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Internal resource waste and centralization degree in two-stage systems: an efficiency analysis

Qingxian An[a], Hong Yan[b], Jie Wu[c*], Liang Liang[c]

[a]School of Business, Central South University, Changsha, 410083, China.

[b]Department of Logistics and Maritime Studies, The Hong Kong Polytechnic University, Hong Kong. China.

[c]School of Management, University of Science and Technology of China, Hefei, 230026, China.

Abstract

Internal resource waste refers to the waste in the intermediate resources between the upstream stage and downstream stage in a production or service system. This study examines a system with a two-stage structure, in which the outputs from the first stage are taken as the inputs for the second stage. Two-stage systems can exist in centralized, decentralized, or mixed organizational modes. In this paper, we propose two-stage DEA models considering a degree of centralization that makes it possible to measure internal resource waste in different system modes. Some managerial insights are tested and verified from the perspective of efficiency analysis. We find that: 1) when there is only one intermediate measure in a centralized two-stage system, internal resource waste can be eliminated completely, and 2) a higher degree of centralization in a two-stage system can lead to less internal resource waste and more expected outputs. Finally, we present a numerical example and two practical real-world examples that illustrate our approach and findings.

Keywords: internal resource waste; data envelopment analysis; centralization degree; two-stage structure

* Corresponding author

1. Introduction

Internal resource waste (IRW) in a production or service system refers to the waste of intermediate resources that is caused by imbalances between the initial and final stages in internal processes. IRW can be an outcome of a number of factors, including asymmetric information, inconsistent production pace, imperfect quality management, and uncertain inbound logistics processes. It occurs when the intermediate products produced by a stage in a process exceed the needs of the next stage. Most of these over-supplied products become waste and are typically stored in the warehouses of either the buyer or the vender, sold to others at a relatively low price, or even disposed of. Thus, it is necessary to investigate the IRW for realizing waste reduction which is one of the primary long-term goals of organizations [1].

A good example of internal resource waste reduction can be seen in the case of the famous Japanese company, Toyota Motor Corporation. After World War II, Toyota experienced a severe shortage of materials, and could not afford the high level of waste that was characteristic of most American companies at that time, in areas such as labor, inventory, space, and processing. Therefore, Toyota created a production mode that it named the Toyota Production System (TPS), which is driven by orders and demand, allowing it to produce only the necessary products within the required time. The physical flow of Toyota's production system is shown as follows. Parts are produced by suppliers and transported to factories based on inbound logistics. Factories use the parts to produce vehicles. Finally, the vehicles are sent to dealers based on outbound logistics. We define the processes moving from suppliers to factories as part of the production stage and the processes moving from factories to dealers as part of the sale stage.

Toyota made a great effort to reduce internal resource waste between two nearby stages. For example, the company tried its best to realize a goal of "zero inventory" in factories by ensuring that they produced only the necessary vehicles to meet customer orders in the sales stage. Because of its ability to reduce internal resource waste, Toyota underwent rapid development and became one of the most competitive motor makers in the world. And in 2012, it was ranked first in the world in automobile sales [2].

Another more traditional example can be found in the banking industry. Yang and Liu [3] analyzed the banking industry in Taiwan, and divided the operations of banks into two sub-stages, the productivity stage and the profitability stage. In the productivity stage, a bank consumes personnel costs, operation costs, and interest costs to produce deposits, which are then used in the profitability stage to bring in interest income, fee income, and fund transfer income. This is a common mechanism among banks. Different kinds of banks' structures may vary slightly, but most take "deposits" as an intermediate measure [4, 5]. Based on this, we can analyze the following real example from the banking industry, which reflects the importance of deposits as an intermediate measure. During the 2008 financial crisis, Iceland was one of the countries in Europe that suffered most heavily. The economic crisis in Iceland involved all three of the country's major banks.[†] The debts of Iceland's banks increased to roughly twelve times the amount of the country's gross domestic product (GDP). These high levels of foreign debt meant excessive deposits for the banks because the banks could not find enough companies, governments, or other organizations in which to properly invest their deposits because of the bad investment environment. Finally, the excessive debts of Iceland's banks led to a collapse of the banking industry, which was the largest experienced by any country in economic history (see http://www.economist.com/node/12762027 for a detailed description [6]). These excessive deposits could not only be seen as increasing the financial risks faced by banks, but also as increasing internal resource waste, because the deposits were not used to produce profits in the latter stages of the banks' processes, and were only held in the bank. Morrison and White [7] suggested that in order to reduce the damage caused by these excessive deposits (waste), banks and governments should consider supporting deposit insurance schemes, as the soundness of the financial sector was uncertain.

As internal resource waste results from bad coordination between stages of production, two possible approaches can be used to reduce it. One is to reduce the products produced in the former stage while still meeting the demands of the latter stage, and the other is to boost the

[†] NBI (commonly referred to as Landsbanki), Arion Bank (formerly Kaupthing Bank) and Islandsbanki (formerly Glitnir).

expected input consumption in the latter stage without exceeding the supply of the former stage [1]. These approaches both suggest that products should be produced only as needed for the next stage in an entire production process. This is also the main goal of the *just in time* (JIT) system [1, 8, 9]. Therefore, in order to measure and decrease or even eliminate the amount of internal resource waste produced in a production system, it is essential to first evaluate the system's actual internal performance and benchmarking.

Evaluating the efficiency of a production system can contribute to the understanding of the system's performance and the factors leading to efficiency. There are several methods for measuring efficiency, such as the stochastic frontier function and data envelopment analysis. Data envelopment analysis (DEA) is a popular non-parameter approach for evaluating the relative efficiency of homogenous decision making units (DMUs), especially with multiple inputs and multiple outputs [10-12]. Through the DEA approach, the inefficiency of resource utilization in a production system can be detected. Since the relative efficiency indicates a gap between the evaluated production system and an efficient one, it can set a benchmark for the production system to improve its performance [13-15]. Because of these advantages, we choose the DEA approach to investigate the performance of production systems.

The single-stage DEA model, which is the conventional DEA model, perceives the internal structure of a production system as a "black box." In other words, it does not take into account the internal structure of the production system, and provides no information about internal resource utilization performance. Because of this, the efficiency of a DMU's performance is often overrated. In this study, a two-stage DEA approach is proposed for examining the relationship between two stages, for example between a supplier in the first stage and a manufacturer in the second stage. So far, a number of studies have been conducted using the two-stage DEA approach to measure the efficiency of two-stage network structure systems where the outputs from the first stage are referred to as intermediate measures and are taken as inputs for the second stage. Cook et al. [16] reviewed the studies on two-stage DEA models and classified them into four categories: standard DEA approach, efficiency decomposition approach, network-DEA approach, and game-theoretic approach. Some latest studies of two-stage DEA can also be classified into these four categories. For example, Du et al. [17] developed a Nash bargaining game model to evaluate the performance of DMUs in a two-stage system, which can be classified as a game-theoretic approach, while Kao and Hwang [18] proposed decomposing overall efficiency into technical efficiency and scale elasticity in two-stage systems, which can be classified as an efficiency decomposition approach. Sahoo et al. [19, 20] estimated the technical efficiency by two approaches, one of which uses a single network technology for two interdependent sub-technologies which can be classified as network-DEA approach and the other uses two independent sub-technology frontiers which can be classified as efficiency decomposition approach. Premachandra et al. [21] devised a two-stage DEA model for efficiency decomposition and applied it to US mutual fund families from 1993 to 2008. However, all of these two-stage DEA models only focus on efficiency measurement and efficiency decomposition [17-25].

