Bentham, Marx and Rawls ethical principles: In search for a compromise ☆

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ABSTRACT

This paper demonstrates how the three basic universal ethical principles widely used in social choice theory, can be deduced as being particular cases of the minimization of a p-metric distance function. Once this logic unity has been shown, it is postulated how the three principles can be combined by formulating an extended goal programming model. In this way, possible clashes between the three principles can be quantified. This quantification could be interpreted as being the degree of sacrifice of some of the principles in order to reach the final consensus. The operational character of the approach is illustrated with the help of a simple numerical example.

Liberté, Égalité, "Fraternité" (Declaration of the Rights of Man)

1. Introduction

When we are dealing with social choice problems, hypothetical conflicts between some ethical principles should not be ignored or covered up, but encouraged. In many general scenarios the principle of majority could be incompatible with the principle of minority, freedom could be incompatible with fairness (fraternity) or with equity, etc. see [7]. The basic question is "is it possible to quantify the degree of conflict between the achievements of these principles in a particular decision making problem"? Or equivalently, "is it possible to compute the degree of sacrifice of some principles in order to reach a final solution"? In short, instead of a prior position being adopted, ethical principles should be brought together until a convergent position is developed. When there are no easy solutions, a very likely situation, then principles must be willing to be sacrificed for the good of the consensus or final solution.

In this paper, we propose an analytical framework for dealing with three basic ethical principles, derived from the universal "Declaration of the Rights of Man". This declaration is considered a basic pillar of the Western culture. It should be clarified from the beginning that the purpose of this research is merely to put some transparency to the possible ethical conflicts and not to solve them.

The three ethical principles, within a context of social choice, can be defined as follows:

a) In social theory the idea of freedom is associated with the Benthamite or utilitarian principle that implies the maximization of the welfare of the society by maximizing the sum of total of the welfare of all the member of the society. Thus, a maximum individual freedom is preserved [1].
b) The idea of fairness is normally associated with the Rawlsian or minimax principle that emanates from the "veil of ignorance", implying the maximization of the welfare of the worst-off individual. In that way, the idea of fraternity or fairness is optimized (see [14], especially pages 75–83).
c) The idea of an even allocation of the total welfare between all the members of society, thus providing a maximum equity is usually associated with partial aspects of the Marxian political perspective.

It is important to be aware that the scientific contribution of these three leading social scientists is much richer and complex than their identification with freedom, fraternity and equity; although in social science literature their names are usually linked to these principles.

The basic idea of this paper is to apply and extend the idea of compromise consensus developed elsewhere [6,7] to the simultaneous optimization of the above ethical principles. To this effect, a model is proposed that permits one to detect the possible clashes between the principles as well as to carry out the quantification of the actual degree of conflict.

Two caveats should be made. First, this paper does not deal with the general issue of merging group decision making methods

with multi-criteria analysis. This orientation is well covered in the literature, especially linking voting methods and multi-criteria analysis as happens with the multicriteria approval method ([12] pages 181–187). A review of this type of method with an applied orientation can be seen in Kangas et al. [11]. Second, recent contributions in the direction of searching for a compromise consensus but with a different orientation with respect the one followed in this paper can be seen in Sun and Ma [18] and Gong et al. [4].

The paper is organized as follows. In Section 2, it is demonstrated how the three ethical principles can be straightforwardly derived from the minimization of a p-metric distance function and how they can be combined into a single optimization model based on extended goal programming. In Section 3,with the help of a numerical example it is illustrated how the proposed models work from a computational perspective. Finally, Section 4 presents the main conclusions derived from this research.

2. Analytical framework

The general setting for the incorporation of the ethical principles commented on in the preceding section is the following. We have a society formed by n members ($i=1, 2,…,n$). Each member of the society has to give judgment values over m objects ($j, k=1, 2, …,m$). The objects can be electoral candidates, criteria, alternatives, etc. It should be noted that we do not impose, a priori, any condition on the type of information in the m objects. In other words, the nature of the measures used by the members of the society to express their judgments values could be ordinal or cardinal, the information could be complete or incomplete, the cardinal nature could be defined by utility functions or by "pairwise" comparison matrices, etc.

