4.24}$$

$$w^H + w^U = 1 \tag{4.25}$$

$$w^P + w^C = 1 \tag{4.26}$$

$$\lambda_{j,H}, \lambda_{j,U}, \lambda_{j,C}, W^t, w^H, w^U, w^P, w^C \ge 0, \quad j=1,\ldots,J, t=1,\ldots,T \tag{4.27}$$

$$S^{(t,t+1),\,free}_{pk,H}, S^{(t,t+1),\,free}_{qk,U} : free, \quad p=1,\ldots,n_p, q=1,\ldots,n_q, t=1,\ldots,T-1 \tag{4.28}$$

where $\beta^t_{k,H}$, $\beta^t_{k,U}$, $\beta^t_{k,C}$, $\lambda^t_{j,H}$, $\lambda^t_{j,U}$, $\lambda^t_{j,C}$, $S^{(t,t+1),\,free}_{pk,H}$, $S^{(t,t+1),\,free}_{qk,U}$, $\mu^t_{cj,H}$, $\gamma^t_{dj,H}$ and $\gamma^t_{dj,U}$, $t=1,\ldots,T$, $j=1,\ldots,J$, $p=1,\ldots,n_p$, $q=1,\ldots,n_q$, $c=1,\ldots,m_c$, $d=1,\ldots,m_d$ are variables of this model. $\beta^t_{k,H}$ and $\beta^t_{k,U}$ are the measures of inefficiencies of the HB and UB production activities for bus transit firm *k* in period *t*, respectively, and $\beta^t_{k,C}$ is the measure of ineffectiveness of the consumption process for bus transit firm *k* in period *t*. $S_{pk,H}^{(t,t+1),\,free}$ and $S_{qk,U}^{(t,t+1),\,free}$ are slack variables. L and U are the lower bound and upper bound given to the various shared inputs. W^t, w^H, w^U, w^P and w^C are the weights on period *t*, the HB production activity, UB production activity, production process and consumption process respectively, and represent the relative importance of these periods, activities and processes.[1] It is assumed that the linking items between production process and consumption process are fixed by constraints (4.2), (4.6), (4.10) and (4.11), and the carry-over items in the production process act as the discretionary link by constraints (4.3), (4.4), (4.7) and (4.8). Constraints (4.3) and (4.7) impose the continuity condition between two consecutive periods, and constraints (4.4) and (4.8) represent the fact that carry-over items can be freely increased or decreased. Note that the linking and carry-over items can be assumed to have other forms based on the characteristics of these items. The related constraints of other forms are shown in Appendix A. In addition, the initial conditions can be accounted for by constraints (4.22)-(4.23) which are given and fixed. Furthermore, the three basic measures, PHBPE, PUBPE and PSEV can be determined from $1-\beta_{k,H}^t$, $1-\beta_{k,U}^t$ and $1-\beta_{k,C}^t$ respectively, while the other seven induced measures, period-production efficiency (PPE), period-operational effectiveness (POEV), production efficiency of the HB activity (HBPE), production efficiency of the UB activity (UBPE), PE, SEV and OEV, can be obtained by $1-(w^H\cdot\beta_{k,H}^t+w^U\cdot\beta_{k,U}^t)$, $1-[w^P(w^H\cdot\beta_{k,H}^t+w^U\cdot\beta_{k,U}^t)+w^C\cdot\beta_{k,C}^t]$, $1-\sum_{t=1}^{T}W^t\cdot\beta_{k,H}^t$, $1-\sum_{t=1}^{T}W^t\cdot\beta_{k,U}^t$, $1-\sum_{t=1}^{T}W^t(w^H\cdot\beta_{k,H}^t+w^U\cdot\beta_{k,U}^t)$, $1-\sum_{t=1}^{T}W^t\cdot\beta_{k,C}^t$ and $1-\beta_k$, respectively. β_k is equal to zero if and only if the bus transit firm is operationally effective and $\beta_{k,H}^t=\beta_{k,U}^t=\beta_{k,C}^t=0, t=1,\ldots,T$. Since the model combines the measures of PHBPE, PUBPE and PSEV to compute the OEV measure, the results can provide further insight into the sources of OEV.

3. Empirical results

3.1 The data

The panel data set used in this paper is obtained from the annual statistical reports

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[1] If the VRS over the reference technology are assumed, we impose $\sum_{j=1}^{J}\lambda_{j,Q}^t=1\,(Q\in\{H,U,C\}), t=1,\ldots,T$, on the technology in Model (4).