In the abovementioned two-stage DEA studies, only very limited attention has been paid to the problem of internal resource waste in a two-stage production system. Sahoo et al. [19] proposed the DEA model based on the assumption of allocative inefficiency that exists between the two stages when the sub-technology managers have the conflicting objectives. The conflicting objectives may result in the internal resource waste when difference exists in how much of intermediate products to produce and consume, which further results in the inefficiency. Chen and Yan [25] have employed network DEA models to measure the efficiency of a parallel two-stage production system. They analyzed the relationship between a production system and two divisions. Their study also attempts to explore the concept of internal resource waste but is limited. In their work, centralized, decentralized and mixed scenarios are analyzed individually, but the relationship between each scenarios are less studied. In this paper, we will integrate these three scenario in a general DEA model, and investigate the quantitative relationship between degree of centralization and internal resource allocation.

Imbalance between different stages in a production system causes IRW. Different kinds of controls on a production system will bring different degrees of imbalance or coordination performance, and will further affect the amount of IRW. Many scholars and managers believe that centralization in an organization can assist managers in integrating and using decentralized and limited resources to improve utilization efficiency and achieve returns to scale [26]. For example, Tomasz [26] indicates that a centralized distribution storage system allows a company to operate at a lower cost while providing customers with better service. This implies that a higher degree of centralization in a production system is more likely to decrease waste. However, thus far, this has never been theoretically proven from an efficiency perspective, which is the focus of this study. Based on degree of centralization, we classify production systems into three categories: centralized, decentralized, and mixed. A centralized system is supervised by a single super decision maker who can arrange the operations of the two stages, while a decentralized production system is one without such a super decision maker. In a mixed production system, a decision maker has decision-making power that is not absolute. Compared to the previous two-stage DEA works, this study firstly measures the quantitative relationship between degree of centralization and internal resource waste in a production system. It examines a generic two-stage system, in which all outputs of the first stage are intermediate measures that can be used as inputs in the second stage. A centralization degree index is then introduced into the two-stage DEA model. The change in the value of the centralization degree index enables us to identify the amount of IRW produced in scenarios with different degrees of centralization, ranging from those that are centralized to those that are decentralized. One interesting finding is that an efficient, centralized DEA production system with only one intermediate measure has no internal resource waste.

The remainder of this paper is organized as follows. Section 2 describes internal resource waste in different kinds of efficient two-stage systems. Section 3 measures the impact of degree of centralization on IRW. Section 4 presents a numerical example and one real-world data sets that are used to illustrate our approach and findings. The conclusions of the study are presented in Section 5.

2. Internal resource waste in two-stage system

In this section, a system with a two-stage structure is described, in which the outputs from the first stage are used as the inputs for the second stage. It is essentially the same as the traditional two-stage system presented by Kao and Hwang [23] and Du et al. [17]. Here, we use the banking system in Seiford and Zhu [27] as an example shown in Figure 1 to illustrate the two-stage structure.

***

[Insert Figure 1 about here]

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In this banking system with the above structure, the process from “employees,” “assets,” and “equity” to “revenue” and “profits” is defined as stage 1, and the process from “revenue” and “profits” to produce “market value,” “earnings per share,” and “returns to investors” is defined as stage 2. For ease of explanation, we assume that *n* DMUs with two-stage structures will be evaluated. We define the notations in the two-stage system as follows. X is the input of the production system, Y is the intermediate measure (i.e., the output of the first stage and the input of the second stage), and Z is the output of the production system. In other words, the first stage takes place when the supplier uses X to produce Y, while the second stage takes place when the manufacturer uses Y to produce Z. As we know, the output-oriented DEA model aims to find the maximum outputs for the system under the current inputs, and the input-oriented DEA model aims to find the minimum inputs for producing the current outputs of the system. In real companies, pursuing more output with limited resources are more common than reducing inputs while maintaining the level of outputs. With this in mind, we focus on building DEA output-oriented models for measuring IRW in a generic two-stage production system, where all outputs from the first stage become the inputs for the second stage. We also study the impact of the degree of centralization on IRW in a two-stage production system. By measuring the quantitative relationship between degree of centralization and internal resource waste, managers can make more reasonable decisions on enhancing, maintaining, or decreasing centralization, based on their goals for reducing waste and their firms' real ability to centralize.

2.1. Internal resource waste in a centralized production system

A centralized production system is a system in which all divisions are controlled by a single decision maker, known as the "super decision maker," who has access to available information. The inputs of the first stage for DMU_j $(j=1,2,...,n)$ are denoted as X_j, the intermediate measures as Y_j, and the outputs as Z_j. We define the DMU that is evaluated in DEA programming as DMU_0.

In a production system that is supervised by a super decision maker, the level of intermediate measures taken as the input of the second stage must not be higher than that taken as the output of the first stage. That is, $\sum_{j=1}^{n} Y_j \lambda_{j1} \geq \sum_{j=1}^{n} Y_j \lambda_{j2}$. This constraint has been popularly applied in previous studies, such as Chen and Yan [25], Chen et al. [28], Chen et al. [29]. Without this constraint, production cannot exist in real organizations. Furthermore, we assume that the initial input and final output all satisfy a condition of strong free-disposability, which is common in DEA models. The production possibility set for the centralized production system under the constant returns to scale assumption is as follows.

$$T_{central} = \left\{ (X,Y,Z) \mid \sum_{j=1}^{n} X_j \lambda_{j1} \leq X, \sum_{j=1}^{n} Y_j \lambda_{j1} \geq \sum_{j=1}^{n} Y_j \lambda_{j2}, \sum_{j=1}^{n} Z_j \lambda_{j2} \geq Z \right\}$$

where λ_{j1} and λ_{j2} stand for unknown variables (often referred to as "structural" or "intensity" variables) for connecting the input and output vectors by a convex combination.

One can evaluate the performance of a decision making unit either in input-oriented manner or in output-oriented manner or in non-oriented manner. In this paper, based on the above production possibility set, we concentrate on out-oriented manner shown in the following output-oriented DEA model. This model can be used to determine the value of the maximum expected outputs by using the current inputs, so that a manager can set a target for DMUs. At the same time, the internal resource waste in a centralized production system can be identified. The model is as follows.

$$\begin{cases} \max \ \delta_{\text{central}} \\ s.t. \quad \sum_{j=1}^{n} X_j \lambda_{j1} \le X_0 \\ \qquad \sum_{j=1}^{n} Y_j \lambda_{j1} \ge \sum_{j=1}^{n} Y_j \lambda_{j2} \\ \qquad \sum_{j=1}^{n} Z_j \lambda_{j2} \ge \delta_{central} Z_0 \\ \qquad \lambda_{j1}, \lambda_{j2} \ge 0, \ j = 1, \dots, n. \end{cases} \tag{1}$$

Denote $\delta^*_{central}$ is the objective function value of model (1). Then, $1/\delta^*_{central}$ is the efficiency of the centralized production system.

