Production and pricing Problems in make-to-order supply chain with cap-and-trade regulation

Xiaoping Xu, Wei Zhang, Ping He, Xiaoyan Xu

PII: S0305-0483(15)00161-9
DOI: http://dx.doi.org/10.1016/j.omega.2015.08.006
Reference: OME1585

To appear in: *Omega*

Received date: 21 April 2014
Revised date: 6 August 2015
Accepted date: 6 August 2015

Cite this article as: Xiaoping Xu, Wei Zhang, Ping He, Xiaoyan Xu, Production and pricing Problems in make-to-order supply chain with cap-and-trade regulation, *Omega,* http://dx.doi.org/10.1016/j.omega.2015.08.006

Production and Pricing Problems in Make-To-Order Supply Chain with Cap-and-Trade Regulation

Xiaoping Xu[a], Wei Zhang[b], Ping He[b*], Xiaoyan Xu[b]

[a] School of Management, University of Science and Technology of China,

96 Jinzhai Road, Hefei, Anhui 230026, PR China

[b] School of Management, Zhejiang University,

866 Yuhangtang Road, Hangzhou, Zhejiang 310058, PR China

Abstract

This paper studies the production and pricing problems in MTO (make-to-order) supply chain containing an upstream manufacturer who produces two products based on MTO production and a downstream retailer. The manufacturer is regulated by cap-and-trade regulation and determines the wholesale prices of the two products. To comply with the regulation, the manufacturer can buy or sell emission permits through an outside market. The retailer determines its order quantities to meet the price-sensitive demands. We derive the optimal total emissions and production quantities of the two products, and based on them, we analyze the impact of emission trading price on the optimal production decisions and the two firms' optimal profits. The emission trading decisions follow a two-threshold policy and the optimal total emissions are increasing in the cap. However, contrary to intuition, the optimal production quantities of the two products may be decreasing in the cap. The manufacturer's optimal profit is decreasing (increasing) in the buying (selling) price of emission permits, and that the retailer's optimal profit is decreasing in the buying (selling) price of emission permits. The optimal total emissions are decreasing in buying or selling price of emission permits, however, the optimal production quantities of the two products may be increasing (decreasing) in the buying (selling) price of emission permits. Numerical examples are conducted to illustrate our findings and some managerial insights are presented.

Keywords: cap-and-trade; supply chain; production; pricing

*Corresponding author. Tel.: +86 571-88206827
E-mail: phe@zju.edu.cn

1. Introduction

It is a global consensus that carbon emissions are the main reasons contributing to global warming. To curb the carbon emissions, some legislations and mechanisms are proposed in many regions and countries. Emissions trading scheme (cap-and-trade regulation) is now generally accepted as one of the most effective market-based mechanisms (Hua et al., 2011). According to the report from European Commission (2013), the earliest and largest EU emissions trading scheme (EU-ETS) has covered 31 countries with more than 11,000 firms and limited around 45% total EU emissions[1]. With cap-and-trade regulation, the manufacturers receive the cap from the government agencies and the emission permits can be traded through an outside market. Since carbon emissions incur in almost all stages of the production process, cap-and-trade regulation has significant effects on the manufacturers' production decisions and it will indirectly affect the partners in their supply chains.

Most manufacturing firms produce multiple products and sell these products through retailers. For example, HeBei (Cheng De) Iron&Steel Group produces 16 kinds of products, such as hot rolled coil, wire products and so on. He produces these products based on MTO (make-to-order) production and these products are sold by sales agencies who order these products from HeBei (Cheng De) Iron&Steel Group[2]. Lots of carbon emissions are generated in producing each of these products. To reduce carbon emissions, China has established 7 carbon trading pilots and is planning to establish a national carbon trading platform before 2015 which contains many manufacturing industries, especially electricity, steel, petrochemical and cement[3]. HeBei Iron&Steel Group is the second largest steel companies in China. So, he needs to determine the products' wholesale prices under cap-and-trade regulation and reasonably allocate emission permits to the products to maximize their profits, and the retailers determine their order quantities to meet market needs based on the manufacturer's wholesale prices. Compared with a single product scenario, the investigation of two products will involve the allocation of the emission permits as well as the product portfolio selection and production. Due to the differences of production technologies or raw materials in the production process, these products have different production costs and carbon emissions intensity (i.e. carbon emission

1 http://ec.europa.eu/clima/publications/docs/factsheet_ets_en.pdf
2 http://www.csteelnews.com/qypd/yygl/201405/t20140513_242430.html
3 http://www.chinairn.com/news/20131216/103140280.html

generated by producing per-unit product). Thus with cap-and-trade regulation, how to make the emission trading decisions as well as the product portfolio selection and production is an urgent problem of firms today and in the future.

In this paper, we investigate the production and pricing problems in MTO supply chain consisting of two risk-neutral firms, an upstream manufacturer who produces two products based on MTO production and a downstream retailer. The manufacturer determines its wholesale prices with cap-and-trade regulation and the retailer determines its order quantities. It is assumed that the two products can be substitutes or complements and the maximal potential profit per unit of emission permit of product 1 is no less than that of product 2. We find that the manufacturer buys (sells) emission permits from (to) the outside market when the cap is lower (higher) than a lower (higher) threshold and neither buy nor sell emission permits for intermediate levels of emission cap. The optimal total emissions and production quantities of product 2 are increasing in the cap while the optimal production quantities of product 1 may be decreasing in the cap. The emission trading prices are exogenous variables to the supply chain and the selling price per unit of emission permit is not higher than the buying price of emission permits, which can be seen in many trading markets, such as ECX (European Climate Exchange). We find that the manufacturer's optimal profit is decreasing (increasing) in the buying (selling) price of emission permits, and that the retailer's optimal profit is decreasing in the buying (selling) price of emission permits. We also find that the optimal total emissions and production quantities of product 2 are decreasing in the buying (selling) prices of emission permits while the production quantities of product 1 may be increasing in the buying (selling) prices of emission permits. Numerical examples are conducted to illustrate our findings and some managerial insights are presented.

The remainder of this paper is organized as follows. The relative literature is reviewed in Section 2. The problem is modeled in Section 3. The main results are presented in Section 4. Numerical examples are given in Section 5. Section 6 concludes the paper with a discussion of the possible future research direction. All proofs are presented in Appendix.

2. Literature review

Much of the literature investigates the problem of operation decision with emissions trading scheme. Here we review the studies highly related to our paper and these studies can be classified into three categories: the first is to explore the production decisions of a single firm with cap-and-trade regulation; the second is to integrate environmental aspect into supply chain; the last is to consider the operations management and environment problems in the supply chain.

Recently, several papers have investigated the problem of the production decisions of a single firm with cap-and-trade regulation. Benjaafar et al. (2013) introduce cap-and-trade regulation to the production problems and point out three simple models concerning inventory management. Based on the classic Arrow-Karlin model, Dobos (2005) analyzes the effect of emission trading system on a firm's production and inventory. Song and Leng (2011) investigate the newsvendor problem under three carbon emissions policies and get the optimal production quantity and profit under each policy. Based on the EOQ model, Hua et al. (2011) investigate the problem of a firm's inventory management that how to manage carbon footprints with carbon emission trading mechanism and they think that the pricing decisions should be considered in the future research. After that, Li and Gu (2012) investigate the impact of tradable emission permits on a firm's production-inventory strategy and compare the optimal production-inventory strategies with those without emission permits. Chen et al. (2013) investigate the optimal order quantities under different environmental regulations and find that a firm can reduce carbon emissions without significantly increasing cost. Zhang and Xu (2013) discuss the optimal production quantities with multi-product by providing an efficient solution method. The above literature studies focus on a firm and do not involve in pricing decisions. Moreover, their studies except Zhang and Xu (2013) are based on only a single product. Although Zhang and Xu (2013) investigate the optimal production quantities of multi-product, they only provide an algorithm to solve their model and give some simple theoretical analysis. Two products which are substitutes or complements are discussed in our study and we can find some managerial insights presented in the paper. For one-product scenario, the optimal production decisions will be determined after knowing the emission trading decisions because all the emission permits obtaining from the government agencies and the outside market will be used to produce this product. However, the discussion of two products will involve not only the emission trading decisions but also the allocation of the emission permits which exist in many manufacturing firms.

Some literature integrates environmental aspect into supply chain by proposing the green/sustainable supply chain network. Pati et al (2008) formulate a mixed integer goal programming model in the waste management of reverse logistics system and use the model to make decisions on the management of reverse distribution network for the reverse logistics system. Diabat and Simchi-Levi (2010) present a green supply chain management model with a method of mixed-integer program under a carbon emission constraint and demonstrate the model with a computational study. Sundarakani et al. (2010) examine the carbon footprints across the supply chain so that the firm can reasonably design the supply chain networks. Combined with life cycle assessment, Chaabane et al. (2012) introduce a mixed-integer linear programming to design the sustainable supply chains with emission trading scheme. Dekker et al (2012) review the contribution of Operations Research on green logistics under cap-and-trade regulation and give a sketch of green supply chain which involves the decisions of transportation, inventory and facility. Eskandarpour et al (2015) analyze the models of 87 papers containing both economic and environmental factors, and present some new avenues of research to consider sustainability into supply chain network design. Their work tells us that how to reasonably design supply chain to reduce carbon emissions. However, their studies do not involve the production and pricing decisions in the supply chain. In addition, the supply chain design may be a long time and costly investments (Zhang and Xu, 2013).

Up to now, the literature containing both the operations management and environment problems in the supply chain is sparse. Cruz (2008) investigates the optimality conditions of manufacturers and retailers based on the supply chain networks by considering their profits, the carbon emissions and risk. Liu et al. (2012) study the optimal wholesale price and retail price in three supply chain network structures. Du et al. (2011) explore the optimal decisions of the emission permit supplier and the emission-dependent firm with cap-and-trade regulation. Du et al. (2013) further investigate the optimal production decisions with cap-and-trade system in a so-called emission-dependent supply chain. Some studies investigate the optimal operation decisions based on the supply chain networks (Cruz, 2008; Liu et al., 2010). However, their work is not based on cap-and-trade regulation. The others investigate the operation decisions with cap-and-trade regulation based on two-stage supply chain. Yet, their studies are based on a single product and do not consider the pricing decisions.

3. Problem Formulation

We consider a make-to-order supply chain consisting of an upstream manufacturer who produces two products and a monopolistic downstream retailer who may be not actually monopolistic but is powerful (Liu et al., 2012) such as HeBei (Cheng De) Iron&Steel Group. We model the production and pricing problems as the manufacturer-Stackelberg game, to which the manufacturer is the game's leader. That is, the manufacturer regulated by cap-and-trade regulation firstly determines its optimal wholesale prices, then the retailer determines its optimal order quantities based on the wholesale prices and finally the manufacturer produces the two products based on MTO production. It is assumed that the two firms in the supply chain have full and symmetric information and have no inventory. With cap-and-trade regulation, the manufacturer is initially allocated free permits C ($C \geq 0$) on its emissions, and is allowed to trade emission permits with other firms or government agencies through an outside market. Since the emission permits is the limited resources, the manufacturer may not buy enough emission permits to produce the two products and there may not be demand when the manufacturer wants to sell surplus emission permits (that is, the limited allowance availability), we assume that the maximal purchases of emission permits from the outside market are T ($T \geq 0$) and the maximal sales of emission permits to the outside market are V ($V \geq 0$). For ease of analysis, we do not consider the limited allowance availability in our model and only show the optimal total emissions and production quantities of the two products in Theorem 1 to 3 which have considered the limited allowance availability. The permits trading prices, that is, the buying and selling prices of emission permits, are b and

s ($0 < s \le b$), respectively (Gong and Zhou, 2013; Hong, Chu and Yu, 2012). Although we call them prices, they actually represent the cost and revenue of buying and selling unit of emission permits, respectively. The permits trading prices can be quite significant and have been studied both empirically and theoretically (Stavins, 1995; Woerdman et al., 2001).

Denote by p_i and d_i the selling price and the deterministic demand of product i ($i = 1, 2$). Based on the study of Goyal and Netessine (2007), we assume that

$$p_i = \alpha_i - q_i - \lambda q_{3-i} \quad (1)$$

where α_i ($\alpha_i > 0$) is the demand curve intercept and represents a measure of market size, and $\lambda \in [-1,1]$ where $0 < \lambda \le 1$ ($-1 \le \lambda < 0$) means that the two products are substitutes (complements). Note that $\lambda = 0$ implies that the demands of the two products are independent, and $\alpha_i \ge p_i > c_{r_i} (i = 1, 2)$ where c_{r_i} is the unit sale cost of product i. The manufacturer supplies the product i with the wholesale prices w_i ($i = 1, 2$). So, the optimal profit of the retailer Π_R can be found by solving the following model:

$$\Pi_R = \max_{q_1, q_2} \{(p_1 - c_{r_1} - w_1) q_1 + (p_2 - c_{r_2} - w_2) q_2\}. \quad (2)$$

Denote by π_i the manufacturer's profit of product i. Based on MTO production, we can get

$$\pi_i = (w_i - c_{m_i}) q_i \ (i = 1, 2).$$

where c_{m_i} is the unit production cost of product i. Carbon dioxide incurs in the production activities. Denote by e_i the incurred emissions of producing per unit product (emission intensity for short) i ($i = 1, 2$). Let E be the total emissions of the manufacturer, i.e., the used emission permits in production activities. So we can obtain $E = e_1 q_1 + e_2 q_2$. Considering the emissions cost, the optimal profit of the manufacturer Π_M can be found by solving the following model:

$$\Pi_M = \max_{w_1, w_2} \{(w_1 - c_{m_1}) q_1 + (w_2 - c_{m_2}) q_2 - b(E - C)^+ + s(C - E)^+\}. \quad (3)$$

The first two terms are the profits when producing the products 1 and 2, respectively. The last two terms are the emission costs. Let w_1^* and w_2^* be the optimal solutions to Model (3) and q_1^* and q_2^* be the corresponding optimal order quantities (optimal production quantities). Note that the optimal selling prices p_1^* and p_2^* can be found based on q_1^*, q_2^* and Equation (1).