published by the National Federation of Bus Passenger Transportation of the Republic of China, and contains 20 bus transit firms for the period 2004-2012. On the input side, we use the number of drivers (DRIVER), the number of vehicles (VEHICLE) and the number of liters of fuel (FUEL) as dedicated inputs for HB and UB services, respectively, and use the number of ticket agents (TICKET) as dedicated inputs for consumption service.[2] The number of technicians (TEC) and the number of management staff (MGT) are selected as shared inputs, in which technicians are shared between HB and UB services and management staff is shared among HB, UB and consumption services. On the output side, vehicle-kms (VEHKM) of HB and UB services are treated as the intermediate output flowing from the HB and UB production activities to the consumption process, respectively. Passenger-kms (PASSKM) and the number of passengers (PASS) are selected as two final desirable outputs, and the number of accidents (ACC) is the final undesirable output for consumption service. In addition, network length (NWLTH) of HB and UB services is used to identify the effect of the carry-over item. Road network is a carry-over item, because it can be treated as an asset and preserved to the next period. We use the network length as a proxy for road network. $\mu^t_{cj,H}$, $\gamma^t_{dj,H}$ and $\gamma^t_{dj,U}$ are endogenously determined by the proposed model. However, the model does not allow these shared inputs to take weights of 0 or 1. The weights for these shared inputs need to be limited. We consider $\mu^t_{cj,H}$ to range from 0.3 to 0.7, while $\gamma^t_{dj,H}$ and $\gamma^t_{dj,U}$ range from 0.2 to 0.8. Although these ranges are determined by the authors, they are deemed reasonable by management. Finally, since the weights on periods, production process and consumption process are exogenously pre-assigned scalars, for simplicity, we assume $W^t = 0.1111$, $t = 1,\dots,9$ and $w^H = w^U = w^P = w^C = 0.5$. Table 2 provides the descriptive statistics of all variables used in this paper. It is notable that the number of drivers is different from the number of vehicles. In the Taiwanese bus transit industry, there are two kinds of driver assignment. One is two drivers sharing a vehicle, and the other is one driver to operate one vehicle. In addition, drivers belong to labor, which is easily increased or decreased, while vehicles are an asset, which is harder to change. Hence, the number of drivers is larger than the number of vehicles on average, and the former is more variable than the latter.

<Table 2 insert here>

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[2] A bus transit firm can have service capability in terms of vehicle availability and network length, but its service delivery performance may be more a function of driver availability and fuel consumption, thus combining the driver availability and fuel consumption with vehicle availability and network length as inputs in the production stage.

3.2 Performance results

Based on the MDNDEA model, the researcher can not only calculate the OEV scores, but also obtain the efficiency and effectiveness scores of individual processes and activities among periods. In order to obtain more information on operational performance in the bus transit industry, it is more appropriate to use the MDNDEA model to gauge the performance of bus transit firms in Taiwan.

Before estimating the various efficiency scores of bus transit firms by the MDNDEA model, one sensitivity test is used to understand how efficiency estimates obtained by the MDNDEA model are likely to change in response to altering the weights for the shared inputs. We change the range of $\mu^{t}_{cj,H}$ from 0.3-0.7 to 0.2-0.8 and re-evaluate the efficiency scores by the MDNDEA model. Then, the degree of similarity between the two settings of ranges is examined by calculating Spearman's rank correlation coefficient (Avkiran and McCrystal, 2012). The results show that correlation coefficients of PE, HBPE, UBPE and SEV are 0.9459, 0.9712, 0.9774 and 0.9968, and all coefficients are significant at the 5% significance level, indicating that the MDNDEA model is more robust to the perturbation of weights.[3]

Table 3 displays the average results of OEV scores and its components of bus transit firms obtained by the MDNDEA model. Looking first at the average OEV score of bus transit firms, its value was 0.8540, with a range from 0.7507 to 0.9871. This indicates that, on average, there is room for bus transit firms to enhance their performance by 14.6% in the study period. Since OEV is defined as the weighted-average performance of production and consumption processes, we can explore the contributions of these two processes. As can be seen from Table 3, the PE (0.7960) was worse than SEV (0.9121), implying that the operational ineffectiveness mainly came from the production process. For the production process, the PE score was determined by the weighted-average of HBPE and UBPE scores. We can further investigate where the production inefficiency comes from. As shown in Table 3, the HBPE (0.7904) was slightly lower than the UBPE (0.8015). This means that inefficiencies of both HB and UB activities lead to production inefficiency.

As for individual bus transit firms, the results show that no bus transit firm was effective in terms of the OEV. Since the OEV score is equal to unity if and only if all production and consumption processes are simultaneously efficient and effective in each period, this result signifies that none of the bus transit firms performed efficiently in terms of all their three services in each period. All bus transit firms should enhance their performance in at least one of these three services. In terms of individual activities, four bus transit firms (e.g., CitiAir, Hualien, Fengyuan and Chiayi) were efficient in HB activity, six firms (e.g., Sanchung,

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[3] The sensitivity tests of divisional weights and period weights are also examined. Due to lack of space, the results are not reported here, but can be obtained from the authors upon request.