If we assume that $\delta^*_{central}$ is the optimal value of $\delta_{central}$ from model (1), $\delta^*_{central} = 1$ implies that the DMU is efficient, while $\delta^*_{central} > 1$ suggests that the DMU is inefficient under the centralized organization mode. The fewer the value $\delta^*_{central}$ is, the more efficient the DMU0 is. $\delta^*_{central}$ can be regarded as representing the maximum output proportion possible in $T_{central}$ by using input X_0. One can also set up the output-oriented network DEA model for measuring the efficiency in multiplier form as follows.

$$\begin{cases} \min \ \sum_{i=1}^{m} \omega_i x_{i0} \\ s.t. \ \ \sum_{i=1}^{m} \omega_i x_{i0} - \sum_{r=1}^{s} \mu_r y_{r0} \geq 0 \\ \quad \sum_{r=1}^{s} \mu_r y_{r0} - \sum_{k=1}^{p} v_k Z_{k0} \geq 0 \\ \quad \sum_{k=1}^{p} v_k Z_{k0} = 1 \\ \quad \omega_i, \mu_r, v_k \geq 0, i = 1,...,n, r = 1,...,s; k = 1,..., p. \end{cases} \tag{2}$$

where ω_i, μ_r, v_k are the dual decision variables to the respective constraints of model (1). Denote the optimal solution of the model (2) by ω_i^*, μ_r^*, v_k^*. One can obtain the overall efficiency, sub-processes' efficiencies as $\sum_{k=1}^{p} v_k^* Z_{k0} / \sum_{i=1}^{m} \omega_i^* x_{i0}$, $\sum_{r=1}^{s} \mu_r^* y_{r0} / \sum_{i=1}^{m} \omega_i^* x_{i0}$, $\sum_{k=1}^{p} v_k^* Z_{k0} / \sum_{r=1}^{s} \mu_r^* y_{r0}$ respectively. As $\sum_{k=1}^{p} v_k^* Z_{k0} / \sum_{i=1}^{m} \omega_i^* x_{i0} \equiv (\sum_{r=1}^{s} \mu_r^* y_{r0} / \sum_{i=1}^{m} \omega_i^* x_{i0}) \times (\sum_{k=1}^{p} v_k^* Z_{k0} / \sum_{r=1}^{s} \mu_r^* y_{r0})$, the overall efficiency is the product of the sub-processes' efficiencies. Moreover, it can be seen that this model is just the output-oriented form of the two-stage model Kao and Hwang [23] for measuring the efficiency, and is also equivalent to the output-oriented form of Sahoo et al. [19] under constant returns to scale.

We can denote $(\lambda_{j1}^*, \lambda_{j2}^*)$ as the optimal solution to model (1). If DMU_0 is not efficient in model (1), we can use the optimal values from model (1) to project this DMU onto the efficient frontier via the following formulas:

$$\text{Stage 1} \begin{cases} \hat{X}_0 = \sum_{j=1}^{n} \lambda_{j1}^* X_j \\ \hat{Y}_0^1 = \sum_{j=1}^{n} \lambda_{j1}^* Y_j \end{cases} \quad \text{and} \quad \text{Stage 2} \begin{cases} \hat{Y}_0^2 = \sum_{j=1}^{n} \lambda_{j2}^* Y_j \\ \hat{Z}_0 = \sum_{j=1}^{n} \lambda_{j2}^* Z_j \end{cases}$$

Thus, ($\hat{X}_0$, $\hat{Y}_0^1$, $\hat{Y}_0^2$, $\hat{Z}_0$) is the benchmark of DMU_0, where ($\hat{X}_0$, $\hat{Y}_0^1$) is for the first stage and ($\hat{Y}_0^2$, $\hat{Z}_0$) is for the second stage. According to the projection of the two stages, we can further rank efficient DMU based on its vector ($\hat{X}_0 - X_0, \hat{Y}_0 - Y_0, \hat{Z}_0 - Z_0$) where some arguments may be positive. The internal resource waste of Y is then $W_{central} = \hat{Y}_0^1 - \hat{Y}_0^2 = \sum_{j=1}^{n} Y_j \lambda_{j1}^* - \sum_{j=1}^{n} Y_j \lambda_{j2}^*$ when DMU_0 becomes efficient. For ease of illustration, in this study, internal resource waste throughout the paper all refers to this kind of internal resource waste which exists on the condition that the production system becomes output efficient.

It is clear that the projections for intermediate measure Z of the first stage $\sum_{j=1}^{n} Y_j \lambda_{j1}^*$ and the second stage $\sum_{j=1}^{n} Y_j \lambda_{j2}^*$ may be not equal even when the evaluated DMU is output efficient, which should be considered as inefficiency of the DMU. We will explain that even though $\sum_{j=1}^{n} Y_j \lambda_{j1}^*$, $\sum_{j=1}^{n} Y_j \lambda_{j2}^*$ obtained from model (1) are not equal, the inputs $\hat{X}_0 = \sum_{j=1}^{n} \lambda_{j1}^* X_j$ cannot be reduced any more while guaranteeing the target outputs $\hat{Z}_0 = \sum_{j=1}^{n} \lambda_{j2}^* Z_j$ can be produced. First, we give the following lemma.

Lemma 1. The projections for intermediate measure Y of the first stage $\sum_{j=1}^{n} Y_j \lambda_{j1}^*$ and the second stage $\sum_{j=1}^{n} Y_j \lambda_{j2}^*$ have the same value on at least one dimension.

Proof. This is easily obtained. If not, that is, all the dimensions of $\sum_{j=1}^{n} Y_j \lambda_{j1}^*$ are larger than that of $\sum_{j=1}^{n} Y_j \lambda_{j2}^*$, the input of the system can be reduced to produce the intermediate measures that are fewer than $\sum_{j=1}^{n} Y_j \lambda_{j1}^*$ but larger than or equal to $\sum_{j=1}^{n} Y_j \lambda_{j2}^*$ according to the monotony property of DEA technology. Therefore, $\sum_{j=1}^{n} Y_j \lambda_{j1}^*$ and $\sum_{j=1}^{n} Y_j \lambda_{j2}^*$ have the same value on at least dimension. □

Theorem 1. The gap between the projections for intermediate measure Y of two stages, $\sum_{j=1}^{n} Y_j \lambda_{j1}^*$ and $\sum_{j=1}^{n} Y_j \lambda_{j2}^*$, cannot be reduced by reducing the inputs $\sum_{j=1}^{n} \lambda_{j1}^* X_j$ while guarantees the outputs $\sum_{j=1}^{n} \lambda_{j2}^* Z_j$ can be produced.

