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A learning ladder toward efficiency: Proposing network-based stepwise benchmark selection ☆

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ABSTRACT

Starting from the knowledge-based view of efficiency improvement, we propose a network-based approach to find the optimal stepwise benchmarking paths toward the efficiency frontier. The approach treats the Data Envelopment Analysis system as a network of teaching and learning firms and calculates the overall shortest paths taking into account both input endowment similarity and the efficiency gap covered in each step. In addition, based on network centrality concepts, the method discriminates between efficient and intermediate units, and highlights possible outliers or specialized units. As a real-world example, the method is applied to a network of Canadian bank branches and practical implications are discussed.

1. Introduction

Efficiency improvement involves technological and organizational changes, and such substantial changes require a carefully oriented inter-organizational knowledge absorbing process [1]. Taking into account that slow learning is a major reason for inefficiency [2], adopting a stepwise benchmarking path not only facilitates the knowledge absorption process but also reduces the risk of failure implied by setting an out-of-reach efficiency target [3,4].

Assuming the Data Envelopment Analysis (DEA) method as a network of units that aim to learn by benchmarking, we propose an application of Social Network Analysis (SNA) in the DEA context to transform the benchmarking information of DEA efficiency measurement into a network of possible efficiency improvements, and calculate the optimal stepwise benchmarking paths.

This paper is grounded on the knowledge-based view of efficiency, which understands efficiency improvement as a learning process [5]. According to the theory of absorptive capacity [6], one firm's ability to learn from another depends on the similarity of the two firms' knowledge base, organizational structures and consumption policies [7]. In a DEA problem, this knowledge overlap and structural relevance can be measured through the similarities in the inputs as well as in the outputs [1,5,8].

Another question also discussed in the paper is the fact that because DEA does not provide stepwise benchmarking paths, it might be risky or not feasible to cover all efficiency gaps in one step [9]. To overcome this problem, several strategies have been proposed to post-process the DEA benchmarking information based on contextual policies in order to provide a reasonable, desirable and feasible stepwise benchmarking path. These procedures can be categorized under the general label of *stepwise benchmarking*.

A recent and relevant trend in the literature [3,4,9,10] uses self-organized map (SOM) input clustering and machine learning techniques to find the optimal path toward efficiency. While some of these techniques provide more appropriate paths than traditional stratification or context-dependent DEA (CD-DEA) methods, some aspects still need improving. First, the existent proposals optimize each step (not the whole path). Using the SNA shortest path makes it possible to optimize the whole path and, when possible, to minimize zigzagging the path toward the efficient frontier. Second, they cannot control the number of steps; the present proposal, by contrast, provides a control parameter and can provide paths with various numbers of steps. Third, clustering based on input similarities reduces the n dimensions input vector data (where n is the number of inputs) into a two dimensional map. Detecting input similarity based on this information—by determining the Euclidean distance of the cluster centers—is not an accurate proxy. In the present paper, this constraint is relaxed and a weighted vector comparison is used to select the most similar Decision Making Units (DMUs) in the path, which seems to be a more realistic and accurate proxy. This comparison indicates

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that a comprehensive model with flexible and comparable policies is still lacking, and SNA has the potential to overcome these shortcomings.

Liu et al. [11] were the first to introduce the novel idea of analyzing DEA problems as a network of efficiency endorsements, and to propose an application of SNA in DEA to obtain a better discrimination in the efficiency ranking of the DMUs. The network structure proposed by Liu et al. [11] is based on lambda values, resulting from a recursive efficiency analysis of the DMUs under study, and discrimination is based on eigenvector centrality of efficient units.

Apart from eigenvector centrality, other powerful SNA concepts can be helpful in efficiency problems, such as proximity, shortest path and alpha centrality. Our proposal combines concepts and methods from SNA and DEA and, for the first time, these concepts are applied in a SNA method with applications such as calculating the optimal benchmarking path, detecting possible outliers, clustering units and highlighting specialized DMUs.

The proposed method has three steps. The first step measures the efficiency scores and results in initial benchmarking peers of DMUs. The DEA type (i.e., radial DEA, hyperbolic DEA, directional distance function—DDF—or slack-based models—SBM) and the possible orientations (input, output or directionally oriented) are exogenous to the method. The second step post-processes the DEA scores and transforms the benchmarking information into a directed and weighted network of all possible efficiency improvement paths. The third step analyzes the resulting network, calculates the optimal benchmarking path, and highlights the potential presence of specialized units as well as the possible outliers.

In the present paper, a path toward the efficiency frontier is considered *optimal* when the unit under evaluation is a relatively good performer, or there are some better intermediate performers with relatively similar input endowments in the middle, promoting the learning process and lowering the risk of failure. Based on this definition, the path shown in Fig. 1 part (III) is a *non-optimal* path, and the one shown in Fig. 1 part (IV) is an *optimal* path. Setting two intermediate efficiency improvement targets not only facilitates the learning process but also indicates that the benchmark is feasible in practical terms.

Despite their shared origin, the method proposed by Liu et al. [11] and our proposal are different. First, as shown in Fig. 1 part I, the nodes in the method proposed by Liu et al. [11] are DMUs and the links between the nodes are the efficiency endorsements directed toward the efficient peers. In the current paper, the nodes are DMUs but the links are not only toward efficient units but also toward any better performer unit (Fig. 1 part II). In this way, the model will include all possible efficiency improvements that are paramount for calculating optimal paths.

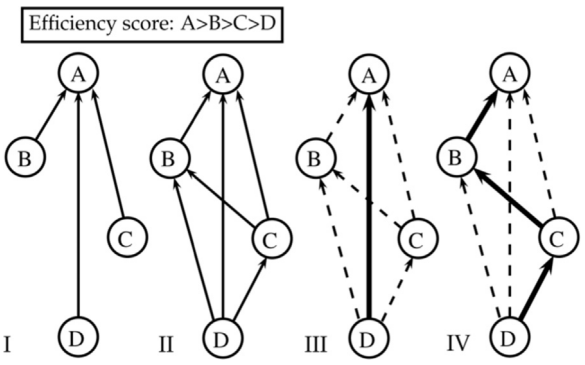

Fig. 1. Illustrative benchmarking paths. (I) DEA benchmarking information. (II) All possible efficiency improvement. (III) Normal benchmarking. (IV) Network-based stepwise benchmarking (optimal path).

The second difference concerns the network construction method. The links in the method proposed by Liu et al. [11] are weighted by summing the lambda values obtained from a recursive DEA efficiency analysis of all possible input/output combinations. This method implies defining several DEA programs with just a partial description of the existing technology, which is difficult to accept, as only a carefully selected dataset with completely substitutable inputs and outputs would be acceptable. Hence, in order to preserve the definition of the technology, in our proposal we avoid using partial technologies.

Finally, considering that it is relatively easier to introduce sophisticated functions of costs and risks as link weights in an SNA network than in a DEA program, our proposal calculates a function of input similarities and benchmarking risks to weight the links. In contrast, the method proposed by Liu et al. [11] does not take into account the input similarities or the efficiency gaps. As a proxy for input similarities, our model calculates the Euclidean distance of normalized inputs, taking advantage of the unit invariant property of inputs in constant return to scale (CRS) and variable return to scale (VRS) [13]. In a similar way, the efficiency gap between the DMUs at the start and end of a link is used as a proxy to benchmark risk of failure. To bonus more appropriate intermediate better performing DMUs, an exponential value of the above-mentioned proxies is taken. It is also possible to introduce a fixed cost to each benchmarking step and a matrix of substitution rates for the inputs.

