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PII: S0305-0483(16)00016-5
DOI: http://dx.doi.org/10.1016/j.omega.2015.11.010
Reference: OME1648

To appear in: *Omega*

Received date: 15 September 2014
Revised date: 15 July 2015
Accepted date: 15 November 2015

Cite this article as: Jerim Kim, Sung-Yong Son, Jung-Min Lee and Hyung-Tae Ha, Scheduling and performance analysis under a stochastic model for electri vehicle charging stations, *Omega,* http://dx.doi.org/10.1016/j.omega.2015.11.01

Scheduling and Performance Analysis under a Stochastic Model for Electric Vehicle Charging Stations

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Abstract

Wide-spread infrastructures for electric vehicle battery charging stations are essential in order to significantly increase the implementation of electric vehicles (EVs) in the foreseeable future. Therefore, we propose a stochastic model and charge scheduling methods for an EV battery charging system. We utilize a flexible Poisson process with a hidden Markov chain for modeling the complexity of the time-varying behavior of the EV stream into the system. Relevant random factors and constraints, which include parking times, requested amounts of electricity, the number of parking lots (charging facilities), and maximal demand level, are considered within the proposed stochastic model. Performance measures for the proposed charge scheduling are analytically derived by obtaining stationary distributions

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of states concerning the number of inbound EVs, waiting time distributions, and the joint distributions of parking time and electricity charged during random parking times.

Keywords. electric vehicles, battery charging station, stochastic modeling, charge scheduling, Markov-modulated Poisson process, performance measures.

1 Introduction

Electric vehicles (EVs) are considered to be the most significant green transportation alternative for the foreseeable future. Recent studies concerning EVs address many aspects, including EV development, the social impact of substituting fossil-fuel vehicles, and public policy that enables the spread of EVs. Although government-provided motivation and environmental benefits exist, EV implementation has not been significantly fast. One of the primary restrictions of the public spread of EVs is the lack of EV battery charging infrastructures [8, 9, 17]. In conventional battery charging technology, slow (regular) chargers require an average of three to six hours in order to fully charge an empty battery for common-size EVs, whereas fast chargers can substantially reduce the charging time to less than half an hour. However, high-speed charging facilities incur significantly higher costs and still require substantially more service time than conventional fuel-based automobile stations. When compared with existing gas stations, the battery charging time required for even fast charging equipment can be considered too long by many drivers. Therefore, the efficient operation of battery charging stations is an important factor in the acceleration of the public spread of EVs.

Some recent publications examine the strategic levels of EV battery charging stations, such as stochastic demands, optimal locations, and spread [6, 12, 13, 14]. However, analytic or computational approaches to operational levels, such as system efficiency and charge scheduling performance, remain relatively unexplored [2, 15, 18].

This paper proposes a more realistic stochastic model for EV battery charging stations. Two typical charge scheduling methods, the first-in-first-served (FIFO) and processor sharing (PS), are considered. The framework for the incoming stream of EVs under the proposed stochastic model addresses the time-varying behavior of EV arrivals by exploiting a flexible Poisson process of the Markov-modulated Poisson process (MMPP). Performance measures for the charging scheduling are analytically derived by obtaining stationary distributions for the states that account for the status of inbound EVs, waiting time distributions, and joint distributions of parking time and charged electricity amount during random parking times.

The remainder of this paper is organized as follows. In section 2, we propose a stochastic framework for the EV changing stations and two charge scheduling methods. In section 3, we introduce and analytically obtain performance measures of the charge scheduling under the proposed stochastic system by deriving stationary distributions of the status of EVs in the parking lot, waiting time distributions and Laplace-Stieltjes transformations of the parking time and amount of electricity charged during the time an EV is parked. Numerical examples in section 4 demonstrate some practical interpretations for the proposed system under stochastic environments. And concluding remarks follow in section 5.

2 Stochastic modeling

2.1 EV charging station

Our scenario represents the likely event that multiple EV drivers arrive a location, such as an apartment building, department store, or office building, within a given time period and immediately plug in their EVs in order to electrically charge them while they are parked. Although the primary reason EV drivers enter a station is to park their EVs, these facilities may decide to provide the additional service of electrical charging because of capacity limitation of EVs' battery and a substantial growth in EV usage. In this paper, we consider a station for EVs that has parking spaces, each of which is connected to the EV battery charging facility. An EV that arrives at the station occupies one parking space, assuming an unoccupied parking space exists, and departs the station after a random parking time. The EV leaves the station after the random parking time, regardless of whether its requested amount of energy has been charged. If all spaces are already occupied when the EV arrives, it will immediately depart the parking facility without waiting. After an EV parks in the station, it immediately plugs into an electrical charger and continues to charge while it is parked. The electrical charge amount requested by an incoming EV varies according to the EVs situation.

In order to effectively model the EV battery charging station system, we must first summarize the system's description and the notation used for the main deterministic and stochastic components.

Parking time (S_n)

The random parking time of the station's n^{th} incoming EV, denoted by $\{S_n,\ n \geq 1\}$, is assumed to have an independent and identical exponential distribution with mean parameter ν^{-1}.

Requested charging electricity amount (Y_n)

The amount of electricity requested by the n^{th} incoming EV, which we represent using $\{Y_n,\ n \geq 1\}$, is an independent and identical exponential distribution with mean parameter μ^{-1}. The variables for parking time and requested amount of charging energy are assumed to be stochastically independent.[2]

[2]We consider in this paper only the cases that it is reasonable to assume that parking time and requested amount of energy are not strongly dependent in EV stations. Dependence assumption could be reasonably acceptable according to circumstances and types of EVs' stations. It should also be mentioned that the analytical analysis for performance measures proposed in this paper will be extremely challenging without the independence assumption.

Maximal charging rate

The maximal charging rates for all chargers are equal. Without loss of generality, the maximal charging rate for an EV is set to be 1. Note that the actual charging rates at specific times can differ according to the charge scheduling.

Charge amount during time parked (X_n)

The amount of electricity charged for the n^{th} incoming EV while it is parked, denoted by $\{X_n,\ n \geq 1\}$, is also stochastically distributed. Note that the amount of electricity charged satisfies the inequality $X_n \leq \min\{S_n,\, Y_n\}$.

Number of battery chargers and parking lots (K)

Slow (regular) EV battery chargers are installed in every parking lot, i.e., the amount of charging equipments equals the number of parking lots.

Maximum demand level (P)

The voltage constraint of the electricity distribution system, which is a typical voltage regulation limit specified by many electricity distribution utilities, is used to set a ceiling limit for the maximum system demand in order to prevent the total power consumption overload produced by the EVs charging at any given time.[3]

[3] Note that random charging activities that do not have ceiling limits for the maximum system demand could significantly stress the electricity distribution grid by causing severe voltage fluctuations. This also affects the dispatching suboptimality of the power generation. It degrades system efficiency and economy, and it increases the likelihood of blackouts due to network overloads [4, 16]. The upper limit of the maximum system demand is often applied to smart grid systems that facilitate bidirectional communication infrastructures in order to address these problems.

2.2 Markov-modulated Poisson process for EV arrival stream

Because the MMPP is widely considered to be a very flexible stochastic framework that can be used to model unusual events and is a well-defined probabilistic model that can accommodate the flexibility of stochastic integer inputs, we employ this process in order to characterize the stochastic inbound process of the arrival of EVs into a battery charging station for our proposed stochastic model.

An intensity function is the sole rate parameter used to determine the Poisson process. We suppose that the arrival rate of EVs to a parking lot is governed by the process $\{J(t) : t \geq 0\}$ with values in the set, $\{1, 2, \ldots, m\}$, of finite elements that represent m cases for the traffic circumstances. For $i = 1, 2, \ldots, m$, we let λ_i be the arrival rate of EVs to the parking lot for traffic state i. That is, when $J(t) = i$, the EV arrival rate is λ_i at time t. Note that $\{J(t) : t \geq 0\}$ is a continuous time Markov process. Also, note that an EV arrives at the parking lot according to a doubly stochastic Poisson process with stochastic intensity $\lambda_{J(t)}$. Here, the EV arrival process is said to be an MMPP with representation (Q, Λ), where Q is the infinitesimal generator of continuous time Markov process $\{J(t) : t \geq 0\}$, and $\Lambda \equiv \text{diag}(\lambda_1, \ldots, \lambda_m)$ is a Poisson arrival rate matrix according to the traffic situations.

2.3 Charge scheduling

FIFO scheduling

Although every EV can be charged to its maximal charging rate when the number of EVs charging in the station is less than P, only P EVs should be charged with the maximal charging rate when the total requested charging amount for the parking lot exceeds the upper limit of electrical power. As the FIFO rule under stochastic models can be seen for instance in [3, 21], the selection criterion is simple. The first P EVs to arrive in the parking facility are chosen, and the remaining EVs wait in the parking lot. From the waiting EVs, the first to arrive is chosen to be charged when an EV completes its charging or leaves without charging completely. Therefore, in the FIFO scheduling, the EVs in a given parking lot are classified into three categories: charging, waiting to charge, or remaining in the lot even though their charge is complete.

PS scheduling

The PS scheduling operates similarly to the FIFO scheduling when the total requested EV charging amount for a given parking lot does not exceed the upper limit of the electrical power. Once this limit is exceeded, the PS scheduling evenly distributes electrical power to all charging EVs by sharing the available total voltage. Consequently, each EV charges with a lower maximal rate. Therefore, the adjusted charging rate for each charging EV is P/n when the number of charging EVs exceeds P.

3 Performance analysis

3.1 Performance measures

This section discusses the performance measures used for two charge scheduling methods, the FIFO and PS scheduling, under a flexible stochastic model of EV battery charging stations. Strategic approaches to the efficient operation of charging stations can be inferred using the analytic results of our proposed performance measures.

