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Measurement and decomposition of the Malmquist productivity index for parallel production systems

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Abstract

The conventional Malmquist productivity index (MPI), which ignores the internal structure of a production system when measuring changes in performance between two periods, may produce misleading results. This paper thus takes the operations of the component processes into account in investigating the MPI of parallel production systems. A relational data envelopment analysis (DEA) model is developed to measure the biennial MPIs of the system and internal processes at the same time, and it is shown that the former is a linear combination of the latter. This decomposition helps identify the processes that cause the decline in performance of the system. An example of thirty-nine branches of a commercial bank, with deposits, sales, and services as the three major functions operating in parallel, is used to illustrate this approach.

Keywords: data envelopment analysis, Malmquist productivity index, parallel system, relational model.

1. Introduction

As competition in the global market is becoming ever more intense, production processes need to become more efficient in order to improve company performance, thus enabling firms to survive. Data envelopment analysis (DEA), developed by Charnes et al. [1], is perhaps the most effective approach for measuring the relative efficiencies of a set of decision making units (DMUs) that apply multiple inputs to produce multiple outputs. Based on the results of this analysis, a DMU is able to identify the sources of inefficiency, and thus work to make the necessary improvements. Since every DMU aims to improve its performance, a measure that shows the relative improvement of one DMU compared to the others is desired, and one way to obtain this is the Malmquist Productivity Index (MPI), which measures changes in the performance of a DMU between two periods. Some typical examples include Tunisian schools [2], Taiwanese scholars [3], and Iranian steam power plants [4]. This is also important for examining the effect of an act or policy over a period of time, such as the Sarbanes-Oxley Act of the US [5], the reorganization of Taiwanese forests [6], and the privatization of the Pakistani cement industry [7].

Conventionally, the DEA technique measures efficiency from the viewpoint of a system, the whole DMU, in which only the inputs consumed and the outputs produced by the system are considered. In other words, the system is treated as a black box, ignoring the operations of the component processes. As a result, a DMU that performs better in all processes than another one may still have a lower efficiency score [8]. If the efficiency measures are incorrect, then the subsequent measure of changes in performance will also be incorrect. For this reason, the operations of the component processes must be considered when the data is available, and this led to development of the network DEA method [9].

The network DEA approach takes the operations of the component processes of a system into account. Many network DEA models have been proposed [10], with most developed for two-stage systems, where all the inputs are supplied in the first stage to produce intermediate products for the second stage, which then produces the final outputs. Kao and Hwang [11] proposed a model to measure the MPI of this type of system, and showed that the system MPI is the product of the individual process MPIs. This property can be extended to multi-stage systems.

The purpose of this paper is to develop a model to measure the MPI of parallel systems, as in practice many systems have this structure. The most typical case is that an organization performs several functions in parallel; for example, teaching and research at universities [12], crime control and protection and maintenance of order at police departments [13], banking, insurance, and securities at financial holding companies [14], and sales and services at banks [15] (see Kao [10] for a detailed review of the related literature). Most of the models developed for parallel systems can only measure the system efficiency, which makes it impossible to examine the relationship between the system and process MPIs. However, the relational model proposed by Kao [16] is able to measure the system and process efficiencies at the same time. Moreover, in this model the system efficiency is a weighted average of the process efficiencies. One benefit of this is that the relatively inefficient processes can be identified. Since the MPI is a ratio of two efficiencies, the relationship between the MPI of the system and those of the processes can be explored based on this model.

This paper thus proposes a relational model to measure the system and process MPIs at the same time for parallel systems, and the relationship between them is also derived. To demonstrate how the proposed model can be applied to real world problems, the performance and changes in performance of thirty-nine branches of a commercial bank, with the data taken from Jahanshahloo et al. [17], are measured.