The rest of paper is organized as follows. Section 2 provides an overview of the previous studies in stepwise efficiency improvement and network-based DEA. Section 3 presents our proposal and revises the application issues. Section 4 provides details of an empirical application evaluating the relative efficiencies of 79 Canadian bank branches and the results are discussed. Finally, Section 5 summarizes the work, states the practical implications and suggests directions for future research.

2. Literature review

2.1. DEA and stepwise efficiency improvement (SEI)

The need for efficiency improvement (EI) is emphasized in the efficiency literature [3,4,9,14]. Scholars mention different but relatively convergent logics and reasons for efficiency improvement, including the learning process, handling data heterogeneity due to size, facilitating efficiency gap removal and dealing with different evaluation contexts.

There are five trends in stepwise efficiency improvement (SEI) methods. Among those, CD-DEA and efficiency improvement path are the most relevant to the present paper. The CD-DEA trend started with Seiford and Zhu [15] and became popular after the improvements made by Seiford and Zhu [16]. Other scholars have extended the CD-DEA method and combined it with other DEA concepts, for example SBM, assurance regions (AR), super and cross efficiency. Although this branch of the literature deals with overall stepwise efficiency improvement, it does not provide a specific path or road-map for inefficient units to remove inefficiency.

The most relevant precedent to the present paper is the efficiency improvement path, which began with the paper by Hong et al. [14] and includes methods that aim to introduce an optimal path toward efficiency. Lozano and Villa [17] and Lozano and Villa [18] provide a sequence of gradual intermediate targets toward efficiency where the targets are not observed units. In contrast, all other papers in this stream calculate an optimal path utilizing the information from observed DMUs. Also, Lozano and Villa [17] and Lozano and Villa [18] only use DEA methods while the other

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proposals enhance the DEA results with self-organized map (SOM) and machine learning techniques.

The method proposed by Hong et al. [14] clusters the DMUs utilizing SOM techniques; inside each cluster it finds the shortest paths for each DMU toward the layered frontiers using SOM methods. Hong et al. [14] were the first to introduce the *stepwise improvement path* and *shortest path* concepts in the literature.

Lozano and Villa [17] state that most DEA models project an inefficient unit onto a more distant target, which makes it more difficult to attain. They advocate determining a sequence of targets that end in the efficient frontier. Obviously, these targets are generally closer to the original unit than the one-step projection. The sequence of targets provided by Lozano and Villa [17] are hypothetical units under CRS. Lozano and Villa [18] went on to improve their method, first for technical inefficient units by calculating a sequence of targets toward the VRS frontier, and then for scale inefficient units. In both cases, the successive targets are obtained by iteratively solving specific DEA models that take into account given bounds on the rates of change in inputs and outputs that the unit can implement in each step (see Lozano and Villa [18]).

In a similar way to Hong et al.'s [14] proposal, Sharma and Yu [10] utilize a SOM method to cluster the DMUs based on input characteristics. For a second time, the method clusters the DMUs based on efficiency. Hong et al. provide a fixed number of efficiency layers. To present a stepwise path, their method finds higher efficiency ranked DMUs inside each input similarity clusters. The method proposed by Sharma and Yu [10] also uses SOM clustering and efficiency-based clustering for stepwise improvement. In both methods the DMUs are presented with benchmarks inside the clusters.

Estrada et al. [3] focused on the practical difficulties of removing all the inefficiency in one step and propose a proximity-based stepwise target selection process. The method first clusters the DMUs based on the input similarities using the SOM clustering technique. The authors then find the shortest paths by moving from one cluster to a better performing unit in the closest neighboring clusters utilizing a reinforcement learning (RL) algorithm. This method improves on [14] in two ways. First, a path in Hong et al. [14] is caged in its input similarity cluster, but the Estrada et al. [3] method lets the path move toward the closest neighboring clusters. Second, the Hong et al. [14] method utilizes the layering technique while the Estrada et al. [3] method relaxes this condition and lets the efficiency improvement take place in a more appropriate way.

Lim et al. [4] attempted to improve the CD-DEA model [16] with an SEI method that first clusters DMUs into several layers according to their efficiency scores, and then establishes a benchmarking path across the sequence of layers. Among the DMUs in the next layer, the most preferable one is selected as the next benchmark target, based on three criteria: attractiveness, progress, and infeasibility [4].

Park et al. [9] state that most of the previously proposed methods for stepwise efficiency improvement only consider the efficiency score in selecting benchmark targets and they propose a stepwise method to take into account preference, direction and similarity at the same time. To select the intermediate benchmark, their proposal takes into account the minimum deviation from the direct path toward the ultimate target, and input similarity. To measure the input similarity, the method categorizes the DMUs into a fixed number of clusters using an SOM method. Then the Euclidean distance between the groups is used as a proxy for input similarity of DMUs. A weighted sum of deviation and input similarity is used to optimize each step (and not the whole path) toward the ultimate path. Although the methods by Sharma and Yu [10], Estrada et al. [3] and Park et al. [9] are relatively similar in their strategies, the proposal by Park et al. [9] has better achievements and fewer shortcomings.

2.2. DEA and Social Network Analysis (SNA)

Liu et al. [11] were the first to explore SNA techniques in the DEA research field and proposed a network-based approach to further increase discrimination among the efficient DMUs. In constructing the network, the observed node is set to point to its referent DMUs as suggested by DEA, and corresponding lambda values for these referent DMUs are taken as the strength of the network link [11]. The important point is that the network is woven not only by the full input/output model, but also by models of all possible input/output combinations. In other words, Liu et al. [11] use all possible partial technologies in the dataset to create more links between the DMUs. Some links will probably appear starting from efficient units to inefficient units because in some partial technologies the efficient units may become inefficient. Incorporating these models into the system basically introduces the merits of each DMU in various situations into the system and thus provides the key information for further discrimination. Once the network has been constructed, the centrality concept commonly used in Social Network Analysis—specifically, eigenvector and alpha centrality—is employed to rank the efficient DMUs [11,20]. The network-based approach by Liu et al. [11] tends to rank high the DMUs that are not specialized and have balanced strengths. In the resulting network, the more central DMUs are the highly ranked efficient units. The link width (thickness) represents the λ values and the direction is toward the efficient units. The links back from efficient units to peripheral units are because of the iterative partial efficiency definition.

In their second paper, Liu and Lu [19] enhance their network-based approach by removing the bias caused by a scale difference among organizations and highlighting its ability to identify the strengths and weaknesses of each organization [19]. This ability is a result of partial technology definition by iteratively solving all possible input/output combinations. The authors then apply the method in a two-stage R&D evaluation model which separates the R&D process into two stages: technology development and technology diffusion.

In their third paper, Liu and Lu [20] extend their network-based approach into the two-stage DEA context, applying their proposal to a dataset of banks to illustrate a real-world problem. In addition to basic ranking, they suggest a benchmark unit for each input/intermediate/output factor, and identify the strengths of each efficient unit.