In order to illustrate the mathematical concepts of the performance measures, it is helpful to analyze the charging station system from the perspective of an arbitrary EV within the system. When at least one EV is charging in the station at time $t = 0$, we choose an arbitrary charging EV and label it the "tagged EV". Following are the random factors that a tagged EV interacts with at the charging station:

$$\begin{aligned}
\mathcal{S} &\equiv \text{ parking time of the tagged EV,} \\
\mathcal{Y} &\equiv \text{ requested amount of energy from the tagged EV,} \\
\mathcal{X} &\equiv \text{ charged amount of energy for the tagged EV during its parking time } S, \\
\mathcal{J} &\equiv \text{ state of the Markov process } J(t) \text{ at a time that the tagged EV arrives.}
\end{aligned}$$

Decision makers may be interested in performance measures of the system such as the probability that an inbound EV will fully charge while it is parked, and the proportion of the expected amount of charged energy and the possible maximum charge amount while it is parked. The probability that an inbound EV will be fully charged during its parking time as the first performance measure, which we represent using $p_i(y)$, will be expressed in terms of the conditional probability with $\mathcal{Y} = y$, $\mathcal{S} \geq y$ and $\mathcal{J} = i$, i.e.,

$$p_i(y) \equiv \mathbb{P}(\mathcal{X} = y | \mathcal{Y} = y, \mathcal{S} \geq y, \mathcal{J} = i).$$

The second measure is the rational expectation of energy consumed considering the time parked and the requested energy. The ratio of the two conditional expectations $\mathcal{X}$ and $\min\{\mathcal{Y}, \mathcal{S}\}$ when $\mathcal{J} = i$ is defined to be

$$m_i = \frac{\mathbb{E}[\mathcal{X} | \mathcal{J} = i]}{\mathbb{E}[\min\{\mathcal{Y}, \mathcal{S}\} | \mathcal{J} = i]}.$$

3.2 Stationary distributions

The stationary distributions of steady states, the waiting time of EVs in parking lots, and the amount of charging energy while the EVs are parked play important roles when analyzing the performance measures under the proposed stochastic model. It should be noted that the stationary probability for steady states is unaffected by the different schemes provided by the two charge scheduling methods.

1) Stationary probabilities for steady states

If we let

$$
\begin{aligned}
N(t) &= \text{number of EVs that are in charge or waiting for charging at a time } t,\\
L(t) &= \text{number of EVs to have completed charging}\\
&\quad\ \text{but be still staying in parking lot at the time } t,
\end{aligned}
$$

then $\{(N(t), L(t), J(t)) : t \geq 0\}$ is a continuous time Markov process with $\frac{(K+1)(K+2)m}{2}$ states. The state space is

$$
E \equiv \{(n, l, j) : n = 0, 1, \ldots, K, \ l = 0, 1, \ldots, K - n, \ j = 1, \ldots, m\}.
$$

A clarification of the possible state changes of the continuous time Markov process $\{(N(t), L(t), J(t)) : t \geq 0\}$, and their transition rates follow:

- $(n, l, i) \to (n + 1, l, i)$: This transition occurs when an EV arrives at the parking lot and finds an unoccupied space. Because the EV arrival rate is λ_i when $J(t) = i$, the transition rate from (n, l, i) to $(n + 1, l, i)$ is λ_i.
- $(n, l, i) \to (n - 1, l, i)$: This transition occurs when an EV that is not fully charged leaves the parking lot. Each charging EV departs the parking lot with a rate of ν. Therefore, the transition rate from (n, l, i) to $(n - 1, l, i)$ is $n\nu$.
- $(n, l, i) \to (n - 1, l + 1, i)$: This transition occurs when an EV completes charging.
  - FIFO scheduling: When $n \leq P$, the transition rate is $n\mu$, and if $n > P$, the transition rate is $P\mu$.
  - PS scheduling: Each EV completes charging with a rate of $r_n\mu$ given n EVs that are charging in the parking lot, where $r_n = \min\{1, P/n\}$.

  Note that the transition rate from (n, l, i) to $(n-1, l+1, i)$ is $nr_n\mu$ for both the FIFO and PS scheduling methods.

- $(n, l, i) \rightarrow (n, l-1, i)$: This transition occurs when a parked EV that has completed charging departs the parking lot. Each of these EVs departs the parking lot with a rate of ν. Therefore, the transition rate from (n, l, i) to $(n, l-1, i)$ is $l\nu$.

- $(n, l, i) \rightarrow (n, l, j)$, $i \neq j$: This transition occurs when $J(t)$ transits from i to j. Hence, the transition rate from (n, l, i) to (n, l, j) is q_{ij}, where q_{ij} is the (i, j)th entry of Q.

Given these possible changes of state and their corresponding transition rates provided by the continuous time Markov process, we can obtain an infinitesimal generator $\tilde{Q}$, which can be expressed in the lexicographic order:

$$\tilde{Q} = \begin{bmatrix} B_0 & A_0 & & & & \\ C_1 & B_1 & A_1 & & & \\ & C_2 & B_2 & A_2 & & \\ & & \ddots & \ddots & \ddots & \\ & & & C_{K-1} & B_{K-1} & A_{K-1} \\ & & & & C_K & B_K \end{bmatrix},$$

where A_n, B_n and C_n are the following matrices.

- Matrix A_n, with $n = 0, 1, \dots, K-1$, is a $(K-n+1)m \times (K-n)m$ matrix given by

$$A_n = \begin{bmatrix} \Lambda & O & \cdots & O \\ O & \Lambda & \cdots & O \\ & \ddots & \ddots & \\ O & \cdots & O & \Lambda \\ O & O & \cdots & O \end{bmatrix}. \tag{1}$$

- Matrix B_n, with $n = 0, 1, \ldots, K$, is a $(K-n+1)m \times (K-n+1)m$ matrix given by

$$B_n = \begin{bmatrix} D_{n0} & & & & \\ \nu I_m & D_{n1} & & & \\ & 2\nu I_m & D_{n2} & & \\ & & \ddots & \ddots & \\ & & & (K-n)\nu I_m & D_{n,K-n} \end{bmatrix},$$

in which

$$D_{nk} = \begin{cases} Q - (nr_n\mu + (n+k)\nu)I_m, & \text{if } k = K-n, \\ Q - \Lambda - (nr_n\mu + (n+k)\nu)I_m, & \text{otherwise,} \end{cases}$$

and, for positive integer k, I_k is the k-dimensional identity matrix.

- Matrix C_n, with $n = 1, 2, \ldots, K$, is a $(K-n+1)m \times (K-n+2)m$ matrix given by

$$C_n = \begin{bmatrix} n\nu I_m & nr_n\mu I_m & & & & \\ & n\nu I_m & nr_n\mu I_m & & & \\ & & \ddots & \ddots & & \\ & & & n\nu I_m & nr_n\mu I_m & \\ & & & & n\nu I_m & nr_n\mu I_m \end{bmatrix}.$$

We note that the continuous time Markov process $\{(N(t), L(t), J(t)) : t \geq 0\}$ is a level dependent Quasi-Birth-and-Death (QBD) process. Therefore, we are interested in the stationary probability that process $(N(t), L(t), J(t))$ remains at (n, l, j). We denote this stationary probability using π_{nlj}, i.e.,

$$\pi_{nlj} = \lim_{t\to\infty} \mathbb{P}\big((N(t), L(t), J(t)) = (n, l, j)\big).$$

If we let

$$\begin{aligned} \boldsymbol{\pi}_{nl} &= (\pi_{nl1}, \ldots, \pi_{nlm}), \quad n = 0, 1, \ldots, K, \ l = 0, 1, \ldots, K-n, \\ \boldsymbol{\pi}_n &= (\boldsymbol{\pi}_{n0}, \ldots, \boldsymbol{\pi}_{n,K-n}), \quad n = 0, 1, \ldots, K, \quad \text{and} \\ \boldsymbol{\pi} &= (\boldsymbol{\pi}_0, \ldots, \boldsymbol{\pi}_K), \end{aligned}$$

then $\boldsymbol{\pi}$ is the stationary probability vector of the continuous time Markov process $\{(N(t), L(t), J(t)) : t \geq 0\}$. Moreover, this stationary probability vector $\boldsymbol{\pi}$ can be calculated using the matrix analytic method [7, 10, 11]. Given the level dependent QBD structure of infinitesimal generator $\tilde{Q}$, stationary probability vector $\boldsymbol{\pi}$ can be obtained by defining a matrix G_n, with $n = 0, 1, \ldots, K-1$, to be

$$G_0 = -B_0^{-1} A_0, \tag{2}$$

$$G_n = -(B_n + C_n G_{n-1})^{-1} A_n, \quad n = 1, 2, \ldots, K-1, \tag{3}$$

and letting $\boldsymbol{\beta}_K$ be the unique m-dimensional row vector that satisfies

$$\boldsymbol{\beta}_K (B_K + C_K G_{K-1}) = \mathbf{0} \quad \text{and} \quad \boldsymbol{\beta}_K \mathbf{1} = 1, \tag{4}$$

where $\mathbf{1}$ and $\mathbf{0}$ are column vectors of appropriate size whose components are all ones and zeros, respectively, in which row vectors $\boldsymbol{\beta}_n$ are

$$\boldsymbol{\beta}_n = -\boldsymbol{\beta}_{n+1} C_{n+1} (B_n + C_n G_{n-1})^{-1}, \quad n = K-1, K-2, \ldots, 1, \tag{5}$$

$$\boldsymbol{\beta}_0 = -\boldsymbol{\beta}_1 C_1 (B_0)^{-1}. \tag{6}$$

Therefore, the stationary probability can be obtained from the following theorem.

Theorem 1. *The stationary probability vector* $\boldsymbol{\pi} = (\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, \ldots, \boldsymbol{\pi}_K)$ *can be expressed by normalizing vector* $(\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_K)$ *as follows:*

$$\boldsymbol{\pi}_n = \frac{1}{\sum_{k=0}^{K} |\boldsymbol{\beta}_k|} \boldsymbol{\beta}_n, \tag{7}$$

where $|\boldsymbol{\beta}_k|$ *is the* l_1*-norm of* $\boldsymbol{\beta}_k$.

In order to have further examination of steady states, we must also introduce a loss probability. When an arriving EV finds all parking spaces already occupied, an EV may immediately depart the station without waiting. In this case, we say that the EV is lost. The loss probability for an arbitrary EV that arrives when process $J(t)$ is at i can easily be obtained using the stationary probability.[4]

Furthermore, if $\boldsymbol{\kappa} \equiv (\kappa_1, \ldots, \kappa_m)$ is the stationary probability vector of the continuous time Markov process $\{J(t) : t \geq 0\}$, i.e., $\boldsymbol{\kappa} Q = \mathbf{0}$ and $\boldsymbol{\kappa}\mathbf{1} = 1$, then the effective (average) EV arrival rate λ can be obtained from $\lambda \equiv \boldsymbol{\kappa}\Lambda\mathbf{1}$, in which $\mathbf{1}$ is the m-dimensional column vector with 1 in every entry.