2. Biennial Malmquist productivity index

Let X_{ij} and Y_{rj} denote the ith input, i=1,…, m, and rth output, r=1,…, s, respectively, of the jth DMU, j=1,…, n. The conventional input-oriented CCR, or black-box, model for measuring the efficiency of the kth DMU under constant returns to scale can be formulated as follows [1]:

$$E_k^{\mathrm{CCR}}=\max.\ \sum_{r=1}^{s} u_r Y_{rk}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_i X_{ik} = 1$$

$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leq 0, \quad j=1,\ldots, n \qquad (1)$$

$$u_r, v_i \geq \varepsilon, \quad r=1,\ldots, s, i=1,\ldots, m$$

where u_r and v_i are virtual multipliers, and ε is a small non-Archimedean number [18] imposed to avoid ignoring any input or output factor in calculating efficiencies. Although the input- and output-oriented CCR models yield the same efficiency measure, this paper adopts the input-oriented one for measuring the MPI. For cases that some inputs are non-discretionary whose values cannot be reduced, one may need to use the output-oriented model.

Suppose each DMU has q processes operating in parallel. Let $X_{ij}^{(p)}$ and $Y_{rj}^{(p)}$ denote the input and output of the pth process, such that the total amount of all q processes is equal to that of the DMU, i.e., $X_{ij} = \sum_{p=1}^{q} X_{ij}^{(p)}$ and $Y_{rj} = \sum_{p=1}^{q} Y_{rj}^{(p)}$. Figure 1 shows the structure of a general parallel system. Theoretically, every process can consume all m inputs and produce all s outputs. In reality, however, a process will consume only certain inputs and produce certain outputs. Here the general notation of $X_{ij}^{(p)}$, i=1,…, m, and $Y_{rj}^{(p)}$, r=1,…, s, is used for simplicity of expression. In applications, many $X_{ij}^{(p)}$ and $Y_{rj}^{(p)}$ are zero. In order to obtain more meaningful results, Kao [16] developed a relational model, which takes the operations of the q processes into account, as follows:

$$E_k^{\text{Kao}} = \max. \sum_{r=1}^{s} u_r Y_{rk}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_i X_{ik} = 1$$

system constraints:

$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leq 0, \quad j=1,\ldots, n \qquad (2)$$

process constraints:

$$\sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \leq 0, \quad p=1,\ldots, q, j=1,\ldots, n$$

$$u_r, v_i \geq \varepsilon, \quad r=1,\ldots, s, i=1,\ldots, m$$

There are two features to be noted in this model. One is that the same factor has the same multiplier associated with it, no matter which process they correspond to. The other is that each of the system constraints corresponding to a DMU is the sum of its q process constraints. They are thus redundant, and can be deleted from the model. Since more constraints are involved in this parallel model, the corresponding efficiency, E_k^{Kao}, is less than that measured from the black-box model, E_k^{CCR}.

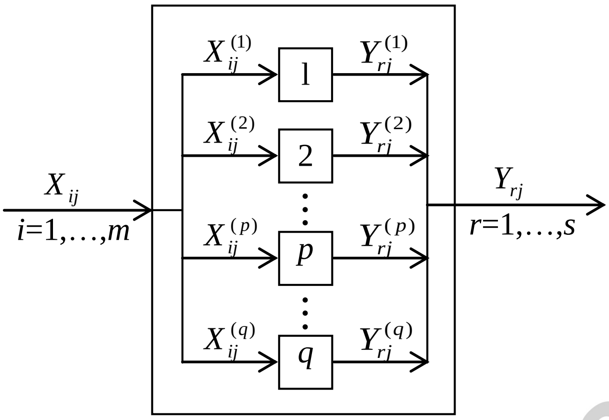

Figure 1. Parallel production system.

When an optimal solution (u_r^*, v_i^*) for Model (2) is obtained, the system (or the DMU) and process efficiencies are calculated as follows:

$$E_k^{\mathrm{Kao}} = \sum_{r=1}^{s} u_r^* Y_{rk} \Big/ \sum_{i=1}^{m} v_i^* X_{ik} = \sum_{r=1}^{s} u_r^* Y_{rk}$$

$$E_k^{(p)} = \sum_{r=1}^{s} u_r^* Y_{rk}^{(p)} \Big/ \sum_{i=1}^{m} v_i^* X_{ik}^{(p)}, \text{ p=1,..., q}$$