Proof. According Lemma 1, there is at least one dimension of the projected intermediate measures of two stages has the same value. The tight constraint(s) makes the reduction of inputs is impossible. Because if it is possible, $\sum_{j=1}^{n} \lambda_{j1}^* X_j$ must not be the optimal solution of model (1). The contradiction occurs. □

2.2. Internal resource waste in a decentralized production system

Unlike a centralized production system, a decentralized production system is one in which each division has its own incentive and strategy, without a super decision maker controlling all divisions. Each stage of production involves pursuing the maximum output needed to achieve goals. In this scenario, the second stage cannot see the detailed situation of the first stage, thus it should make decision based on the bygone intermediate measure from stage 1. The production possibility set corresponding to a decentralized production system is as follows.

$$T_{decentral} = \left\{ (X, Y, Z) \mid \sum_{j=1}^{n} X_j \lambda_{j1} \le X, \sum_{j=1}^{n} Y_j \lambda_{j1} \ge \sum_{j=1}^{n} Y_j \lambda_{j2}, \sum_{j=1}^{n} Y_j \lambda_{j2} \le Y, \sum_{j=1}^{n} Z_j \lambda_{j2} \ge Z \right\}$$

The internal resource waste of DMU_0 in a decentralized production system can be identified through the following model.

$$\begin{cases} \max \ \delta_{decentral} \\ s.t. \ \ \sum_{j=1}^{n} X_j \lambda_{j1} \le X_0 \\ \sum_{j=1}^{n} Y_j \lambda_{j1} \ge \sum_{j=1}^{n} Y_j \lambda_{j2} \\ \sum_{j=1}^{n} Y_j \lambda_{j2} \le Y_0 \\ \sum_{j=1}^{n} Z_j \lambda_{j2} \ge \delta_{decentral} Z_0 \\ \lambda_{j1}, \lambda_{j2} \ge 0, \\ i = 1,...,m; p = 1,...,t; r = 1,...,s; j = 1,...,n \end{cases} \tag{3}$$

By using model (3), we can determine the efficiency of DMU_0 and its IRW. Denote that $\delta^*_{decentral}$ by the optimal value of $\delta_{decentral}$ from model (3). Then, $1/\delta^*_{decentral}$ is the efficiency of the centralized production system. $\delta^*_{decentral} = 1$ implies that DMU_0 is efficient, while $\delta^*_{decentral} > 1$ suggests that the DMU is inefficient under the decentralized organization mode. $\delta^*_{decentral}$ can be regarded as representing the maximum output proportion possible in decentralized mode by using input X_0.

Similarly, one can set up the DEA model for estimating the efficiency in multiplier form as

$$\begin{cases} \min \ \sum_{i=1}^{m} \omega_i x_{i0} + \sum_{r=1}^{s} u_r y_{r0} \\ s.t. \ \ \sum_{i=1}^{m} \omega_i x_{i0} - \sum_{r=1}^{s} \mu_r y_{r0} \ge 0 \\ \sum_{r=1}^{s} \mu_r y_{r0} + \sum_{r=1}^{s} u_r y_{r0} - \sum_{i=1}^{p} v_k Z_{k0} \ge 0 \\ \sum_{i=1}^{p} v_k Z_{k0} = 1 \\ \omega_i, \mu_r, v_k \ge 0, i = 1,...,n, r = 1,...,s; k = 1,..., p. \end{cases} \tag{4}$$

We can denote $(\lambda^*_{j1}, \lambda^*_{j2})$ as the optimal solution to model (3). If DMU_0 is not efficient in model (3), we can use the optimal values from model (3) to gain the benchmark for the two stages.

$$\text{Stage 1} \begin{cases} \hat{X}_0 = \sum_{j=1}^{n} \lambda_{ij}^* X_j \\ \hat{Y}_0^1 = \sum_{j=1}^{n} \lambda_{j1}^* Y_j \end{cases} \quad \text{and} \quad \text{Stage 2} \begin{cases} \hat{Y}_0^2 = \sum_{j=1}^{n} \lambda_{j2}^* Y_j \\ \hat{Z}_0 = \sum_{j=1}^{n} \lambda_j^* Z_j \end{cases}$$

The internal resource waste of Y is then $W_{decentral} = \hat{Y}_0^1 - \hat{Y}_0^2 = \sum_{j=1}^{n} Y_j \lambda_{j1}^* - \sum_{j=1}^{n} Y_j \lambda_{j2}^*$ when DMU_0 becomes efficient.

Assuming that production systems under centralized and decentralized organization modes require the same inputs, intermediate measures, and outputs, we can then obtain the following results regarding the two efficiencies of DMU_0 under centralized and decentralized organization modes through the above models.

Theorem 2: $\delta_{central}^* \geq \delta_{decentral}^*$.

The proof is in the Appendix.

Theorem 2 shows that the efficiency, the reciprocal of optimal δ^*, of a DMU under a centralized organization mode is not higher than that with same inputs, intermediate measures, and outputs under the decentralized organization mode. This indicates that a production system can produce more outputs under a centralized organization mechanism than under a decentralized organization mechanism, on the condition that the production system becomes efficient in each situation. It should be noted that the result does not suggest that a decentralized production system performs better than a centralized one, and only involves a comparison of the performance of two production systems with identical inputs, intermediate measures, and outputs under two organizational mechanisms. Because the performance of an efficient centralized production system is not worse than that of an efficient decentralized production system for a group of DMUs, the efficient DMUs in a centralized organization should also be efficient in a decentralized one. However, DMUs that are efficient in a decentralized production system may not be efficient in a centralized one. Therefore, DMUs on a centralized production frontier must be on a decentralized production frontier but DMUs on a decentralized production frontier may not be on a centralized production frontier. If a DMU on a decentralized production frontier is not on a centralized production frontier, it implies that if the DMU wants to be efficient under a centralized organization mechanism, it should make a greater effort, because more outputs are needed for efficiency.

Based on the illustration of the internal resource waste of two-stage systems in decentralized and centralized organization mechanisms when the system becomes DEA efficient, we regard the relationship between them as follows.

Theorem 3: *If all evaluated DMUs transform their organization mechanisms from decentralized to centralized, the internal resource waste of each DMU will not increase. That is,* $W_{central} \leq W_{decentral}$.

The proof is in the Appendix.

Theorem 3 shows that under a centralized organization mechanism, the flexibility of the DMU in allocating intermediate measures increases. In a decentralized organization mechanism, each stage of the DMU has its own incentive and it is essential that the adjustment of resources to Y_0 for the second stage does not exceed the output produced in the first stage. However, this restriction does not apply to the DMU in a centralized organization mechanism, in which the level taken as the input of the second stage should not exceed that taken as the expected output of the first stage. Under a centralized production system, more resources can be invested into the second stage to reduce IRW. Subsequently, the DMU under a centralized organization mechanism does not produce more waste than under the decentralized organization mechanism.

Furthermore, we can develop the following theorem about internal resource waste in a centralized production system.

Theorem 4: *If all evaluated DMUs with only one intermediate measure are under centralized organization mechanisms, the internal resource waste of these DMUs will not exist* *when they become DEA efficient.*

The proof is in the Appendix.

In the presence of a single intermediate measure, Theorem 4 shows that the internal waste can be avoided in some simple production systems that have only one intermediate measure, by controlling the degree of centralization.

3. Measure the effect of centralization degree on internal resource waste

In this section, the influence of degree of centralization on internal resource waste is examined. Based on Theorem 3, we know that internal resource waste under a centralized organization mechanism does not exceed IRW under a decentralized one, meaning that the degree of centralization of a production system can affect the level of waste produced. It is widely accepted that few production systems are under absolute centralized control or decentralized control and most production systems operate under a certain degree of centralization, so achieving balance between decentralization and centralization is an important challenge for many organizations [30]. In many production systems, stages have their own decision authorities that aim to achieve the goals of the overall production system. In particular, almost all production systems in holding sub-companies are under the control of both their holding companies and themselves, as is the case for some sub-companies of the Shanghai Automotive Industry Corporation Group Motor Corporation Limited in China (SAIC). Therefore, it is essential to study the quantitative relationship between the degree of centralization and internal resource waste in production systems, in order to determine the proper degree of centralization for managers according to waste requirement, or to set waste targets according to degree of centralization.