Lemma 1. *The loss probability, denoted by p_i^{loss}, that an arbitrary EV which arrives when process $J(t)$ is at i, will immediately leave without parking can straightforwardly be obtained:*

$$p_i^{\text{loss}} = \frac{1}{\kappa_i} \sum_{n=0}^{K} \pi_{n,K-n,i} \,. \tag{8}$$

The loss probability p_i^{loss} is important as a required quantity to derive performance measures $p_i(y)$ and m_i for both the FIFO and PS scheduling methods. Note that the loss probability p_i^{loss}, which is expressed as a weighted average of the stationary distributions for steady states, can be determined from capability and status of the parking facility in the EV station, but not from charge scheduling or status of incoming EVs.

[4] Note that if we define a new continuous Markov process, the loss probability can be derived differently. Consider a continuous time Markov process $\{(N(t) + L(t), J(t)) : t \geq 0\}$, which has $(K+1)m$ states with an infinitesimal generator $\hat{Q}$, provided by

$$\hat{Q} = \begin{bmatrix} \hat{B}_0 & \hat{A}_0 & & & & \\ \hat{C}_1 & \hat{B}_1 & \hat{A}_1 & & & \\ & \hat{C}_2 & \hat{B}_2 & \hat{A}_2 & & \\ & & \ddots & \ddots & \ddots & \\ & & & \hat{C}_{K-1} & \hat{B}_{K-1} & \hat{A}_{K-1} \\ & & & & \hat{C}_K & \hat{B}_K \end{bmatrix},$$

where $\hat{A}_n = \Lambda$, $\hat{B}_n = Q - \min\{n, P\}\mu I_m - \Lambda$ and $\hat{C}_n = \min\{n, P\}\mu I_m$. Because $\psi_{ni} = \sum_{l=0}^{n} \pi_{l,n-l,i}$, we can let ψ_{ni} be the stationary probability of $\{(N(t) + L(t), J(t)) : t \geq 0\}$, $p_i^{\text{loss}} = \phi_{Ki}/\kappa_i$.

2) Conditional waiting time distribution for the FIFO scheduling

In addition to the stationary probabilities for steady states, we also consider the following random factors, which affect the tagged EV under the systems circumstances.

$$\begin{aligned}\mathcal{W} &= \text{waiting time of the tagged EV for charging,}\\ \mathcal{N}_0 &= \text{number of EVs that are in charge or waiting for charging}\\ &\quad\ \text{at the time that the tagged EV arrives.}\end{aligned}$$

We see that $\mathcal{W} + \mathcal{X} \leq \mathcal{S}$, and

$$\mathcal{X} \stackrel{d}{=} \begin{cases} E_{\nu+\mu} & \text{if } \mathcal{W} < \mathcal{S}, \\ 0 & \text{if } \mathcal{W} \geq \mathcal{S}, \end{cases} \tag{9}$$

in which E_λ is exponentially distributed with mean λ^{-1} and $\stackrel{d}{=}$ denotes the equality in distribution. The probability that waiting time $\mathcal{W}$ of the tagged EV remains under the parking time $\mathcal{S}$ is the primary interest of this subsection. Since $\{\mathcal{W} < \mathcal{S}\}$ and $\mathcal{J}$ are independent, the conditional waiting time distribution is expressed in terms of products of two probabilities of waiting time and number of EVs that are in charge or waiting for charging, that is,

$$\begin{aligned}\mathbb{P}(\mathcal{W} < \mathcal{S} | \mathcal{J} = i) &= \sum_{n=0}^{K-1} \mathbb{P}(\mathcal{W} < \mathcal{S} | \mathcal{J} = i, \mathcal{N}_0 = n) \mathbb{P}(\mathcal{N}_0 = n | \mathcal{J} = i) \\ &= \sum_{n=0}^{K-1} \mathbb{P}(\mathcal{W} < \mathcal{S} | \mathcal{N}_0 = n) \mathbb{P}(\mathcal{N}_0 = n | \mathcal{J} = i). \end{aligned} \tag{10}$$

Since, when $\mathcal{N}_0 \leq P - 1$, the tagged EV can be charged immediately after arriving the charging station, i.e., $\mathcal{W} = 0$, $\mathcal{X}$ is identical to $\min\{\mathcal{S}, \mathcal{Y}\}$ with exponential distribution with mean $(\nu + \mu)^{-1}$. And in the case of $\mathcal{N}_0 \geq P$, the tagged EV in FIFO scheduling should wait for charging until all the EVs in the queue that arrived before the tagged EV start to be charged. We further denote $\hat{\tau}_n$ and $\tilde{\tau}$ a first passage time that the number of EVs which arrive earlier than the tagged EV, but are still waiting for charging becomes n, and waiting time that the tagged EV starts to be charged if $\mathcal{N}_0 = P$ and $\mathcal{S} = \infty$, respectively, i.e.,

$$\begin{aligned}\hat{\tau}_n &= \inf\{t \geq 0 : \text{there exist } n \text{ number of EVs that arrive earlier than the tagged EV,} \\ &\qquad \text{but are still waiting for charging}\}, \\ \tilde{\tau} &= \mathcal{W}, \quad \text{given } \mathcal{N}_0 = P \text{ and } \mathcal{S} = \infty.\end{aligned}$$

Note that a probability that the waiting time is less than a parking time when $\mathcal{N}_0 = n$, denoted by w_n, i.e.,

$$w_n = \mathbb{P}(\mathcal{W} < \mathcal{S} | \mathcal{N}_0 = n) = \mathbb{P}(\tilde{\tau} + \hat{\tau}_0 < \mathcal{S} | \mathcal{N}_0 = n),$$

can be obtained from the following recursive relationship between the waiting time and the first passage time

$$w_n = \mathbb{P}(\hat{\tau}_{n-P-1} < \mathcal{S} | \mathcal{N}_0 = n) w_{n-1} \quad \text{for } n > P,$$

and $w_P = \frac{P(\mu+\nu)}{P(\mu+\nu)+\nu}$ because $\mathcal{W} \overset{d}{=} E_{P(\mu+\nu)}$. Therefore, using $\mathbb{P}(\tilde{\tau}_{n-1} < \mathcal{S} | \mathcal{N}_0 = n) = \frac{P(\mu+\nu)+n\nu}{P(\mu+nu)+n\nu+\nu}$ for $n \geq 1$ from $\hat{\tau}_{n-1} \overset{d}{=} E_{nP(\mu+\nu)}$ when $\mathcal{N}_0 = n$, the waiting time w_n can be explicitly determined as

$$w_n = \begin{cases} 1 & \text{for } n < P, \\ \frac{P(\mu+\nu)}{P\mu+n\nu+\nu} & \text{for } n \geq P. \end{cases} \tag{11}$$

And the probability of number of EVs that are in charge or waiting for charging under the stochastic environment $\mathcal{J} = i$ is easily obtained by making use of the conditional Poisson arrivals see time averages (conditional PASTA) property;

$$\mathbb{P}(\mathcal{N}_0 = n | \mathcal{J} = i) = \frac{1}{\kappa_i (1 - p_i^{\text{loss}})} \sum_{0 \leq l \leq K-1-n} \pi_{nli}. \tag{12}$$

Finally, on substituting (12) and (11) into (10), the probability that the tagged EV is served before leaving when $\mathcal{J} = i$ can be obtained in the following theorem.

Theorem 2. *The probability that the tagged EV is served before leaving when $\mathcal{J} = i$ is given by*

$$\mathbb{P}(\mathcal{W} < \mathcal{S} | \mathcal{J} = i) = \frac{1}{\kappa_i (1 - p_i^{\text{loss}})} \Big(\sum_{\substack{0 \leq n < P \\ 0 \leq l < K-n}} \pi_{nli} + \sum_{\substack{P \leq n < K \\ 0 \leq l < K-n}} \pi_{nli} \frac{P(\mu+\nu)}{P\mu + n\nu + \nu} \Big). \tag{13}$$

3) Conditional joint distribution of $\mathcal{X}$ and $\mathcal{S}$ for the PS scheduling

In order to obtain the performance measures of the PS scheduling, we must derive the conditional joint distribution of charged amount of energy and parking time. We further denote

$$\begin{aligned} \tilde{\mathcal{Y}} &\equiv \text{ remained charging amount from the requested amount of energy by the tagged EV,} \\ \sigma &\equiv \text{ epoch that the tagged EV departs the parking lot,} \\ \chi(t_1, t_2) &\equiv \text{ charged amount of energy during } [t_1, t_2] \text{ for the tagged EV.} \end{aligned}$$

We can also define the joint stationary distribution of the charged amount and parking time and its Laplace-Stieltjes transform, which are represented by $\Phi_{nli}(x, s)$ and $\phi_{nli}(w, z)$, respectively, to be

$$\Phi_{nli}(x, s) = \mathbb{P}(\chi(0, \sigma) \le x, \sigma \le s | N(0) = n, L(0) = l, J(0) = i, \tilde{\mathcal{Y}} = \infty),$$

and

$$\begin{aligned} \phi_{nli}(w, z) &\equiv \int_0^\infty \int_0^\infty e^{-wx-zs} \Phi_{nli}(dx, ds), \\ &= \mathbb{E}[e^{-w\chi(0,\sigma)-z\sigma} | N(0) = n, L(0) = l, J(0) = i, \tilde{\mathcal{Y}} = \infty] \end{aligned}$$

where $n = 1, 2, \ldots, K$, $l = 0, 1, \ldots, K - n$, $i = 1, \ldots, m$. In order to derive $\phi_{nli}(w, z)$, we introduce a first passage time τ_n

$$\tau_n \equiv \inf\{t > 0 : N(t) = n\}, \qquad n = 1, 2, \ldots, K.$$

Henceforth, we use $\tau_{K+1} \equiv \infty$ for notation convenience. Given $n = 1, 2, \ldots, K - 1$, let $G^*_n(w, z)$ be the $(K - n + 1)m \times (K - n)m$ matrix whose $((l, i), (k, j))$th entry is given by

$$\begin{aligned} &(G^*_n(w, z))_{(l,i),(k,j)} \\ &\equiv \mathbb{E}[e^{-w\chi(0,\tau_{n+1})-z\tau_{n+1}} \mathbb{1}_{\{\tau_{n+1}<\sigma, L(\tau_{n+1})=k, J(\tau_{n+1})=j\}} \mid N(0) = n, L(0) = l, J(0) = i, \tilde{\mathcal{Y}} = \infty], \end{aligned}$$

$0 \le l \le K - n$, $0 \le k \le K - n - 1$, $1 \le i, j \le m$. Furthermore, if $n = 1, 2, \ldots, K$, we let $\boldsymbol{h}^*_n(w, z)$ signify a $(K - n + 1)m$-dimensional column vector whose (l, i)th component is

$$(\boldsymbol{h}^*_n(w, z))_{(l,i)} \equiv \mathbb{E}[e^{-w\chi(0,\sigma)-z\sigma} \mathbb{1}_{\{\tau_{n+1}\ge\sigma\}} \mid N(0) = n, L(0) = l, J(0) = i, \tilde{\mathcal{Y}} = \infty],$$

$0 \le l \le K-n$, $1 \le i \le m$. Consequently, $G_n^*(w,z)$ and $\boldsymbol{h}_n^*(w,z)$ can be recursively calculated using the following proposition.