3. The proposed model

In this section we explain our proposed model. First we explain the theoretical framework and the overall research approach. Then we detail and formally present the proposed steps. Finally, we introduce the dataset and its characteristics.

3.1. Theoretical framework

The proposed method post-analyzes and transforms the DEA benchmarking information into a directed and weighted network of learning DMUs. Fig. 2 illustrates the theoretical framework of the present paper. The method is divided into two stages: i) Data Envelopment Analysis and ii) DEA post-analysis. In the first stage, the decision maker introduces the input and output data that affect the inter-organizational process.

In the second stage, the method constructs the network based on the DEA benchmarking information and efficiency scores. The

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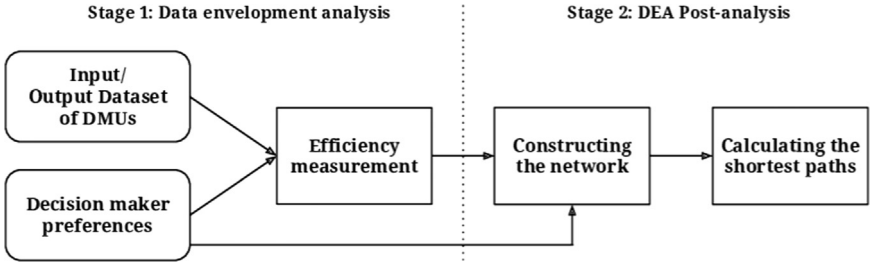

Fig. 2. Theoretical framework.

network contains all possible efficiency improvement paths between DMUs. The starting node is an inefficient unit and the ending node is a better performer (not necessarily an efficient unit).

The links are established by using a weight function that takes into account two factors for each link: i) the input changes, ii) the efficiency gap covered. To bonus the shorter links the exponential summation of the factors is taken into account. In addition the decision maker can adjust the weight of each factor and assign a constant to the function to control the number of steps.

Dijkstra's algorithm (see Xu et al. [21]) is used to calculate the shortest path. The method presents the shortest stepwise paths to all peers marked by DEA and highlights the most preferable path based on the decision maker's preferences.

Based on the resulting information, other applications are introduced: i) highlighting the possible outliers and specialized units, ii) clustering the DMUs based on their network neighborhoods.

The following sub-section formally presents the model.

3.2. Model

3.2.1. Step 1: Efficiency measurement and finding the ultimate targets

The first step measures the DMU efficiency scores using a Data Envelopment Analysis (DEA) model. To eliminate the effects of units of measurement on the magnitudes of distance comparison, the method normalizes all inputs and outputs using the invariant property of DEA, and replaces the normalized inputs with the original values[1]. It has been demonstrated that normalizing only the inputs does not change the efficiency calculations [22]. The normalized values are calculated by

$$x^i = \frac{x_i}{\max_{i=1,\ldots,n} x_i} \backslash\backslash ; i = 1,\ldots,n, x_i \leftarrow x^i i = 1,\ldots,n. \quad (1)$$

The DEA model, orientation, and type of return to scale are exogenous to the present model and, based on the operating conditions, are decided according to the decision maker's preferences. In the present paper, we use Banker et al.'s [23] proposal, assuming the input orientation and variable returns to scale (VRS) as technological assumption:

$$\begin{aligned} \text{Min} \quad & \theta_k \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_k x_{ik}, \quad i = 1,\ldots,m, \\ & \backslash - t \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rk}, \quad r = 1,\ldots,s, \\ & \backslash - t \sum_{j=1}^{n} \lambda_j = 1, \\ & \backslash - t \theta_k, \lambda_j \ge 0; \quad \forall j, \end{aligned} \quad (2)$$

where n is the number of DMUs; m and s are the respective number of inputs and outputs; and x_{ij} and y_{rj} are the amount of the ith input consumed and the amount of the rth output produced by the jth DMU, respectively. The problem is to solve technical efficiency (TE) θ_k and lambda value λ_j for an observed DMU. By varying the index k over all DMUs, we obtain TE in each DMU. If TE is equal to one, then the DMU is technically efficient. If TE is smaller than one, then the DMU is technically inefficient [11]. The DEA model provides a set of inefficient units, their respective benchmarking peers and efficiency scores.

3.2.2. Step 2: Constructing the network

The method utilizes the benchmarking information of DEA efficiency measurement to construct the benchmarking network. Fig. 3 part I shows an illustrative DEA program with input orientation. Unit A and B are the efficient units and form the best practice VRS frontier. In this example $\theta_A = \theta_B > \theta_C > \theta_D > \theta_E$.

The benchmarking network is defined as $G=(N,L,w)$ as follows:

$$\begin{aligned} \backslash - N &= \{1,2,\ldots,n\}, \\ \backslash 5pcL &= \{(u,v) | \theta_u < \theta_v\}, \\ \backslash 5pcd(u,v) &= \sqrt{\textstyle\sum_{i=1}^{m} (x_{iv} - x_{iu})^2}, \\ \backslash - e(u,v) &= \theta_v - \theta_u, \\ \backslash 5pcp(f) &= \text{base}^{(f/\max(f))}, \\ \backslash 3pc1 &= w_0 + w_1 + w_2, \\ w(u,v) &= w_0 + w_1 \times p(d(u,v)) + w_2 \times p(e(u,v)). \end{aligned} \quad (3)$$

N is the set of all DMUs and the set of all pairs of nodes (u,v) is present in L if DMU_u is more efficient than DMU_v. The function d calculates the input similarities and as a proxy we propose distance between the input values of unit u and v in vector space. The resulting distance is scalar. The function e calculates the benchmarking risk of failure and as a proxy proposes the efficiency gap. The function p assigns an exponential penalty for longer steps. The latter will bonus the shorter steps better than longer ones. The base is to be optimized by the decision maker's preferences for shorter steps. Also w_i are the preference weights decided by the decision maker's preferences, where larger base values will provide smaller jumps in terms of input/efficiency changes and smaller base values provide steps with relatively higher jumps. In the present paper, we used base=*100*. w_0 as the fixed cost of each step. The decision maker can control the number of steps by modifying this parameter. w_1 is the relative importance of input similarity and w_2 is the relative importance of benchmarking risk of failure. Finally the w function calculates the weights for links.

3.2.3. Step 3: Calculating the shortest paths

Given the directed and weighted network of $G=(N,L,w)$, the method utilizes Dijkstra's shortest path algorithm to calculate the shortest paths from each inefficient unit to its peers on the frontier. The parameter ∂_v for every $v \in L$ such that $\partial_v = \text{dist}(s,v)$, v is the ending and s is the starting node. Summing up, the algorithm follows the corresponding process:

[1] Since we are looking for input similarities, the proposed normalization should be maintained irrespective of the orientation taken.