4. Main Results

In this section, we firstly develop the optimal total emissions and production quantities of two products with cap-and-trade regulation in the supply chain, and then discuss the impact of the buying and selling prices of emission permits on the optimal total emission, optimal production quantities and optimal profits.

Based on Model (1) and Model (2), we have that

$$\Pi_R = \max_{q_1,q_2}\{(\alpha_1 - q_1 - \lambda q_2 - c_{r_1} - w_1)q_1 + (\alpha_2 - q_2 - \lambda q_1 - c_{r_2} - w_2)q_2\} \quad (4)$$

Lemma 1. *The optimal order quantity of product* i *to Model (4) satisfies*:

$$q_i = [\alpha_i - c_{r_i} - w_i - (\alpha_{3-i} - c_{r_{3-i}} - w_{3-i})\lambda]\big/[2(1-\lambda^2)]\ (i=1,2)$$

The proof of Lemma 1 (and the subsequent results) can be found in Appendix. Lemma 1 shows the relationship of the optimal order quantities with the wholesale prices. We find that the optimal order quantities are linear with the wholesale prices.

From Lemma 1, we know that the retailer-price response function to the wholesale prices is $q_i = [\alpha_i - c_{r_i} - w_i - (\alpha_{3-i} - c_{r_{3-i}} - w_{3-i})\lambda]\big/[2(1-\lambda^2)]$. We find that there is one-to-one relationship between the order quantities and the wholesale prices. We change the form of reaction function as follows: $w_i = \alpha_i - c_{r_i} - 2(q_i + \lambda q_{3-i})$ ($i=1,2$). For simplicity, we let q_i ($i=1,2$) be the decisions variable in the Model (3). So we change the Model (3) as follows:

$$\Pi_M = \max_{q_1,q_2}\{(w_1 - c_{m_1})q_1 + (w_2 - c_{m_2})q_2 - b(E-C)^+ + s(C-E)^+\}. \quad (4)$$

Denote by E_1 and E_2 the emissions of products 1 and 2, respectively. Then $E_1 = e_1 q_1$, $E_2 = e_2 q_2$ and $E = E_1 + E_2$. We have

$$\pi_i = \gamma_i E_i - \eta_i E_i^2 - \bar{\lambda} E_i E_{3-i},\ \ i=1,2, \quad (5)$$

where $\gamma_i = (\alpha_i - c_{r_i} - c_{m_i})\big/e_i$, $\eta_i = 2\big/e_i^2 > 0$ and $\bar{\lambda} = 2\lambda/(e_1 e_2)$. Here γ_i can be interpreted as the maximal potential profit per unit of emission permit of product i. It is easy to verify that $\eta_1 + \eta_2 \geq 2\bar{\lambda}$ and $\eta_1\eta_2 \geq \bar{\lambda}^2$. Similarly, we have the optimal profit of the retailer $\Pi_R = (\eta_1 E_1^2)\big/2 + (\eta_2 E_2^2)\big/2 + \bar{\lambda} E_1 E_2$. Since the optimal wholesale prices w_1^* and w_2^* can be found based on q_1^*, q_2^* and Lemma 1, we just show the optimal total emissions and production quantities of the two products in the optimal decisions.

Define $R(E_1,E_2) = \gamma_1 E_1 - \eta_1 E_1^2 + \gamma_2 E_2 - \eta_2 E_2^2 - 2\bar{\lambda} E_1 E_2$. It is clear that $R(E_1,E_2)$ is the manufacturer's profit without cap-and-trade regulation. The first and second derivatives of $R(E_1, E_2)$ with respect to E_1 and E_2 is as follows:

$$\partial R(E_1, E_2)/\partial E_1 = \gamma_1 - 2\eta_1 E_1 - 2\bar{\lambda} E_2 \quad (6)$$

$$\partial R(E_1, E_2)/\partial E_2 = \gamma_2 - 2\eta_2 E_2 - 2\bar{\lambda} E_1 \quad (7)$$

Model (6) and Model (7) show that the reduction of the marginal profit of product 1(2) is $2\eta_1$ ($2\eta_2$) when the unit emission permit is used to produce product 1(2); $2\bar{\lambda}$ is the reduction of the marginal profit of product 1(2) when the unit emission permit is used to produce product 2(1). So, $\eta_1 > \bar{\lambda}$ indicates that the reduction of Model (6) is larger than that of Model (7) when the unit emission permit is used to produce product 1 and $\eta_2 > \bar{\lambda}$ indicates that the reduction of Model (6) is less than that of Model (7) when the unit emission permit is used to produce product 2. Without loss of generality, we assume that $\gamma_1 \geq \gamma_2$. Please note that γ_i is determined by the potential market size, the unit production cost, the unit sale cost and the unit emissions. $\gamma_1 \geq \gamma_2$ does not imply that product 1 incurs the lower unit emissions.

Define $f(E) = \max_{E_1, E_2}\{\gamma_1 E_1 - \eta_1 E_1^2 + \gamma_2 E_2 - \eta_2 E_2^2 - 2\bar{\lambda} E_1 E_2 : E_1 + E_2 = E\}$. It is clear that $f(E)$ is the manufacturer's optimal total profit of the two products under given total emissions E. Let E_1^* and E_2^* be the optimal solutions to $f(E)$ under given total emissions E. Based on $f(E)$, Model (4) can be rewritten as:

$$\Pi_M = \max\{\Pi_M^b = \max_{E \geq C \geq 0}\{f(E) - bE + bC\}, \Pi_M^s = \max_{0 \leq E \leq C}\{f(E) - sE + sC\}\}, \quad (8)$$

where Π_M^b is the optimal profit of the manufacturer for the case with buying emission permits, and Π_M^s is the optimal profit of the manufacturer for the case with selling emission permits. Let E^* be the optimal total emissions after considering the limited allowance availability. After knowing E^*, E_1^* and E_2^* will also be the optimal emissions of the two products after considering the limited allowance availability. From Model (8), we know that the optimal decisions can be derived based on the property of the function $f(E)$. Define $\underline{C}_0 = (\gamma_1 - \gamma_2)/2(\eta_1 - \bar{\lambda})$ and $\underline{C}^0 = (\gamma_1 - \gamma_2)/2(\bar{\lambda} - \eta_2)$ for the total emissions, where $\underline{C}_0$ ($\underline{C}^0$) is the minimal (maximal) total emission to make Model (6) and Model (7) equal to each other, $f(E)$ and its optimal solutions have the following properties as shown in Lemma 2.

Lemma 2. *The function* $f(E)$ *is strictly concave*.

(1) *When* $\eta_1 > \bar{\lambda}$ *and* $\eta_2 \geq \bar{\lambda}$, $f(E)$ *and its optimal solutions satisfy*:

(i) *If* $E \leq \underline{C}_0$, *then* $f(E) = \gamma_1 E - \eta_1 E^2$. *The optimal solutions satisfy* $E_1^* = E$

and $E_2^* = 0$;

(ii) *If* $E \geq \underline{C}_0$, *then* $f(E) = \{(\gamma_1 - \gamma_2)^2 + 4[\gamma_1(\eta_2 - \bar{\lambda}) + \gamma_2(\eta_1 - \bar{\lambda})]E - 4(\eta_1\eta_2 - \bar{\lambda}^2)E^2\} / [4(\eta_1 + \eta_2 - 2\bar{\lambda})]$. *The optimal solutions satisfy* $E_1^* = [\gamma_1 - \gamma_2 + 2(\eta_2 - \bar{\lambda})E] / [2(\eta_1 + \eta_2 - 2\bar{\lambda})]$ *and* $E_2^* = (\gamma_2 - \gamma_1 + 2(\eta_1 - \bar{\lambda})E) / [2(\eta_1 + \eta_2 - 2\bar{\lambda})]$.

(2) *When* $\eta_1 > \bar{\lambda}$ *and* $\eta_2 < \bar{\lambda}$, $f(E)$ *and its optimal solutions satisfy*:

(i) *If* $E \leq \underline{C}_0$, *then* $f(E) = \gamma_1 E - \eta_1 E^2$. *The optimal solutions satisfy* $E_1^* = E$ *and* $E_2^* = 0$;

(ii) *If* $\underline{C}_0 \leq E \leq \underline{C}^0$ *then* $f(E) = \{(\gamma_1 - \gamma_2)^2 + 4[\gamma_1(\eta_2 - \bar{\lambda}) + \gamma_2(\eta_1 - \bar{\lambda})]E - 4(\eta_1\eta_2 - \bar{\lambda}^2)E^2\} / [4(\eta_1 + \eta_2 - 2\bar{\lambda})]$. *The optimal solutions satisfy* $E_1^* = [\gamma_1 - \gamma_2 + 2(\eta_2 - \bar{\lambda})E] / [2(\eta_1 + \eta_2 - 2\bar{\lambda})]$ *and* $E_2^* = (\gamma_2 - \gamma_1 + 2(\eta_1 - \bar{\lambda})E) / [2(\eta_1 + \eta_2 - 2\bar{\lambda})]$;

(iii) *If* $E > \underline{C}^0$, *then* $f(E) = \gamma_2 E_2 - \eta_2 E_2^2$. *The optimal solutions satisfy* $E_1^* = 0$ *and* $E_2^* = E$.

(3) *When* $\eta_1 \leq \bar{\lambda}$ *and* $\eta_2 \geq \bar{\lambda}$, $f(E) = \gamma_1 E - \eta_1 E^2$.

Lemma 2 shows the optimal total profit and emissions of the two products under given total emissions E. Lemma 2 implies that whether the manufacturer produces product i not only depends on the total emissions but also the exogenous parameters (η_1, η_2 and $\bar{\lambda}$). Lemma 2 (1) indicates that the manufacturer firstly uses the emission permits to only produce product 1 when the total emissions is less than the lower threshold ($\underline{C}_0$), then to produce product 2 for the rest total emissions. This occurs because the marginal profit per unit of emission permit of product 1 is always larger than that of product 2 when the total emissions are less than the lower threshold and can be equal to that of product 2 when the total emissions is no less than the lower threshold. Note that the higher threshold is not existed in this situation. Lemma 2(2) implies that the manufacturer firstly only produces product 1 and then produces the two products and finally only produces product 2, which means that more emission permits will be allocated to produce product 2 as the emission permits increases. It is because the marginal profit per unit of emission permit of product 1(2) is larger than that of product 2(1) when the total emission is less than the lower (higher) threshold and they can be equal to each other for intermediate levels of the total emission.

Lemma 2 (3) means that the marginal profit per unit of emission permit of product 1 is always larger than that of product 2 so that the manufacturer only produces product 1.

Define $M_1 = (\gamma_2\bar{\lambda} - \gamma_1\eta_2)/(\bar{\lambda} - \eta_2)$ and $M_2 = (\gamma_2\eta_1 - \gamma_1\bar{\lambda})/(\eta_1 - \bar{\lambda})$. M_1 can be interpreted as the maximal potential profit per unit of emission permit when the manufacturer only wants to produce product 2 or the minimal potential profit per unit of emission permit when the manufacturer produces the two products simultaneously. M_2 can be interpreted as the minimal potential profit per unit of emission permit when the manufacturer only wants to produce product 1 or the maximal potential profit per unit of emission permit when the manufacturer produces the two products. Note that $M_1 \le 0$ means that the manufacturer will never only produce product 2 and $M_2 \le 0$ means that the manufacturer will never produce the two products simultaneously. We define the following thresholds for Theorem 1 to Theorem 3, $C_b^1 = \max\{(\gamma_1 - b)/(2\eta_1), 0\}$, $C_b^2 = [(\gamma_1 - b)(\eta_2 - \bar{\lambda}) + (\gamma_2 - b)(\eta_1 - \bar{\lambda})]/[2(\eta_1\eta_2 - \bar{\lambda}^2)]$, $C_b^3 = (\gamma_2 - b)/(2\eta_2)$; $C_s = [\gamma_1(\eta_2 - \bar{\lambda}) + \gamma_2(\eta_1 - \bar{\lambda})]/[2(\eta_1\eta_2 - \bar{\lambda}^2)]$, $C_s^1 = (\gamma_1 - s)/(2\eta_1)$, $C_s^2 = [(\gamma_1 - s)(\eta_2 - \bar{\lambda}) + (\gamma_2 - s)(\eta_1 - \bar{\lambda})]/[2(\eta_1\eta_2 - \bar{\lambda}^2)]$, $C_s^3 = (\gamma_2 - s)/(2\eta_2)$, then the optimal total emissions and production quantities of the two products under different situations can be shown in the following three theorems.