Proposition 1. *Given complex number (w,z) with $\mathrm{Re}(w,z) \ge 0$, matrix $G_n^*(w,z)$ and column vector $\boldsymbol{h}_n^*(w,z)$ are provided by*

$$
\begin{aligned}
G_1^*(w,z) &= \big((wr_1+z)I_{Km} - B_1^*\big)^{-1} A_1, && (14)\\
G_n^*(w,z) &= \big((wr_n+z)I_{(K-n+1)m} - B_n^* - C_n^* G_{n-1}^*(w,z)\big)^{-1} A_n, \quad n=2,3,\ldots,K-1, && (15)\\
\boldsymbol{h}_1^*(w,z) &= \nu\big((wr_1+z)I_{Km} - B_1^*\big)^{-1}\mathbf{1}, && (16)\\
\boldsymbol{h}_n^*(w,z) &= \big((wr_n+z)I_{(K-n+1)m} - B_n^* - C_n^* G_{n-1}^*(w,z)\big)^{-1}(\nu\mathbf{1} + C_n^*\boldsymbol{h}_{n-1}^*(w,z)),\\
&\qquad\qquad n=2,3,\ldots,K, && (17)
\end{aligned}
$$

where A_n is the $(K-n+1)m \times (K-n)m$ matrix given by (1), B_n^* is a $(K-n+1)m \times (K-n+1)m$ matrix given by

$$
B_n^* = \begin{bmatrix}
D_{n0}^* & & & & \\
\nu I_m & D_{n1}^* & & & \\
 & 2\nu I_m & D_{n2}^* & & \\
 & & \ddots & \ddots & \\
 & & & (K-n)\nu I_m & D_{n,K-n}^*
\end{bmatrix}
$$

with

$$
D_{nk}^* = \begin{cases}
Q - ((n-1)r_n\mu + (n+k)\nu)I_m, & \text{if } k = K-n,\\
Q - \Lambda - ((n-1)r_n\mu + (n+k)\nu)I_m, & \text{if } 1 \le k \le K-n-1,
\end{cases}
$$

and C_n^* *is a* $(K-n+1)m \times (K-n+2)m$ *matrix given by*

$$C_n^* = \begin{bmatrix} (n-1)\nu I_m & (n-1)r_n\mu I_m & & & & \\ & (n-1)\nu I_m & (n-1)r_n\mu I_m & & & \\ & & \ddots & \ddots & & \\ & & & (n-1)\nu I_m & (n-1)r_n\mu I_m & \\ & & & & (n-1)\nu I_m & (n-1)r_n\mu I_m \end{bmatrix}.$$

Proof. The proof of this is provided in Appendix A. □

We are finally able to obtain the Laplace-Stieltjes transform $\boldsymbol{\phi}(w,z)$ using the following theorem.

Theorem 3. *Given complex number* (w,z) *with* $\mathrm{Re}(w,z) \geq 0$, *the vector representation of the Laplace-Stieltjes transforms of the stationary distributions for charged amount* $\boldsymbol{\phi}_n(w,z)$ *can be recursively obtained:*

$$\boldsymbol{\phi}_K(w,z) = \boldsymbol{h}_K^*(w,z), \tag{18}$$

$$\boldsymbol{\phi}_n(w,z) = \boldsymbol{h}_n^*(w,z) + G_n^*(w,z)\boldsymbol{\phi}_{n+1}(w,z), \quad n = K-1, K-2, \ldots, 1, \tag{19}$$

$$\text{where} \quad \boldsymbol{\phi}_{nl}(w,z) = (\phi_{nl1}(w,z), \ldots, \phi_{nlm}(w,z))^\top, \quad n = 1,2,\ldots,K, \ l = 0,1,\ldots,L-n,$$

$$\text{and} \quad \boldsymbol{\phi}_n(w,z) = ((\boldsymbol{\phi}_{n0}(w,z))^\top, \ldots, (\boldsymbol{\phi}_{n,K-n}(w,z))^\top)^\top, \quad n = 1,2,\ldots,K.$$

Proof. The proof of this can be found in Appendix B. □

3.3 Calculating performance measures

In order to obtain the performance measures for the FIFO scheduling, we present the following theorem.

Theorem 4. *In the FIFO scheduling, the performance measures are obtained as follows:*

1. *The conditional probability* $p_i(y)$ *given* $\mathcal{Y} = y$, $\mathcal{S} \geq y$ *and* $\mathcal{J} = i$ *is obtained as*

$$p_i(y) = \frac{1}{\kappa_i(1-p_i^{\mathrm{loss}})}\Big(\sum_{\substack{n<P \\ 0\leq l<K-n}} \pi_{nli} + \sum_{\substack{P\leq n<K \\ 0\leq l<K-n}} \pi_{nli}\frac{P(\mu+\nu)}{P\mu+n\nu+\nu}\Big).$$

2. The expectation of $\mathcal{X}$ given $\mathcal{J} = i$ is given by

$$\mathbb{E}[\mathcal{X}|\mathcal{J} = i] = \frac{1}{\kappa_i(1-p_i^{\mathrm{loss}})}\frac{1}{\nu+\mu}\Big(\sum_{\substack{n<P\\0\le l<K-n}}\pi_{nli} + \sum_{\substack{P\le n<K\\0\le l<K-n}}\pi_{nli}\frac{P(\mu+\nu)}{P\mu+n\nu+\nu}\Big).$$

Therefore, we have

$$m_i = \frac{1}{\kappa_i(1-p_i^{\mathrm{loss}})}\Big(\sum_{\substack{n<P\\0\le l<K-n}}\pi_{nli} + \sum_{\substack{P\le n<K\\0\le l<K-n}}\pi_{nli}\frac{P(\mu+\nu)}{P\mu+n\nu+\nu}\Big).$$

Proof. Because $P(\mathcal{X} \ge y \mid \mathcal{Y} = y, \mathcal{J} = i) = P(\mathcal{X} \ge y \mid \mathcal{Y} = \infty, \mathcal{J} = i)$, the performance measure $p_i(y)$ can be written as

$$\begin{aligned}
p_i(y) &= \frac{\mathbb{P}(\mathcal{X} = y, \mathcal{S} \ge y \mid \mathcal{Y} = y, \mathcal{J} = i)}{\mathbb{P}(\mathcal{S} \ge y \mid \mathcal{Y} = y, \mathcal{J} = i)}\\
&= \frac{\mathbb{P}(\mathcal{X} \ge y \mid \mathcal{Y} = y, \mathcal{J} = i)}{\mathbb{P}(\mathcal{S} \ge y)}\\
&= e^{\nu y}\mathbb{P}(\mathcal{X} \ge y \mid \mathcal{Y} = \infty, \mathcal{J} = i)\\
&= e^{\nu y}\mathbb{P}(\mathcal{X} \ge y, \mathcal{W} < \mathcal{S} \mid \mathcal{Y} = \infty, \mathcal{J} = i)\\
&= e^{\nu y}\mathbb{P}(\mathcal{X} \ge y \mid \mathcal{W} < \mathcal{S}, \mathcal{Y} = \infty, \mathcal{J} = i)\mathbb{P}(\mathcal{W} < \mathcal{S} \mid \mathcal{Y} = \infty, \mathcal{J} = i)\,.
\end{aligned}$$

And, because $\mathcal{X}$ is exponentially distributed with mean ν^{-1} when $\mathcal{Y} = \infty$ and $\mathcal{W} < \mathcal{S}$, and $\{\mathcal{Y} = \infty\}$ and $\mathcal{W} < \mathcal{S}$ are independent, the conditional probability $p_i(y)$ becomes identical to the conditional waiting time distribution in Theorem 2;

$$\begin{aligned}
p_i(y) &= \mathbb{P}(\mathcal{W} < \mathcal{S} \mid \mathcal{Y} = \infty, \mathcal{J} = i)\\
&= \mathbb{P}(\mathcal{W} < \mathcal{S} \mid \mathcal{J} = i).
\end{aligned}$$

Moreover, given (9), we can calculate the following expectation:

$$\begin{aligned}
\mathbb{E}[\mathcal{X}|\mathcal{J} = i] &= \mathbb{E}[E_{\nu+\mu}|\mathcal{W} < \mathcal{S}, \mathcal{J} = i]\mathbb{P}(\mathcal{W} < \mathcal{S}|\mathcal{J} = i)\\
&= \frac{1}{\nu+\mu}\mathbb{P}(\mathcal{W} < \mathcal{S}|\mathcal{J} = i).
\end{aligned}$$

and

$$\mathbb{E}[\min(\mathcal{Y},\ \mathcal{S})|\mathcal{J} = i] = \frac{1}{\nu+\mu}.$$

Therefore, $m_i = p_i(y) = \mathbb{P}(\mathcal{W} < \mathcal{S}|\mathcal{J} = i)$ in the FIFO scheduling. The proof is completed. $\square$

In order to analyze the PS scheduling, we need to establish a conditional probability. This probability represents the tagged EVs request of energy amount $\mathcal{Y} = y$ in the $\mathcal{J} = i$ environment of an inbound stream that can charge up to energy amount $\mathcal{X} = x$ within the time parked $\mathcal{S} = s$. This conditional probability can be expressed as

$$\begin{aligned} F_i(x, s|y) &= \mathbb{P}(\mathcal{X} \le x, \mathcal{S} \le s | \mathcal{Y} = y, \mathcal{J} = i). \\ &= \begin{cases} F_i(x, s|\infty) & \text{if } x < y, \\ 1 - e^{-\nu s} & \text{if } x \ge y. \end{cases} \end{aligned} \tag{20}$$