Taipei, Kuang-hua, Tansui, Chungli and Chiayi) were efficient in UB activity, and eight firms (e.g., Sanchung, Capital, Taipei, Chih-nan, Taoyuan, Hsinchu, Hualien and Ubus) were effective in consumption process. However, some firms with efficiency in one dimension were relatively inefficient in others. For example, Taoyan was effective in the consumption process, but had lower PE in both production activities. Taoyan should improve its resource utilization. The ranking of bus transit firms is also listed in Table 3. Hualien was the best one among all in the OEV, with the first ranking in HBPE and SEV, and the ninth ranking in UBPE. On the other hand, Geya had the lowest OEV. The main reason is that Geya had an extraordinarily low UBPE (0.3013). Although Kuang-hua and Taoyuan ranked first in UBPE and SEV, respectively, they were the sixteenth and seventeenth in OEV. This result indicates that the ranking of bus transit firms in different dimensions of performance measures is inconsistent. In other words, the sources of operational ineffectiveness of bus transit firms are different. Hence, compared with the conventional DEA model, the proposed MDNDEA model can reveal inefficiency in individual activities/processes and provide operators more information about where to improve performance.

<Table 3 insert here>

One of the merits of the MDNDEA model is that it can measure the period performance in a unified model, so that it can provide an overall trend of performance change. This is why the MDNDEA model is superior to the static multi-activity network DEA model. Hence, we can further investigate the average trend of performance change over the period 2004-2012. Fig. 2 indicates that the average POEV scores maintained stable variance over the sample period. Similarly, the POEV can be decomposed into PPE and PSEV. From Fig. 2, it can be found that the PPE scores were lower than the PSEV scores during 2004-2011, while the PSEV score was worse than the PPE score in 2012. It is worth noting that the average PSEV scores revealed higher levels over the sample period. This implies that these bus transit firms performed well in the consumption process over the sample period. We further explore the PPE between the HB and UB activities. As can be seen in Fig. 2, the PHBPE scores were better than the PUBPE scores during 2004-2007, while the PUBPE scores were greater than the PHBPE scores during 2008-2012. However, PHBPE and PUBPE appeared to have similar patterns over the sample period.

<Figure 2 insert here>

The correlation coefficients among OEV, PE and SEV in respective periods are shown in Table 4. The correlation coefficients between OEV and PE are significantly positive, and those between OEV and SEV are insignificantly positive over the period 2004-2012. This indicates that the production side is more important than the consumption side in terms of the variances in OEV of bus transit firms. Bus transit firms should focus on a decrease in their various inputs. The correlation coefficient between PE and SEV is not significant over the period 2004-2012, indicating that a higher PE does not guarantee a lower SEV.

As for respective periods, there are significantly positive correlations between OEV and PE every year, while there are only significantly positive correlations between OEV and SEV in 2006 and 2012. This means that the input allocation decision has a closer relation with OEV than service efforts. In addition, most correlation coefficients between PE and SEV are insignificantly negative over the sample period, but the coefficients are significantly negative in 2007 and 2011. This implies the existence of a substitute relationship between these two processes in these two years.

<Table 4 insert here>

Table 5 further shows the pairwise comparisons of PE, HBPE and UBPE. Over the period 2004-2012, the correlation coefficients between PE and HBPE are significantly positive, as are those between PE and UBPE, implying that the PE is achieved by both HBPE and UBPE. Bus transit firms should efficiently utilize and share resources between HB and UB activities. In addition, the correlation coefficient between HBPE and UBPE is insignificantly negative over the period 2004-2012. This means that the enhancement of efficiency in the HB activity does not necessarily decrease efficiency in the UB activity.

In terms of respective periods, HBPE and UBPE have a significantly positive relationship with PE every year, while the correlation between HBPE and UBPE is only significantly positive in 2008. This indicates that if the bus transit firms can improve their efficiencies in the HB and UB activities, they will bring direct effects on production efficiency gains. On the other hand, the relationship between HBPE and UBPE is significantly complementary in 2008.

<Table 5 insert here>

In addition, the proposed MDNDEA model is not a simple dynamic performance measurement model. It considers the impacts of carry-over items. Hence, it can help bus transit firms to modify their long-term planning and investments by investigating changes in carry-over items. Table 6 shows what average of network lengths of HB and UB can be reduced or expanded during 2004-2012.[4] Only one bus transit firm (e.g., Chiayi) shows no changes in both carry-over items. Two firms (e.g., Sanchung and Kuang-hua) should increase the network length of highway bus service, while two firms (e.g., Taipei and Chungli) should decrease the network length of highway bus service. Three firms (e.g., CitiAir, Hualien and Fengyuan) require a reduction of the network length of urban bus service. Four firms (e.g., Taoyuan, Hsinchu, Taichung and Kaohsiung) need to expand both carry-over items, while

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[4] Since network lengths of HB and UB have been defined as the discretionary link in this application, they can be freely increased or decreased.

three firms (e.g., Chih-nan, Changhua and Geya) need to reduce both carry-over items. Five firms (e.g., Capital, Chung-shing, Tansui, Ubus and Pingtung) should enlarge their network length of highway bus service, but contract their network length of urban bus service.