In the paper, we use “increasing” and “decreasing” in the non-strict sense to mean “non-decreasing” and “non-increasing”, respectively.

Theorem 1. *When* $\eta_1 > \bar{\lambda}$ *and* $\eta_2 \ge \bar{\lambda}$, *the optimal total emissions and production quantities of the two products over all possible levels of cap* C *in three cases are shown in Table 1. Pleases see Table 1 in Appendix.*

Theorem 1 shows that the emission trading decisions depend on the received cap and follow a two-threshold policy. The lower threshold is equal to the amount of emission permits to satisfy that the marginal profits of the two products are equal to the buying price of emission permits. The higher threshold is equal to the amount of emission permits to satisfy that the marginal profits of the two products are equal to the selling price of emission permits. If the cap is lower than the lower threshold, then the manufacturer buys emission permits from the outside market; if the cap is larger than the higher threshold, then the manufacturer sells surplus emission permits to the outside market and if the cap is between the two thresholds, the manufacturer will neither buy nor sell emission permits. For example, in the case with $M_2 > b$, when $0 \le C < C_b^2$, the manufacturer buys $\min\{C_b^2 - C, T\}$ unit emission permits; when $C_b^2 \le C \le C_s^2$, the manufacturer neither buys nor sells emission permits; when $C > C_s^2$, the manufacturer sells $\min\{C - C_s^2, V\}$ unit emission permits.

Theorem 1 also shows that the optimal total emissions and production quantities of the two products are increasing in the cap. This result is rather intuitive because the firm tends to produce more products when more emission permits are received. Note that the optimal total emissions and production quantities may remain constant as the cap increases. For example, in the case with $M_2 > b$, the optimal total emissions and production quantities of the two products remain constant as the cap increases when $(C_b^2 - T) < C < C_b^2$ or $C_s^2 < C \le C_s^2 + V$. They remain constant because the manufacturer allocates the emission permits to the two products to make the marginal profits of the two products equal to the emission permits prices. After considering the limited allowance availability, the marginal profits of the two products are equal to zero when $C > C_s + V$ because the manufacturer cannot sell all the surplus emission permits so that he will use the emission permits to produce the two products.

Theorem 1 indicates that product 1 is the "dominant product" which means that the manufacturer always produces this product. However, whether the manufacturer produces product 2 depends on M_2 and the cap. When $M_2 \le s$, which indicates that the manufacturer will not simultaneously produce the two products, he only produces product 1 because the maximal potential profit per unit of emission permit of product 1 is no less than that of product 2; when $s < M_2 \le b$, which indicates that the manufacturer will not buy emission permits to produce the two products, he produces product 2 under high cap; when $M_2 > b$, which indicates that the manufacturer always produces product 2, he produces product 2 under any cap.

Theorem 2. *When $\eta_1 > \bar{\lambda}$ and $\eta_2 < \bar{\lambda}$, the optimal total emissions and production quantities of the two products over all possible levels of cap C in six cases are shown in Table 2. Please see Table 2 in Appendix.*

Similar to Theorem 1, Theorem 2 shows that the emission trading decisions also depend on the received cap and follow a two-threshold policy. However, there is some difference in the two thresholds. When $M_1 \le b$, which indicates that the manufacturer will never buy emission permits to only produce product 2, the lower threshold is the same to that in Theorem 1; When $M_1 > b$, which indicates that the manufacturer will buy emission permits to only produce product 2, the lower threshold is equal to the amount of emission permits to satisfy that the marginal profit of product 2 is equal to the buying price of emission permits. When $M_1 \le s$, the higher threshold is the same to that in Theorem 1; When $M_1 > s$, the higher threshold is equal to the amount of emission permits to satisfy that the marginal profit of product 2 is equal to the selling price of emission permits. If the cap is lower than the lower threshold, then the manufacturer buys emission permits from the outside market; if the cap is larger than the higher threshold, then the manufacturer sells surplus emission permits to the outside market and if the cap is between the two thresholds, the manufacturer will neither buy nor sell emission permits.

Theorem 2 also shows that the optimal total emissions and production quantities of product 2 are increasing in the cap. However, as the cap increases, (1) the optimal production quantities of product 1 firstly increase and then decrease when $M_2 > s$; (2) the optimal production quantities of product 1 increases when $M_2 \le s$. The optimal total emissions and production quantities of the two products increases as the cap increases because (i) the manufacturer cannot buy enough emission permits from the outside market; (ii) he will only uses the received cap to produce products; (iii) he cannot sell all the surplus emission permits to the outside market. And the optimal production quantities of product 1 are decreasing in the cap because the manufacturer uses more emission permits to produce product 2 when $M_2 > s$. Based on Model (6) and (7), we know the marginal profit of each product per unit of emission permit decreases when more emission permits are used to produce the two products. Theorem 2 indicates that, if the reduction of the marginal profit of product 1 is larger than that of product 2 when the unit emission permit is used to produce product 1 (product 2), then the manufacturer will allocate more emission permits to product 2 as the cap increases. So, the manufacturer reduces the emission permits to produce product 1 and increases the emission permits to produce product 2. Theorem 2 also indicates that, even though product 1 has larger maximal potential profit per unit of emission permit than product 2, the manufacturer allocate more emission permits to product 2 for the reason of the substitutability and complementarity of the two products.

Note that the optimal total emissions and production quantities of the two products remain constant as the cap increases because the manufacturer produces the two products to make the marginal profits of the two products equal to the emission trading prices or make the marginal profits of product 2 equal to the selling price of emission permits. After considering the limited allowance availability, we find that the manufacturer produces the two products to make the marginal profits of the two products equal to zero or make the marginal profits of product 2 equal to zero when there are enough surplus emission permits. It is because the manufacturer cannot sell surplus emission permits to obtain profits and he has the motivation to produce more products to make their marginal profit equal to zero.

Similar to Theorem 1, we find that the manufacturer may produce product 1 or both of the products. However, under high value of T, the manufacturer may only produce product 2 because the marginal profit of product 2 is always larger than that of product 1 which is not exited in two independent products. For example, when $s \le b < M_1 \le M_2$, the manufacturer only produces product 2. We know that $\eta_1 > \bar{\lambda} > \eta_2$. Recall that $\eta_i = 2/e_i^2$ ($i = 1,2$). We can easily verify that $e_1 < e_2$. So, cap-and-trade regulation may not have the ability to induce the manufacturer to produce low-carbon products which have less carbon emissions per unit of product in production process. It is because the profit-maximizing manufacturer produces the products which can obtain more profit and he has no motivation to care other aspects.

Theorem 3. *When* $\eta_1 \le \bar{\lambda}$ *and* $\eta_2 \ge \bar{\lambda}$, *we have* (i) *if* $0 \le C < C_b^1$, *the optimal total emissions are* $E^* = \min\{T + C, C_b^1\}$, and *the optimal production quantities are* $q_1^* = \min\{T + C, C_b^1\}/e_1$ *and* $q_2^* = 0$; (ii) *if* $C_b^1 \le C < C_s^1$, *the optimal total emissions are* $E^* = C$, *and the optimal production quantities are* $q_1^* = C/e_1$ *and* $q_2^* = 0$; (iii) *when* $C_s^1 \le C < C_s^1 + V$, *the optimal total emissions are* $E^* = C_s^1$, *and the optimal production quantities are* $q_1^* = C_s^1/e_1$ *and* $q_2^* = 0$; (iv) *when* $C_s^1 + V \le C < \gamma_1/(2\eta_1) + V$, *the optimal total emissions are* $E^* = C - V$, *and the optimal production quantities are* $q_1^* = (C - V)/e_1$ *and* $q_2^* = 0$; (v) *when*

$C \geq \gamma_1/(2\eta_1)+V$, *the optimal total emissions are* $E^* = \gamma_1/(2\eta_1)$, *and the optimal production quantities are* $q_1^* = \gamma_1/(2e_1\eta_1)$ *and* $q_2^* = 0$.

Theorem 3 shows that the emission trading decisions depend on the received cap and follow a two-threshold policy. Note that only product 1 will be produced (that is, the emission permits are allocated only to product 1). The lower threshold is equal to the amount of emission permits to satisfy that the marginal profit of product 1 is equal to the buying price of emission permits. The higher threshold is equal to the amount of emission permits to satisfy that the marginal profit of product 1 is equal to the selling price of emission permits. If the cap is lower than the lower threshold, then the manufacturer buys emission permits from the outside market; if the cap is larger than the higher threshold, then the manufacturer sells surplus emission permits to the outside market and if the cap is between the two thresholds, the manufacturer will neither buy nor sell emission permits.

Theorem 3 also shows that the optimal total emissions and production quantities of product 1 are increasing in the cap. It occurs because the firm tends to produce more products when more emission permits are received. Note that the optimal total emissions and production quantities of product 1 may remain constant as the cap increases. For example, when $C_s^1 \leq C < C_s^1 + V$, the optimal total emissions and production quantities of product 1 is C_s^1 and C_s^1/e_1 , respectively. It is because the manufacturer produces product 1 to make the marginal profits of product 1 equal to the emission permits prices. Theorem 3 indicates that, if the reduction of marginal profit of product 1 is always less than that of product 2 when the unit emission permit is used to produce product 1 or product 2, then the manufacturer will allocate all the emission permits to product 1 because the marginal profit of product 1 is always larger than that of product 2. We can easily verify that $e_1 > e_2$ because of $\eta_1 < \eta_2$. So, cap-and-trade regulation may not have the ability to induce the manufacturer to produce low-carbon products.

From Theorem 1 to 3, we can easily find that the cap has significant impact on the manufacturer's optimal production decisions. For example, when $\eta_1 \leq \bar{\lambda}$ and $\eta_2 \geq \bar{\lambda}$, the optimal total emissions are $\min\{T+C, C_b^1\}$ if $0 \leq C < C_b^1$; C if $C_b^1 \leq C < C_s^1$ and $C-V$ if $C_s^3 + V \leq C < \gamma_2/(2\eta_2)+V$. The result is different from that of Hua et al. (2011) and Benjaafar et al. (2013), who hold that the cap cannot be used as a direct lever to control carbon emissions. However, for the reason of the limit of the purchases (sales) of emission permits from (to) the outside market and the difference of the buying and selling prices of emission permits, the cap can directly affect the optimal production decisions, which implies that the cap can be used as a direct lever to control carbon emissions.

If the manufacturer produces a single product, suppose that he will only produce product 1, Theorem 3 shows the optimal total emissions and production quantities of product 1. Based on the above theoretical results, we can find that there are some differences between one product setting and two-product setting. (1) the production of two products will involve the allocation of emission permits which makes the problem more complex. For example, when $\eta_1 > \bar{\lambda}$ and $\eta_2 \geq \bar{\lambda}$, the firm should allocate the emission permits on the two products if $M_2 > b$; (2) it will involve the product portfolio selection and production. Theorem 2 indicates the manufacturer may produce product 1(product 2) or both of the two products. (3) the optimal total emissions under one-product scenario are no larger than that under two-product scenario while the optimal production quantities of product 1 under one-product scenario are no less than that under two-product scenario. It indicates that the optimal total emissions increase in the kinds of products while the production of the existing products will be reduced after adding one or more kinds of products. In practice, firms produce multi-product to face various market demands. Since it is too complex to investigate multi-product setting by considering the substitutability and complementarity, we select two products as the representation of multi-product. Note that, if $\bar{\lambda} = 0$, the demands of the two products will be independent which is a special case of Theorem 1. The optimal production decisions can be extended to multiple independent products because they have similar model structure. So, there is no difference in emission trading decisions and production decision between the two independent products setting and multiple independent products setting. That is, for multiple products, the emission trading decisions follows a two-threshold policy and the manufacturer produces multiple products to make the marginal profits of the multiple products equal to the emission trading prices. However, the only difference between the two settings is the increase of thresholds to determine the kinds of products.

Corollary 1. *The manufacturer's optimal profit is decreasing in* b *and is increasing in* s. *The retailer's optimal profit is decreasing in* b (s).

Corollary 1 shows that the increase of the buying price of emission permits will reduce the manufacturer's profit because it increases the manufacturer's buying cost in the case that he buys emission permits from the outside market, and that the increase of the selling price per unit of emission permit will increase the manufacturer's profit for the reason that he can generate more profit when selling surplus emission permits to the outside market.

According to Theorem 1 to 3, we can obtain a clear result that the increase of the emission permits prices will induce the manufacturer to produce less. As a result, the order quantities are also reduced. Thus based on the retailer's profit $\Pi_R = (\eta_1 E_1^2)/2 + (\eta_2 E_2^2)/2 + \bar{\lambda} E_1 E_2$, we can find that its profit will decrease. Note that the two firms' optimal profits may remain constant as the increase of b (s) under the given cap, for example, in the case when the optimal total emissions are equal to the received cap.