Because $(\mathcal{X}, \mathcal{Y}, \mathcal{S}, \mathcal{J})$ and $(\chi(0,\sigma), \tilde{\mathcal{Y}}, \sigma, J(0))$ have the same distribution when applying the PS scheduling, $F_i(x, s|\infty)$ can be expressed as

$$\begin{aligned} F_i(x, s|\infty) &= \mathbb{P}(\mathcal{X} \le x, \mathcal{S} \le s | \mathcal{Y} = \infty, \mathcal{J} = i) \\ &= \mathbb{P}(\chi(0,\sigma) \le x, \sigma \le s | \tilde{\mathcal{Y}} = \infty, J(0) = i). \end{aligned}$$

Therefore, $F_i(x, s|\infty)$ can be obtained by employing the stationary distributions π_{nli} and $\Phi_{n+1,l,i}(x)$:

$$F_i(x, s|\infty) = \frac{1}{\kappa_i(1 - p_i^{\text{loss}})} \sum_{n=0}^{K-1} \sum_{l=0}^{K-n-1} \pi_{nli} \, \Phi_{n+1,l,i}(x). \tag{21}$$

It should be noted that the Laplace-Stieltjes transform of conditional distribution $F_i(x, s|\infty)$ substantially helps us determine the explicit functional expressions of the proposed performance measures. We define the Laplace-Stieltjes transform of conditional distribution $F_i(x, s|\infty)$ to be

$$\begin{aligned} f_i(w, z|\infty) &\equiv \int_0^\infty \int_0^\infty e^{-wx - zs} F_i(dx, ds|\infty), \\ &= \mathbb{E}[e^{-w\chi(0,\sigma) - z\sigma} | J(0) = i, \tilde{\mathcal{Y}} = \infty], \quad i = 1, \dots, m. \end{aligned}$$

Finally, we have the following proposition for the Laplace-Stieltjes transformation of conditional distribution $F_i(x, s|\infty)$ based on the results of equations (14)-(19) by utilizing concept of the Laplace-Stieltjes transformation method for waiting time distributions in queueing models [5].

Proposition 2. *The Laplace-Stieltjes transform of $F_i(x, s|\infty)$ is given by*

$$f_i(w, z|\infty) = \frac{1}{\kappa_i(1 - p_i^{\text{loss}})} \sum_{n=0}^{K-1} \sum_{l=0}^{K-n-1} \pi_{nli} \phi_{n+1,l,i}(w, z) \tag{22}$$

for complex numbers w and z with $\mathrm{Re}(w), \mathrm{Re}(z) \geq 0$, where $\phi_{nli}(w,z)$ is obtained using (18) and (19) with $G_n^*(w,z)$ and $\boldsymbol{h}_n^*(w,z)$, which can be determined using (14)-(17).

Conditional distribution $F_i(x,s|\infty)$ can be calculated using the numerical inversion of Laplace-Stieltjes transform $f_i(w,z|\infty)$. Many numerical inversion algorithms for Laplace transforms have been developed in order to obtain probability distributions in stochastic models. The survey presented by [1], indicates that the Gaver-Stehfest, Euler, and Talbot algorithms are some of the popular one dimensional inversion techniques. Furthermore, given the Fourier-series method based on the Euler summation, we expect the Euler algorithm to adequately calculate the conditional distribution $F_i(x,s|\infty)$ by exploiting its Laplace-Stieltjes transform $f_i(w,z|\infty)$, which was presented in Proposition 2. The inversion formula of the Euler algorithm is

$$F_i(x,s|\infty) \approx \frac{1}{xs}\sum_{k=0}^{n_k}\sum_{j=0}^{n_j}\mathrm{Re}\Big\{\omega_k f_i(\frac{\alpha_k}{x},\frac{\beta_j}{s}|\infty)\Big\}, \quad 0 < x, s < \infty, \tag{23}$$

where weights ω_k and nodes α_k and β_j are complex numbers, which depend on n, but do not depend on the transform f_i, x or s.[5] The weights ω_k and nodes α_k and β_k are needed to specify according to the desirable error bound. More details can be found in [19] and [20].

Finally, applying the previously proposed theorems, lemma, and propositions allows us to calculate the measures of the PS scheduling according to the following theorem.

Theorem 5. *In the PS scheduling, the performance measures are obtained as follows:*

1. *The conditional probability $p_i(y)$ when $\mathcal{Y} = y$, $\mathcal{S} \geq y$ and $\mathcal{J} = i$, is given by*

$$p_i(y) \quad = \quad e^{\nu y}\big(1 - F_i(y,\infty|\infty)\big).$$

[5]Because we only need $F_i(x,\infty|\infty)$ in order to obtain the performance measures in Theorem 5, one-dimensional Euler-Laplace inversion, which is described in the following formula, is necessary:

$$F_i(x,\infty|\infty) \approx \frac{1}{x}\sum_{k=0}^{n_k}\mathrm{Re}\Big\{\tilde{\omega}_k f_i(\frac{\tilde{\alpha}_k}{x},0|\infty)\Big\}, \quad 0 < x < \infty, \tag{24}$$

in which weights $\tilde{\omega}_k$ and nodes $\tilde{\alpha}_k$ are complex numbers that depend on n but not on the Laplace transform f_i or variable x.

2. The expectation of $\mathcal{X}$ given $\mathcal{J} = i$ is given by

$$\mathbb{E}[\mathcal{X}|\mathcal{J}=i] = \frac{1-f_i(\mu,0|\infty)}{\mu}.$$

Therefore, we have the rational expectation

$$m_i = \frac{\mu+\nu}{\mu}(1-f_i(\mu,0|\infty)).$$

Proof. Performance measure $p_i(y)$ can be rewritten in terms of a conditional distribution $F_i(y,\infty|\infty)$;

$$\begin{aligned} p_i(y) &= \frac{\mathbb{P}(\mathcal{X}=y,\mathcal{S}\geq y \mid \mathcal{Y}=y,\mathcal{J}=i)}{\mathbb{P}(\mathcal{S}\geq y \mid \mathcal{Y}=y,\mathcal{J}=i)} \\ &= \frac{\mathbb{P}(\mathcal{X}\geq y \mid \mathcal{Y}=y,\mathcal{J}=i)}{\mathbb{P}(\mathcal{S}\geq y)} \\ &= e^{\nu y}\big(1-F_i(y,\infty|\infty)\big). \end{aligned}$$

For m_i, we have

$$\begin{aligned} \mathbb{E}[\mathcal{X}|\mathcal{J}=i] &= \mathbb{E}[\mathcal{X}\mathbb{1}_{\{\mathcal{X}=\mathcal{Y}\}}|\mathcal{J}=i] + \mathbb{E}[\mathcal{X}\mathbb{1}_{\{\mathcal{X}<\mathcal{Y}\}}|J(0)=i] \\ &= \int_0^\infty \mu e^{-\mu y}\mathbb{E}[y\mathbb{1}_{\{\mathcal{X}=y\}}|\mathcal{J}=i,\mathcal{Y}=y]dy \\ &\quad + \int_0^\infty \mu e^{-\mu y}\mathbb{E}[\mathcal{X}\mathbb{1}_{\{\mathcal{X}<y\}}|\mathcal{J}=i,\mathcal{Y}=y]dy. \end{aligned} \tag{25}$$

In addition, the first term in the right-hand side of (25) can be written as

$$\begin{aligned} &\int_0^\infty \mu e^{-\mu y}\mathbb{E}[y\mathbb{1}_{\{\mathcal{X}=y\}}|\mathcal{J}=i,\mathcal{Y}=y]dy \\ &= \int_0^\infty \mu y e^{-\mu y}\mathbb{E}[\mathbb{1}_{\{\mathcal{X}\geq y\}}|J(0)=i,\mathcal{Y}=\infty]dy \\ &= \mathbb{E}[\int_0^{\mathcal{X}} \mu y e^{-\mu y}dy|J(0)=i,\mathcal{Y}=\infty] \\ &= \mathbb{E}[-\mathcal{X}e^{-\mu\mathcal{X}} + \frac{1-e^{-\mu\mathcal{X}}}{\mu}|\mathcal{J}=i,\mathcal{Y}=\infty] \\ &= -\int_0^\infty e^{-\mu x}xF_i(dx,\infty|\infty) + \frac{1-f_i(\mu,0|\infty)}{\mu} \end{aligned} \tag{26}$$

Moreover, the second term in the right-hand side of (25) can be written as

$$\begin{aligned}
&\int_0^\infty \mu e^{-\mu y}\mathbb{E}[\mathcal{X}\mathbb{1}_{\{\mathcal{X}<y\}}|\mathcal{J}=i,\mathcal{Y}=y]dy \\
&= \int_0^\infty \mu e^{-\mu y}\mathbb{E}[\mathcal{X}\mathbb{1}_{\{\mathcal{X}<y\}}|\mathcal{J}=i,\mathcal{Y}=\infty]dy \\
&= \int_0^\infty \mu e^{-\mu y}\int_0^y x dF_i(dx,\infty|\infty)dy \\
&= \int_0^\infty \int_x^\infty \mu e^{-\mu y} dy x dF_i(dx,\infty|\infty) \\
&= \int_0^\infty e^{-\mu x} x dF_i(dx,\infty|\infty). \qquad (27)
\end{aligned}$$

Substituting (26) and (27) into (25) provides

$$\mathbb{E}[\mathcal{X}|J(0)=i] = \frac{1-f_i(\mu,0|\infty)}{\mu}.$$

Therefore

$$m_i = \frac{\mu+\nu}{\mu}(1-f_i(\mu,0|\infty)).$$

This completes the proof. □

4 Numerical Examples

In this section, we perform numerical examples to demonstrate the practical interpretations via comparing the two proposed charge scheduling methods from the various aspects of the flexible stochastic modeling. The useful view points will be discussed in terms of each incoming EV, the operations of a charging station, and two performance measures.