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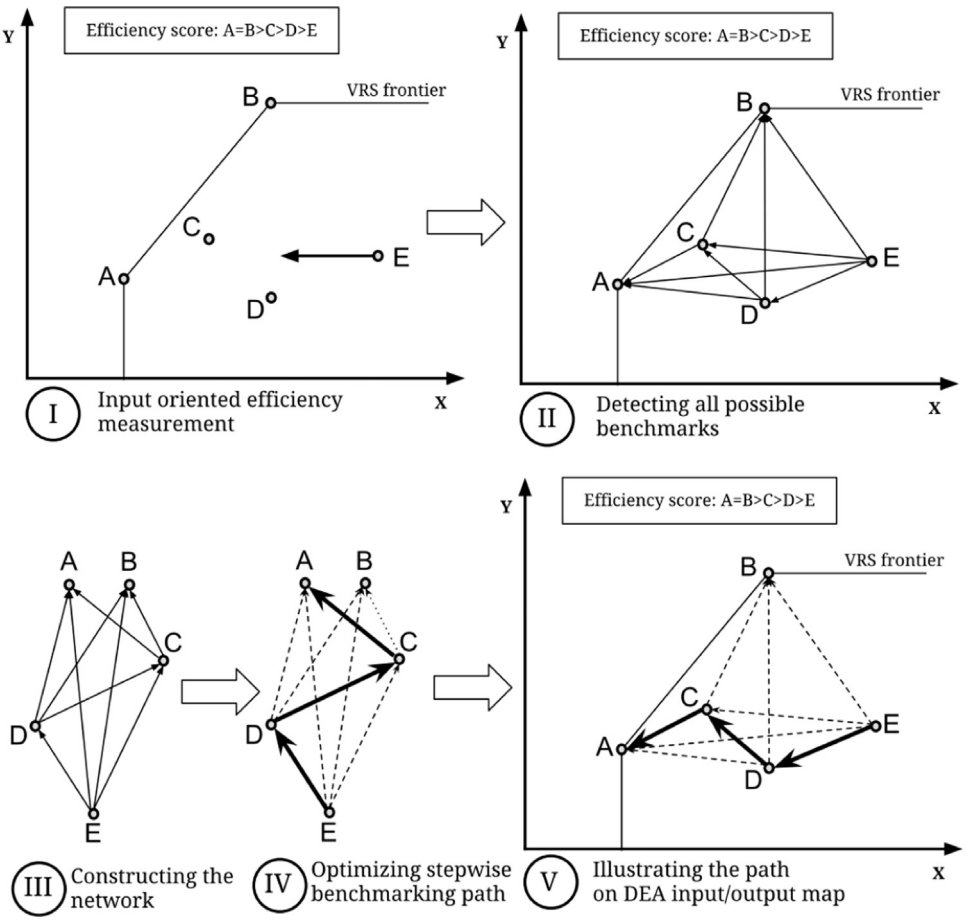

Fig. 3. Network construction steps.

1. Set the $P:=0$, $Q:=V$ and $\partial_s:=\infty$ for every $v \in Vs$.
2. While Q is not empty do the following.
   (a) Find a vertex $v \in Q$ such that $\partial_v=\min\{\partial_u: u \text{ in } Q\}$.
   (b) Set $Q:=Q-v$, $P := P \cup v$. $\partial_v:=\min\{\partial_u,\partial_v+w(v,u)\}$ for every $u \in Q \cap N+(v)$.

For an inefficient unit, the shortest paths to all its efficient peers are calculated and the shortest path is highlighted. It is possible to use the DEA calculated target as an additional, virtual ultimate target. This way the method provides the shortest paths toward efficiency not only for the real observed peers but also for the DEA calculated target. From one hand, this modification provides intermediate benchmarks that are more compatible with the optimization orientation. From the other, since the final target is probably not an observed unit, the final benchmark of such path will not be a real benchmark. In the present work, we include only the efficient peers of each unit and not its DEA calculated target. The shortest path algorithm finds the paths with minimum weight summations. This way the method finds paths with minimum level of zigzagging based on the available sample in the dataset.

3.2.4. Clustering the DMUs

The aforementioned method takes into account the decision maker's preferences in terms of maximum allowed changes for inputs in each step to cluster the DMUs. The decision maker can limit the maximum percentage change of inputs for DMUs in each step by defining the matrix:

$$\mathbf{J_I}=\begin{pmatrix} |\%\Delta x_1| & |\%\Delta x_2| & \cdots & |\%\Delta x_m| \\ \delta_{11} & \delta_{12} & \cdots & \delta_{1m} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2m} \\ \vdots & \vdots & \ddots & \cdots \\ \delta_{n1} & \delta_{n2} & \cdots & \delta_{nm} \end{pmatrix} \begin{matrix} \\ \text{DMU}_1 \\ \text{DMU}_2 \\ \vdots \\ \text{DMU}_n \end{matrix}, \tag{4}$$

where m is the number of inputs, n is the number of units and δ_{ij} is the maximum percentage that the input i can increase or decrease for unit j in each step.

In a similar way, the decision maker can imply the preferences for the maximum percentage output change by defining the matrix:

$$\mathbf{J_O}=\begin{pmatrix} |\%\Delta y_1| & |\%\Delta y_2| & \cdots & |\%\Delta y_s| \\ \delta_{11} & \delta_{12} & \cdots & \delta_{1s} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2s} \\ \vdots & \vdots & \ddots & \cdots \\ \delta_{n1} & \delta_{n2} & \cdots & \delta_{ns} \end{pmatrix} \begin{matrix} \\ \text{DMU}_1 \\ \text{DMU}_2 \\ \vdots \\ \text{DMU}_n \end{matrix}. \tag{5}$$

Each pair of $(\mathbf{J_I}, \mathbf{J_O})$ can break down the network $G=(N,L,w)$ into several components and each component can be treated as a cluster of benchmarking units. Comparing the amount of changes in inputs and outputs in each step by the given functions, it is possible to filter the non-matching links and provide a subset of the initial network. A component may or may not include an efficient unit but at each cluster there is at least one unit that is the

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best performer connected to others. Each cluster shows the maximum benchmark that is possible for inefficient units to take, given the maximum amount of changes in inputs and outputs. The shortest path to the best performer of the group shows the time needed to the frontier.

Analyzing a series of slightly increasing intervals of ($\mathbf{J_I}$, $\mathbf{J_O}$) provides an overview of the efficiency evolution during the benchmarking steps.

3.2.5. Highlighting possible outliers and specialized units

To determine the presence of possible outliers and specialized units, the method provides a graphical network layout in which the node sizes represent the alpha centrality of the nodes and the outliers and specialized units stand out. Alpha centrality is a descriptive measurement of nodes in a network. It is a generalized eigenvector measure of centrality [12] adaptable for directed and weighted networks. In eigenvector-like centrality measurements, the centrality status of a node is not only based on its own location and status, but also on the recursive location and status of its neighbors. In our context, an efficient unit is assigned a higher alpha centrality score when the units which have chosen it is as a benchmark, are recursively chosen as benchmarks by more intermediate, better performer units.

Alpha centrality scores are calculated as $\boldsymbol{c} = (\mathrm{diag}\ (1_n) - \alpha.\mathrm{adj}(G)^{\mathrm{T}})\boldsymbol{W}$ where $\mathrm{diag}(1_n)$ is the identity matrix, $\mathrm{adj}(G)^{\mathrm{T}}$ is the transposed adjacency matrix of network G, and $\boldsymbol{W}$ is a weight matrix where the elements are calculated using the w function. We apply alpha-centrality at α equaling 0.05. These scores are used to visually highlight more central units. Moreover, in a force-directed layout with $\boldsymbol{W}$ weights, the efficient frontier takes place on the outer boarder of the graph which is more consistent to the efficient frontier concept.