If we define the weight for process p as $w^{(p)} = \sum_{i=1}^{m} v_i^* X_{ik}^{(p)} / \sum_{i=1}^{m} v_i^* X_{ik}$, i.e., the proportion of the aggregate input of process p in that of all processes, then we have $\sum_{p=1}^{q} w^{(p)} = 1$ and $w^{(p)} \ge 0$, p=1,..., q. The average of the q process efficiencies weighted by $w^{(p)}$ is:

$$\sum_{p=1}^{q} w^{(p)} E_k^{(p)} = \sum_{p=1}^{q} \left(\frac{\sum_{i=1}^{m} v_i^* X_{ik}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}} \right) \left(\frac{\sum_{r=1}^{s} u_r^* Y_{rk}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}^{(p)}} \right) = \sum_{p=1}^{q} \left(\frac{\sum_{r=1}^{s} u_r^* Y_{rk}^{(p)}}{\sum_{i=1}^{m} v_i^* X_{ik}} \right) = \frac{\sum_{r=1}^{s} u_r^* Y_{rk}}{\sum_{i=1}^{m} v_i^* X_{ik}} = E_k^{\mathrm{Kao}}$$

A property that the system efficiency (E_k^{Kao}) is a weighted average of the q process efficiencies ($E_k^{(p)}$, p=1,..., q) is obtained. By comparing the efficiencies of the processes of a DMU, the processes that perform less satisfactorily than others are identified. When there is more than one optimal solution to Model (2), the efficiencies of a specific process of different DMUs are not comparable. In this case, the idea of prioritizing processes, as proposed by Kao and Hwang [19], can be applied to address this issue. Suppose the efficiencies of Process t of DMUs a and b are to be compared. Model (2) is applied first to obtain the system efficiencies E_a^{Kao} and E_b^{Kao}. Then the following model is used to find the efficiency of Process t for DMU a, given its system efficiency is E_a^{Kao}:

$$
\begin{aligned}
E_a^{(t)} = \max.\quad & \sum_{r=1}^{s} u_r Y_{ra}^{(t)} \\
\text{s.t.}\quad & \sum_{i=1}^{m} v_i X_{ia}^{(t)} = 1 \\
& \sum_{r=1}^{s} u_r Y_{ra} = E_a^{Kao} \sum_{i=1}^{m} v_i X_{ia} \\
& \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0, \quad j=1,\ldots,n \\
& \sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \le 0, \quad p=1,\ldots,q,\ j=1,\ldots,n \\
& u_r, v_i \ge \varepsilon, \quad r=1,\ldots,s,\ i=1,\ldots,m
\end{aligned}
\qquad (3)
$$

The efficiency of Process t for DMU b can be calculated similarly. If the efficiencies of another process w are to be compared, then the efficiency of Process w of DMU a is measured via the above model, while additionally requiring the efficiency of Process t to be $E_a^{(t)}$. This process can be continued for all processes. The process considered the most important is the first to measure its efficiency, the second important is measured next, and so forth.

The ratio of the efficiencies of a DMU in two different periods serves as a measure of changes in performance. Caves et al. [20] proposed using the technology of one of the two periods to calculate the efficiency of both the earlier and later periods. If the ratio of the efficiency of the later period to that of the earlier period for a DMU is greater than unity, then the performance of this DMU has improved; otherwise, it has declined. Since using different periods as the base period may obtain inconsistent results [6], Färe et al. [21] suggested using the geometric mean of the MPIs calculated from the two base periods as the final MPI. Another approach is to use data of the two periods to construct an aggregate technology. Based on this, the efficiencies of the same DMU in the two periods are calculated, and the ratio of that of the later period to that of the earlier one is the MPI, referred to as the biennial MPI [22]. The biennial MPI is a special case of the more general global MPI [23], which involves more than two periods. In this paper, the biennial MPI is used to measure changes in performance.

Denote X_{ij} and Y_{rj} as the data of the system of an earlier period, and $\hat{X}_{ij}$ and $\hat{Y}_{rj}$ as that of a later period. The conventional black-box model for measuring the efficiency of DMU k of the earlier period based on the aggregate technology of the two periods, which ignores the operations of the component processes, can be formulated as:

$$\begin{aligned} E_k^{\mathrm{BB}} = \max.\ & \sum_{r=1}^{s} u_r Y_{rk} \\ \text{s.t.}\ & \sum_{i=1}^{m} v_i X_{ik} = 1 \\ & \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0, \quad j=1,\dots,n \\ & \sum_{r=1}^{s} u_r \hat{Y}_{rj} - \sum_{i=1}^{m} v_i \hat{X}_{ij} \le 0, \quad j=1,\dots,n \\ & u_r, v_i \ge \varepsilon, \quad r=1,\dots,s,\ i=1,\dots,m \end{aligned} \tag{4}$$