The following notation will be used throughout the paper:

- $w_i \equiv$ weight or social influence of the ith member or social group;e.g., the size of the social group or the case of qualified majority.
- $R_{jk}^i \equiv$ judgment value provided by the ith member of the society when he/she compares the jth and the kth objects (i.e., the data for our exercise).
- $R_{jk}^S \equiv$ final judgment value assigned by the society as a whole to the jth object when it is compared with the kth one (i.e., the unknowns for our exercise).
- $F \equiv$ set of conditions that R_{jk}^S must be met; these conditions depend on the nature of the measures used by the n members of the society.
- $p \equiv$ topological metric; i.e., a real number belonging to the closed interval $[1, \infty]$.

It should be indicated that the set of conditions F defined above depend heavily on the characteristics of the preferential information provided by the n members of the society. In [6] a precise characterization of the feasible set F for the following types of preferential information can be seen: ordinal and complete, ordinal and partial, and cardinal and complete, respectively.

From the above setting, we have introduced the following "generator" of social choice functions (see [5,6,7], for technical details, and [19,20], for the mathematical and preferential theoretical foundations of this type of distance function).

$$U_p = -\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1 k \neq j}^{m} w_i^p \left|R_{jk}^i - R_{jk}^S\right|^p\right]^{1/p} \tag{1}$$

Function (1) is optimized over a feasible set such as:
$R_{jk}^S \in F$ (set of conditions)

Model (1) minimizes for a p-metric distance functions the deviation between the preferential information provided by the n members of the society (data of the problem) and the final social consensus (unknowns of the model). Now we will see how from model (1) several social choice functions can be straightforwardly obtained, leading to the solutions implied by the three ethical social principles commented on above.

Let us start by particularizing (1) for metric $p=1$, this yields the following equation:

$$U_B = -\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1 k \neq j}^{m} w_i \left|R_{jk}^i - R_{jk}^S\right|\right] \tag{2}$$

The optimal value of (2) over the feasible set F provides the "best social optimum" from the point of view of the majority; that is, the Benthamite or utilitarian solution that best preserves individual freedom.

Lets us now particularize (1) for metric $p=\infty$, this yields the following equation:

$$U_R = -\left[Max_{i,j,k} w_i \left|R_{jk}^i - R_{jk}^S\right|\right] \tag{3}$$

The optimal value of (3) over the feasible set F implies the minimization of the disagreement of the member of the society most displaced with respect to the majority solution defined by (2). This solution represents the "best social optimum" from the point of view of minority; that is, from the perspective of the worst-off member of the society according to Rawl's principle, leading to the point of maximum fairness.

The Marxian solution or point of maximum equity derives straightforwardly from model (2) as follows:

$$w_1\left|R_{jk}^1 - R_{jk}^S\right| = w_2\left|R_{jk}^2 - R_{jk}^S\right| = … = w_n\left|R_{jk}^n - R_{jk}^S\right| \tag{4}$$

In fact (4) implies a chain of $n(n-1)/2$ equations which, in combination with the equation establishing the feasible conditions, allows the determination of the social allocation of maximum equity.

It is important to note that all the above models present computational problems. In fact, the existence of absolute values implies the optimization of non-smooth functions. Moreover, the system of $n(n-1)/2$ equations given by (4) represent a very strong condition that will be met in very few occasions in real applications. To address both problems the following change in variables is proposed (for the seminal idea, see [3]):

$$n_{jk}^i = \frac{1}{2}\left[\left|R_{jk}^i - R_{jk}^S\right| + (R_{jk}^i - R_{jk}^S)\right] \tag{5}$$

$$p_{jk}^i = \frac{1}{2}\left[\left|R_{jk}^i - R_{jk}^S\right| - (R_{jk}^i - R_{jk}^S)\right] \tag{6}$$

By adding (5) and (6), and then by subtracting (6) from (5) we obtain:

$$n_{jk}^i + p_{jk}^i = \left|R_{jk}^i - R_{jk}^S\right| \tag{7}$$

$$n_{jk}^i - p_{jk}^i = R_{jk}^i - R_{jk}^S \tag{8}$$

By using (7) and (8), the optimization problem given by (1) turns into the following Archimedean GP model (e.g., [9,10]):

$$U_p = -\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1 k \neq j}^{m} w_i^p (n_{jk}^i + p_{jk}^i)^p\right]$$

subject to :

$$R_{jk}^S + n_{jk}^i - p_{jk}^i = R_{jk}^i \qquad \forall i,j,k$$
$$R_{jk}^S \in F \text{ (set of conditions)} \tag{9}$$