<Table 6 insert here>

3.3 Managerial implications

Based on the analysis of the relationship between the PE and SEV, we can gain further insights into the performance of bus transit firms and then provide direction for improvement. In order to construct the performance matrix, we divide these 20 bus transit firms into four groups by average PE and SEV scores as shown in Fig. 3.[5]

Those bus transit firms located in the first group represent higher PE and SEV. They are Tansui, Chungli and Hualien. Those firms provide benchmarks for others. Bus transit firms located in the second group have lower PE, but higher SEV. The priority for these firms is to reduce the usage of input resources. Ten bus transit firms have worse PE but better SEV, including Sanchung, Capital, Taipei, Chih-nan, Taoyuan, Hsinchu, Taichung, Changhua, Ubus and Geya. Two bus transit firms, Chung-shing and Fengyuan, are located in the third group and have both lower PE and SEV. Their operators should simultaneously consider means to improve production and consumption. CitiAir, Kuang-hua, Kaohsiung, Pingtung and Chiayi belong to the fourth group. They have higher PE but lower SE. These firms should expand or adjust their services to attract more passengers and pay more attention to reducing accidents.

<Figure 3 insert here>

Since PE can be further decomposed into HBPE and UBPE, we can investigate the direction for improving PE. Similarly, we separate these 20 bus transit firms into four groups by average HBPE and UBPE scores. Fig. 4 shows the efficiency matrix.[6]

First, six bus transit firms have higher HBPE and UBPE. Those best-performing firms should maintain their HBPE and UBPE. They are CitiAir, Kuang-hua, Tansui, Chungli, Hualien and Chiayi. Second, those bus transit firms with lower HBPE and higher UBPE should focus on their HB service and improve the input resources utilization in this activity. Sanchung, Capital, Taipei, Chih-nan, Taichung and Kaohsiung belong to this group. Tayuan experiences lower HBPE and UBPE. It should adjust the usage of inputs in HB and UB activities simultaneously. Fourth, seven bus transit firms have higher HBPE, but lower UBPE. They are Chung-shing, Hsinchu, Fengyuan, Changhua, Ubus, Geya and Pingtung. The

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[5] PE is taken as the horizontal axis and SEV is taken as the vertical axis. This performance matrix is divided into four quadrants by the average scores of PE and SEV of 20 bus transit firms.

[6] HBPE is taken as the horizontal axis and UBPE is taken as the vertical axis. This efficiency matrix is divided into four quadrants by the average scores of HBPE and UBPE of 20 bus transit firms.

priority for these firms is to improve the UBPE by controlling the usage of input resources in the UB activity. Specifically, although Kaohsiung's HBPE score and Pingtung's UBPE score are lower than the average scores, their PE scores are higher than the average score. This result indicates that one of these two activities contributes to efficient production, and the other should be improved.

<Figure 4 insert here>

4. Conclusions

This paper proposes the MDNDEA model, which combines the multi-activity, network and dynamic DEA models, to identify all efficiencies and effectiveness in each activity and process in each period. The main advantage of this model is that the linkage between activities/processes, the shared inputs among activities/processes, and the effects of carry-over items are included in this unified model so as to provide more appropriate measures of performance. We use data on 20 bus transit firms in Taiwan over the period 2004-2012 to illustrate the empirical application. Based on the operational characteristics of bus transit firms, both desirable and undesirable outputs are also incorporated into this model.

Our empirical results show that the average POEV scores maintained stable variance over the 9-year period. HB and UB services appeared to have similar patterns of period efficiency. Moreover, transit bus firms performed well in the consumption process over the sample period. In terms of individual firms, the results indicate that none of the bus transit firms was effective in terms of the OEV, and the sources of operational ineffectiveness among bus transit firms were different. Some firms needed to improve their efficiencies in the HB service, some needed to improve their efficiencies in the UB service, and others needed to improve their effectiveness in the consumption process. Hence, the proposed MDNDEA model can reveal inefficiency/ineffectiveness in individual activities/processes, and provide operators more information about the trend of performance. However, there are some limitations in this paper. One is the assumption of weights. We simply set $W^t = 0.1111$, $t = 1, \ldots, 9$ and $w^H = w^U = w^P = w^C = 0.5$. Future research can relax this assumption and investigate the impacts of different weights. In addition, environmental factors are not taken into consideration. Future research can further explore environmental factors affecting the operational process of bus transit firms.

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available the data.

Appendix A

Referring to Tone and Tsutsui (2014), the linking items can be divided into fixed link and free link, and the carry-over items can be classified into desirable link, undesirable link, discretionary link and non-discretionary link. If the linking items are freely adjustable, the fixed linking constraints (4.2) and (4.10) between HB production activity and consumption process can be replaced with constraint (A.1), and constraints (4.6) and (4.11) between UB production activity and consumption process can be replaced with constraint (A.2) (Tone and Tsutsui, 2014):

$$\sum_{j=1}^{J} \lambda_{j,H}^{t} m_{ij,HC}^{t} = \sum_{j=1}^{J} \lambda_{j,C}^{t} m_{ij,HC}^{t}, \quad i = 1,\ldots,n_i,\ t = 1,\ldots,T \tag{A.1}$$

$$\sum_{j=1}^{J} \lambda_{j,U}^{t} m_{lj,UC}^{t} = \sum_{j=1}^{J} \lambda_{j,C}^{t} m_{lj,UC}^{t}, \quad l = 1,\ldots,n_l,\ t = 1,\ldots,T \tag{A.2}$$