The decision makers can then apply their preferences by modifying the weight parameters to discriminate input endowment similarities or efficiency gap differences between DMUs. By changing the weight parameters, the weight scores of the links change and the network layout forces the units with highest scores to be far from the center of the network. Using this technique the decision maker can distinguish the specialized DMUs that are relatively further away from other DMUs in terms of input endowments, or possible outliers that are at a distance from others in terms of efficiency scores.

4. Empirical application, results and discussion

4.1. Data

We evaluate the proposed method by applying it to a network of a Canadian bank which has 79 branches (DMUs). The data was originally published by Estrada et al. [3]. The fact that the data comes from centrally planned units that compete in the same context certifies the homogeneity of the DMUs, which is a crucial condition for the validity of the DEA model. There are three inputs: the number of employees (full-time equivalent) in services, sales and other activities; and four outputs: loans, mortgages, registered retirement saving plans and letters of credit. As indicated previously, the input variables are normalized so the vector of distances and the summation of the distances do not affect the quality of the results.

The data is also suitable for finding the most appropriate intermediate benchmarks to facilitate the inter-organizational learning process. The units with a higher percentage of employee support are more likely to learn from better performers with the same portion of input endowments.

Table 1
Efficiency measurement results.

DMU	Efficiency scores	Benchmark target	DMU	Efficiency scores	Benchmark target
1	1	Efficient unit	41	0.445	42, 49, 65
2	0.378	4, 34, 36, 54, 65	42	1	Efficient unit
3	0.706	4, 36, 39, 59	43	0.744	29, 36, 39, 55, 69
4	1	Efficient unit	44	0.639	34, 54, 65
5	1	Efficient unit	45	0.577	34, 42, 54
6	0.274	34, 42, 54, 65	46	0.607	29, 34, 49, 65, 69
7	0.537	29, 36, 39, 65, 69	47	0.641	4, 29, 32, 36, 55
8	0.461	36, 49, 65, 69	48	0.478	36, 49, 55, 69
9	0.617	21, 34, 54	49	1	Efficient unit
10	0.329	36, 49, 55, 69	50	0.636	34, 49, 54, 65
11	1	Efficient unit	51	1	Efficient unit
12	0.531	34, 36, 65, 69	52	0.985	21, 29, 36, 54
13	0.769	42, 49, 65	53	0.914	23, 29, 39, 55, 74
14	1	Efficient unit	54	1	Efficient unit
15	0.909	29, 36, 49, 65, 69	55	1	Efficient unit
16	0.575	49, 65	56	0.384	42, 49, 54, 70
17	0.593	29, 36, 39, 54	57	0.798	34, 54
18	0.675	36, 42, 49	58	0.462	29, 36, 49, 62
19	1	Efficient unit	59	1	Efficient unit
20	0.633	42, 49, 54, 65	60	0.615	4, 29, 36, 39, 55, 69
21	1	Efficient unit	61	0.511	34, 42, 54
22	0.549	36, 49, 55, 69	62	1	Efficient unit
23	1	Efficient unit	63	1	Efficient unit
24	1	Efficient unit	64	0.689	42, 49, 65
25	0.759	29, 34, 36, 49, 65, 69	65	1	Efficient unit
26	0.792	29, 34, 36, 65, 69	66	0.650	34, 36, 42, 65
27	0.645	34, 36, 49, 65, 69	67	0.475	42, 49, 65
28	0.778	34, 54	68	0.802	29, 36, 49, 55, 69
29	1	Efficient unit	69	1	Efficient unit
30	0.879	23, 59, 63	70	1	Efficient unit
31	0.623	19, 34, 36, 55, 69	71	1	Efficient unit
32	1	Efficient unit	72	0.782	4, 14, 23, 36, 74
33	0.808	29, 36, 39, 55	73	0.991	23, 24, 36, 51
34	1	Efficient unit	74	1	Efficient unit
35	0.751	34, 42, 49, 54, 65	75	0.380	Efficient unit
36	1	Efficient unit	76	1	Efficient unit
37	0.450	34, 54, 65	77	1	Efficient unit
38	0.364	34, 42, 49, 54, 65	78	1	Efficient unit
39	1	Efficient unit	79	0.498	34, 36, 49, 65, 69
40	0.553	34, 36, 42, 49, 54, 65			

4.2. Empirical results and discussions

4.2.1. Efficiency measurement and network construction

We evaluated the efficiency of DMUs using a VRS DEA model assuming the input orientation. Table 1 shows the original results with 32 efficient DMUs and 47 inefficient DMUs, with their corresponding benchmark peers. The measurement results show a high frequency of firms with high efficiency scores.

We then constructed the network based on expression 3 and using the igraph [24] in the R-project [25] statistical package. The final network has 79 nodes and 2646 links that show a huge number of possible benchmarking path selection options. The resulting network has one component and is a weighted directed acyclic network. Fig. 4 shows the resulting network with the

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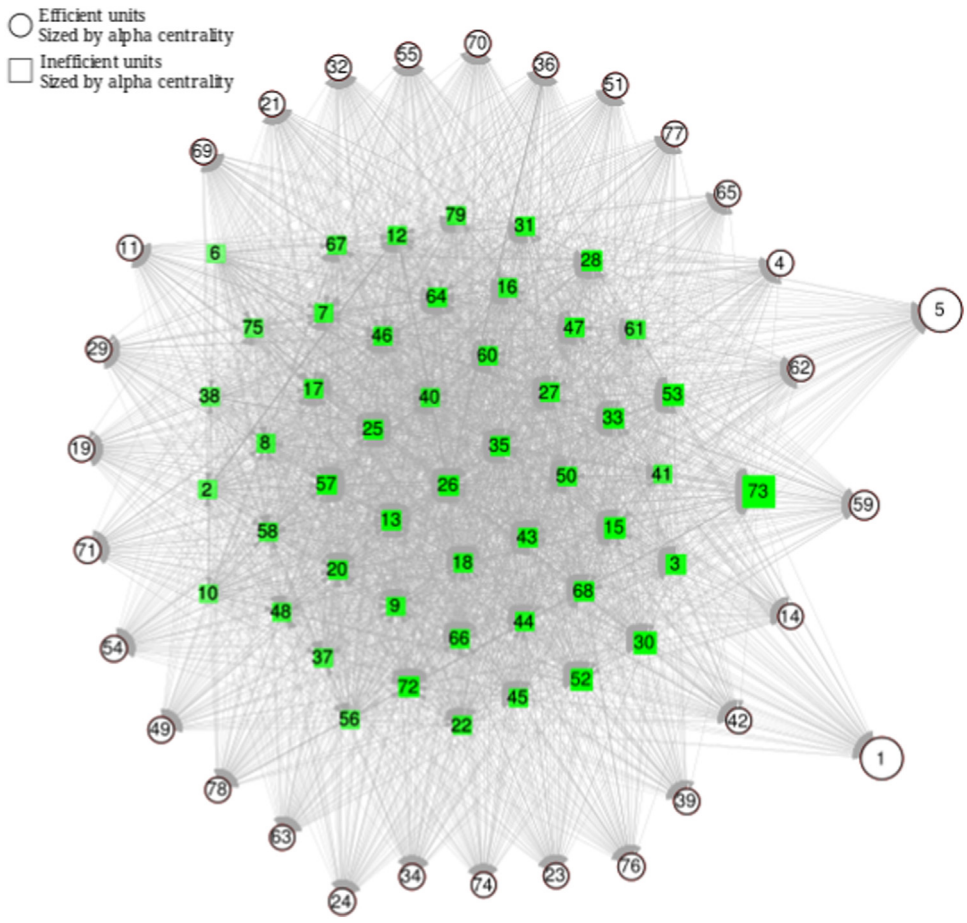

Fig. 4. Network of all possible efficiency connections.

decision maker's preferences as $w_0=0$, $w_1=0.5$ and $w_2=0.5$. The circles represent the efficient units and the squares, the inefficient DMUs.