A comparison between $\sum_{j=1}^{n} Y_j \lambda_{j1}^*$ in model (1) (i.e., the centralized production system) and intermediate product value Y_o can lead to two possible scenarios, which are as follows. For ease of illustration, we represent $\sum_{j=1}^{n} Y_j \lambda_{j1}^{*}$ from model (1) as R_0.

Scenario I: All arguments of Y_o satisfy $\max\{R_{r0}, Y_{r0}\} = Y_{r0}$, $r = 1, ..., m.$ m is the number of intermediate measures. In this scenario, model (3) is completely equivalent to model (1), because if all arguments satisfy $\max\{R_{r0}, Y_{r0}\} = Y_{r0}$, then the constraints of $\sum_{j=1}^{n} Y_j \lambda_{j2} \le Y_0$ have no function in model (3). This implies that centralized and decentralized production systems have no differences in terms of internal resource waste.

Scenario II: Not all arguments of Y_0 satisfy $\max\{R_{r0}, Y_{r0}\} = Y_{r0}$ (i.e., some or all arguments satisfy $\max\{R_{r0}, Y_{r0}\} = R_{r0}$). In this scenario, the performance of the centralized production system is different from that of the decentralized production system in terms of IRW, in that the waste of the centralized production system does not exceed that of the decentralized one. When index r satisfies $\max\{R_{r0}, Y_{r0}\} = Y_{r0}$, the constraint of $\sum_{j=1}^{n} Y_{rj} \lambda_{j2} \le Y_0$ in model (3) has no function with regard to waste difference, and therefore we only adjust the constraints of the arguments that satisfy $\max\{R_{r0}, Y_{r0}\} = R_{r0}$ when we change the degree of centralization.

Because model (3) is equivalent to model (1) in the first scenario, the degree of centralization does not affect internal resource waste. For the second scenario, the following model is applied to measure the relationship between degree of centralization and internal resource waste.

$$\begin{cases} \max \ \delta_\kappa \\ s.t. \ \ \sum_{j=1}^{n} X_j \lambda_{j1} \le X_0 \\ \quad \sum_{j=1}^{n} Y_j \lambda_{j1} \ge \sum_{j=1}^{n} Y_j \lambda_{j2} \\ \quad \sum_{j=1}^{n} Y_j \lambda_{j2} \le Y_0 + (C_0 - Y_0) \times \kappa \\ \quad \sum_{j=1}^{n} Z_j \lambda_{j2} \ge \delta Z_0 \\ \quad \lambda_{j1}, \lambda_{j2} \ge 0, \ j = 1, \dots, n \end{cases} \tag{5}$$

where $C_0 = \max\left\{R_{r0}, Y_{r0}\right\}$. R_{r0} is gained from model (1). κ is a constant given before we solve the programming problem, with a value that lies between zero and one. This constant is used to indicate the difference between a given production system and the decentralized production system, in order to reflect the degree of centralization. Thus, we define κ by the centralization degree. When $\kappa = 0$, no difference can be found between the given production system and the decentralized one, meaning that the given production system is a decentralized production system. When $\kappa = 1$, the difference between the given production system and the decentralized one is at a maximum, meaning that the given production system is a centralized production system. When $0 < \kappa < 1$, the production system is under a mixed organizational mechanism. The IRW of Y for any κ is also calculated by $\sum_{j=1}^{n} Y_j \lambda_{j1}^* - \sum_{j=1}^{n} Y_j \lambda_{j2}^*$, where $(\lambda_{j1}^*, \lambda_{j2}^*)$ is the optimal solution for model (5). In fact, model (5) is also applicable to the first scenario. In this case, $C_0 = Y_0$, then $(C_0 - Y_0) = 0$ and κ has no function in model (5), suggesting that model (5) can be used to measure IRW in any scenario. Through computer programming, we can calculate the results for different degrees of centralization κ, ranging from 0 to 1. Internal resource waste in a production system under different degrees of centralization κ can thus be found by employing model (5).

When different degrees of centralization κ are assigned to different production systems, the following theorem regarding these production systems' efficiency can be derived.

Theorem 5: *For two centralization degree index* κ_1 *and* κ_2*, if* $\kappa_1 \geq \kappa_2$*, then* $\delta^*_{\kappa_1} \geq \delta^*_{\kappa_2}$.

The proof is in the Appendix.

Theorem 5 shows that a production system with a higher degree of centralization is relatively less efficient than one with a lower degree of centralization when the inputs, intermediate measures, and outputs are same. The reason for this is similar to the explanation given for Theorem 2, which indicates that an efficient production system with more centralization can produce more outputs. Because there are often multiple optimal solutions to the equation in model (5), internal resource waste as calculated by $\sum_{j=1}^{n} Y_j^* \lambda_{j1} - \sum_{j=1}^{n} Y_j^* \lambda_{j2}$ may vary when different computer software is used, which makes comparison among systems with different centralization degree fairly difficult. To address this problem, we set a second objective to further optimize the optimal solutions. After gaining the related efficiency value through model (5), we maximize the slack to gain unique results for internal resource waste when efficiency remains unchanged.

The steps involved in this approach are shown below. First, as the optimal value of model (1) is equivalent to model (5) when $\kappa = 1$, the following model is applied to determine the IRW of a centralized production system, and R_0 is used to set the degree of centralization.

$$
\begin{cases}
\max \ S_0^- \\
s.t. \quad \sum_{j=1}^{n} X_j \lambda_{j1} \leq X_0 \\
\qquad \sum_{j=1}^{n} Y_j \lambda_{j1} = \sum_{j=1}^{n} Y_j \lambda_{j2} + S_0^- \\
\qquad \sum_{j=1}^{n} Z_j \lambda_{j2} \geq \delta^*_{central} Z_0 \\
\qquad \lambda_{j1}, \lambda_{j2}, S_0^- \geq 0, \ j = 1, \ldots, n
\end{cases}
\tag{6}
$$

This is a multiple objective program when the number of intermediate measures is multiple. To solve the program, we sum up the slack as a new objective function, in order to transform the multiple objectives program into a single objective program. Then, the model becomes:

$$\begin{cases} \max \ e^{T} S_0^- \\ s.t. \ \ \sum_{j=1}^{n} X_j \lambda_{j1} \le X_0 \\ \quad \sum_{j=1}^{n} Y_j \lambda_{j1} = \sum_{j=1}^{n} Y_j \lambda_{j2} + S_0^- \\ \quad \sum_{j=1}^{n} Z_j \lambda_{j2} \ge \delta_{central}^{*} Z_0 \\ \quad \lambda_{j1}, \lambda_{j2}, S_0^- \ge 0, j = 1, \dots, n \end{cases} \tag{7}$$

where $\delta_{central}^{*}$ is the optimal value of model (1). Through model (7), $\sum_{j=1}^{n} Y_j \lambda_{j1}^{*}$ (i.e., R_0 in a centralized production system) can be obtained. When the production system is mixed, the optimal value δ_{κ}^{*} can be obtained by applying model (5). We can then measure internal resource waste with the following model.

$$\begin{cases} \max \ e^{T} S_0^- \\ s.t. \ \ \sum_{j=1}^{n} X_j \lambda_{j1} \le X_0 \\ \quad \sum_{j=1}^{n} Y_j \lambda_{j1} - \sum_{j=1}^{n} Y_j \lambda_{j2} = S_0^- \\ \quad \sum_{j=1}^{n} Y_j \lambda_{j2} \le Y_0 + (C_0 - Y_0) \times \kappa \\ \quad \sum_{j=1}^{n} Z_j \lambda_{j2} \ge \delta_{\kappa}^{*} Z_0 \\ \quad \lambda_{j1}, \lambda_{j2}, S_0^- \ge 0, j = 1, \dots, n \end{cases} \tag{8}$$

where δ_{κ}^{*} is the optimal value of model (5). S_0^- represents the IRW in a production system with κ degree of centralization. The other variables represent the same meanings as those in model (5). Through the change of parameter κ in model (8), we can identify the level of IRW during the process of centralization.