If the carry-over items are desirable, they are treated as outputs, and target values cannot be less than the observed values. Hence, the discretionary linking constraints (4.4) and (4.8) must be substituted for constraints (A.3) and (A.4) (Tone and Tsutsui, 2014):

$$\sum_{j=1}^{J} \lambda_{j,H}^{t} d_{pj,H}^{(t,t+1)} \geq d_{pj,H}^{(t,t+1)}, \quad p = 1,\ldots,n_p, t = 1,\ldots,T-1 \tag{A.3}$$

$$\sum_{j=1}^{J} \lambda_{j,U}^{t} d_{qj,U}^{(t,t+1)} \geq d_{qj,U}^{(t,t+1)}, \quad q = 1,\ldots,n_q, t = 1,\ldots,T-1 \tag{A.4}$$

In contrast with desirable links, undesirable links are considered inputs, and target values cannot be greater than the observed values. Then, the discretionary linking constraints (4.4) and (4.8) must be replaced with constraints (A.5) and (A.6) (Tone and Tsutsui, 2014):

$$\sum_{j=1}^{J} \lambda_{j,H}^{t} d_{pj,H}^{(t,t+1)} \leq d_{pj,H}^{(t,t+1)}, \quad p = 1,\ldots,n_p, t = 1,\ldots,T-1 \tag{A.5}$$

$$\sum_{j=1}^{J} \lambda_{j,U}^{t} d_{qj,U}^{(t,t+1)} \leq d_{qj,U}^{(t,t+1)}, \quad q = 1,\ldots,n_q, t = 1,\ldots,T-1 \tag{A.6}$$

Finally, if the carry-over items are non-discretionary, their values are unchanged. Hence, the discretionary linking constraints (4.4) and (4.8) must be substituted for constraints (A.7) and (A.8) (Tone and Tsutsui, 2014):

$$\sum_{j=1}^{J} \lambda_{j,H}^{t} d_{pj,H}^{(t,t+1)} = d_{pj,H}^{(t,t+1)}, \quad p = 1,\ldots,n_p, t = 1,\ldots,T-1 \tag{A.7}$$

$$\sum_{j=1}^{J} \lambda_{j,U}^{t} d_{qj,U}^{(t,t+1)} = d_{qj,U}^{(t,t+1)}, \quad q = 1,\ldots,n_q, t = 1,\ldots,T-1 \tag{A.8}$$

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Table 1. Notations

	Production process		Consumption process
	HB service	UB service	
Dedicated inputs			
	$X^{t}_{aj,H}$		
		$X^{t}_{bj,U}$	
			$X^{t}_{ej,C}$
Shared inputs			
	$\mu^{t}_{cj,H}X^{t}_{cj,S}$	$(1-\mu^{t}_{cj,H})X^{t}_{cj,S}$	
	$\gamma^{t}_{dj,H}X^{t}_{dj,SC}$	$\gamma^{t}_{dj,U}X^{t}_{dj,SC}$	$(1-\gamma^{t}_{dj,H}-\gamma^{t}_{dj,U})X^{t}_{dj,SC}$
Intermediate inputs/outputs			
	$M^{t}_{ij,HC}$		$M^{t}_{ij,HC}$
		$M^{t}_{lj,UC}$	$M^{t}_{lj,UC}$
Carry-over items			
	$D^{(t,t+1)}_{pj,H}$		
		$D^{(t,t+1)}_{qj,U}$	
Desirable output			
			$Y^{t}_{fj,C}$
Undesirable output			
			$B^{t}_{gj,C}$

Table 2. Summary statistics of inputs and outputs

	Average	Std. Dev.	Max	Min
Dedicated Inputs				
Process of HB service				
DRIVER (persons)	193.21	209.45	1,060.78	7.61
VEHICLE (vehicles)	188.29	202.66	1,084.00	8.00
FUEL (1000 liters)	5,373.71	8,759.20	52,778.48	236.17
Process of UB service				
DRIVER (persons)	207.31	255.73	1,131.91	1.49
VEHICLE (vehicles)	184.54	203.28	818.00	2.00
FUEL (1000 liters)	4,600.52	5,870.03	21,700.87	16.44
Process of consumption				
TICKET (persons)	150.42	237.56	1,188.00	4.00
Shared inputs				
TEC (persons)	66.59	96.31	524.00	2.00
MGT (persons)	43.86	39.72	222.00	2.00
Intermediate inputs/outputs				
Process of HB service				
VEHKM (1000 vehicle-kms)	10,620.36	6,879.48	26,869.91	158.12
Process of UB service				
VEHKM (1000 vehicle-kms)	8,547.89	13,908.03	52,742.08	14.01
Carry-over item				
Process of HB service				
NWLTH (kms)	937.92	922.25	3,569.90	24.50
Process of UB service				
NWLTH (kms)	290.27	367.54	2,114.00	1.00
Outputs				
Desirable outputs				
PASSKM (1000 passenger-kms)	90,979.30	96,278.58	401,809.89	74.03
PASS (1000 persons)	29,831.10	48,054.13	213,565.56	18.68
Undesirable output				
ACC (times)	88.19	154.86	1,015.00	1.00

Note: The average is made by firm over the entire 9 years.