The edge thickness shows the appropriateness of the ending point for selection as the benchmark for the starting node. The node size is based on alpha centrality score of each unit. We apply alpha centrality with alpha equaling 0.05 which is determined by [12] The alpha centrality can be interpreted as popularity in terms of number of units that prefer this node as a benchmark. In other words, it is more important for a unit to be selected by a popular node than by an unpopular node. The node size captures the amount of changes in input endowments or efficiency level to benchmark the specified unit which are calculated by w function. If the node is relatively far from other units (in terms of inputs or efficiency), it receives more weight from benchmarking firms and grows bigger. The intermediate firms toward such a dissimilar firm will also gain some node size. In the network presented here, unit 73 is the only inefficient unit that is similar to units 1 and 5, and has gained a bigger node size and lies on the periphery. The nodes at the center are the most inefficient firms that have several efficient and inefficient units with similar endowments. The inefficient units are attracted by their most preferable benchmarks. This attraction is done using a spring embedder and force directed graph to draw what are known as Fruchterman–Reingold [26] algorithms.

4.2.2. Finding the shortest stepwise benchmarking paths

We use the *R* package *igraph* [24], which applies Dijkstra's algorithm to find the shortest paths. For each unit, we calculate the shortest paths to the set of peers. Using the initial weights $w_0=0$, $w_1=0.5$ and $w_2=0.5$ the method provides shortest paths for unit 2 toward the efficiency frontier as shown in Fig. 5a. The source node is on the bottom of the network and the paths are toward the nodes at the top of the graph. The efficiency increases by moving toward top nodes. The numbers in parenthesis are the input endowment distance and the percentages of efficiency improvement in each step. As shown in Fig. 5b, the method has the ability to provide relatively similar steps in terms of input and efficiency changes. The decision maker can adjust the relative importance of input or efficiency changes in each step by changing the decision maker weight preferences. It is also possible to control the number of steps based on the available data by adding a fixed cost to each step.

Table 2 demonstrates the shortest path proposed for the first 10 inefficient DMUs. The shortest paths to all peers are provided and the one with the best score is highlighted. The path length shows the number of benchmarking steps that a DMU must take to reach the efficiency frontier. As shown in that table, there are fewer steps for DMUs with higher efficiency scores than for the less efficient DMUs. The scores are the sum of weights generated by the multi-factor weight function for each step. For each DMU the paths to all peers are calculated by the method and the minimum score has been decided as the optimal shortest path.

Fig. 5 shows the effect of fixed cost on the number of steps of the shortest stepwise paths. From the figure we can see that the stepwise benchmarking path from unit 2 to unit 36 has a length of four in Fig. 5 a, three in Fig. 5 b, and two in Fig. 5 c (having $w_0=0$, 0.01, 0.03 respectively while $w_1=w_2$). Although the method provides paths with less average length, the effect of length reduction is not equal for all paths, since it is determined by the available

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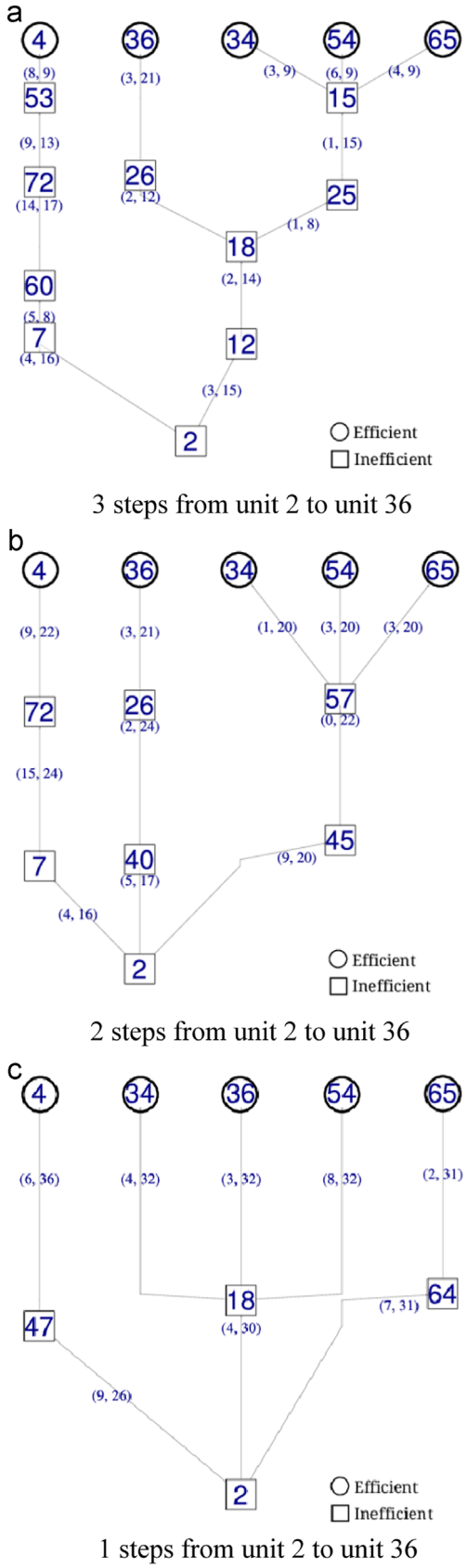

Fig. 5. Shortest path for sample unit 2 toward efficient peers. The numbers in parenthesis are, respectively, the input endowment distance and the percentages of efficiency improvement in each one, both rounded down. (a) 3 steps from unit 2 to unit 36. (b) 2 steps from unit 2 to unit 36. (c) 1 step from unit 2 to unit 36.

Table 2
Shortest stepwise benchmarking paths for the first 10 inefficient units.