By implementing the same type of substitutions, the Bentham point or solution of maximum freedom given by (2) will be obtained

by solving the following Weighted GP model (e.g., [9,10]):

$$U_B = - \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1 k \neq j}^{m} w_i(n_{jk}^i + p_{jk}^i) \right]$$

subject to :

$$R_{jk}^S + n_{jk}^i - p_{jk}^i = R_{jk}^i \qquad \forall i,j,k$$
$$R_{jk}^S \in \mathbf{F}(\text{set of conditions}) \qquad (10)$$

Similarly, the Rawls point or solution of maximum fairness given by (3), will be obtained by solving the following GP Chebyshev model (e.g., [9,10]):

$$U_R = -D$$

subject to :

$$w_i \sum_{j=1}^{m} \sum_{k=1 k \neq j}^{m} (n_{jk}^i + p_{jk}^i) - D \leq 0$$
$$R_{jk}^S + n_{jk}^i - p_{jk}^i = R_{jk}^i \qquad \forall i,j,k$$
$$R_{jk}^S \in \mathbf{F}(\text{set of conditions}) \qquad (11)$$

where D represents the disagreement of the member of the society with the opinions most displaced (i.e., the worst-off member in a Rawlsian language) from the solution obtained.

By implementing the same changes, the Marxian solution or solution of perfect equity given by (4), will be obtained by solving the following system of equations:

$$w_i(n_{jk}^i + p_{jk}^i) - w_t(n_{jk}^t + p_{jk}^t) = 0 \quad i = 1,...,n-1 \quad t = i+1,...,n$$
$$R_{jk}^S + n_{jk}^i - p_{jk}^i = R_{jk}^i \quad i = 1,...,n \qquad \forall j,k$$
$$R_{jk}^S \in \mathbf{F}(\text{set of conditions}) \qquad (12)$$

The equation system (12) is very demanding and there is likely to be no solution. However, a solution can be approximated by transforming system (12) in the following meta-goal programming model (e.g., [2]):

$$U_M = - \sum_{i=1}^{n-1} \sum_{t=i+1}^{n} (\eta_{it} + \rho_{it})$$

Subject to :

$$w_i(n_{jk}^i + p_{jk}^i) - w_t(n_{jk}^t + p_{jk}^t) + \eta_{it} - \rho_{it} = 0 \quad i = 1,...,n-1 \quad t = i+1,...,n$$
$$R_{jk}^S + n_{jk}^i - p_{jk}^i = R_{jk}^i \quad i = 1,...,n \forall j,k$$
$$R_{jk}^S \in \mathbf{F}(\text{set of conditions}) \qquad (13)$$

It should be noted that if the equation system (12) has a solution, the optimal value of achievement function (13) will be zero, and both models will provide the same solution.

In short, model (10) provides the Bentham's solution, model (11) the Rawlsian's solution, and model (13) the Marxian solution, respectively. However, each one of these three solutions represents equilibriums underpinned by three different significant social foundations. For that reason, it is tempting to merge the three solutions in order to test compatibilities, trades-off, etc between the three ethical orientations considered. We can undertake that task by implementing a linear convex combination of models (10), (11) and (13), which leads to the following extended goal programming formulation (e.g., [15,16]):

$$U(\lambda_1, \lambda_2,) = -\lambda_1 D - \lambda_2 \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1 k \neq j}^{m} w_i(n_{jk}^i + p_{jk}^i) \right]$$
$$- (1-\lambda_1) \sum_{i=1}^{n-1} \sum_{t=i+1}^{n} (\eta_{it} + \rho_{it})$$

Subject to :

$$w_i(n_{jk}^i + p_{jk}^i) - w_t(n_{jk}^t + p_{jk}^t) + \eta_{it} - \rho_{it} = 0 \quad i = 1,...,n-1 \quad t = i+1,...,n$$

$$w_i \sum_{j=1 k}^{m} \sum_{k=1 k \neq j}^{m} (n_{jk}^i + p_{jk}^i) - D \leq 0 \quad i = 1,...,n \forall j,k$$
$$R_{jk}^S + n_{jk}^i - p_{jk}^i = R_{jk}^i \quad i = 1,...,n$$
$$R_{jk}^S \in \mathbf{F}(\text{set of conditions}), \text{ being } \lambda_1 + \lambda_2 \in [0,1] \qquad (14)$$

where λ_1 and λ_2 play the role of control parameters. Thus, when $\lambda_1 = \lambda_2 = 0$, we obtain the Marxian solution (**maximum equity**), when $\lambda_1 = 1$ and $\lambda_2 = 0$ the Rawlsian solution is derived (**maximum fairness**) and finally when $\lambda_1 = 0$ and $\lambda_2 = 1$ the Bentahmite solution is elicited (**maximum freedom**). On the other hand, for values of control parameters λ_1 and λ_2 such as $\lambda_1 + \lambda_2 \in (0,1)$, any compromises, if they exist between the above three solutions can be determined. In this way, λ_1 and λ_2 trades-off the amount of social achievement among the solutions derived from the three classic social principles considered. In short, these parameters quantify the marginal rate of substitution among the social equilibriums derived from different social principles. The interest of this proposal and how the method functions will be illustrated in the next section, with the help of a numerical example.