Table 3. Operational effectiveness and its components of individual bus transit firms

Firm	OEV	PE	HBPE	UBPE	SEV

Sanchung	0.8729 (9)	0.7459 (14)	0.4917 (19)	1.0000 (1)	1.0000 (1)
Capital	0.8816 (8)	0.7633 (13)	0.5770 (17)	0.9496 (8)	1.0000 (1)
Taipei	0.8469 (12)	0.6937 (17)	0.3874 (20)	1.0000 (1)	1.0000 (1)
Chih-nan	0.8919 (5)	0.7837 (10)	0.5875 (16)	0.9799 (7)	1.0000 (1)
CitiAir	0.8180 (15)	0.9715 (3)	1.0000 (1)	0.9431 (10)	0.6644 (19)
Chung-shing	0.7566 (18)	0.6880 (18)	0.8069 (12)	0.5692 (17)	0.8252 (16)
Kuang-hua	0.7930 (16)	0.9229 (5)	0.8458 (9)	1.0000 (1)	0.6631 (20)
Tansui	0.9347 (4)	0.9358 (4)	0.8717 (8)	1.0000 (1)	0.9336 (13)
Chungli	0.9444 (2)	0.8966 (6)	0.7932 (13)	1.0000 (1)	0.9923 (10)
Taoyuan	0.7776 (17)	0.5553 (20)	0.4997 (18)	0.6108 (15)	1.0000 (1)
Hsinchu	0.8519 (11)	0.7038 (16)	0.9372 (6)	0.4704 (19)	1.0000 (1)
Hualien	0.9871 (1)	0.9743 (2)	1.0000 (1)	0.9485 (9)	1.0000 (1)
Fengyuan	0.8217 (14)	0.7650 (12)	1.0000 (1)	0.5300 (18)	0.8783 (15)
Taichung	0.8846 (6)	0.7961 (9)	0.7897 (14)	0.8025 (12)	0.9731 (11)
Changhua	0.8818 (7)	0.7679 (11)	0.9628 (5)	0.5729 (16)	0.9958 (9)
Ubus	0.8618 (10)	0.7236 (15)	0.8306 (10)	0.6166 (14)	1.0000 (1)
Geya	0.7507 (20)	0.5564 (19)	0.8114 (11)	0.3013 (20)	0.9451 (12)
Kaohsiung	0.7511 (19)	0.8145 (8)	0.6884 (15)	0.9406 (11)	0.6877 (18)
Pingtung	0.8284 (13)	0.8609 (7)	0.9269 (7)	0.7949 (13)	0.7959 (17)
Chiayi	0.9440 (3)	1.0000 (1)	1.0000 (1)	1.0000 (1)	0.8880 (14)
Average	0.8540	0.7960	0.7904	0.8015	0.9121
Std. Dev.	0.0681	0.1280	0.1907	0.2250	0.1207

Note: Rankings are provided in parentheses.

Table 4. Spearman correlation coefficients between operational effectiveness and its components

Year		OEV	PE	SEV
2004	OEV	1.0000		
	PE	0.9580*	1.0000	
	SEV	0.4103	0.1052	1.0000
2005	OEV	1.0000		
	PE	0.8266*	1.0000	
	SEV	0.2557	-0.2844	1.0000
2006	OEV	1.0000		
	PE	0.6064*	1.0000	
	SEV	0.7068*	-0.0306	1.0000
2007	OEV	1.0000		
	PE	0.4468*	1.0000	
	SEV	0.3565	-0.6414*	1.0000
2008	OEV	1.0000		
	PE	0.6646*	1.0000	
	SEV	0.4043	-0.3156	1.0000
2009	OEV	1.0000		
	PE	0.8586*	1.0000	
	SEV	0.3132	-0.1951	1.0000
2010	OEV	1.0000		
	PE	0.8301*	1.0000	
	SEV	0.1742	-0.3405	1.0000
2011	OEV	1.0000		
	PE	0.5863*	1.0000	
	SEV	0.3613	-0.4996*	1.0000
2012	OEV	1.0000		
	PE	0.5401*	1.0000	
	SEV	0.6718*	-0.2130	1.0000
2004-2012	OEV	1.0000		
	PE	0.5248*	1.0000	
	SEV	0.4234	-0.4405	1.0000

Note: * is significant at the 5% level.