Corollary 2. (1) *When* $\eta_1 > \bar{\lambda}$ *and* $\eta_2 \geq \bar{\lambda}$, E^*, q_1^* *and* q_2^* *are decreasing in* b (s); (2) *When* $\eta_1 > \bar{\lambda}$ *and* $\eta_2 < \bar{\lambda}$, E^* *and* q_2^* *are decreasing in* b (s) *and* q_1^* *is firstly increasing and then decreasing in* b (s); (3) *When* $\eta_1 \leq \bar{\lambda}$ *and* $\eta_2 \geq \bar{\lambda}$, E^* *and* q_1^* *are decreasing in* b (s).

Corollary 2 (1) and Corollary 2 (3) show that the optimal total emissions and production quantities of the two products are decreasing in the buying or selling price of emission permits. This result is intuitive because the increase of the buying or selling price of emission permits induces the manufacturer to buy less emission permits or sell more emission permits so that the optimal production quantities of the two products will be reduced. Corollary 2 (2) shows that the optimal total emissions and production quantities of product 2 are decreasing in the buying or selling price of emission permits and the optimal production quantities of product 1 is firstly increasing and then decreasing in the buying or selling price of emission permits. Based on Lemma 2(2), we know that the manufacturer has the motivation to produce product 1 when the given total emissions are low and to produce product 2 when the given total emissions are high. So, when the buying (selling) price of emission permits increases which implies that the manufacturer will buy less (sell more) emission permits, he will reduce the production quantities of product 2 and increase the production quantities of product 1. When the buying (selling) price of emission permits is larger enough, the manufacturer may only produce product 1 which means that it will not involve the allocation of emission permits so that the production quantities of product 1 will also be reduced. Corollary 2 indicates that, if the reduction of the marginal profit of product 1 is larger than that of product 2 when the unit emission permit is used to produce product 1 (product 2), then the manufacturer will allocate more emission permits to product 1 as the buying (selling) price of emission permits increases.

5. Numerical Study

In this section, we conduct two numerical examples to illustrate part of the above theoretical results. The first example shows the optimal total emissions and production quantities of the two products over all the levels of cap; the second example demonstrates the impact of the buying price of emission permits on the optimal total emissions and production quantities of the two products. The related parameters are set as follows: $\alpha_1 = 380$, $\alpha_2 = 350$, $c_{r_1} = 30$, $c_{r_2} = 10$, $c_{m_1} = 50$, $c_{m_2} = 7$, $e_1 = 2$, $e_2 = 3$, $s = 8$, $b = 40$, $T = 70$, $V = 50$. We have $\gamma_1 = 150$, $\eta_1 = 0.5$, $\gamma_2 = 111$, $\eta_2 = 2/9$. Note that $-1/3 \le \bar{\lambda} \le 1/3$ because of $\bar{\lambda}^2 \le \eta_1\eta_2$. We know that the substitutability and complementarity of products exist in many industries and have significant impact on the manufacturer's optimal production decisions. Since $\bar{\lambda}$ represents the relationship of the two products (substitutability and complementarity), we let $\bar{\lambda} = 0, 1/6, 5/18$ in the following two examples. $\bar{\lambda} = 0$ means that the demands of the two products are independent (Theorem 1 indicates that $\bar{\lambda} = 0$ and $\bar{\lambda} < 0$ have no difference in production decisions so that we do not consider the complementarity in the numerical study); in order to illustrate Theorem 1, we let $\bar{\lambda} = 1/6$ so that $\eta_1 > \bar{\lambda}, \eta_2 > \bar{\lambda}$; in order to illustrate Theorem 2, we let $\bar{\lambda} = 5/18$ so that $\eta_1 > \bar{\lambda} > \eta_2$.

In the first example, we present the following three figures to show the optimal total emissions and production quantities of the two products over all the levels of cap under each value of $\bar{\lambda}$.

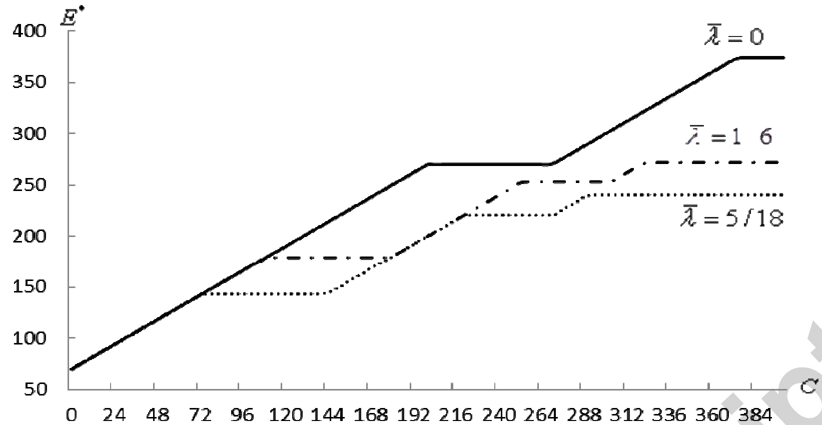

Fig 1.1 The optimal total emissions over different levels of cap

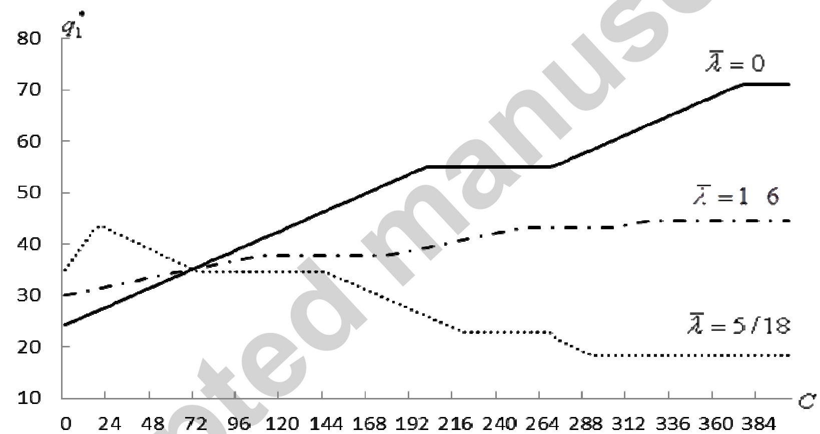

Fig 1.2 The optimal production quantities of product 1 over different levels of cap

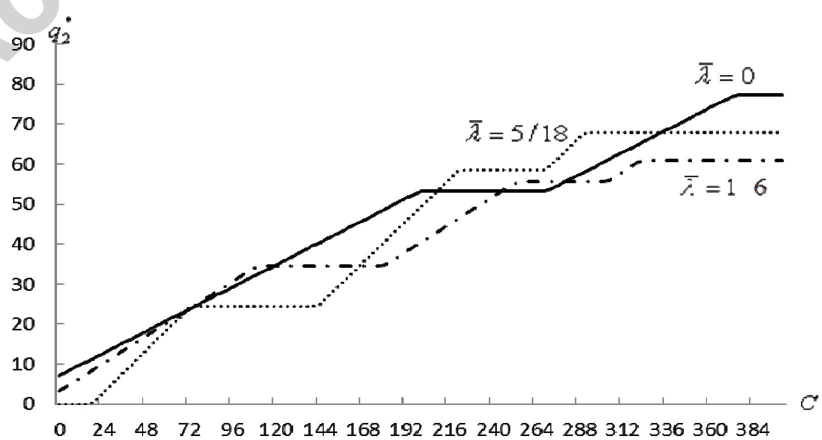

Fig 1.3 The optimal production quantities of product 2 over different levels of cap

Fig 1.1 shows that the emission permits follows a two-threshold policy which have been found in Theorem 1 to 3. For example, if $\bar{\lambda}=0$, the manufacturer buys emission permits when the cap is in the interval of $[0, 269.75)$, then neither buys nor sells emission permits when the cap is in the interval of $[269.75, 373.75]$ and finally sells the surplus emission permits when the cap is in the interval of $(373.75, +\infty)$.

Figs 1.1 to 1.3 show that the optimal total emissions and production quantities of product 2 are increasing in the cap. The optimal production quantities of product 1 are also increasing in the cap if $\bar{\lambda}=0$ or $\bar{\lambda}=1/6$, however, if $\bar{\lambda}=5/18$, they are increasing in the cap when the cap is in the interval of $[0,17.75)$ and are decreasing in the cap when the cap is in the interval of $(17.75,+\infty)$. We know that $\eta_1>\bar{\lambda}>\eta_2$ if $\bar{\lambda}=5/18$. Based on Model (6) and (7), we can verify that the marginal profit of product 1 is less than that of product 2 when enough emission permits are received to produce the two products. So, the manufacturer will reduce the production quantities of product 1 and increase the production of product 2.

Moreover, if $\bar{\lambda}=5/18$, we can easily verify that the marginal profits of the two products are equal to $b=40$ when the cap is in the interval of $[72.4,142.4)$ and are equal to $s=8$ when the cap is in the interval of $(220.9, 271)$ which indicates that the manufacturer buys or sells emission permits to ensure that its marginal profits of the two products equals the emission permits prices. Please note that (i) the marginal profits of the two products are equal to zero when the cap is in the interval of $(290.55,+\infty)$; (ii) the marginal profits of the two products is between zero and $s=8$ when the cap is in the interval of $(271, 290.55)$. They occur because of the limited allowance availability (the manufacturer use the unsold emission permits to produce the two products so that their marginal profits are less than the selling price of emission permits and finally are equal to zero under high cap). Similarly, we can also find that the marginal profits are equal to the emission permits prices or zero if $\bar{\lambda}=0$ or $\bar{\lambda}=1/6$.

In the second example, we let $C=50$, and b varies from 8 to 120 with one-step size. The following three figures shows the impact of the buying emission permits on the optimal total emissions and production quantities of the two products under each value of $\bar{\lambda}$.

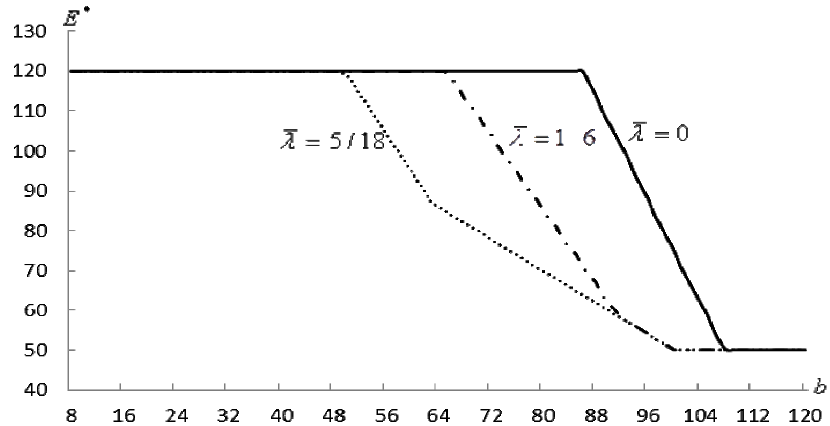

Fig 2.1 The impact of the buying prices of emission permits on the total emissions

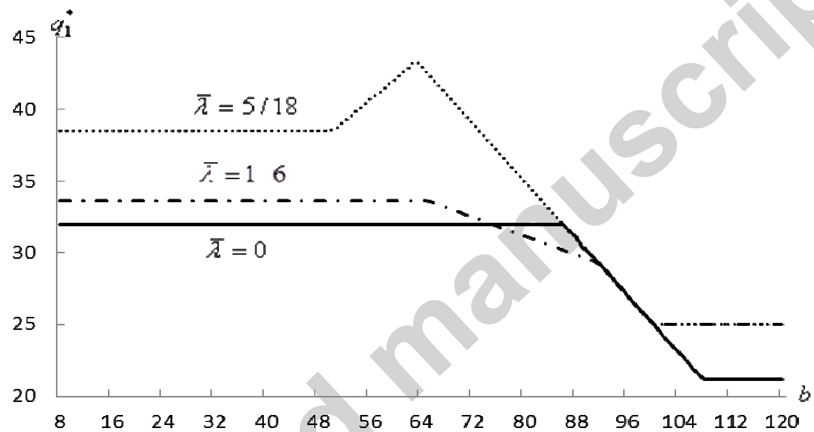

Fig 2.2 The impact of the buying prices of emission permits on the optimal production quantities of product 1