	From	To	Penalty score[a]	Path	Length	Optimal
1	2	36	0.050	2–12–26–36	3	*
2	2	34	0.055	2–58–18–15–34	4	
3	2	65	0.060	2–58–18–15–65	4	
4	2	54	0.066	2–45–57–54	3	
5	2	4	0.092	2–47–4	2	
6	3	39	0.090	3–39	1	*
7	3	59	0.096	3–59	1	
8	3	4	0.100	3–4	1	
9	3	36	0.146	3–36	1	
10	6	34	0.066	6–58–18–15–34	4	*
11	6	42	0.067	6–58–18–15–42	4	
12	6	65	0.071	6–58–18–15–65	4	
13	6	54	0.079	6–56–45–57–54	4	
14	7	36	0.049	7–68–36	2	*
15	7	29	0.050	7–68–29	2	
16	7	69	0.054	7–26–69	2	
17	7	65	0.069	7–13–65	2	
18	7	39	0.073	7–33–39	2	
19	8	36	0.053	8–43–36	2	*
20	8	69	0.054	8–13–69	2	
21	8	65	0.065	8–13–65	2	
22	8	49	0.072	8–50–35–49	3	
23	9	21	0.022	9–21	1	*
24	9	34	0.027	9–57–34	2	
25	9	54	0.035	9–57–54	2	
26	10	55	0.059	10–22–68–55	3	*
27	10	36	0.069	10–22–68–36	3	
28	10	69	0.071	10–22–13–69	3	
29	10	49	0.090	10–22–13–49	3	
30	12	69	0.033	12–26–69	2	*
31	12	36	0.035	12–26–36	2	
32	12	34	0.041	12–18–15–34	3	
33	12	65	0.045	12–18–15–65	3	
34	13	65	0.020	13–65	1	*
35	13	42	0.023	13–42	1	
36	13	49	0.028	13–49	1	
37	15	69	0.009	15–69	1	*
38	15	29	0.011	15–29	1	
39	15	65	0.020	15–65	1	
40	15	36	0.020	15–36	1	
41	15	49	0.027	15–49	1	

[a] Penalty scores are calculated using function p in Formula (3). The penalty score is higher for benchmark paths with less input similarity or more efficiency score gaps covered in individual steps of each path. The optimal path (*) describes the minimum "penalty score" from the specific unit to the frontier.

data. While having overall control over the path length can be useful for the CD-DEA, it also seems that forcing the method to provide paths with fixed length is not optimal. In general, it can be seen that the method is able to control the length of benchmarking paths.

4.2.3. *Clustering DMUs based on benchmarking segments*

To provide benchmark segmentation or clustering of DMUs we assume that the DMUs can change the amount of inputs and outputs by 10% at each step. Given this assumption, $\mathbf{J_I}$ is a matrix of $m \times n$ and $\mathbf{J_O}$ is a matrix of $s \times n$ with all elements equal to 10. Utilizing these matrices, we filter the graph generated in the previous step and the resulting network is shown in Fig. 6, where the components with more than two connected members are highlighted.

The graph has several unconnected inefficient units. We put them in one cluster and this cluster shows the inefficient units that cannot be connected to an intermediate benchmark with maximum 10% change in inputs and outputs. It is also obvious that none of the units can reach the efficient unit having the given possible changes. There are several components that demonstrate

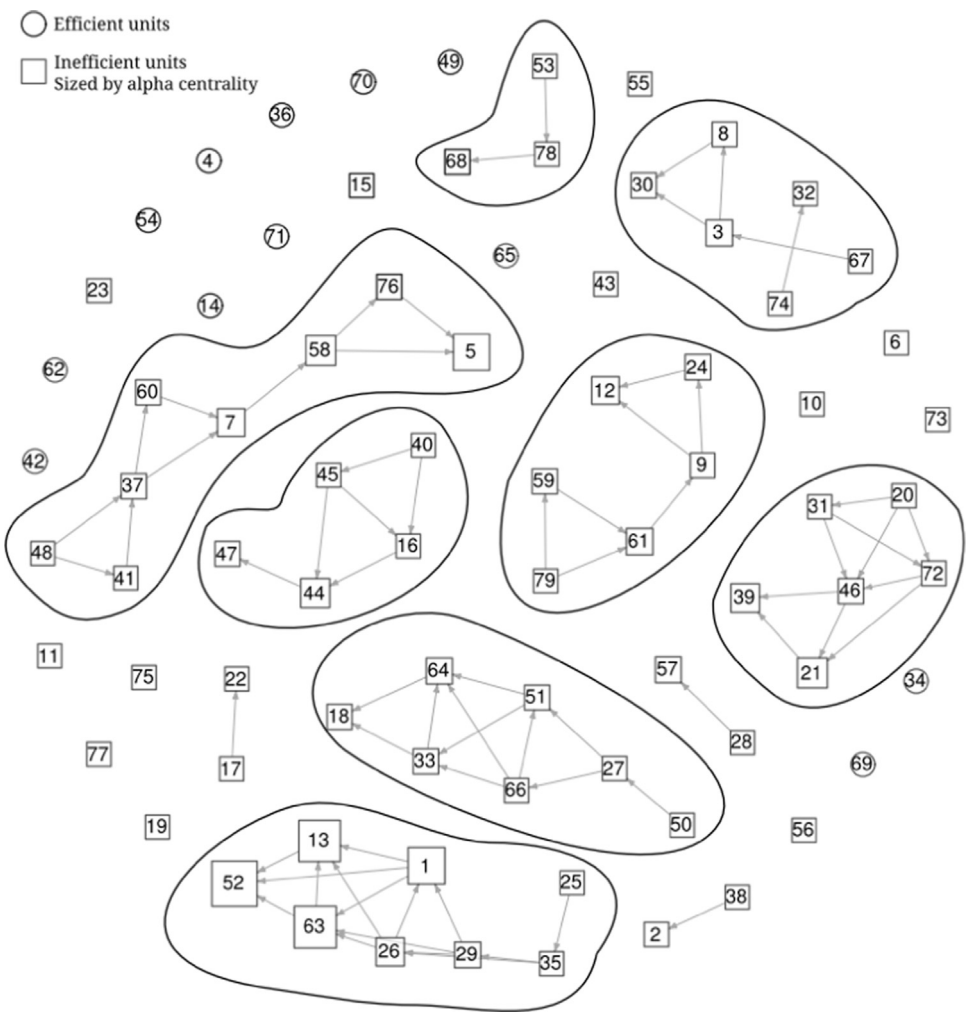

Fig. 6. Clustering DMUs based on their benchmarking status.

the possibilities of stepwise benchmarking inside the allowed interval. As shown in Fig. 6, unit 1 can benchmark unit 52, directly or through other units. The node sizes here show the popularity of the DMUs in term of the possibility of their being benchmarked by other DMUs, taking into account the maximum allowed changes.

By slightly increasing the input and output change intervals, the network evolves and its evolution reveals some important information. For example the isolated units connect to other better performers and make the clusters bigger.

Gradually, the clusters join together until all units are connected. The units that connect later are more isolated, inefficient units and are potential choices for shutdown, since they are so inefficient that pushing them toward efficiency requires much effort and time.

4.2.4. Highlighting the possible outliers and special units

Deviation in the network may be due to the existence of outliers. To exaggerate the effect of input similarities, we change the weight preferences into $w_0=0$, $w_1=0.9$ and $w_2=0.1$. Fig. 7 shows the network after the change. The outcome is that there are four units with significant different input endowments. This may be due to heterogeneity in scale or an indication of the presence of outliers (units 1, 5 and 59). Since it is not possible to disentangle what type of error it is, we assume that units 1 and 5 are outliers and unit 59 is a very large unit. To solve the outliers problem we replace them with unit 4, the second most similar unit to unit 1 and 5. We construct the network with $w_0=0$, $w_1=0.5$ and $w_2=0.5$. This time the network shows a smoother deviation in terms of node sizes. We apply alpha-centrality fixing alpha at 0.05.