In addition, for fixed values λ_1 and λ_2, the utility function $U(\lambda_1, \lambda_2)$ could be interpreted as being the total sacrifice of ethical principles in order to reach the final consensus. This kind of interpretation can be extended to the quotients

$$\frac{D}{U(\lambda_1, \lambda_2)}, \quad \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1 k \neq j}^{m} w_i(n_{jk}^i + p_{jk}^i)}{U(\lambda_1, \lambda_2)}, \quad \frac{\sum_{i=1}^{n-1} \sum_{t=i+1}^{n} (\eta_{it} + \rho_{it})}{U(\lambda_1, \lambda_2)}$$

that could represent the marginal sacrifice corresponding to fairness, freedom and equity, respectively. It is important to note that the concept of degree of sacrifice reinforces the basic purpose of our work on the group decision making model. In fact, the satisficing paradigm underlying our research is more appropriated for a collective decision making problem than the traditional optimizing paradigm. Thus, in a solution coming from the optimizing approach the idea of sacrifice does not make not too much sense, but it does make plenty of sense when considering a satisficing perspective

3. A numerical illustration

In order to illustrate how the models proposed above can be computed and the solutions interpreted we have resorted to an example taken from the literature [13].Thus, in the context of a participatory forest plan problem in Lycksele (Sweden) the preferences of several stakeholders with respect to several criteria were requested. As a part of the exercise, seven recreationists were interviewed about the relative importance attached to five criteria, following a "pairwise" comparison format. From that, the following seven "pairwise" comparison matrices of dimension 5×5 were obtained:

$$\mathbf{M}^1 \equiv \begin{bmatrix} 1 & 7 & 7 & 7 & 5 \\ 0.14 & 1 & 5 & 1 & 1 \\ 0.14 & 0.2 & 1 & 1 & 7 \\ 0.14 & 1 & 1 & 1 & 7 \\ 0.2 & 1 & 0.14 & 0.14 & 1 \end{bmatrix} \quad \mathbf{M}^2 \equiv \begin{bmatrix} 1 & 1 & 1 & 3 & 1 \\ 1 & 1 & 3 & 3 & 3 \\ 1 & 0.33 & 1 & 3 & 3 \\ 0.33 & 0.33 & 0.33 & 1 & 3 \\ 1 & 0.33 & 0.33 & 0.33 & 1 \end{bmatrix}$$

$$\mathbf{M}^3 \equiv \begin{bmatrix} 1 & 5 & 1 & 7 & 0.33 \\ 0.2 & 1 & 5 & 7 & 0.2 \\ 1 & 0.2 & 1 & 0.2 & 0.33 \\ 0.14 & 0.14 & 5 & 1 & 1 \\ 3 & 5 & 3 & 1 & 1 \end{bmatrix} \quad \mathbf{M}^4 \equiv \begin{bmatrix} 1 & 9 & 1 & 1 & 5 \\ 0.11 & 1 & 5 & 1 & 5 \\ 1 & 0.20 & 1 & 1 & 7 \\ 1 & 1 & 1 & 1 & 1 \\ 0.2 & 0.20 & 0.14 & 1 & 1 \end{bmatrix}$$

Table 1
Aggregated matrices for the Bentham, Rawls and Marx ethical principles.

Va	R_{12}^S	R_{13}^S	R_{14}^S	R_{15}^S	R_{23}^S	R_{24}^S	R_{25}^S	R_{34}^S	R_{35}^S	R_{45}^S	U_B	U_R	U_M
$\lambda_1=0,\ \lambda_2=1$	5	1	3	5	5	5	3	1	3	3	130.15	26.00	156.74
$\lambda_1=1,\ \lambda_2=0$	5	1	5	1.40	5	5	1	3	7	3	151.74	23.60	68.24
$\lambda_1=\lambda_2=0$	5	1	3	1	5	3	2	1	5.40	3	172.20	24.60	0
$\lambda_1=\lambda_2=0.33$	5	1	5	5	5	3.74	1	1.27	5.27	3	140.87	28.47	58.38