Table 5. Spearman correlation coefficients between production efficiency and its components

Year		PE	HBPE	UBPE
2004	PE	1.0000		
	HBPE	0.5524*	1.0000	
	UBPE	0.7000*	-0.1843	1.0000
2005	PE	1.0000		
	HBPE	0.5473*	1.0000	
	UBPE	0.8351*	0.0085	1.0000
2006	PE	1.0000		
	HBPE	0.7058*	1.0000	
	UBPE	0.5544*	-0.1654	1.0000
2007	PE	1.0000		
	HBPE	0.5328*	1.0000	
	UBPE	0.8248*	0.0458	1.0000
2008	PE	1.0000		
	HBPE	0.9770*	1.0000	
	UBPE	0.6443*	0.4751*	1.0000
2009	PE	1.0000		
	HBPE	0.6382*	1.0000	
	UBPE	0.6216*	-0.1554	1.0000
2010	PE	1.0000		
	HBPE	0.5655*	1.0000	
	UBPE	0.5795*	-0.2217	1.0000
2011	PE	1.0000		
	HBPE	0.7579*	1.0000	
	UBPE	0.7339*	0.1414	1.0000
2012	PE	1.0000		
	HBPE	0.9693*	1.0000	
	UBPE	0.5761*	0.4341	1.0000
2004-2012	PE	1.0000		
	HBPE	0.5479*	1.0000	
	UBPE	0.5494*	-0.2482	1.0000

Note: * is significant at the 5% level.

Table 6. Average slack value of network lengths of HB and UB during 2004-2012

Firm	Network length of HB	Network length of UB
Sanchung	1,976.51	0
Capital	400.56	-18.83
Taipei	-24.78	0
Chih-nan	-49.53	-46.54
CitiAir	0	-1.95
Chung-shing	624.24	-12.67
Kuang-hua	662.66	0
Tansui	44.06	-0.60
Chungli	-38.14	0
Taoyuan	2,320.81	172.87
Hsinchu	1,217.62	11.89
Hualien	0	-3.40
Fengyuan	0	-15.16
Taichung	1,437.29	77.26
Changhua	-5.98	-14.58
Ubus	27.97	-3.73
Geya	-11.50	-4.31
Kaohsiung	769.82	6.61
Pingtung	345.25	-2.85
Chiayi	0	0
Average	484.84	7.20
Std. Dev.	719.95	44.83

- The multi-activity dynamic network data envelopment analysis model is proposed.
- Multiple activities, multiple processes and multiple periods are considered in a unified model.
- Both desirable and undesirable outputs are considered.
- The performance of bus transit firms in Taiwan is assessed.
- The sources of operational ineffectiveness among bus transit firms are different.