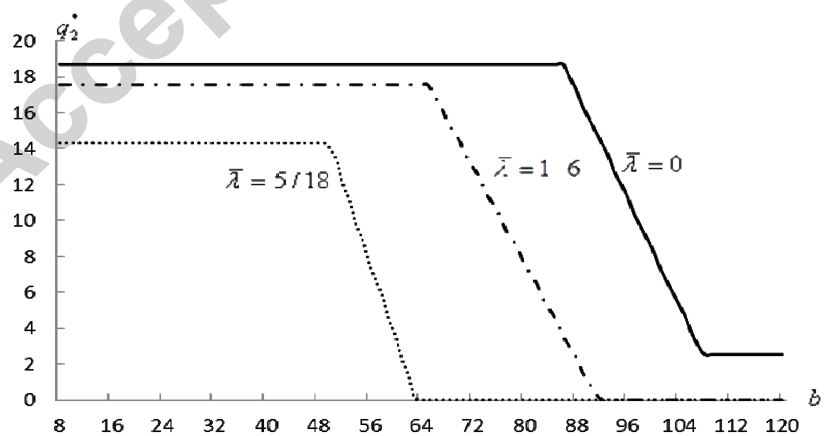

Fig 2.3 The impact of the buying prices of emission permits on the optimal production quantities of product 2

Figs 2.1 to 2.3 demonstrate the impact of the buying prices of emission permits on the optimal total emissions and production quantities of the two products which is shown in Corollary 2. Fig 2.1 and Fig 2.3 show that the optimal total emissions and production quantities of product 2 are decreasing in the buying prices of emission permits. Fig 2.2 shows that, if $\bar{\lambda}=0$ or $\bar{\lambda}=1/6$, the optimal production quantities of product 1 are also decreasing in the buying prices of emission permits, however, if $\bar{\lambda}=5/18\,(\eta_1>\bar{\lambda}>\eta_2)$ the optimal production quantities of product 1 are increasing in the buying price of emission permits when $b\in[8,62.25]$ because more emission permits are used to produce product 1 after less emission permits are obtained which can be seen in Lemma 2(2); they are decreasing in the buying price of emission permits when $b\in(62.25,+\infty)$ because only product 1 will be produced so that it will not involve the allocation of emission permits. Figs 2.1 to 2.3 also show that, if the reduction of the marginal profit of product 1 is larger than that of product 2 when the unit emission permit is used to produce product 1 (product 2), then (i) the manufacturer will allocate more emission permits to product 1 as the buying price of emission permits increases when they produce two products; (ii) he will reduce the production of product 1 as the buying price of emission permits increases when he only produce product 1.

6. Conclusion

With the imposed regulations and legislation on carbon emission control, firms have to manage carbon emissions in their supply chains. In this paper, we derive the optimal total emissions and production quantities of the two products over all possible levels of cap, and explore the impact of the emission trading prices on the optimal production decisions and the optimal profits.

By analyzing the impact of the cap and emission trading prices on the production decisions, we find some managerial insights: (1) the emission trading decisions follow a two-threshold policy. If the cap is lower (higher) than one (the other) threshold, then the manufacturer buys (sells) emission permits from (to) the outside market and if the cap is between the two thresholds, the manufacturer will neither buy nor sell emission permits; (2) if the reduction of the marginal profit of product 1 is larger than that of product 2 when the unit emission permit is used to produce product 1 (product 2), then the manufacturer will allocate more emission permits to product 2 as the cap increases, otherwise, the manufacturer will allocate all the emission permits to product 1. So, the manufacturer may produce product 1 (product 2) or both of the two products which indicates that cap-and-trade regulation may not have the ability to induce the manufacturer to produce low-carbon products; (3) the optimal total emissions and production quantities of product 2 are decreasing in buying or selling price of emission permits, while the optimal production quantities of product 1 may be increasing in buying or selling price of emission permits.

In this paper, we assume that the emission trading prices are exogenous. It is possible that the emission trading prices varies with the cap allocated by the government agencies because the amount of the cap will affect the emission permits' supply and demand in the outside market. Moreover, it is interesting to consider multi-period production problem where the surplus emission permits can be used in the next period.

Acknowledgements

The authors are grateful to the editor and the anonymous referees for their constructive comments, which substantially helped the authors improve the quality of the manuscript. This work is supported by the national Natural Science Foundation of China (Nos. 71201153, 71371176), NSFC Major international cooperation program (No. 71320107004).

References

Benjaafar S, Li YZ, Daskin M. Carbon Footprint and the Management of Supply Chains: Insights from Simple Models. IEEE Transactions on Automation Science and Engineering 2013; 10(1): 99-116.

Chaabane A, Ramudhin A, Paquet M. Design of sustainable supply chains under the emission trading scheme. International Journal of Production Economics 2010; 135: 37-49.

Chen X, Benjaafar S, Elomri A. The carbon-constrained EOQ. Operations Research Letters 2013; 41(2):172-179

Cruz JM. Dynamics of supply chain networks with corporate social responsibility through integrated environmental decision-making. European Journal of Operational Research 2008; 184:

1005-1031.

Dekker R, Bloemhof J, Mallidis I. Operations Research for green logistics–An overview of aspects, issues, contributions and challenges. European Journal of Operational Research 2012; 219(3): 671-679.

Diabat A, Simchi-Levi D. A Carbon-Capped Supply Chain Network Problem. IEEE International Conference on Industrial Engineering and Engineering Management 2010; doi.org/10.1109/IEEM.2009.5373289.

Dobos I. The effects of emission trading on production and inventories in the Arrow–Karlin model. International Journal of Production Economics 2005; 93–94: 301–308.

Du SF, Zhu LL, Liang L, Ma F. Emission-dependent supply chain and environment-policy-making in the 'cap-and-trade' system. Energy Policy 2013; 57: 61-67.

Du SF, Ma F, Fu ZJ, Zhu LL, Zhang JJ. Game-theoretic analysis for an emission-dependent supply chain in a 'cap-and-trade' system. Annals of Operations Research 2011; doi.org/10.1007/s10479-011-0964-6.

Eskandarpour M, Dejax P, Miemczyk J, Peton O. Sustainable supply chain network design: An optimization-oriented review. Omega: International Journal of Management Science 2015 ; 54: 11-32.

Gong X, Zhou SX. Optimal production planning with emission trading. Operations Research 2013; 61(4):908-924.

Goyal M, Netessine S. Strategic technology choice and capacity investment under demand uncertainty. Management Science 2007, 53(2): 192-207.

Hong ZF, Chu CB, Yu YG. Optimization of Production Planning for Green Manufacturing. 2012 9th IEEE International Conference on Networking Sensing and Control 2012; 193-196.

Hua GW, Cheng TCE, Wang SY. Managing carbon footprints in inventory management. International Journal of Production Economics 2011; 132:178–185.

Li SD, Gu MD. The effect of emission permit trading with banking on firm's production–inventory strategies. International Journal of Production Economics 2012; 137: 304–308.

Liu ZL, Anderson TD, Cruz JM. Consumer environmental awareness and competition in two-stage supply chains. European Journal of Operational Research 2012; 218: 602-613.

Pati RK, Vrat P, Kumar P. A goal programming model for paper recycling system. Omega: International Journal of Management Science 2008; 36(3): 405-417.

Song JP, Leng MM. Analysis of the Single-Period Problem under Carbon Emissions Policies. International Series in Operations Research & Management Science 2011; 176: 297-313.

Stavins RN. Transaction costs and tradeable permits. Journal of Environmental Economics and Management 1995; 29:133-148.

Sundarakani B, Souza RD, Goh M, Wagner SM, Manikandan S. Modeling carbon footprints across the supply chain. International Journal of Production Economics 2010; 128: 43-50.

Woerdman E. Emissions trading and transaction cost: analyzing the flaws in the discussion. Ecological Economics 2001; 38: 293-304.

Zhang B, Xu L. Multi-item production planning with carbon cap and trade mechanism. International Journal of Production Economics 2013; 144: 118–127.

Appendix

Proof of Lemma 1. Based on the Model (4), we let $F(q_1,q_2)=(\alpha_1-q_1-\lambda q_2-c_{r_1}-w_1)q_1+(\alpha_2-q_2-\lambda q_1-c_{r_2}-w_2)q_2$, then the first partial derivatives of $F(q_1,q_2)$ with regard to q_1 and q_2 are $\partial F(q_1,q_2)/\partial q_1=\alpha_1-c_{r_1}-w_1-2q_1-2\lambda q_2$ and $\partial F(q_1,q_2)/\partial q_2=\alpha_2-c_{r_2}-w_2-2q_2-2\lambda q_1$, respectively, and the second partial derivatives of $F(q_1,q_2)$ with regard to q_1 and q_2 are that $\partial^2 F(q_1,q_2)/\partial q_1{}^2=-2$ and $\partial^2 F(q_1,q_2)/\partial q_2{}^2=-2$, and the second mixed partial derivative of $F(q_1,q_2)$ with regard to q_1 and q_2 is $\partial^2 F(q_1,q_2)/\partial q_1\partial q_2=-2\lambda$.

We let $H_1=\partial^2 F(q_1,q_2)/\partial q_1{}^2=-2$, $H_2=\partial^2 F(q_1,q_2)/\partial q_2{}^2=-2$, and $H_3=\partial^2 F(q_1,q_2)/\partial q_1\partial q_2=-2\lambda$, then we can get that $H_1H_2-(H_3)^2>0$ and $H_1<0$. So we can obtain the optimal solution of Π_R when the first partial derivatives of $F(q_1,q_2)$ with regard to q_1 and q_2 are equal to zero, that is, $q_i=[\alpha_i-c_{r_i}-w_i-(\alpha_{3-i}-c_{r_{3-i}}-w_{3-i})\lambda]/[2(1-\lambda^2)]$ ($i=1,2$).

Proof of Lemma 2. Since $E_1+E_2=E$, $f(E)$ can be rewritten as the following equation

$$f(E)=\max_{E_1\le E}\{\gamma_1E_1-\eta_1E_1^2+\gamma_2(E-E_1)-\eta_2(E-E_1)^2-2\bar{\lambda}E_1(E-E_1)\} \tag{A.1}$$

Let $g(E_1)=\gamma_1E_1-\eta_1E_1^2+\gamma_2(E-E_1)-\eta_2(E-E_1)^2-2\bar{\lambda}E_1(E-E_1)$. Denote by $g'(E_1)$ and $g''(E_1)$ the first and second derivatives of $g(E_1)$, respectively. It can be verified that $g'(E_1)=\gamma_1-\gamma_2+2\eta_2E-2\bar{\lambda}E+4\bar{\lambda}E_1-2\eta_1E_1-2\eta_2E_1$ and $g''(E_1)=-2(2\bar{\lambda}-\eta_1-\eta_2)<0$. Note that $g''(E_1)<0$ implies that (i) $g'(E_1)$ is strictly decreasing in E_1, (ii) $g(E_1)$ is strictly concave, and (3) $f(E)$ is also strictly concave because strict concavity is preserved under maximization.

We know that $g'(0)=\gamma_1-\gamma_2+2\eta_2 E-2\bar{\lambda}E$ and $g'(E)=\gamma_1-\gamma_2-2\eta_1 E+2\bar{\lambda}E$. It is obvious that the optimal solution E_1^* to Eq.(A.1) depends on the sign of $g'(0)$ and $g'(E)$. Note that $\gamma_2 \le \gamma_1$. (1) when $\eta_1>\bar{\lambda}$ and $\eta_2 \ge \bar{\lambda}$, it is clear that $g'(0)=\gamma_1-\gamma_2+2\eta_2 E-2\bar{\lambda}E \ge 0$. So the optimal solution E_1^* to Eq.(A.1) depends on the sign of $g'(E)=\gamma_1-\gamma_2-2\eta_1 E+2\bar{\lambda}E$. If $g'(E)=\gamma_1-\gamma_2-2\eta_1 E+2\bar{\lambda}E \ge 0$, then $E_1^*=E$. Otherwise (i.e., if $g'(E)=\gamma_1-\gamma_2-2\eta_1 E+2\bar{\lambda}E<0$), $E_1^*=(\gamma_1-\gamma_2+2\eta_2 E)/2(\eta_1+\eta_2)$. Hence, if $E \le \underline{C}_0$, then we have $g'(E)\ge 0$. So, the optimal solutions satisfy $E_1^*=E$ and $E_2^*=0$, and $f(E)=\gamma_1 E-\eta_1 E^2$; if $E \ge \underline{C}_0$, then $g'(E)\le 0$. So, the optimal solutions satisfy $E_1^*=[\gamma_1-\gamma_2+2(\eta_2-\bar{\lambda})E]/[2(\eta_1+\eta_2-2\bar{\lambda})]$ and $E_2^*=(\gamma_2-\gamma_1+2(\eta_1-\bar{\lambda})E)/[2(\eta_1+\eta_2-2\bar{\lambda})]$, and $f(E)=\{(\gamma_1-\gamma_2)^2+4[\gamma_1(\eta_2-\bar{\lambda})+\gamma_2(\eta_1-\bar{\lambda})]E-4(\eta_1\eta_2-\bar{\lambda}^2)E^2\}/[4(\eta_1+\eta_2-2\bar{\lambda})]$. (2) when $\eta_1>\bar{\lambda}$ and $\eta_2<\bar{\lambda}$, we can find that $0<\underline{C}_0 \le \underline{C}^0$ because of $\eta_1+\eta_2>2\bar{\lambda}$. If $g'(E)=\gamma_1-\gamma_2-2\eta_1 E+2\bar{\lambda}E \ge 0$, then $g'(0)=\gamma_1-\gamma_2+2\eta_2 E-2\bar{\lambda}E \ge 0$. So, $E_1^*=E$ when $E \le \underline{C}_0$. If $g'(0)=\gamma_1-\gamma_2+2\eta_2 E-2\bar{\lambda}E \le 0$, then $g'(E)=\gamma_1-\gamma_2-2\eta_1 E+2\bar{\lambda}E \le 0$. So, $E_1^*=0$ when $E \ge \underline{C}^0$. When $\underline{C}_0 \le E \le \underline{C}^0$, then $g'(0)=\gamma_1-\gamma_2+2\eta_2 E-2\bar{\lambda}E \ge 0$ and $g'(E)=\gamma_1-\gamma_2-2\eta_1 E+2\bar{\lambda}E \le 0$. So, $E_1^*=[\gamma_1-\gamma_2+2(\eta_2-\bar{\lambda})E]/[2(\eta_1+\eta_2-2\bar{\lambda})]$. Since $E_1+E_2=E$, we can easily get E_2^*. (3) when $\eta_1 \le \bar{\lambda}$ and $\eta_2 \ge \bar{\lambda}$, we can find that $g'(0)\ge g'(E)\ge 0$ because of $\eta_1+\eta_2>2\bar{\lambda}$. So, $E_1^*=E$ and $E_2^*=0$.