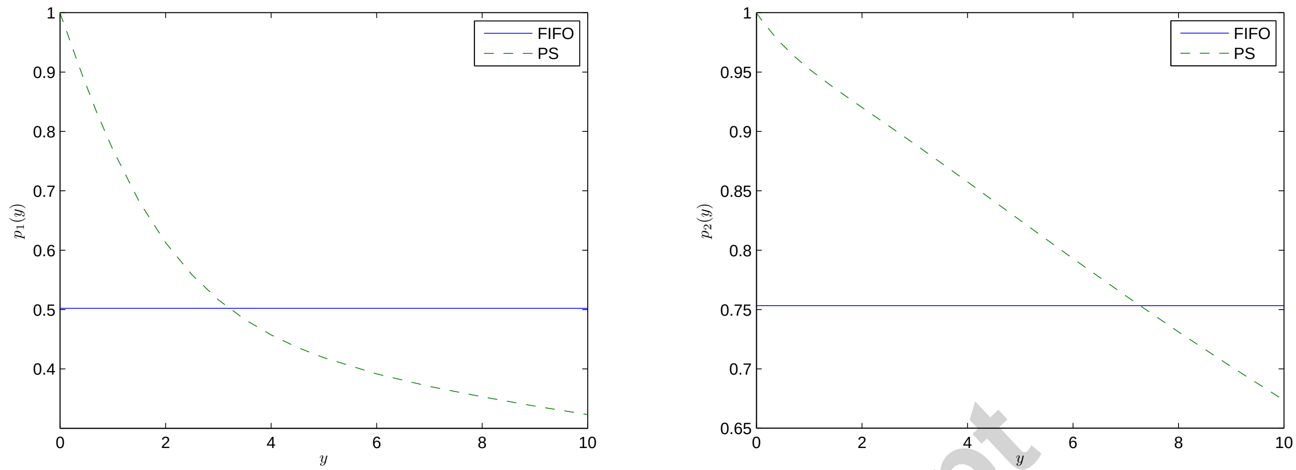
We consider a simple case with the parameters

$$Q = \begin{bmatrix} -0.25 & 0.25 \\ 0.1 & -0.1 \end{bmatrix}, \ \Lambda = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}, \ \mu = \frac{1}{4}, \ \nu = \frac{1}{6}.$$

Note from $\lambda_1 = 5$ and $\lambda_2 = 1$ that the five times as many as EVs on average arrive the charging station when the traffic circumstance $J(0) = 1$, compared with the case in which $J(0) = 2$. Once the loss probability p_i^{loss}, the stationary distribution π_{nli} and the conditional distribution $F_i(y, \infty|\infty)$ are calculated, the conditional probability measure $p_i(y) = \mathbb{P}(\mathcal{X} = y|\mathcal{Y} = y, \mathcal{S} \geq y, \mathcal{J} = i)$ can be numerically obtained. Note again the rational expectation measure $m_i = p_i(y) = \mathbb{P}(\mathcal{W} < \mathcal{S}|\mathcal{J} = i)$ in the FIFO scheduling.

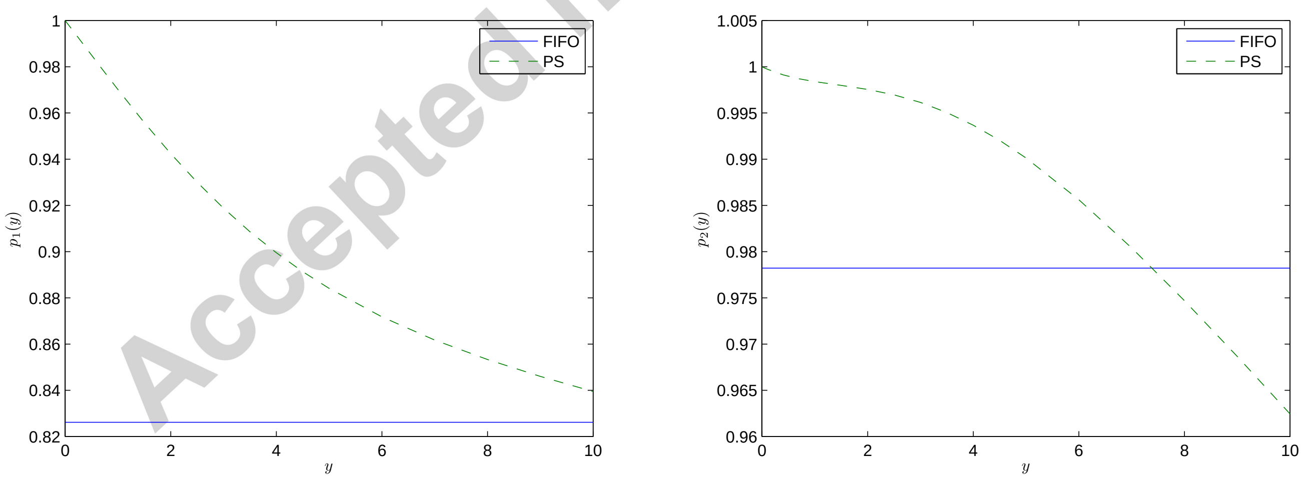
Figure 1 plots the performance measure $p_i(y)$ of the FIFO and the PS charge scheduling methods with respect to the requested charging electricity amount $\mathcal{Y} = y$ when the number of chargers and the maximum demand level are 30 and 5, respectively, i.e., $K = 30$ and $P = 5$. It is seen from the left panel of Figure 1 that in the case of $J(0) = 1$, while the conditional probability measure $p_i(y)$ for the FIFO rule is constant because it is independent of the requested charging electricity amount $\mathcal{Y} = y$, the conditional probability measure $p_i(y)$ for the PS rule is a decreasing function. The PS charge scheduling is superior to the FIFO charge scheduling up to a certain point of the requested charging electricity amount, but after the certain point, the FIFO charge scheduling outperforms to the PS charge scheduling. The right panel of Figure 1 shows that the switching point that the FIFO charge scheduling starts to be superior to the PS charge scheduling becomes greater in the traffic circumstance $J(0) = 2$ that relatively fewer EVs arrive. In addition, Figure 2 shows the performance comparison between the FIFO and the PS scheduling methods when the maximum demand level P increases to be $P = 10$. The switching points when $P = 10$ become greater than those when $P = 5$ in both stochastic circumstances of $J(t) = 1, 2$. It should be noted that interestingly, unlike when $P = 5$, the switching point in $J(0) = 1$ is bigger than the switching point in $J(0) = 2$. Therefore, it is observed that the uniform superiority between the FIFO and the PS charge scheduling methods doesn't exist. The superiority depends on many circumstances such as the capacity of a charging station, the requested charging electricity amounts and the stochastic incoming environment.

Now, consider the operational aspects of a charging station with respect to the change of the maximum demand level P. Figure 3 shows that there also exist switching points for the superiority of the FIFO and the PS charge scheduling methods. The FIFO charge scheduling is superior to the PS charge scheduling only when the maximum demand level P is lower than



(a) $J(0) = 1$ (b) $J(0) = 2$

Figure 1: The conditional probability measure $p_i(y)$ with respect to $\mathcal{Y}$ when $P = 5$ and $K = 30$.



(a) $J(0) = 1$ (b) $J(0) = 2$

Figure 2: The conditional probability measure $p_i(y)$ with respect to $\mathcal{Y}$ when $P = 10$ and $K = 30$.

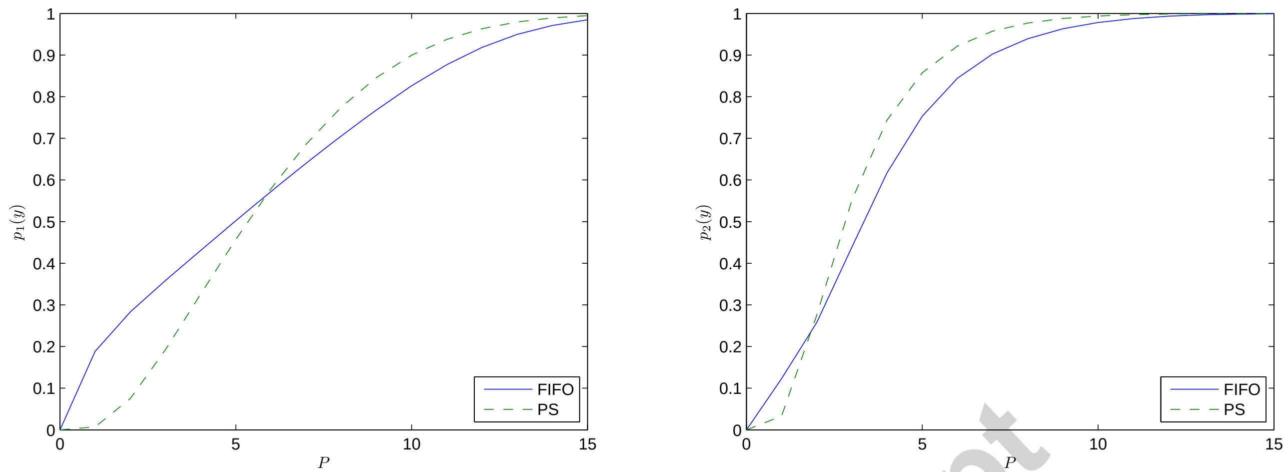
a certain switching point, and after the maximum demand level P gets higher than the certain switching point, the PS charge scheduling outperforms to the PS charge scheduling. It is also interesting that the switching point of the maximum demand level P becomes lower when the number of EVs in charge is stochastically smaller. It is seen once again that the uniform superiority between the FIFO and the PS charge scheduling methods doesn't exist. Therefore, we suggest that practitioners need find optimal charging strategies depending on their own circumstances and goal functions. It should also be mentioned that, when a charging station makes a contract with an electricity distribution company, the optimal maximal demand level can be determined on the basis of the proposed stochastic model and its performance analysis.

Lastly, Figure 4 shows that the rational expectation measure m_i of the FIFO and the PS charge scheduling methods is very close, from which one should note that, although the rational expectation measure m_i should be useful to quantify the performance of a charging station, it is limited to characterize the performance differences between the FIFO and the PS charge scheduling methods.