The efficiency of the later period, $\hat{E}_k^{\mathrm{BB}}$, is measured by replacing Y_{rk} in the objective function and X_{ik} in the first constraint with $\hat{Y}_{rk}$ and $\hat{X}_{ik}$, respectively. The biennial MPI of the system is the ratio of $\hat{E}_k^{\mathrm{BB}}$ to E_k^{BB}: $MPI_k^{\mathrm{BB}} = \hat{E}_k^{\mathrm{BB}} / E_k^{\mathrm{BB}}$. In this model, only the system efficiency can be measured.

When operations of the processes are also considered, a parallel model of the following form is obtained for measuring the system efficiency:

$$\begin{aligned} E_k^{\mathrm{Parallel}} = \max.\ & \sum_{r=1}^{s} u_r Y_{rk} \\ \text{s.t.}\ & \sum_{i=1}^{m} v_i X_{ik} = 1 \\ & \sum_{r=1}^{s} u_r Y_{rj}^{(p)} - \sum_{i=1}^{m} v_i X_{ij}^{(p)} \le 0, \quad p=1,\dots,q,\ j=1,\dots,n \\ & \sum_{r=1}^{s} u_r \hat{Y}_{rj}^{(p)} - \sum_{i=1}^{m} v_i \hat{X}_{ij}^{(p)} \le 0, \quad p=1,\dots,q,\ j=1,\dots,n \\ & u_r, v_i \ge \varepsilon, \quad r=1,\dots,s,\ i=1,\dots,m \end{aligned} \tag{5}$$

where $X_{ij}^{(p)}$ and $Y_{rj}^{(p)}$ are the data of the earlier period, and $\hat{X}_{ij}^{(p)}$ and $\hat{Y}_{rj}^{(p)}$ are that of the later period. Similar to the case of the black-box model, the efficiencies of the later period, $\hat{E}_k^{\text{Parallel}}$ and $\hat{E}_k^{(p)}$, can be measured by replacing Y_{rk} and X_{ik} with $\hat{Y}_{rk}$ and $\hat{X}_{ik}$, respectively. At optimality, the system and process efficiencies of the two periods are calculated as:

$$\begin{aligned}
&E_k^{\text{Parallel}} = \sum_{r=1}^{s} u_r^* Y_{rk} \Big/ \sum_{i=1}^{m} v_i^* X_{ik} \\
&E_k^{(p)} = \sum_{r=1}^{s} u_r^* Y_{rk}^{(p)} \Big/ \sum_{i=1}^{m} v_i^* X_{ik}^{(p)}, \text{ p=1,..., q} \\
&\hat{E}_k^{\text{Parallel}} = \sum_{r=1}^{s} u_r^* \hat{Y}_{rk} \Big/ \sum_{i=1}^{m} v_i^* \hat{X}_{ik} \\
&\hat{E}_k^{(p)} = \sum_{r=1}^{s} u_r^* \hat{Y}_{rk}^{(p)} \Big/ \sum_{i=1}^{m} v_i^* \hat{X}_{ik}^{(p)}, \text{ p=1,..., q}
\end{aligned} \tag{6}$$

Obviously, the system efficiency is still a weighted average of the process efficiencies for both periods. That is,

$$\begin{aligned}
&E_k^{\text{Parallel}} = \sum_{p=1}^{q} w^{(p)} E_k^{(p)}, \text{ where } \; w^{(p)} = \sum_{i=1}^{m} v_i^* X_{ik}^{(p)} \Big/ \sum_{i=1}^{m} v_i^* X_{ik} \\
&\hat{E}_k^{\text{Parallel}} = \sum_{p=1}^{q} \hat{w}^{(p)} E_k^{(p)}, \text{ where } \; \hat{w}^{(p)} = \sum_{i=1}^{m} v_i^* \hat{X}_{ik}^{(p)} \Big/ \sum_{i=1}^{m} v_i^* \hat{X}_{ik}
\end{aligned}$$