Before proving Theorem 1, we first show the following lemma:

Lemma A.1. *Suppose that* A *and* B *are constants. The following results hold*:

$$\arg\max_E\{f(E)-pE : A \le E \le B \le \underline{C}_0\}=\max\{A,\min\{(\gamma_1-p)/(2\eta_1),B\}\}, \tag{A.2}$$

$$\arg\max_E\{f(E)-pE : \underline{C}_0 \le A \le E \le B\}$$
$$=\max\{A,\min\{[(\gamma_1-p)(\eta_2-\bar{\lambda})+(\gamma_2-p)(\eta_1-\bar{\lambda})]/[2(\eta_1\eta_2-\bar{\lambda}^2)],B\}\} \tag{A.3}$$

Proof of Theorem 1. When $E \le \underline{C}_0$, Lemma 2(1) indicates that $f(E)=\gamma_1 E-\eta_1 E^2$. From the strict concavity of $f(E)$, it is easy to verify that $f(E)-pE$ attains its maximum over $A \le E \le B \le \underline{C}_0$ at point $\max\{A,\min\{(\gamma_1-p)/2\eta_1,B\}\}$. That is,

$$\arg\max_E\{f(E)-pE : A \le E \le B \le \underline{C}_0\}=\max\{A,\min\{(\gamma_1-p)/(2\eta_1),B\}\}.$$

When $E \ge \underline{C}_0$, we have $f(E)=\{(\gamma_1-\gamma_2)^2+4[\gamma_1(\eta_2-\bar{\lambda})+\gamma_2(\eta_1-\bar{\lambda})]E-4(\eta_1\eta_2-\bar{\lambda}^2)E^2\}/[4(\eta_1+\eta_2-2\bar{\lambda})]$ from Lemma 2 (1). The strict concavity of $f(E)$ implies that $f(E)-pE$ attains its maximum over $\underline{C}_0 \le A \le E \le B$ at point $\max\{A,\min\{[(\gamma_1-p)(\eta_2-\bar{\lambda})+(\gamma_2-p)(\eta_1-\bar{\lambda})]/[2(\eta_1\eta_2-\bar{\lambda}^2)],B\}\}$. That is,

$$\arg\max_E\{f(E)-pE:\underline{C}_0 \le A \le E \le B\}$$
$$=\max\{A,\min\{[(\gamma_1-p)(\eta_2-\bar{\lambda})+(\gamma_2-p)(\eta_1-\bar{\lambda})]/[2(\eta_1\eta_2-\bar{\lambda}^2)],B\}\}.$$

We now develop the optimal decisions in the case with $(\gamma_2\eta_1-\gamma_1\bar{\lambda})/(\eta_1-\bar{\lambda}) \le s$. Recall that M_2 is the maximal profit per unit of emission permit of product 2. $M_2 \le s$ indicates that the manufacturer will never produce product 2. So, the optimal total emissions are no more than $\underline{C}_0$ based on Lemma 2(1). So in finding the optimal total emissions, we can directly restrict them with a maximum $\underline{C}_0$. We next show the optimal total emissions and the optimal production quantities in two cases:

Case 1 $M_2 \le s < \gamma_1 \le b$

In this case, $C_b^1=0$. The interval $0 \le C < C_b^1$ is empty. Recall that γ_1 is the maximal profit per unit of emission permit of product 1. $\gamma_1 \le b$ indicates that the manufacturer will never buy emission permits to produce product 1.

Subcase 1. $C_b^1 \le C \le C_s^1$

$C_b^1 \le C \le C_s^1$ implies that $0 \le C \le (\gamma_1-s)/2\eta_1 \le (\gamma_1-\gamma_2)/[2(\eta_1-\bar{\lambda})]=\underline{C}_0$.

Since $\Pi_M^s=\max_{E\le C}\{f(E)-sE+sC\}=\max_{0\le E\le C\le \underline{C}_0}\{f(E)-sE+sC\}$, we have

$$\arg\max_E\{f(E)-sE+sC:E\le C\}=\arg\max_E\{f(E)-sE+sC:0\le E\le C\le \underline{C}_0\}.$$

Equation (A.2) implies that

$$\arg\max_E\{f(E)-sE+sC:0\le E\le C\le \underline{C}_0\}=\max\{0,\min\{(\gamma_1-s)/(2\eta_1),C\}\}=C, \quad (A.4)$$

Equations (A.4) indicate that the optimal total emissions are $E^*=C$, and the manufacturer neither buys nor sells emission permits. Recall that $q_2^*\equiv 0$. So, the optimal production quantity of product 1 is $q_1^*=C/e_1$.

Subcase 2. $C > C_s^1$

$C > C_s^1$ implies that $C > (\gamma_1-s)/(2\eta_1)$. Note that $C_s^1 \le \underline{C}_0$. Similar to Subcase 1, we have

$$\arg\max_E\{f(E)-sE+sC:0\le E\le C\}=\arg\max_E\{f(E)-sE+sC:0\le E\le \min(\underline{C}_0,C)\}.$$

Equation (A.2) implies that

$$\arg\max_E\{f(E)-sE+sC:0\le E\le C\}=\max\{0,\min\{(\gamma_1-s)/(2\eta_1),\min(\underline{C}_0,C)\}\}=C_s^1 \tag{A.5}$$

Since C is a feasible solution to $\Pi_M^s=\max_{0\le E\le C}\{f(E)-sE+sC\}$, Equations (A.5) indicate that the optimal total emissions are $E^*=C_s^1$, the manufacturer sells $C-C_s^1$ unit emission permits. However, the maximal sales of emission permits to outside market are V. We have that (i) if $C\le C_s^1+V$, then $E^*=C_s^1$; (ii) if $C_s^1+V\le C<\gamma_1/(2\eta_1)+V$, then $E^*=C-V$; (iii) if $C\ge\gamma_1/(2\eta_1)+V$, then $E^*=\gamma_1/(2\eta_1)$. Recall that $q_2^*\equiv 0$. So, the optimal production quantity of product 1 is $q_1^*=C_s^1/e_1$.

<u>Case 2</u> $M_2\le s<b<\gamma_1$

In this case, $C_b^1=(\gamma_1-b)/(2\eta_1)$. $0\le C<C_b^1$ implies that $0\le C\le(\gamma_1-b)/(2\eta_1)\le(\gamma_1-s)/(2\eta_1)\le(\gamma_1-\gamma_2)/[2(\eta_1-\overline{\lambda})]=\underline{C}_0$. Recall that $\Pi_M^b=\max_{E\ge C\ge 0}\{f(E)-bE+bC\}=\max_{C\le E\le\underline{C}_0}\{f(E)-bE+bC\}$. Based on Equation (A.2), we have

$$\arg\max_E\{f(E)-bE+bC:E\ge C\}=\max\{C,\min\{(\gamma_1-b)/(2\eta_1),\underline{C}_0\}\}=C_b^1, \tag{A.6}$$

because $(\gamma_1-b)/(2\eta_1)=\mathrm{C}_b^1$ and $C\le(\gamma_1-b)/(2\eta_1)\le\underline{C}_0$. Since $C_b^1\le\underline{C}_0$, Equation (A.2) implies that

$$\arg\max_E\{f(E)-sE+sC:0\le E\le C\}=\max\{0,\min\{(\gamma_1-s)/(2\eta_1),C\}\}=C, \tag{A.7}$$

because $0\le C\le(\gamma_1-s)/(2\eta_1)$.

Since C is a feasible solution to $\Pi_M^b=\max_{E\ge C\ge 0}\{f(E)-bE+bC\}$, Equations (A.6) and (A.7) indicate that the optimal total emissions are C_b^1, and the manufacturer buys C_b^1-C unit emission permits. However, the emission permits buying from carbon trade market are no more than T, so $E^*=\min\{T+C,C_b^1\}$. Recall that $q_2^*\equiv 0$. So, the optimal production quantity of product 1 is $q_1^*=\min\{T+C,C_b^1\}/e_1$.

Following the same process in proving Subcases 1 and 2 of Case 1, we can similarly prove the optimal total emissions and the optimal production quantities for the case with $\gamma_1>b$ when $C_b^1\le C\le C_s^1$ and $C>C_s^1$, respectively. □

Similarly, we can get the optimal total emissions and production quantities in the case with $s<M_2\le b$ and $M_2>b$, respectively.

Before proving Theorem 2, we first show the following lemma:

Lemma A.2. *Suppose that* A *and* B *are constants. The following results hold*:

$$\arg\max_E\{f(E)-pE: A\le E\le B\le \underline{C}_0\}=\max\{A,\min\{(\gamma_1-p)/(2\eta_1),B\}\}, \quad \text{(A.8)}$$

$$\arg\max_E\{f(E)-pE: \underline{C}_0\le A\le E\le B\le \underline{C}^0\}$$
$$=\max\{A,\min\{[(\gamma_1-p)(\eta_2-\bar{\lambda})+(\gamma_2-p)(\eta_1-\bar{\lambda})]/[2(\eta_1\eta_2-\bar{\lambda}^2)],B\}\} \quad \text{(A.9)}$$

$$\arg\max_E\{f(E)-pE: \underline{C}^0\le A\le E\le B\}=\max\{A,\min\{(\gamma_2-p)/(2\eta_2),B\}\} \quad \text{(A.10)}$$

Proof of Theorem 2. Similar to the proof of Theorem 1, we now develop the optimal decisions in the case with $s\le b<M_1\le M_2$. It is easy to verify that $M_2\le\gamma_1$ because of $\gamma_2\le\gamma_1$. Recall that M_1 is the maximal profit per unit of emission permit when the manufacturer only wants to produce product 2. $M_1>b$ indicates that the manufacturer will only produce product 2. So, the optimal total emissions are no less than $\underline{C}^0$ based on Lemma 2(2). So in finding the optimal total emissions, we can directly restrict them with a minimum $\underline{C}^0$. We next show the optimal total emissions as follows:

<u>Case 1</u> $0\le C<C_b^3$

In this case, $C_b^3=(\gamma_2-b)/(2\eta_2)$. $0\le C<C_b^3$ implies that $0\le C\le(\gamma_2-b)/(2\eta_2)\le(\gamma_2-s)/(2\eta_2)$. Note that $C_b^3>\underline{C}^0$. Similar to Theorem 1, we have

$$\arg\max_E\{f(E)-sE+sC: 0\le E\le C\}=\arg\max_E\{f(E)-sE+sC: \underline{C}^0\le E\le C\}.$$

Equation (A.10) implies that

$$\arg\max_E\{f(E)-sE+sC: \underline{C}^0\le E\le C\}=\max\{\underline{C}^0,\min\{(\gamma_2-s)/(2\eta_2),C\}\}=C. \quad \text{(A.11)}$$

Recall that $\Pi_M^b=\max_{E\ge C\ge 0}\{f(E)-bE+bC\}=\max_{\max(C,\underline{C}^0)\le E}\{f(E)-bE+bC\}$. Based on Equation (A.10), we have

$$\arg\max_E\{f(E)-bE+bC: E\ge\max\{C,\underline{C}^0\}\}$$
$$=\max\{\max\{C,\underline{C}^0\},\min\{(\gamma_2-b)/(2\eta_2),+\infty\}\}=C_b^3 \quad \text{(A.12)}$$

Since C is a feasible solution to $\Pi_M^b=\max_{E\ge C\ge 0}\{f(E)-bE+bC\}$, Equations (A.11) and Equations (A.12) indicate that the optimal total emissions are C_b^3, and the manufacturer buys C_b^3-C unit emission permits. However, the emission permits buying from carbon trade market are no more than T, so ① if $\min\{C+T,C_b^3\}<\underline{C}_0$, the optimal emissions are $E^*=\min\{C+T,C_b^3\}$, and the optimal production quantities are $q_1^*=\min\{C+T,C_b^3\}/e_1$ and $q_2^*=0$; ② if $\underline{C}_0\le\min\{C+T,C_b^2\}\le\underline{C}^0$, the optimal emissions are $E^*=\min\{C+T,C_b^3\}$, the optimal production quantities are $q_1^* = [\gamma_1 - \gamma_2 + 2(\eta_2 - \bar{\lambda})\min\{C+T, C_b^3\}]/[2e_1(\eta_1 + \eta_2 - 2\bar{\lambda})]$ and $q_2^* = (\gamma_2 - \gamma_1 + 2(\eta_1 - \bar{\lambda})\min\{C+T, C_b^3\})/[2e_2(\eta_1 + \eta_2 - 2\bar{\lambda})]$; ③ if $\min\{C+T, C_b^2\} > \underline{C}^0$, the optimal emissions are $E^* = \min\{C+T, C_b^3\}$, the optimal production quantities are $q_1^* = 0$ and $q_2^* = \min\{C+T, C_b^3\})/e_2$.