5 Concluding remarks

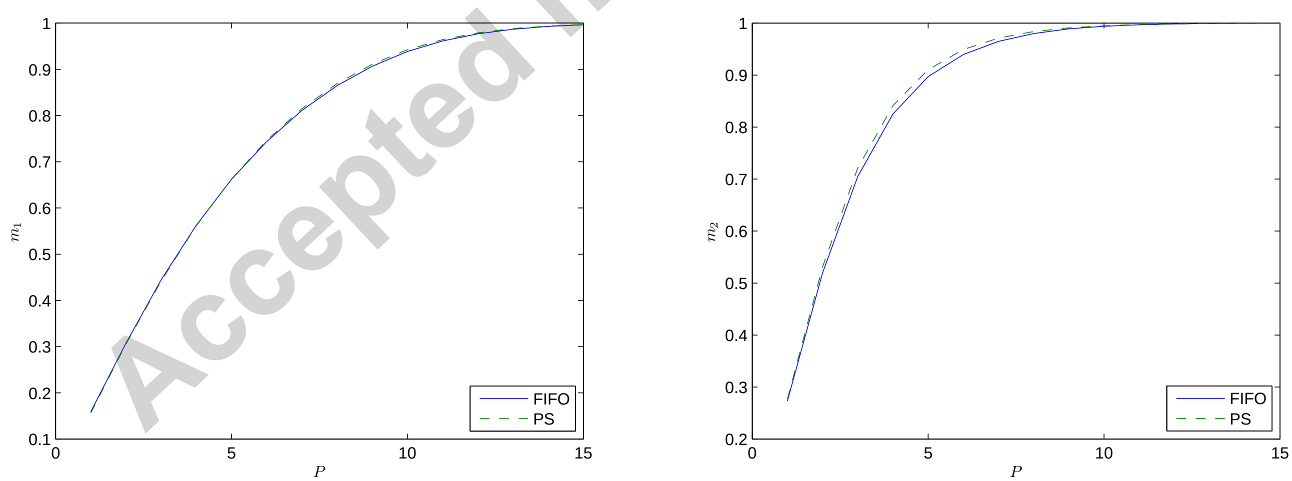
We proposed a realistic stochastic model for EV battery charging stations and analyzed performance measures of two typical charge scheduling methods: the first-in-first-served (FIFO) and the processor sharing (PS). After obtaining stationary distribution for steady states and some conditional distributions, we then derived two performance measures for the charge scheduling methods: the probability of fully charging within the system and the rational expectation of the charged energy given the parking time and requested energy. Furthermore, we employed a flexible Poisson process of a Markov-modulated Poisson process for an incoming stream of EVs under the proposed stochastic model, which incorporates the time-varying behavior of EV arrivals into the parking lot.

Strategic approaches to the efficient operation of charging stations can be inferred using the



(a) $J(0) = 1$ (b) $J(0) = 2$

Figure 3: The conditional probability measure $p_i(4)$ with respect to P when $K = 30$.



(a) $J(0) = 1$ (b) $J(0) = 2$

Figure 4: The rational expectation measure m_i with respect to P when $K = 30$.

analytic results of our proposed performance measures. The results of this research may be of interest when developing the strategic operations of charging stations in the likely event that numerous EV drivers will frequent apartment buildings, department stores, or office buildings within a certain time period and immediately plug in their EVs in order to charge during their visit. Parameters of the random factors and constraints of the proposed model can be estimated or modified according to the real charging station environment. Modifications based on two typical charge scheduling methods can also be considered, and their performance analyses can be conducted accordingly.

Many other issues can also be addressed in future research. For example, strategic operational decisions, such as the optimal number of fast chargers to use when both fast and slow charging equipment are available to install or the impact of energy storage systems. Designing scheduling methods that include the cost of the charging system and the satisfaction of EV drivers could also be valuable. Because the unit cost of the electricity consumed in the charging station fluctuates according to the change in electricity price at the charging time, especially during peak demand, a cost-benefit analysis for EV charging stations connected to smart grid system could also be of interest. This situation, which we call "demand response", will likely be realized in many countries in the near future.

Acknowledgements

We thank our anonymous referees for their constructive comments and suggestions concerning our paper, which have significantly improved its overall quality. We also thank Prof. Bara Kim for kindly sharing his insights during the mathematical development of this paper. The research of Hyung-Tae Ha was supported by Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology (NRF-2013R1A1A2059794).

APPENDICES

A Proof of Proposition 1

For the process $\{(N(t), L(t), J(t) : t \geq 0\}$, let t_0 be the first transition epoch out of the initial state , i.e., let

$$t_0 = \inf\{t > 0 : (N(t), L(t), J(t)) \neq (N(0), L(0), J(0))\}.$$

Let U be defined as

$$U \equiv \begin{cases} 1 & \text{if the tagged EV departs the parking lot at } t_0, \\ 2 & \text{if an EV in charge, excluding the tagged one,} \\ & \text{departs the parking lot at } t_0, \\ 3 & \text{if an EV in charge, excluding the tagged one,} \\ & \text{completes charging its requested amount at } t_0, \\ 4 & \text{if an EV in park departs the parking lot at } t_0, \\ 5 & \text{if } J(t) \text{ transits at } t_0, \\ 6 & \text{if an EV arrives at the parking lot at } t_0, \end{cases}$$

and

$$\mathbb{E}_{nli}[\cdot] = \mathbb{E}[\cdot | N(0) = n, L(0) = l, J(0) = i, \tilde{\mathcal{Y}} = \infty].$$

For $1 \leq n \leq K-1$, $0 \leq l \leq K-n$, $0 \leq k \leq K-n+1$, $1 \leq i,j \leq m$,

$$\begin{aligned}
(G_n^*(w,z))_{(l,i),(k,j)} &= \sum_{u=1}^{6} \mathbb{P}(U=u \mid N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty) \\
&\quad \times \mathbb{E}_{nli}[e^{-w\chi(0,\tau_{n+1})-z\tau_{n+1}} \mathbb{1}_{\{\tau_{n+1}<\sigma, L(\tau_{n+1})=k, J(\tau_{n+1})=j\}} | U=u] \\
&= \sum_{u=1}^{6} \mathbb{P}(U=u \mid N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty) \\
&\quad \times \mathbb{E}_{nli}[e^{-w\chi(0,t_0)-z\tau_0} \mathbb{1}_{\{\tau_{n+1}<\sigma, L(\tau_{n+1})=k, J(\tau_{n+1})=j\}} | U=u] \\
&\quad \times \mathbb{E}_{nli}[e^{-w\chi(t_0,\tau_{n+1})-z(\tau_{n+1}-t_0)} \mathbb{1}_{\{\tau_{n+1}<\sigma, L(\tau_{n+1})=k, J(\tau_{n+1})=j\}} | U=u],
\end{aligned} \tag{28}$$

where the last equality follows from the fact that $\big(\chi(0,t_0), t_0\big)$ and $\big(\chi(t_0,\tau_{n+1}), \tau_{n+1}-t_0\big)$ are independent, given $\{N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty, U=u\}$. Given $\{N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty\}$, t_0 has the exponential distribution with mean $\big((n-1)r_n\mu + (n+l)\nu + (1-\delta_{n+l,K})\lambda_i - q_{ii}\big)^{-1}$ and is independent of U, where δ_{ij} is the Kronecker delta. Hence,

$$\begin{aligned}
&\mathbb{E}_{nli}[e^{-w\chi(0,t_0)-zt_0} \mathbb{1}_{\{\tau_{n+1}<\sigma, L(\tau_{n+1})=k, J(\tau_{n+1})=j\}} \mid U=u] \\
&= \mathbb{E}_{nli}[e^{-(wr_n+z)t_0} \mid U=u] \\
&= \frac{a_{nli}}{r_n w + z + a_{nli}},
\end{aligned} \tag{29}$$

where $a_{nli} = (n-1)r_n\mu + (n+l)\nu + (1-\delta_{n+l,K})\lambda_i - q_{ii}$. Furthermore, given $\{N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty\}$, U has the distribution:

$$\mathbb{P}(U=1 \mid N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty) = \frac{\nu}{a_{nli}}; \tag{30}$$

$$\mathbb{P}(U=2 \mid N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty) = \frac{(n-1)\nu}{a_{nli}}; \tag{31}$$

$$\mathbb{P}(U=3 \mid N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty) = \frac{(n-1)r_n\mu}{a_{nli}}; \tag{32}$$

$$\mathbb{P}(U=4 \mid N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty) = \frac{l\nu}{a_{nli}}; \tag{33}$$

$$\mathbb{P}(U=5 \mid N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty) = \frac{-q_{ii}}{a_{nli}}; \tag{34}$$

$$\mathbb{P}(U=6 \mid N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty) = \frac{(1-\delta_{n+l,K})\lambda_i}{a_{nli}}. \tag{35}$$

Now we deal with the conditional expectation

$$\mathbb{E}_{nli}[e^{-w\chi(t_0,\tau_{n+1})-z(\tau_{n+1}-t_0)}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}|U=u]$$

separately for $u=1,2,\ldots,6$:

- If $N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty$ and $U=1$, then $t_0=\sigma$. Hence

$$\mathbb{E}_{nli}[e^{-w\chi(t_0,\tau_{n+1})-z(\tau_{n+1}-t_0)}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}|U=1] \quad = \quad 0. \qquad (36)$$

- If $N(0)=n, n\geq 2,\ L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty$ and $U=2$, then $N(t_0)=n-1,\ L(t_0)=l$ and $J(t_0)=i$. Hence by the Markov property

$$\begin{aligned}&\mathbb{E}_{nli}[e^{-w\chi(t_0,\tau_{n+1})-z(\tau_{n+1}-t_0)}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}|U=2]\\ =\quad &\mathbb{E}_{n-1,l,i}[e^{-w\chi(0,\tau_{n+1})-z\tau_{n+1}}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}].\end{aligned}$$