$$\mathbf{M}^5 \equiv \begin{bmatrix} 1 & 1 & 1 & 1 & 0.20 \\ 1 & 1 & 0.2 & 5 & 0.20 \\ 1 & 5 & 1 & 5 & 0.20 \\ 1 & 0.20 & 0.20 & 1 & 3 \\ 5 & 5 & 5 & 0.33 & 1 \end{bmatrix} \quad \mathbf{M}^6 \equiv \begin{bmatrix} 1 & 5 & 1 & 3 & 5 \\ 0.2 & 1 & 3 & 5 & 3 \\ 1 & 0.33 & 1 & 0.33 & 0.33 \\ 0.33 & 0.2 & 3 & 1 & 0.33 \\ 0.2 & 0.33 & 3 & 3 & 1 \end{bmatrix}$$

$$\mathbf{M}^7 \equiv \begin{bmatrix} 1 & 5 & 5 & 5 & 07 \\ 0.2 & 1 & 5 & 5 & 05 \\ 0.2 & 0.2 & 1 & 5 & 07 \\ 0.2 & 0.2 & 0.2 & 1 & 05 \\ 0.14 & 0.2 & 0.14 & 0.2 & 01 \end{bmatrix}$$

It should be noted that [17] was used for the valuation of the entries of the seven matrices. For instance, $R_{12}^1 = 7$, means that for the first stakeholder the first criterion is of *demonstrated importance* with respect to the second criterion. This verbal judgment is transformed by the scale into the digit 7. It is also important to note that for this type of preferential information, the set of conditions **F** only implies the fulfilment of some limit conditions, like the following:

$$t_1 \le R_{jk}^S \le t_2 \quad \forall j, k$$

t_1 and t_2 being the bounds of the scale. Thus, for Saaty´s scale, we have $t_1 = 0.11$ and $t_2 = 9$. However, the use of this scale or any other one is of theoretical irrelevance for the proposed analytical framework.

By applying model (14) to the above seven matrices and for several values of control parameters λ_1 and λ_2, the results shown in Table 1 were obtained. Thus, this table shown the aggregate matrix according to the three social principles used. Also, the last row of the table displays the results for $\lambda_1 = \lambda_2 = 0.333$; that is, the aggregate solution when the three social principles weight the same. It is not surprising that the three classic and sensible social principles considered clash significantly with each other. Thus, the best solution from the Rawls perspective (maximum fairness) implies the second worst solution for Bentham (maximum freedom) and Marx (maximum equity). On the other hand, the Marxian perfect equity is feasible, but it implies the worst solution in terms of Bentham (freedom) and so on and so forth. These particular results are obviously highly dependent on the case study analyzed. However the approach proposed in the paper allows the quantification of the possible clash between these three classic social principles for any type of situation.

It should be noted that the use of a "pairwise" comparison format as well as the scale used are basic ingredients of the well-known Analytic Hierarchy Process (AHP) proposed by Saaty [17]. However, neither the approach introduced in this paper nor the example used to illustrate it have any basic relationship with the AHP procedure.

4. Conclusions

There is a noteworthy consilience underlying the proposed approach. In fact, the best-known ethical principles proposed by leading social scientists at very different moments of time and which are universally accepted, can be those precisely derived from the minimization of a *p*-metric distance function. Once this fact was incorporated into our analysis, it was quite simple to combine the three principles in order to quantify possible compatibilities, conflicts, trades-off, etc. It is also important to note that the proposed approach is very efficient from a computational point of view, since it only requires solving a small number of linear programming models of a moderate size.

On the other hand, it should be noted that the main purpose of this research is of a theoretical nature. Despite of that, the following tentative remarks with a more applied orientation can be made:

a) The different compromise consensuses obtained on balancing the three ethical principles considered can be used, among other scenarios, as the initial point for a general negotiation process.
b) The proposed methodology can be used to aggregate experts' judgments having taken into account a combination of three different collective perspectives.
c) The proposed aggregation system possesses good theoretical properties [8]. This fact, makes it easier for the solutions derived from the different models to be accepted by the group/society.

Finally, it should be noted that this article might help in the direction of building sound, new bridges between social sciences and the operational research/management science (OR/MS) discipline. In fact, social sciences can be invigorated with new formulations and analytical procedures taken from the OR/MS field, whereas this discipline can be enriched with the introduction of new problems, challenges and areas of application.

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