<u>Case 2</u> $C_b^3 \le C < C_s^3$

$C_b^3 \le C \le C_s^3$ implies that $C \ge (\gamma_1 - \gamma_2)/[2(\bar{\lambda} - \eta_2)] = \underline{C}^0$.

Since $\Pi_M^s = \max_{E \le C}\{f(E) - sE + sC\} = \max_{\underline{C}^0 \le E \le C}\{f(E) - sE + sC\}$, we have

$\arg\max_E\{f(E) - sE + sC : E \le C\} = \arg\max_E\{f(E) - sE + sC : \underline{C}^0 \le E \le C\}$.

Equation (A.10) implies that

$$\arg\max_E\{f(E) - sE + sC : \underline{C}^0 \le E \le C\} = \max\{\underline{C}^0, \min\{(\gamma_2 - s)/(2\eta_2), C\}\} = C, \tag{A.13}$$

Recall that $\Pi_M^b = \max_{E \ge C \ge 0}\{f(E) - bE + bC\} = \max_{\underline{C}^0 \le C \le E}\{f(E) - bE + bC\}$. Based on Equation (A.10), we have

$$\arg\max_E\{f(E) - bE + bC : E \ge C \ge \underline{C}^0\} = \max\{C, \min\{(\gamma_2 - b)/(2\eta_2), +\infty\}\} = C \tag{A.14}$$

Equations (A.13) and Equations (A.14) indicate that the optimal total emissions are $E^* = C$, and the manufacturer neither buys nor sells emission permits. Recall that $q_1^* \equiv 0$. So, the optimal production quantity of product 2 is $q_2^* = C/e_2$.

<u>Case 3</u> $C \ge C_s^3$

$C \ge C_s^3$ implies that $C \ge (\gamma_2 - s)/(2\eta_2) \ge (\gamma_2 - b)/(2\eta_2) \ge (\gamma_1 - \gamma_2)/[2(\bar{\lambda} - \eta_2)] = \underline{C}^0$.

Similar to Case 1, we have

$\arg\max_E\{f(E) - sE + sC : \underline{C}^0 \le E \le C\} = \arg\max_E\{f(E) - sE + sC : \underline{C}^0 \le E \le C\}$.

Equation (A.10) implies that

$$\arg\max_E\{f(E) - sE + sC : \underline{C}^0 \le E \le C\} = \max\{\underline{C}^0, \min\{(\gamma_2 - s)/(2\eta_2), C\}\} = C_s^3 \tag{A.15}$$

Recall that $\Pi_M^b = \max_{E \ge C \ge 0}\{f(E) - bE + bC\} = \max_{\underline{C}^0 \le C \le E}\{f(E) - bE + bC\}$. Based on Equation (A.10), we have

$$\arg\max_E\{f(E) - bE + bC : E \ge C \ge \underline{C}^0\} = \max\{C, \min\{(\gamma_2 - b)/(2\eta_2), +\infty\}\} = C \tag{A.16}$$

Since C is a feasible solution to $\Pi_M^b = \max_{E \ge C \ge 0}\{f(E) - bE + bC\}$, Equations

(A.15) and (A.16) indicate that the optimal total emissions are C_s^3, and the manufacturer sells $C-C_s^3$ unit emission permits. However, the maximal sales of emission permits to outside market are V. We have that (i) if $C_s^3 \le C < C_s^3 + V$, then $E^* = C_s^3$; (ii) if $C_s^3 + V \le C < \gamma_2/(2\eta_2) + V$, then $E^* = C - V$; (iii) if $C \ge \gamma_2/(2\eta_2) + V$, then $E^* = \gamma_2/(2\eta_2)$. Recall that $q_1^* \equiv 0$. So, the optimal production quantity of product 2 is $q_2^* = C_s^3/e_2$.

Similarly, we can get the optimal total emissions and production quantities in the case with $s < M_1 \le b < M_2$, $s < M_1 \le M_2 \le b$, $M_1 \le s \le b < M_2$, $M_1 \le s < M_2 \le b$ and $M_1 \le M_2 \le s \le b$.

Proof of Theorem 3.

Follow the same process in proving Theorem 1, we can easily get the results shown in Theorem 3.

Proof of Corollary 1.

We now explore the relationship of the manufacturer's optimal profit and the buying price of emission permits in the case with $\eta_1 > \bar{\lambda}$ and $\eta_2 \ge \bar{\lambda}$. When $M_2 > b$, we can verify that $\partial \Pi_M/\partial b = C - C_b^2$ or $\partial \Pi_M/\partial b = -T$ if $0 \le C < C_b^2$. It is obvious that $\partial \Pi_M/\partial b \le 0$. Note that the manufacturer's optimal profit keeps unchanged if $C > C_b^2$. When $s \le M_2 \le b$, we can verify that $\partial \Pi_M/\partial b = (-b)/(2\eta_1) \le 0$ or $\partial \Pi_M/\partial b = -T \le 0$ if $0 \le C < C_b^1$. Note that the manufacturer's optimal profit remains constant if $C > C_b^1$. It is obvious that the function of the manufacturer's optimal profit is continuous about the buying price of emission permits. So, the manufacturer's optimal profit is decreasing in b when $\eta_1 > \bar{\lambda}$ and $\eta_2 \ge \bar{\lambda}$. Following the same process, we can get that the manufacturer's optimal profit is increasing in s when $\eta_1 > \bar{\lambda}$ and $\eta_2 \ge \bar{\lambda}$. Similarly, we can get that the manufacturer's optimal profit is decreasing in b and is increasing in s when (1) $\eta_1 > \bar{\lambda}$ and $\eta_2 < \bar{\lambda}$ or (2) $\eta_1 \le \bar{\lambda}$ and $\eta_2 \ge \bar{\lambda}$.

We explore the relationship of the retailer's optimal profit and the buying price of emission permits in the case with $\eta_1 > \bar{\lambda}$ and $\eta_2 \ge \bar{\lambda}$. We know that the retailer's optimal profit is $\Pi_R = [\eta_1 (E_1^*)^2]/2 + [\eta_2 (E_2^*)^2]/2 + \bar{\lambda} E_1^* E_2^*$. When $M_2 > b$, we can verify that $\partial \Pi_R/\partial b = -1/(2C_b^2) < 0$ or $\partial \Pi_R/\partial b = 0$ if $0 \le C < C_b^2$. Note that the retailer's optimal profit remains constant as the buying price of emission permits increases if $C > C_b^2$. When $s < M_2 \le b$, we can verify that $\partial \Pi_R / \partial b = -(\gamma_1 - b)/(4\eta_1) < 0$ or $\partial \Pi_R / \partial b = 0$ if $0 \le C < C_b^1$. Note that the retailer's optimal profit remains constant if $C > C_b^1$. It is obvious that the function of the retailer's optimal profit is continuous about the buying price of emission permits. So, the retailer's optimal profit is decreasing in b when $\eta_1 > \bar{\lambda}$ and $\eta_2 \ge \bar{\lambda}$. Following the same process, we can get that the retailer's optimal profit is decreasing in s when $\eta_1 > \bar{\lambda}$ and $\eta_2 \ge \bar{\lambda}$. Similarly, we can get that the retailer's optimal profit is decreasing in b (s) when (1) $\eta_1 > \bar{\lambda}$ and $\eta_2 < \bar{\lambda}$ or (2) $\eta_1 \le \bar{\lambda}$ and $\eta_2 \ge \bar{\lambda}$.

Proof of Corollary 2.

(1) From Theorem 1, we can easily get that E^*, q_1^* and q_2^* are decreasing in b (s).

(2) From Theorem 2, we know that $q_1^* \equiv 0$ when $s \le b < M_1 \le M_2$ and $q_1^* = \dfrac{\gamma_1 - \gamma_2 + 2(\eta_2 - \bar{\lambda}) \min\{C+T, C_b^2\}}{2e_1(\eta_1 + \eta_2 - 2\bar{\lambda})}$ when $s < M_1 \le b < M_2$ and $0 \le C < C_b^1$. It is easy to verify that q_1^* is increasing in b because of $\eta_2 < \bar{\lambda}$. $q_1^* = \min\{T+C, C_b^1\}/e_1$ when $s < M_1 \le M_2 \le b$ and $0 \le C < C_b^1$. It is easy to verify that q_1^* is decreasing in b. So, q_1^* is firstly increasing and then decreasing in b. Similarly, we can have q_1^* is firstly increasing and then decreasing in s. From Theorem 2, we can easily get that E^* and q_2^* are decreasing in b (s).

(3) From Theorem 3, we can get that E^* and q_1^* are decreasing in b (s).

Table 1: When $\eta_1 > \bar{\lambda}$ and $\eta_2 \ge \bar{\lambda}$, the optimal total emissions and production quantities of the two products over different caps

	C	E^*	q_1^* and q_2^*
$M_2 \le s$	$0 \le C < C_b^1$	$\min\{T+C, C_b^1\}$	$q_1^* = \min\{T+C, C_b^1\}/e_1$, $q_2^* = 0$
	$C_b^1 \le C < C_s^1$	C	$q_1^* = C/e_1$, $q_2^* = 0$
	$C_s^1 \le C < C_s^1 + V$	C_s^1	$q_1^* = C_s^1/e_1$, $q_2^* = 0$

	$C_s^1+V \le C < \gamma_1/(2\eta_1)+V$	$C-V$	$q_1^*=(C-V)/e_1$, $q_2^*=0$
	$C \ge \gamma_1/(2\eta_1)+V$	$\gamma_1/(2\eta_1)$	$q_1^*=\gamma_1/(2e_1\eta_1)$, $q_2^*=0$
$s<M_2\le b$	$0\le C<C_b^1$	$\min\{T+C,C_b^1\}$	$q_1^*=\min\{T+C,C_b^1\}/e_1$, $q_2^*=0$
	$C_b^1\le C\le \underline{C}_0$	C	$q_1^*=C/e_1$, $q_2^*=0$
	$\underline{C}_0\le C\le C_s^2$	C	$q_1^*=\frac{\gamma_1-\gamma_2+2(\eta_2-\bar{\lambda})C}{2e_1(\eta_1+\eta_2-2\bar{\lambda})}$ $q_2^*=\frac{\gamma_2-\gamma_1+2(\eta_1-\bar{\lambda})C}{2e_2(\eta_1+\eta_2-2\bar{\lambda})}$
	$C_s^2<C\le C_s^2+V$	C_s^2	$q_1^*=\frac{\gamma_1\eta_2-\gamma_2\bar{\lambda}-s\eta_2+s\bar{\lambda}}{2e_1(\eta_1\eta_2-\bar{\lambda}^2)}$ $q_2^*=\frac{\gamma_2\eta_1-\gamma_1\bar{\lambda}-s\eta_1+s\bar{\lambda}}{2e_2(\eta_1\eta_2-\bar{\lambda}^2)}$
	$C_s^2+V<C\le C_s+V$	$C-V$	$q_1^*=\frac{\gamma_1-\gamma_2+2(\eta_2-\bar{\lambda})(C-V)}{2e_1(\eta_1+\eta_2-2\bar{\lambda})}$ $q_2^*=\frac{\gamma_2-\gamma_1+2(\eta_1-\bar{\lambda})(C-V)}{2e_2(\eta_1+\eta_2-2\bar{\lambda})}$
	$C>C_s+V$	C_s	$q_1^*=(\gamma_1\eta_2-\gamma_2\bar{\lambda})/[2e_1(\eta_1\eta_2-\bar{\lambda}^2)]$ $q_2^*=(\gamma_2\eta_1-\gamma_1\bar{\lambda})/[2e_2(\eta_1\eta_2-\bar{\lambda}^2)]$
$M_2>b$	$0\le C<C_b^2$: $\min\{C+T,C_b^2\}<\underline{C}_0$	$C+T$	$q_1^*=\min\{C+T,C_b^2\}/e_1$, $q_2^*=0$
	$0\le C<C_b^2$: $\min\{C+T,C_b^2\}\ge\underline{C}_0$	$\min\{C+T,C_b^2\}$	$q_1^*=\frac{\gamma_1-\gamma_2+2(\eta_2-\bar{\lambda})\min\{C+T,C_b^2\}}{2e_1(\eta_1+\eta_2-2\bar{\lambda})}$ $q_2^*=\frac{\gamma_2-\gamma_1+2(\eta_1-\bar{\lambda})\min\{C+T,C_b^2\}}{2e_2(\eta_1+\eta_2-2\bar{\lambda})}$
	$C_b^2\le C\le C_s^2$	C	$q_1^*=\frac{\gamma_1-\gamma_2+2(\eta_2-\bar{\lambda})C}{2e_1(\eta_1+\eta_2-2\bar{\lambda})}$