  Given $N(0)=n-1$, $\tau_n<\tau_{n+1}$ and $(\chi(0,\tau_n),\tau_n)$ is independence of $(\chi(\tau_n,\tau_{n+1}),\tau_{n+1}-\tau_n)$. Therefore the above equation can be written as

$$\begin{aligned}&\mathbb{E}_{nli}[e^{-w\chi(t_0,\tau_{n+1})-z(\tau_{n+1}-t_0)}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}|U=2]\\
=\ &\sum_{l'=0}^{K-n}\sum_{i'=1}^{m}\mathbb{E}_{n-1,l,i}[e^{-w\chi(0,\tau_n)-z\tau_n}\mathbb{1}_{\{\tau_n<\sigma,L(\tau_n)=l',J(\tau_n)=i'\}}]\\
&\times\mathbb{E}[e^{-w\chi(\tau_n,\tau_{n+1})-z(\tau_{n+1}-\tau_n)}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}|N(\tau_n)=n,L(\tau_n)=l',J(\tau_n)=i',\tilde{\mathcal{Y}}=\infty]\\
=\ &\sum_{l'=0}^{K-n}\sum_{i'=1}^{m}\mathbb{E}_{n-1,l,i}[e^{-w\chi(0,\tau_n)-z\tau_n}\mathbb{1}_{\{\tau_n<\sigma,L(\tau_n)=l',J(\tau_n)=i'\}}]\\
&\times\mathbb{E}_{nl'i'}[e^{-w\chi(0,\tau_{n+1})-z\tau_{n+1}}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}]\\
=\ &\sum_{l'=0}^{K-n}\sum_{i'=1}^{m}(G^*_{n-1}(w,z))_{(l,i),(l',i')}(G^*_n(w,z))_{(l',i'),(k,j)}=\big(G^*_{n-1}(w,z)G^*_n(w,z)\big)_{(l,i),(k,j)}. \qquad (37)\end{aligned}$$

- If $N(0)=n\geq 2, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty$ and $U=3$, then $N(t_0)=n-1,\ L(t_0)=l+1$

and $J(t_0) = i$. Hence by the Markov property,

$$\begin{aligned}
&\mathbb{E}_{nli}[e^{-w\chi(t_0,\tau_{n+1})-z(\tau_{n+1}-t_0)}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}|U=3] \\
&= \mathbb{E}[e^{-w\chi(t_0,\tau_{n+1})-z(\tau_{n+1}-t_0)}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}} \\
&\qquad |N(t_0)=n-1, L(t_0)=l+1, J(t_0)=i, \tilde{\mathcal{Y}}=\infty, U=3] \\
&= \mathbb{E}[e^{-w\chi(0,\tau_{n+1})-z\tau_{n+1}}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}|N(0)=n-1, L(0)=l+1, J(0)=i, \tilde{\mathcal{Y}}=\infty] \\
&= \mathbb{E}_{n-1,l+1,i}[e^{-w\chi(0,\tau_{n+1})-z\tau_{n+1}}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}].
\end{aligned}$$

By using the same way as (37), we obtain

$$\begin{aligned}
&\mathbb{E}_{nli}[e^{-w\chi(t_0,\tau_{n+1})-z(\tau_{n+1}-t_0)}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}|U=3] \\
&= \left(G^*_{n-1}(w,z)G^*_n(w,z)\right)_{(l+1,i),(k,j)}. \qquad (38)
\end{aligned}$$

- If $N(0) = n, L(0) = l, l \geq 1, J(0) = i, \tilde{\mathcal{Y}} = \infty$ and $U = 4$, then $N(t_0) = n$, $L(t_0) = l - 1$ and $J(t_0) = i$. Hence

$$\begin{aligned}
&\mathbb{E}_{nli}[e^{-w\chi(t_0,\tau_{n+1})-z(\tau_{n+1}-t_0)}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}|U=4] \\
&= \mathbb{E}[e^{-w\chi(t_0,\tau_{n+1})-z(\tau_{n+1}-t_0)}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}|N(t_0)=n, L(t_0)=l-1, J(t_0)=i, \tilde{\mathcal{Y}}=\infty] \\
&= \mathbb{E}_{n,l-1,i}[e^{-w\chi(0,\tau_{n+1})-z\tau_{n+1}}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}] \\
&= (G^*_n(w,z))_{(l-1,i),(k,j)}. \qquad (39)
\end{aligned}$$

- If $N(0) = n, L(0) = l, J(0) = i, \tilde{\mathcal{Y}} = \infty$ and $U = 5$, then $N(t_0) = n$, $L(t_0) = l$ and $J(t_0) \neq i$. Hence

$$\begin{aligned}
&\mathbb{E}_{nli}[e^{-w\chi(t_0,\tau_{n+1})-z(\tau_{n+1}-t_0)}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}|U=5] \\
&= \sum_{i'\neq i}\mathbb{P}(J(t_0)=i'|N(0)=n, L(0)=l, J(0)=i', \tilde{\mathcal{Y}}=\infty, U=5) \\
&\quad \times\mathbb{E}[e^{-w\chi(t_0,\tau_{n+1})-z(\tau_{n+1}-t_0)}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}|N(t_0)=n, L(t_0)=l, J(t_0)=i', \tilde{\mathcal{Y}}=\infty] \\
&= \frac{1}{-q_{ii}}\sum_{i'\neq i} q_{ii'}\mathbb{E}_{nli'}[e^{-w\chi(0,\tau_{n+1})-z\tau_{n+1}}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}] \\
&= \frac{1}{-q_{ii}}\sum_{i'\neq i} q_{ii'}(G^*_n(w,z))_{(l,i'),(k,j)}. \qquad (40)
\end{aligned}$$

- If $N(0) = n, L(0) = l, J(0) = i, \tilde{\mathcal{Y}} = \infty$ and $U = 6$, then $t_0 = \tau_{n+1} < \sigma$. Hence

$$\mathbb{E}_{nli}[e^{-w\chi(t_0,\tau_{n+1})-z(\tau_{n+1}-t_0)}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}|U=6] \;=\; 1. \tag{41}$$

Substituting (29)-(41) into (28) yields, for $n = 1, 2, \ldots, K-1$,

$$\begin{aligned}(G_n^*(w,z))_{(l,i),(k,j)} \;=\; & \frac{1}{r_n w + z + a_{nli}}\Big((n-1)\nu(G_{n-1}^*(w,z)G_n^*(w,z))_{(l,i),(k,j)} \\ & +(n-1)r_n\mu(G_{n-1}^*(w,z)G_n^*(w,z))_{(l+1,i),(k,j)} + l\nu(G_n^*(w,z))_{(l-1,i),(k,j)} \\ & +\sum_{i'\neq i} q_{ii'}(G_n^*(w,z))_{(l,i'),(k,j)} + (1-\delta_{n+l,K})\lambda_i\Big)\end{aligned} \tag{42}$$

with convention $G_0^* = O$. This is written in a matrix form

$$(r_n w + z)G_n^*(w,z) - B_n^* G_n^*(w,z) \;=\; C_n^* G_{n-1}^*(w,z)G_n^*(w,z) + A_n, \quad n = 1, 2, \ldots, K-1.$$

This proves (14) and (15).

In a similar way to the derivation of (42), we obtain, for $n = 1, 2, \ldots, K$,

$$\begin{aligned}(\boldsymbol{h}_n^*(w,z))_{(l,i)} \;=\; & \frac{1}{r_n w + z + a_{nli}}\Big(\nu + (n-1)\nu\big\{(G_{n-1}^*(w,z)\boldsymbol{h}_n^*(w,z))_{(l,i)} + (\boldsymbol{h}_{n-1}^*(w,z))_{(l,i)}\big\} \\ & +(n-1)r_n\mu\big\{(G_{n-1}^*(w,z)\boldsymbol{h}_n^*(w,z))_{(l+1,i)} + (\boldsymbol{h}_{n-1}^*(w,z))_{(l+1,i)}\big\} \\ & +l\nu(\boldsymbol{h}_n^*(w,z))_{(l-1,i)} + \sum_{i'\neq i} q_{ii'}(\boldsymbol{h}_n^*(w,z))_{(l,i')}\Big)\end{aligned}$$

with convention $\boldsymbol{h}_0^*(w,z) = \mathbf{0}$. This is written in a matrix form

$$(r_n w + z)\boldsymbol{h}_n^*(w,z) - B_n^*\boldsymbol{h}_n^*(w,z) \;=\; \nu\mathbf{1} + C_n^*\big(G_{n-1}^*(w,z)\boldsymbol{h}_n^*(w,z) + \boldsymbol{h}_{n-1}^*(w,z)\big), \quad n = 1, 2, \ldots, K,$$

which proves (16) and (17). □

B Proof of Theorem 3

Equation (18) immediately follows from its definition. And in order to derive (19), we can write, for $n = K-1, K-2, \ldots, 1$,

$$\begin{aligned}\phi_{nli}(w,z) \;=\; & \mathbb{E}[e^{-w\chi(0,\sigma)-z\sigma}\mathbb{1}_{\{\sigma\le\tau_{n+1}\}} \mid N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty] \\ & + \sum_{k=0}^{K-n-1}\sum_{j=1}^{m}\mathbb{E}[e^{-w\chi(0,\tau_{n+1})-z\tau_{n+1}}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}e^{-w\chi(\tau_{n+1},\sigma)-z(\sigma-\tau_{n+1})} \\ & \qquad \mid N(0)=n, L(0)=l, J(0)=i, \tilde{\mathcal{Y}}=\infty].\end{aligned} \tag{43}$$

The first expectation on the right-hand side of (43) is $(\boldsymbol{h}_n^*(w,z))_{(l,i)}$. The second expectation on the right-hand side of (43) becomes

$$\begin{aligned}
&\mathbb{E}[e^{-w\chi(0,\tau_{n+1})-z\tau_{n+1}}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}}e^{-w\chi(\tau_{n+1},\sigma)-z(\sigma-\tau_{n+1})}\\
&\qquad\qquad \mid N(0)=n,L(0)=l,J(0)=i,\tilde{\mathcal{Y}}=\infty]\\
&= \mathbb{E}[e^{-w\chi(0,\tau_{n+1})-z\tau_{n+1}}\mathbb{1}_{\{\tau_{n+1}<\sigma,L(\tau_{n+1})=k,J(\tau_{n+1})=j\}} \mid N(0)=n,L(0)=l,J(0)=i,\tilde{\mathcal{Y}}=\infty]\\
&\quad \times\mathbb{E}[e^{-w\chi(0,\sigma)-z\sigma} \mid N(0)=n+1,L(0)=k,J(0)=j,\tilde{\mathcal{Y}}=\infty]\\
&= (G_n^*(w,z))_{(l,i),(k,j)}\phi_{n+1,k,j}(w,z).
\end{aligned}$$

Therefore (43) can be written as $\phi_{nli}(w,z) = (\boldsymbol{h}_n^*(w,z))_{(l,i)} + \sum_{l=0}^{K-n-1}\sum_{j=1}^{m}(G_n^*(w,z))_{(l,i),(k,j)}\phi_{n+1,k,j}(w,z)$, which is the component-wise expression of (19). □

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