Note that by removing the hypothetical outliers, the mean efficiency score of the sample increases by 2%.

5. Conclusion and implications

The main purpose of this paper is to develop a network-based method of stepwise benchmarking that takes into account the input similarities and efficiency gap of each step. Based on the absorptive capacity, and the similarity of the teaching and learning organizations, it facilitates the knowledge transfer and inter-organizational learning process. The study starts from the impossibility of a specific unit removing all the existent inefficiencies in a single step.

Therefore, we consider the benchmarking process as a network of learning organizations and try to provide the most appropriate stepwise benchmarking paths. The proposed method first evaluates the efficiency scores and then, using a multi-factor weight function, transforms the initial DEA efficiency scores and benchmarking information into a directed and weighted network. Then the method provides the shortest stepwise benchmarking path. To the best of our knowledge, this is the first time that an SNA method has been proposed for benchmarking purposes in the DEA context.

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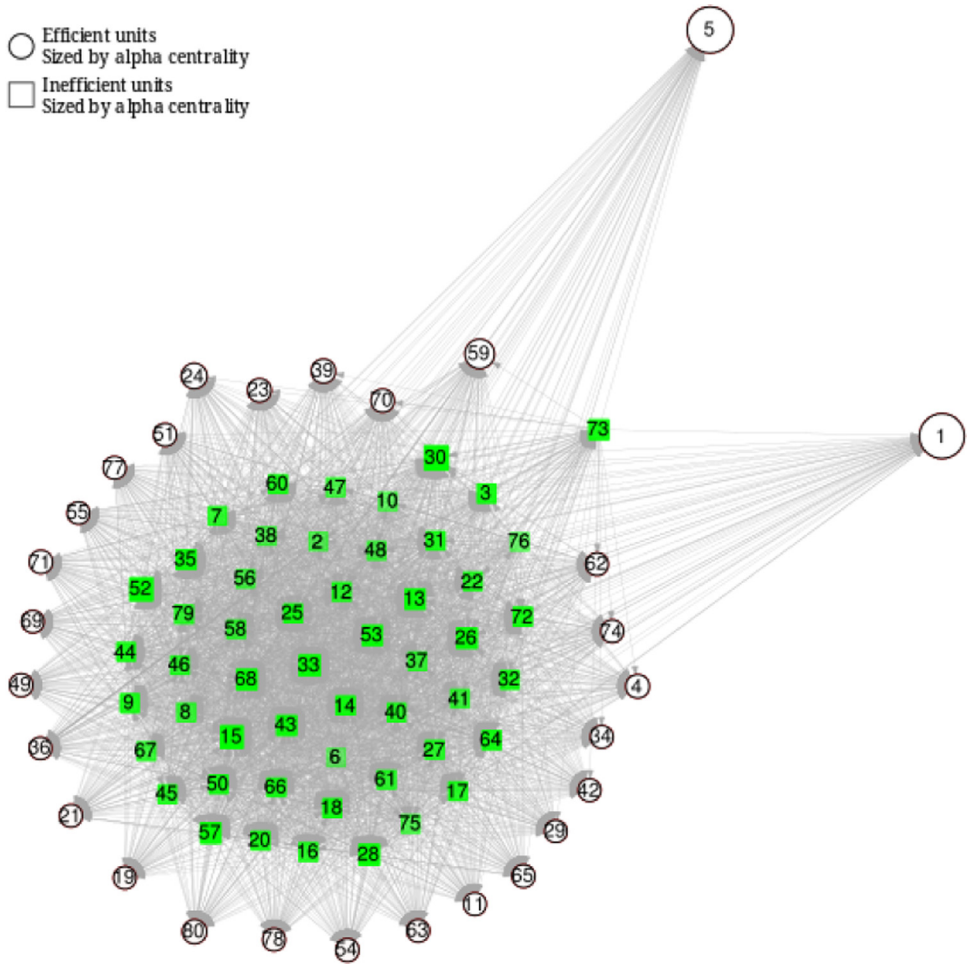

Fig. 7. Highlighting possible outlier or specialized DMUs.

Using the empirical showcase of 79 Canadian bank branches, we explored several characteristics of the method. The model transformed the input/output information into a network of learning DMUs. Then, using the shortest path concept, we calculated the optimal and shortest stepwise benchmarking path for each DMU. Afterwards, using the alpha centrality concept, we highlighted the possible outliers or specialized DMUs and clustered the DMUs based on their benchmarking status in the bounded change network. The final conclusion is that our proposal, based on the knowledge-based view of efficiency, facilitates the knowledge transfer and inter-organizational learning process. It is flexible enough to allow the decision maker to decide on the appropriate stepwise benchmarking paths for learning DMUs.

In addition to its uses in a range of organizations, this method has a direct application in education. Teachers can use it to set desirable and achievable targets for students based on already existing performance of other students. The similarity of students' personal, family and social characteristics can be taken as inputs. The method finds the most similar but better performer benchmarks in the sample.

Unlike other proposals in the field of efficiency and effectiveness, the singularity of this method lies in the way it optimizes the whole path of benchmarking and can control the number of steps. This ability is crucial for short- and long-term planning.

Practitioners can also apply the clustering method to group DMUs and their internal managers to design educational packages. They can cluster the DMUs and design specific training courses for each group. The method has the flexibility to provide variety of group size and numbers based on maximum allowed changes in each step.

The present method has some limitations. First, the ability of benchmarking paths depends on the breadth of the generated network. Thus, for a very inefficient isolated network with no close better performer, the first step of real benchmarks contains a relatively large change in inputs, and efficiency gap removal. This limitation affects the control of method over the exact number of steps.

Second, the method has three decision-maker preference weight parameters, which means there is a trade-off between the flexibility and capabilities of the method, and its ease of use. On the one hand, it provides an impressive tool to distinguish benchmarking conditions of the network and, on the other, it is relatively complex to adjust all three at the same time.

Apart from the eigenvector and alpha centralities, other applications of SNA concepts and techniques in DEA are unexplored. We propose two directions for future research. One possibility is to extend the present paper in other areas of DEA concepts such as dynamic or network DEA. Network DEA opens the organizational 'black box' to evaluate the efficiency of internal interconnected processes, and must not be confused with the network-based approach. We also aim to extend the present research in other DEA models or technological assumption such as Free Disposal Hull (FDH) and Slack-Based Models (SBM) models in dynamic and network DEA. Specifically, we will attempt to combine this method with the proposal by Lozano and Villa [18] to provide a sequence of hypothetical targets that will resolve the first research limitation noted above.

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The second possibility for future research is to introduce other SNA techniques such as dual networks and network evolution into the DEA concept. We will also try to improve the method for sensitivity analysis of decision-making parameters to provide some guidelines for the decision maker. Finally, there are several other DEA concepts that could potentially be applied in the DEA concept, e.g., some other centrality concepts, network flows, connectivity and structural balance.

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