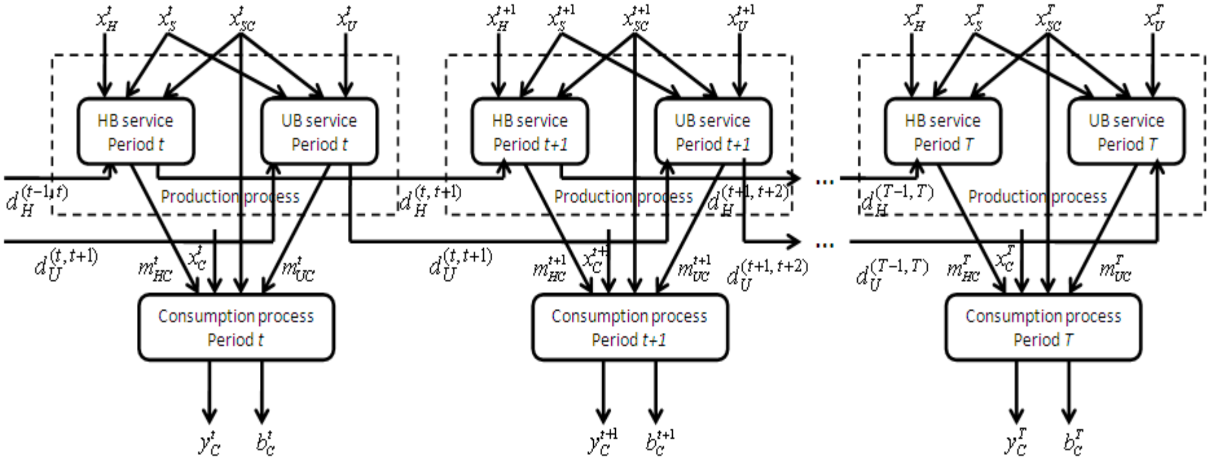

Fig. 1. The operational framework

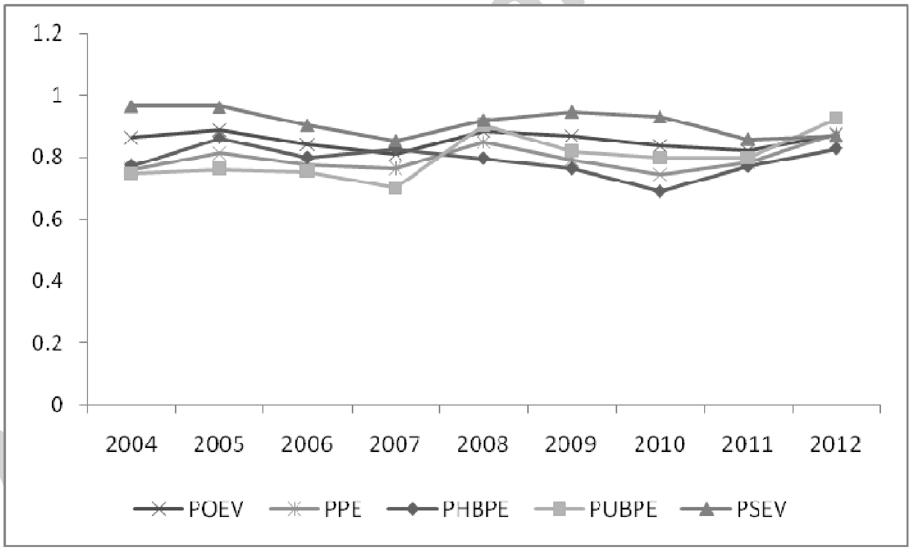

Fig. 2. Period performance

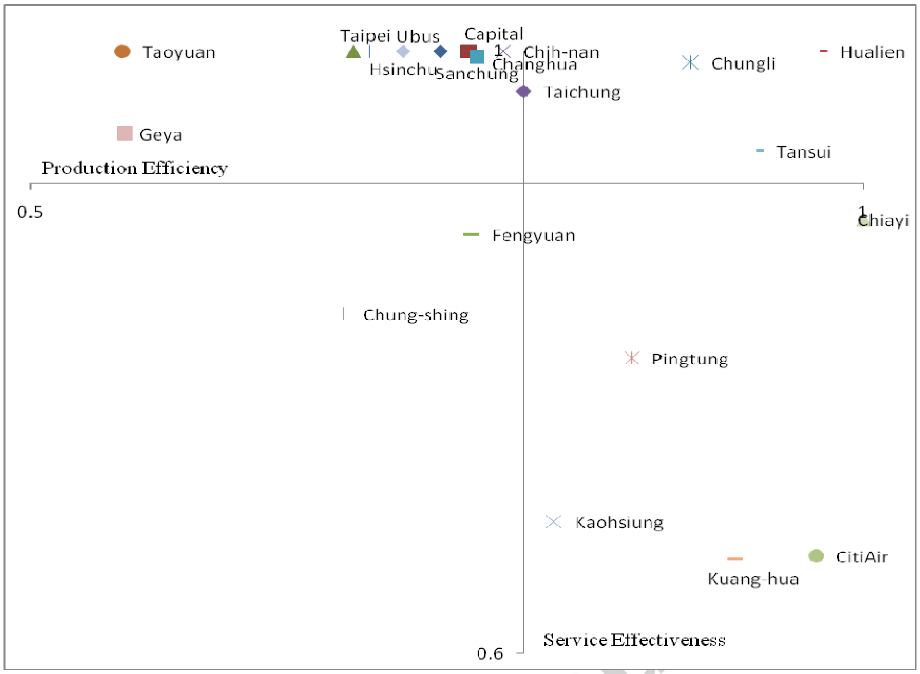

Fig. 3. Production efficiency vs. service effectiveness

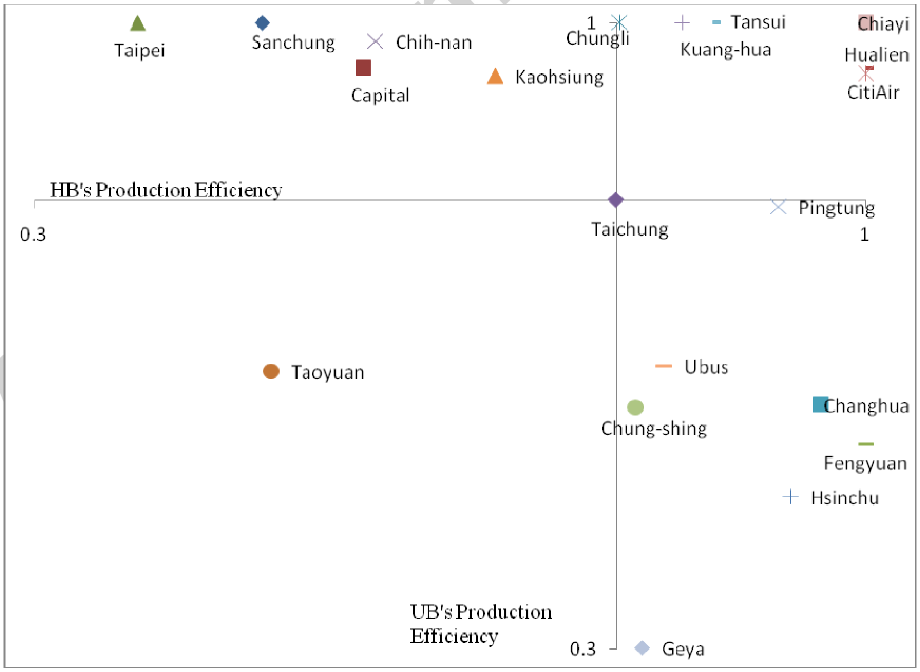

Fig. 4. HB’s production efficiency vs. UB’s production efficiency

Graphical Abstract (for review)