Theorem 6: *Internal resource waste does not increase as the degree of centralization* *increases in a two-stage system.*

The proof is in the Appendix.

4. Applications

In this section, we consider two data sets. The first one is a numerical example that was used in Chen and Yan [25]. The other one is about non-life insurance companies involving two intermediate measures, which was first used in Kao and Hwang [23].

4.1. A Numerical example

In this subsection, we use numerical data to examine the relationship between degree of centralization and internal resource waste in a production system. A total of nine production systems with different degrees of centralization (ranging from decentralized to centralized) are applied to reflect changes in internal resource waste. Both tables and figures are used to describe the relationship between centralization degree and IRW. The numerical data is from Chen and Yan [25]. As the original data has no practical implications, we redefine the input, intermediate measure, and output to demonstrate our approach. We take X as the input, Y_1 and Y_2 as the intermediate measures, and Z_1 and Z_2 as the outputs for the two-stage structure presented in Figure 1. The raw data are as follows in Table 1.

***

[Insert Table 1 here]

***

According to model (1) and model (7), we can measure the efficiencies of these DMUs and their projections under centralized organization mechanism which are given in the following Table.

***

[Insert Table 2 here]

***

Nine DMUs, A, B, …, I are used as samples. By increasing the centralization degree value κ from 0 to 1 by 0.1 for each step, the internal waste of Y_1 and Y_2 for different degrees of centralization can be identified. When $\kappa = 0$, it represents a decentralized organization mechanism. When $\kappa = 1$, it represents a centralized organization mechanism. The IRW of Y_1 is presented in Table 2 and Figure 2.

***

[Insert Table 3 about here]

***

***

[Insert Figure 2 about here]

***

In Figure 2, the value of centralization degree κ lies on the x-axis, which represents the centralization of the production system, while the internal resource waste of Y lies on the y-axis. The higher the value of κ is, the higher the degree of centralization is in the production system. It can be seen from Table 3 and Figure 2 that internal resource waste does not increase as degree of centralization increases. In other words, the correlation between the internal resource waste of Y_1 and the centralization degree κ is negative.

Among the DMUs shown in the sample, DMUs A, G, and I involve no internal resource waste under any degree of centralization in the production system, while the waste of all other DMUs drops when κ changes from 0 (decentralized) to 1 (centralized). DMUs B, D, and E involve no waste when the centralization degree reaches a certain level. Although the elimination of waste cannot be completely achieved in DMUs C, F, and H, the level of waste falls during the centralization process.

Similarly, by using models (5) and (8), we can identify the IRW of Y_2.

***

[Insert Table 4 about here]

***

***

[Insert Figure 3 about here]

***

It can be seen from Table 4 and Figure 3 that the internal resource waste of Y_2 drops in all DMUs as degree of centralization increases. When the production systems are under a centralized organization mechanism, no internal resource waste is found. Among all the DMUs, DMU I is a special case in which almost no waste is produced under any degree of centralization.

With the examples given above, we can further verify that a system with a higher degree of centralization should result in less internal resource waste, and our approach can be used to measure the quantitative relationship between degree of centralization and internal resource waste. Our findings suggest that centralization could benefit DMUs by reducing their internal resource waste in production systems.

Based on our analysis, decision makers can determine suitable degrees of centralization according to their organizations' targets for waste reduction and level of centralization ability. In some cases, centralization increases cost, so decision makers should consider this in increasing degree of centralization. Moreover, Figure 2 and 3 suggest that most DMUs' internal resource waste does not have a linear relationship with degree of centralization. In addition, some DMUs' internal resource waste decreases greatly in the initial stage of centralization and then slowly afterward, while some DMUs' IRW decreases slowly initially and then at a much faster pace. These non-linear relationships should be used as guidance by managers in deciding whether it is worthwhile to increase or decrease the degree of centralization, based on current levels of centralization and the expected costs of increases or decreases.

4.2. Non-life issuance companies

The data in this section is from Kao and Hwang [23], who examined the performance of 24 non-life insurance companied in Taiwan. They divided the insurance industry into two sub-processes: premium acquisition and profit generation. "Operation expenses", "insurance expenses" are the inputs of the first stage, "direct written premiums" and "reinsurance premiums" are the intermediate measures, and "underwriting profit", "investment profit" are the outputs of the second stage. The same with this work, we also assume the constant returns to scale for these companies.

In this system, we show the balanced situation of supply and demand. If the "direct written premiums" and "reinsurance premiums" used in the profit generation stage are smaller than those produced in the premium acquisition stage, it implies that more intermediate measures are produced for the final products. These unused resources will become waste in this system. We can measure the IRW of these intermediate measures under different degrees of centralization κ from 0 to 1. The IRW for "direct written premiums" and "reinsurance premiums" are shown in Figures 4 and 5, respectively.

***

[Insert Figure 4 about here]

***

Figure 4 shows a similar trend to other figures above, in that IRW reduces to zero when the issuance system becomes a centralized one. According to our results, DMUs 2, 9, 11, 12, 15 and 19 produce no waste at any degree of centralization. For most other DMUs, waste is reduced at a variable speed as degree of centralization increases. Taking DMU 17 (Newa) as an example, the slope of the curve increases after $\kappa = 0.7$, meaning that waste begins to decrease after κ increases to 0.8. When centralization degree κ increases from 0 to 0.7, the waste decreases by the same value (204005.5) for each step, but from $\kappa = 0.7$ to $\kappa = 0.8$, it decreases by 473825.4. From $\kappa = 0.8$ to $\kappa = 0.9$ and from $\kappa = 0.9$ to $\kappa = 1$, it decreases by 491564.2. Moreover, we find that there is no linear relationship between the degree of centralization and internal resource waste for most DMUs, expect in the case of DMUs 3, 6, 8, 16, 21 and 23.

***

[Insert Figure 5 about here]

***

Figure 5 also shows that there is no linear relationship between degree of centralization and internal resource waste for most DMUs. The waste in this figure changes in a manner that is different from that of waste of "direct written premiums". Many DMUs have waste levels that are always zero, such as DMUs 1, 3, 6, 9, 10, 11, 12, 15, 20, 21, 22, and 23. In addition, the waste of "reinsurance premiums" shows different trends for different companies. Most companies' waste of "reinsurance premiums" decreases more greatly in the initial step from decentralization to centralization. Two special cases are DMU 4 and DMU 10 which decreases more slowly in the initial step. Most DMUs' waste of "direct written premiums" are reduced to zero when the degree of centralization κ increases to 0.8, except in DMUs 2, 4, 10, 19, 26, and 24. It should be noted that there is no waste of this intermediate product for all DMUs employing centralized organization mechanisms.