The biennial MPIs of the system and q processes for DMU k between the two periods are:

$$\begin{aligned}
&MPI_k^{\text{Parallel}} = \hat{E}_k^{\text{Parallel}} / E_k^{\text{Parallel}} \\
&MPI_k^{(p)} = \hat{E}_k^{(p)} / E_k^{(p)}, \text{ p=1,..., q}
\end{aligned} \tag{7}$$

Based on the above relationships between the system and process efficiencies, the following derivation can be made for the system MPI:

$$\begin{aligned}
MPI_k^{\text{Parallel}} &= \frac{\hat{E}_k^{\text{Parallel}}}{E_k^{\text{Parallel}}} = \frac{\sum_{p=1}^{q} \hat{w}^{(p)} \hat{E}_k^{(p)}}{E_k^{\text{Parallel}}} = \sum_{p=1}^{q} \left(\frac{\hat{w}^{(p)} (\hat{E}_k^{(p)} / E_k^{(p)})}{E_k^{\text{Parallel}} / E_k^{(p)}} \right) \\
&= \sum_{p=1}^{q} \left(\frac{\hat{w}^{(p)}}{E_k^{\text{Parallel}} / E_k^{(p)}} \right) MPI_k^{(p)} = \sum_{p=1}^{q} \omega^{(p)} MPI_k^{(p)}
\end{aligned} \tag{8}$$

where $\omega^{(p)} = \hat{w}^{(p)} / (E_k^{\text{Parallel}} / E_k^{(p)})$ is the coefficient associated with $MPI_k^{(p)}$ in the linear combination. This relationship shows that the system MPI is a linear combination of the process MPIs. Since the sum of all $\omega^{(p)}$ need not be equal to one, the system MPI is not necessarily a weighted average of the process MPIs. It is a weighted average when all process efficiencies of the earlier period are the same.

In the next section we shall illustrate how to calculate the system and process

MPIs, and discuss the relationship between them, using an example of thirty-nine branches of an Iranian commercial bank.

3. An example

The example discussed in this section is taken from Jahanshahloo et al. [17], with thirty-nine branches of a commercial bank in Iran performing three major functions of deposits, sales, and services. The three functions share the common inputs of employees (X_1) and operating expenses (X_2) to produce five outputs, in that deposits collected (Y_1), loans (Y_2) and profit (Y_3), and service charges (Y_4) are attributed to deposits, sales, and services, respectively, and the output of customer satisfaction (Y_5) is a result of all three functions. Figure 2 shows the structure of this system, and Table 1 shows the mean and standard deviation of each input-output factor of the thirty-nine branches in the first six months of 2000 and 2001. The complete data can be found in Jahanshahloo et al. [17].

Since the two inputs are shared by the three functions, it is assumed that the proportion of α_i of input X_1, i=1, 2, 3 with $\alpha_1+\alpha_2+\alpha_3=1$, and the proportion of β_i of input X_2, i=1, 2, 3 with $\beta_1+\beta_2+\beta_3=1$, are used by the ith function. Similarly, the proportion of γ_i of output Y_5, with $\gamma_1+\gamma_2+\gamma_3=1$, are assumed to be attributed to the ith function. These proportions are determined by each branch such that the highest efficiency will be produced for them. Specifically, the model for measuring the efficiency of the kth branch, based on Model (5), is as follows:

$$
\begin{aligned}
E_k^{\text{Parallel}} = \max.\ & u_1Y_{1k} + u_2Y_{2k} + u_3Y_{3k} + u_4Y_{4k} + u_5Y_{5k} \\
\text{s.t.}\ & v_1X_{1k} + v_2X_{2k} = 1 \\
& u_1Y_{1j} + u_5\gamma_1Y_{5j} - (v_1\alpha_1X_{1j} + v_2\beta_1X_{2j}) \le 0, \quad j=1,\ldots,39 \\
& u_2Y_{2j} + u_3Y_{3j} + u_5\gamma_2Y_{5j} - (v_1\alpha_2X_{1j} + v_2\beta_2X_{2j}) \le 0, \quad j=1,\ldots,39 \\
& u_4Y_{4j} + u_5\gamma_3Y_{5j} - (v_1\alpha_3X_{1j} + v_2\beta_3X_{2j}) \le 0, \quad j=1,\ldots,39 \qquad (9) \\
& u_1\hat{Y}_{1j} + u_5\gamma_1\hat{Y}_{5j} - (v_1\alpha_1\hat{X}_{1j} + v_2\beta_1\hat{X}_{2j}) \le 0, \quad j=1,\ldots,39 \\
& u_2\hat{Y}_{2j} + u_3\hat{Y}_{3j} + u_5\gamma_2\hat{Y}_{5j} - (v_1\alpha_2\hat{X}_{1j} + v_2\beta_2\hat{X}_{2j}) \le 0, \quad j=1,\ldots,39 \\
& u_4\hat{Y}_{4j} + u_5\gamma_3\hat{Y}_{5j} - (v_1\alpha_3\hat{X}_{1j} + v_2\beta_3\hat{X}_{2j}) \le 0, \quad j=1,\ldots,39 \\
& \alpha_1+\alpha_2+\alpha_3=1,\ \beta_1+\beta_2+\beta_3=1,\ \gamma_1+\gamma_2+\gamma_3=1
\end{aligned}
$$

$$u_1, u_2, u_3, u_4, u_5, v_1, v_2, \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3 \geq \varepsilon$$

This model is nonlinear. However, it can be linearized by substituting the nonlinear terms of $v_1\alpha_1$, $v_1\alpha_2$, and $v_1\alpha_3$, by a_1, a_2, and a_3, respectively, and replacing the constraint of $\alpha_1+\alpha_2+\alpha_3=1$ with $a_1+a_2+a_3=v_1$ accordingly. The nonlinear terms corresponding to β and γ are handled similarly. As what value of the non-Archimedean number ε to use in computation, this paper follows the suggestion of Charnes and Cooper [18] of 10^{-5}, when the input and output data are expressed in the range of 1 to 100. At optimality, the system and process efficiencies are calculated based on Equation (6), and the system and process MPIs are calculated based on Equation (7).

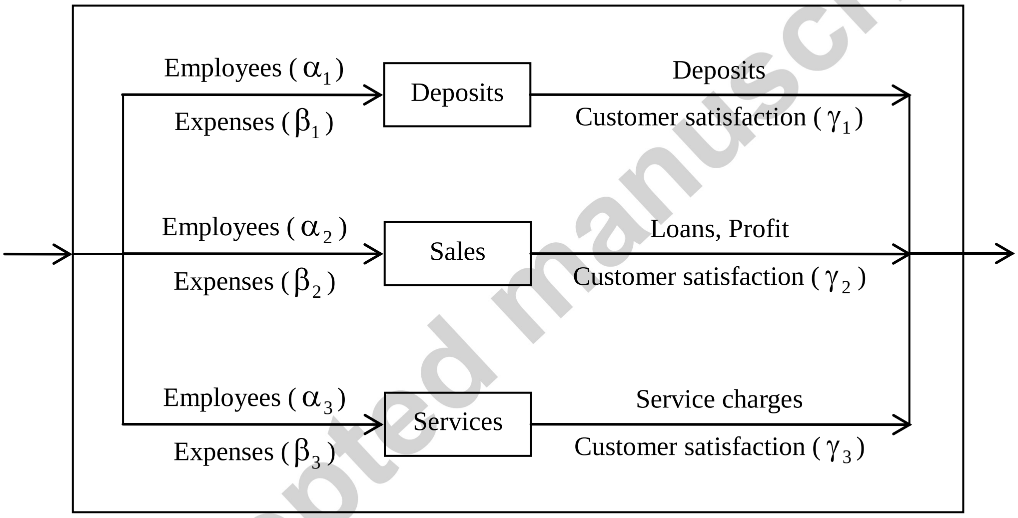

Figure 2. Functional structure of an Iranian commercial bank.

Table 1. Some statistics for the illustrative example.