			$q_2^* = \dfrac{\gamma_2-\gamma_1+2(\eta_1-\bar{\lambda})C}{2e_2(\eta_1+\eta_2-2\bar{\lambda})}$
	$C_s^2 < C \le C_s^2 + V$	C_s^2	$q_1^* = \dfrac{\gamma_1\eta_2-\gamma_2\bar{\lambda}-s\eta_2+s\bar{\lambda}}{2e_1(\eta_1\eta_2-\bar{\lambda}^2)}$ $q_2^* = \dfrac{\gamma_2\eta_1-\gamma_1\bar{\lambda}-s\eta_1+s\bar{\lambda}}{2e_2(\eta_1\eta_2-\bar{\lambda}^2)}$
	$C_s^2 + V < C \le C_s + V$	$C-V$	$q_1^* = \dfrac{\gamma_1-\gamma_2+2(\eta_2-\bar{\lambda})(C-V)}{2e_1(\eta_1+\eta_2-2\bar{\lambda})}$ $q_2^* = \dfrac{\gamma_2-\gamma_1+2(\eta_1-\bar{\lambda})(C-V)}{2e_2(\eta_1+\eta_2-2\bar{\lambda})}$
	$C > C_s + V$	C_s	$q_1^* = (\gamma_1\eta_2-\gamma_2\bar{\lambda})/[2e_1(\eta_1\eta_2-\bar{\lambda}^2)]$ $q_2^* = (\gamma_2\eta_1-\gamma_1\bar{\lambda})/[2e_2(\eta_1\eta_2-\bar{\lambda}^2)]$

Table 2: When $\eta_1 > \bar{\lambda}$ and $\eta_2 < \bar{\lambda}$, the optimal total emissions and production quantities of the two products over different caps

	C		E^*	q_1^* and q_2^*
$s \le b < M_1 \le M_2$	$0 \le C < C_b^3$	$\min\{C+T, C_b^3\} < \underline{C}_0$	$\min\{C+T, C_b^3\}$	$q_1^* = \min\{C+T, C_b^3\}/e_1$, $q_2^* = 0$
		$\underline{C}_0 \le \min\{C+T, C_b^3\} \le \underline{C}^0$		$q_1^* = \dfrac{\gamma_1-\gamma_2+2(\eta_2-\bar{\lambda})\min\{C+T, C_b^3\}}{2e_1(\eta_1+\eta_2-2\bar{\lambda})}$ $q_2^* = \dfrac{\gamma_2-\gamma_1+2(\eta_1-\bar{\lambda})\min\{C+T, C_b^3\}}{2e_2(\eta_1+\eta_2-2\bar{\lambda})}$
		$\min\{C+T, C_b^3\} > \underline{C}^0$		$q_1^* = 0$, $q_2^* = \min\{C+T, C_b^3\})/e_2$
	$C_b^3 \le C < C_s^3$		C	$q_1^* = 0, q_2^* = C/e_2$
	$C_s^3 \le C < C_s^3 + V$		C_s^3	$q_1^* = 0, q_2^* = C_s^3/e_2$
	$C_s^3 + V \le C < \gamma_2/(2\eta_2) + V$		$C-V$	$q_1^* = 0, q_2^* = (C-V)/e_2$

	$C \ge \gamma_2/(2\eta_2)+V$		$\gamma_2/(2\eta_2)$	$q_1^*=0, q_2^*=\gamma_2/(2e_2\eta_2)$
$s<M_1 \le b<M_2$	$0 \le C < C_b^2$	$\min\{C+T, C_b^2\} < \underline{C}_0$	$\min\{C+T, C_b^2\}$	$q_1^*=\min\{C+T, C_b^2\}/e_1$， $q_2^*=0$
		$\min\{C+T, C_b^2\} \ge \underline{C}_0$		$q_1^*=\dfrac{\gamma_1-\gamma_2+2(\eta_2-\bar{\lambda})\min\{C+T, C_b^2\}}{2e_1(\eta_1+\eta_2-2\bar{\lambda})}$ $q_2^*=\dfrac{\gamma_2-\gamma_1+2(\eta_1-\bar{\lambda})\min\{C+T, C_b^2\}}{2e_2(\eta_1+\eta_2-2\bar{\lambda})}$
	$C_b^2 \le C \le \underline{C}^0$		C	$q_1^*=\dfrac{\gamma_1-\gamma_2+2(\eta_2-\bar{\lambda})C}{2e_1(\eta_1+\eta_2-2\bar{\lambda})}$ $q_2^*=\dfrac{\gamma_2-\gamma_1+2(\eta_1-\bar{\lambda})C}{2e_2(\eta_1+\eta_2-2\bar{\lambda})}$
	$\underline{C}^0 \le C \le C_s^3$		C	$q_1^*=0$， $q_2^*=C/e_2$
	$C_s^3 \le C < C_s^3+V$		C_s^3	$q_1^*=0$， $q_2^*=C_s^3/e_2$
	$C_s^3+V \le C < \gamma_2/(2\eta_2)+V$		$C-V$	$q_1^*=0, q_2^*=(C-V)/e_2$
	$C \ge \gamma_2/(2\eta_2)+V$		$\gamma_2/(2\eta_2)$	$q_1^*=0, q_2^*=\gamma_2/(2e_2\eta_2)$
$s<M_1 \le M_2 \le b$	$0 \le C < C_b^1$		$\min\{T+C, C_b^1\}$	$q_1^*=\min\{T+C, C_b^1\}/e_1$， $q_2^*=0$
	$C_b^1 \le C \le \underline{C}_0$		C	$q_1^*=C/e_1$， $q_2^*=0$
	$\underline{C}_0 \le C \le \underline{C}^0$		C	$q_1^*=\dfrac{\gamma_1-\gamma_2+2(\eta_2-\bar{\lambda})C}{2e_1(\eta_1+\eta_2-2\bar{\lambda})}$， $q_2^*=\dfrac{\gamma_2-\gamma_1+2(\eta_1-\bar{\lambda})C}{2e_2(\eta_1+\eta_2-2\bar{\lambda})}$
	$\underline{C}^0 < C \le C_s^3$		C	$q_1^*=0$， $q_2^*=C/e_2$

	$C_s^3 \le C < C_s^3 + V$		C_s^3	$q_1^* = 0$， $q_2^* = C_s^3 / e_2$
	$C_s^3 + V \le C < \gamma_2/(2\eta_2) + V$		$C - V$	$q_1^* = 0, q_2^* = (C - V)/e_2$
	$C \ge \gamma_2/(2\eta_2) + V$		$\gamma_2/(2\eta_2)$	$q_1^* = 0, q_2^* = \gamma_2/(2e_2\eta_2)$
$M_1 \le s \le b < M_2$	$0 \le C < C_b^2$	$\min\{C+T, C_b^2\} < \underline{C}_0$	$\min\{C+T, C_b^2\}$	$q_1^* = (C+T)/e_1$， $q_2^* = 0$
		$\min\{C+T, C_b^2\} \ge \underline{C}_0$		$q_1^* = \dfrac{\gamma_1 - \gamma_2 + 2(\eta_2 - \overline{\lambda})\min\{C+T, C_b^2\}}{2e_1(\eta_1 + \eta_2 - 2\overline{\lambda})}$ $q_2^* = \dfrac{\gamma_2 - \gamma_1 + 2(\eta_1 - \overline{\lambda})\min\{C+T, C_b^2\}}{2e_2(\eta_1 + \eta_2 - 2\overline{\lambda})}$
	$C_b^2 \le C \le C_s^2$		C	$q_1^* = \dfrac{\gamma_1 - \gamma_2 + 2(\eta_2 - \overline{\lambda})C}{2e_1(\eta_1 + \eta_2 - 2\overline{\lambda})}$ $q_2^* = \dfrac{\gamma_2 - \gamma_1 + 2(\eta_1 - \overline{\lambda})C}{2e_2(\eta_1 + \eta_2 - 2\overline{\lambda})}$
	$C_s^2 < C \le C_s^2 + V$		C_s^2	$q_1^* = \dfrac{\gamma_1\eta_2 - \gamma_2\overline{\lambda} - s\eta_2 + s\overline{\lambda}}{2e_1(\eta_1\eta_2 - \overline{\lambda}^2)}$ $q_2^* = \dfrac{\gamma_2\eta_1 - \gamma_1\overline{\lambda} - s\eta_1 + s\overline{\lambda}}{2e_2(\eta_1\eta_2 - \overline{\lambda}^2)}$
	$C_s^2 + V < C \le C_s + V$		$C - V$	$q_1^* = \dfrac{\gamma_1 - \gamma_2 + 2(\eta_2 - \overline{\lambda})(C - V)}{2e_1(\eta_1 + \eta_2 - 2\overline{\lambda})}$ $q_2^* = \dfrac{\gamma_2 - \gamma_1 + 2(\eta_1 - \overline{\lambda})(C - V)}{2e_2(\eta_1 + \eta_2 - 2\overline{\lambda})}$
	$C > C_s + V$		C_s	$q_1^* = (\gamma_1\eta_2 - \gamma_2\overline{\lambda})/[2e_1(\eta_1\eta_2 - \overline{\lambda}^2)]$ $q_2^* = (\gamma_2\eta_1 - \gamma_1\overline{\lambda})/[2e_2(\eta_1\eta_2 - \overline{\lambda}^2)]$
$M_1 \le s < M_2 \le b$	$0 \le C < C_b^1$		$\min\{T + C, C_b^1\}$	$q_1^* = \min\{T + C, C_b^1\}/e_1$， $q_2^* = 0$
	$C_b^1 \le C \le \underline{C}_0$		C	$q_1^* = C/e_1$ ， $q_2^* = 0$

	$\underline{C}_0 \le C \le C_s^2$	C	$q_1^* = \dfrac{\gamma_1 - \gamma_2 + 2(\eta_2 - \overline{\lambda})C}{2e_1(\eta_1 + \eta_2 - 2\overline{\lambda})}$ $q_2^* = \dfrac{\gamma_2 - \gamma_1 + 2(\eta_1 - \overline{\lambda})C}{2e_2(\eta_1 + \eta_2 - 2\overline{\lambda})}$
	$C_s^2 < C \le C_s^2 + V$	C_s^2	$q_1^* = \dfrac{\gamma_1\eta_2 - \gamma_2\overline{\lambda} - s\eta_2 + s\overline{\lambda}}{2e_1(\eta_1\eta_2 - \overline{\lambda}^2)}$ $q_2^* = \dfrac{\gamma_2\eta_1 - \gamma_1\overline{\lambda} - s\eta_1 + s\overline{\lambda}}{2e_2(\eta_1\eta_2 - \overline{\lambda}^2)}$
	$C_s^2 + V < C \le C_s + V$	$C - V$	$q_1^* = \dfrac{\gamma_1 - \gamma_2 + 2(\eta_2 - \overline{\lambda})(C-V)}{2e_1(\eta_1 + \eta_2 - 2\overline{\lambda})}$ $q_2^* = \dfrac{\gamma_2 - \gamma_1 + 2(\eta_1 - \overline{\lambda})(C-V)}{2e_2(\eta_1 + \eta_2 - 2\overline{\lambda})}$
	$C > C_s + V$	C_s	$q_1^* = (\gamma_1\eta_2 - \gamma_2\overline{\lambda})/[2e_1(\eta_1\eta_2 - \overline{\lambda}^2)]$ $q_2^* = (\gamma_2\eta_1 - \gamma_1\overline{\lambda})/[2e_2(\eta_1\eta_2 - \overline{\lambda}^2)]$
$M_1 \le M_2 \le s \le b$	$0 \le C < C_b^1$	$\min\{T + C, C_b^1\}$	$q_1^* = \min\{T + C, C_b^1\}/e_1$， $q_2^* = 0$
	$C_b^1 \le C < C_s^1$	C	$q_1^* = C/e_1$， $q_2^* = 0$
	$C_s^1 \le C < C_s^1 + V$	C_s^1	$q_1^* = C_s^1/e_1$， $q_2^* = 0$
	$C_s^1 + V \le C < \gamma_1/(2\eta_1) + V$	$C - V$	$q_1^* = (C - V)/e_1$， $q_2^* = 0$
	$C \ge \gamma_1/(2\eta_1) + V$	$\gamma_1/(2\eta_1)$	$q_1^* = \gamma_1/(2e_1\eta_1)$， $q_2^* = 0$

Highlights

- **We model a make-to-order supply chain with a manufacturer and a retailer.**
- **The production and pricing problems under cap-and-trade regulation are analyzed.**
- **Two substitutable or complementary products are discussed in our study.**
- **Cap-and-trade regulation may not induce producing low-carbon products.**