5. Conclusions

In this paper, we prove that a centralized production system leads to equal or reduced internal resource waste in a decentralized production system. We also examine the quantitative relationship between degree of centralization and internal resource waste in a production system, and find that a higher level of centralization should result in less internal resource waste and more expected outputs. In addition, we find that an efficient centralized production system creates no waste when it has only one intermediate measure, which indicates that production systems should reduce their numbers of intermediate measures and unnecessary processes as much as possible. This insight supports management approaches that aim to simplify production systems' internal processes.

We began by reviewing other works related to internal resource waste in production systems, and by defining degree of centralization based on the results obtained for production systems under centralized and decentralized organizational mechanisms. Then, we developed two-stage DEA models for measuring IRW in production systems with different degrees of centralization. Because the optimal solutions of the models were arbitrary, a second objective was proposed for further optimizing slack, and the efficiency of DMUs with different degrees of centralization was measured before slacks was further optimized. A numerical example and a practical example were employed to illustrate our approach, and the results show that the internal resource waste of certain production systems changes as the degree of centralization changes. From the numerical example, we find that internal resource waste may still exist even when a production system becomes centralized. From the practical example, we find that resource waste can be reduced to zero by employing a centralized organizational mechanism.

As mentioned above, the internal resource waste in model (4) is a multiple objective program when the number of intermediate measures involved is multiple. In Section 3, we provide a means of summing up waste and transforming a multiple objective program into a single objective program. While different approaches can achieve the same purpose, such as summing up the proportion of slack to actual intermediate measures, in this study, only a traditional two-stage structure is examined, in which outputs from the first stage are referred to as intermediate measures and are taken as inputs for the second stage. Our approach is also suitable for analyzing more complex systems. For example, our models can be easily extended to solve two-supplier problems in which two suppliers are parallel in the first stage. Under such a two-stage system, we only need to consider the two suppliers simultaneously (i.e., a constraint regarding the added supplier should be added to our internal waste model). Describing degree of centralization with a parameter in the model can be a commonly used approach to addressing similar internal resource waste problems.

In fact, in addition to internal resource waste, there are many factors affecting the degree of centralization for managers, such as the willingness of top organizational members to delegate power and the availability of management talent. This paper only measures the appropriate degree of centralization for internal resource waste, without considering such factors. In the real world, when deciding on the appropriate degree of centralization in an organizational structure, many significant factors should be comprehensively considered, in order to make the correct decision on the degree of centralization. In future work, we feel that the waste of a production system should be studied with imprecise data, such as bounded values, ordinal values, and ratio bounded values. Further studies regarding the strategy of organizations and the detailed expectations of each body in a two-stage structure should also be conducted, in order to make the results more precise to the true situations of organizations. Another point is extending the models in this paper to variable returns to scale scenario so as to analyze more real examples. At last, this paper only considers internal resource waste that occurs when supply in a first stage is larger than need in a second stage. When supply in a first stage is smaller than need in a second stage, a shortage will occur, rather than waste. Predicting such shortages with DEA models is an area that we hope to address in our future work.

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Appendix

Proof of Theorem 2: If we suppose that ($\delta^*_{decentral}$, λ^*_{j1} and λ^*_{j2}) is the optimal solution for model (3), this satisfies formula (9).

$$\begin{cases} \sum_{j=1}^{n} X_j \lambda^*_{j1} \le X_0 \\ \sum_{j=1}^{n} Y_j \lambda^*_{j1} \ge \sum_{j=1}^{n} Y_j \lambda^*_{j2} \\ \sum_{j=1}^{n} Y_j \lambda^*_{j2} \le Y_0 \\ \sum_{j=1}^{n} Z_j \lambda^*_{j2} \ge \delta^*_{decentral} Z_0 \\ j = 1,\ldots,n. \end{cases} \tag{9}$$

If this is the case, it must also satisfy formula (10).

$$\begin{cases} \sum_{j=1}^{n} X_j \lambda^*_{j1} \le X_0 \\ \sum_{j=1}^{n} Y_j \lambda^*_{j1} \ge \sum_{j=1}^{n} Y_j \lambda^*_{j2} \\ \sum_{j=1}^{n} Z_j \lambda^*_{j2} \ge \delta^*_{decentral} Z_0 \\ j = 1,\ldots,n. \end{cases} \tag{10}$$

Therefore, the optimal solution for model (1) is a feasible solution to model (3) and thus, $\delta^*_{central} \ge \delta^*_{decentral}$. □

Proof of Theorem 3: When the patterns of DMUs change from those of a centralized production system to those of a decentralized one, according to proposition 1, efficiency should not decrease, and so $\delta^*_{decentral} \le \delta^*_{central}$. We can then verify that λ_{j1} and λ_{j2} in the two patterns satisfy the formulas (11) and (12), respectively, which are as follows.

$$\max \quad (\sum_{j=1}^{n} Y_j \lambda_{j1} - \sum_{j=1}^{n} Y_j \lambda_{j2})$$
$$s.t. \quad \begin{cases} \sum_{j=1}^{n} X_j \lambda_{j1} \le X_0 \\ \sum_{j=1}^{n} Y_j \lambda_{j1} \ge \sum_{j=1}^{n} Y_j \lambda_{j2} \\ \sum_{j=1}^{n} Z_j \lambda_{j2} \ge \delta^*_{central} Z_0 \\ \lambda_{j1}, \lambda_{j2} \ge 0, \; j = 1,...,n \end{cases} \quad (11)$$

and

$$\max \quad (\sum_{j=1}^{n} Y_j \lambda_{j1} - \sum_{j=1}^{n} Y_j \lambda_{j2})$$
$$s.t. \quad \begin{cases} \sum_{j=1}^{n} X_j \lambda_{j1} \le X_0 \\ \sum_{j=1}^{n} Y_j \lambda_{j1} \ge \sum_{j=1}^{n} Y_j \lambda_{j2} \\ \sum_{j=1}^{n} Y_j \lambda_{j2} \le Y_0 \\ \sum_{j=1}^{n} Z_j \lambda_{j2} \ge \delta^*_{decentral} Z_0 \\ \lambda_{j1}, \lambda_{j2} \ge 0, \; j = 1,...,n \end{cases} \quad (12)$$

Because $\delta^*_{decentral} \le \delta^*_{central}$ and because of the constraint $\sum_{j=1}^{n} Y_j \lambda_{j2} \le Y_0$ of (12), a centralized production system will require a higher $\sum_{j=1}^{n} Y_j \lambda_{j2}$ in order to generate more outputs $\delta^*_{central} Z_0$. Thus, $\sum_{j=1}^{n} Y_j \lambda^*_{j2}$ in a centralized production system should not be smaller than that in a decentralized production system.

λ^*_{j1} from model (1) satisfies the condition of formula (12), and therefore $\sum_{j=1}^{n} Y_j \lambda^*_{j1}$ in formula (12) is not smaller than in formula (11) when we further maximize internal resource waste based on the efficiency found. As the internal resource waste is $\sum_{j=1}^{n} Y_j \lambda^*_{j1} - \sum_{j=1}^{n} Y_j \lambda^*_{j2}$ for each model, we can then verify that the internal resources of DMUs under a centralized production system do not exceed those under a decentralized production system. It should be noted that the objective function is a vector in both model (11) and model (12). Also, the two models are single-objective programs when the number of intermediate measures involved is only one, but they become multiple-objective programs when the number of intermediate measures is more than one. □

Proof of Theorem 4: If there is internal resource waste in a centralized production system, the expected outputs of the first stage must be smaller than the expected inputs of the second stage. In other words, $\sum_{j=1}^{n} Y_j \lambda_{j1}^* > \sum_{j=1}^{n} Y_j \lambda_{j2}^*$, where λ_{j1}^* and λ_{j2}^* are the optimal solutions of centralized model (1). We assign an optimal value of $\delta_{central}^*$. Also, because the intermediate measure is only one variable, we can easily find another feasible solution $\lambda_{j2}^{'}$, satisfying $\sum_{j=1}^{n} Y_j \lambda_{j2}^{'} = \sum_{j=1}^{n} Y_j \lambda_{j1}^*$ and $\sum_{j=1}^{n} Z_j \lambda_{j2}^{'} > \sum_{j=1}^{n} Z_j \lambda_{j2}^*$. As the constraint of $\sum_{j=1}^{n} Z_j \lambda_{j2} \geq \delta_{central} Z_0$ in model (1), the feasible subjective function value $\delta'_{\mathrm{central}}$ corresponding to λ_{j1}^*, $\lambda_{j2}^{'}$ will be greater than $\delta_{central}^*$. This contradicts the idea that $\delta_{central}^*$ is the optimal value of model (1). Thus, if only one intermediate measure exists in the centralized production system, internal resource waste will be non-existent. Thus, theorem 3 is true. □

Proof of Theorem 5: If we suppose that $\delta_{decentral}^*$, λ_{j1}^* and λ_{j2}^* are an optimal solutions of model (6) when $\kappa = \kappa_2$, and thus satisfy formula (13).

$$\begin{cases} \sum_{j=1}^{n} X_j \lambda_{j1} \leq X_0 \\ \sum_{j=1}^{n} Y_j \lambda_{j1} \geq \sum_{j=1}^{n} Y_j \lambda_{j2} \\ \sum_{j=1}^{n} Y_j \lambda_{j2} \leq Y_0 + (C_0 - Y_0) \times \kappa_2 \\ \sum_{j=1}^{n} Z_j \lambda_{j2} \geq \delta Z_0 \\ \lambda_{j1}, \lambda_{j2} \geq 0,\ j = 1, \ldots, n \end{cases} \tag{13}$$

As $\kappa_1 \geq \kappa_2$, $Y_0 + (C_0 - Y_0) \times \kappa_2 \leq Y_0 + (C_0 - Y_0) \times \kappa_1$, for any λ_{j1}^* and λ_{j2}^* from model

(5) when $\kappa = \kappa_2$, they also satisfy formula (14).

$$\begin{cases} \sum_{j=1}^{n} X_j \lambda_{j1} \le X_0 \\ \sum_{j=1}^{n} Y_j \lambda_{j1} \ge \sum_{j=1}^{n} Y_j \lambda_{j2} \\ \sum_{j=1}^{n} Y_j \lambda_{j2} \le Y_0 + (C_0 - Y_0) \times \kappa_1 \\ \sum_{j=1}^{n} Z_j \lambda_{j2} \ge \delta Z_0 \\ \lambda_{j1}, \lambda_{j2} \ge 0, j = 1, \ldots, n \end{cases} \tag{14}$$

The region of model (5) where $\kappa = \kappa_2$ is not larger than that where $\kappa = \kappa_1$, and therefore the optimal value of model (5) when $\kappa = \kappa_2$ is not larger than that when $\kappa = \kappa_1$, i.e., $\delta_{\kappa_1}^* \ge \delta_{\kappa_2}^*$.

□

Proof of Theorem 6: When the centralization degree of DMU in model (5) and (13) increases, according to proposition 1, the efficiency value does not decrease, $\delta_{\kappa_1}^* \ge \delta_{\kappa_2}^*$. We can then verify that λ_{j1} and λ_{j2} in the two patterns satisfy formulas (15) and (16), respectively, which are as follows.

$$\begin{cases} \max \ e^{\mathrm{T}} S_0^- \\ s.t. \ \ \sum_{j=1}^{n} X_j \lambda_{j1} \le X_0 \\ \quad \sum_{j=1}^{n} Y_j \lambda_{j1} - \sum_{j=1}^{n} Y_j \lambda_{j2} = S_0^- \\ \quad \sum_{j=1}^{n} Y_j \lambda_{j2} \le Y_0 + (C_0 - Y_0) \times \kappa_1 \\ \quad \sum_{j=1}^{n} Z_j \lambda_{j2} \ge \delta_{\kappa_1}^* Z_0 \\ \quad \lambda_{j1}, \lambda_{j2}, S_0^- \ge 0, \ j = 1, \dots, n \end{cases} \quad (15) \quad \text{and} \quad \begin{cases} \max \ e^{\mathrm{T}} S_0^- \\ s.t. \ \ \sum_{j=1}^{n} X_j \lambda_{j1} \le X_0 \\ \quad \sum_{j=1}^{n} Y_j \lambda_{j1} - \sum_{j=1}^{n} Y_j \lambda_{j2} = S_0^- \\ \quad \sum_{j=1}^{n} Y_j \lambda_{j2} \le Y_0 + (C_0 - Y_0) \times \kappa_2 \\ \quad \sum_{j=1}^{n} Z_j \lambda_{j2} \ge \delta_{\kappa_2}^* Z_0 \\ \quad \lambda_{j1}, \lambda_{j2}, S_0^- \ge 0, \ j = 1, \dots, n \end{cases} \quad (16)$$

Due to $\delta_{\kappa_1}^* \ge \delta_{\kappa_2}^*$, and $Y_0 + (C_0 - Y_0) \times \kappa_2 \le Y_0 + (C_0 - Y_0) \times \kappa_1$, a higher value of $\sum_{j=1}^{n} Y_j \lambda_{j2}$ is needed in the production system (κ_1) to increase the outputs $\delta_{\kappa_1}^* Z_0$. Thus, $\sum_{j=1}^{n} Y_j \lambda_{j2}^*$ in a κ_1 centralization degree production system is not smaller than that in a κ_2 production system.

λ_{j1}^* from model (15) when $\kappa = \kappa_2$ satisfies the constraints of formula (16), and therefore $\sum_{j=1}^{n} Y_j \lambda_{j1}^*$ in formula (16) is not smaller than in formula (15). When the internal resource waste is $\sum_{j=1}^{n} Y_j \lambda_{j1}^* - \sum_{j=1}^{n} Y_j \lambda_{j2}^*$, the internal resource waste of DMU under a κ_1 production system does not exceed that under a κ_2 production system. □

Highlights

1. Building models for measuring the IRW in different kinds of supply chains.

2. Investigating the relationship between centralization degree and IRW.

3. More centralization brings less internal resource waste and more outputs.

4. Centralized supply chain with only one intermediate measure could eliminate IRW.

5. Conducting manager to determine centralization degree for waste reducing.