	Employees (X_1)	Expenses (X_2)	Deposits (Y_1)	Loans (Y_2)	Profit (Y_3)	Charges (Y_4)	Satisfaction (Y_5)
2000							
Average	343.59	13.33	139208.47	86534.56	1380970691	77630074.44	70047.18
S.D.	253.20	3.73	149932.87	149827.94	3066843165	171891394.32	72529.86
2001							
Average	311.28	13.32	141257.85	80877.07	3955543895	54448668.51	785.44
S.D.	186.44	3.87	150912.27	125989.18	18320618046	115625305.21	1429.85

If the system is treated as a black box, ignoring the operations of the three functions, then the model is as follows:

$$
\begin{aligned}
E_k^{\mathrm{BB}} = \max.\quad & \sum_{r=1}^{5} u_r Y_{rk} \\
\text{s.t.}\quad & \sum_{i=1}^{2} v_i X_{ik} = 1 \\
& \sum_{r=1}^{5} u_r Y_{rj} - \sum_{i=1}^{2} v_i X_{ij} \le 0, \quad j=1,\ldots,39 \\
& \sum_{r=1}^{5} u_r \hat{Y}_{rj} - \sum_{i=1}^{2} v_i \hat{X}_{ij} \le 0, \quad j=1,\ldots,39 \\
& u_r,\ v_i \ge \varepsilon, \quad r=1,\ldots,5,\ i=1,2
\end{aligned} \tag{10}
$$

where the constraints corresponding to the three functions in Model (9) are aggregated.

Columns two to four of Table 2 show the results obtained from the black-box model. There are five branches which are efficient in the first period, and only one in the second. The MPIs indicate that there are seven branches, Numbers 3, 9, 19, 20, 23, 25, and 28, whose performances have improved, and one, Number 22, for which it remains the same in the two periods. The performances of the thirty-nine branches in general have declined from 2000 to 2001.

When the operations of the three processes, i.e., functions, are considered, only two branches remain efficient in the first period, and none in the second, as indicated by the E_k^{Parallel} and $\hat{E}_k^{\text{Parallel}}$ values in columns eight and twelve. As expected, the system efficiencies measured from the parallel model, E_k^{Parallel} and $\hat{E}_k^{\text{Parallel}}$, are less than or equal to those measured from the black-box model, E_k^{BB} and $\hat{E}_k^{\mathrm{BB}}$, for the same branch. The averages in the last row show that the performances of the three processes (columns five to seven) are similar in the first period, and process one performs better than process two, which in turn performs better than process three in the second period (columns nine to eleven). The numbers in parentheses under each efficiency score are the associated weights. Clearly, the weighted average of the three process efficiencies is equal to the system efficiency. Using branch Number 1 to illustrate this, we have $0.0039\times0.2676+0.9899\times0.2744+0.0062\times0.2721=0.2744$ for the first period and $0.9861\times0.1227+0.0130\times0.0054+0.0009\times0.0252=0.1210$ for the second.

Table 2. Results for the thirty-nine branches of a commercial bank calculated from the black-box and parallel models.

Branch	E_k^{BB}	$\hat{E}_k^{BB}$	MPI_k^{BB}	$E_k^{(1)}$ ($w^{(1)}$)	$E_k^{(2)}$ ($w^{(2)}$)	$E_k^{(3)}$ ($w^{(3)}$)	$E_k^{Parallel}$	$\hat{E}_k^{(1)}$ ($\hat{w}^{(1)}$)	$\hat{E}_k^{(2)}$ ($\hat{w}^{(2)}$)	$\hat{E}_k^{(3)}$ ($\hat{w}^{(3)}$)	$\hat{E}_k^{Parallel}$	$MPI_k^{(1)}$ ($\omega^{(1)}$)	$MPI_k^{(2)}$ ($\omega^{(2)}$)	$MPI_k^{(3)}$ ($\omega^{(3)}$)	$MPI_k^{Parallel}$ (Total)
1	0.2858	0.1378	0.4823	0.2676 (0.0039)	0.2744 (0.9899)	0.2721 (0.0062)	0.2744	0.1227 (0.9861)	0.0054 (0.0130)	0.0252 (0.0009)	0.1210	0.4583 (0.9619)	0.0198 (0.0130)	0.0926 (0.0008)	0.4412 (0.9757)
2	0.2611	0.1464	0.5607	0.2502 (0.0043)	0.2452 (0.0205)	0.2515 (0.9752)	0.2513	0.1423 (0.9885)	0.0038 (0.0108)	0.0370 (0.0007)	0.1407	0.5687 (0.9841)	0.0154 (0.0105)	0.1472 (0.0007)	0.5599 (0.9953)
3	0.4663	0.4784	1.0259	0.1697 (0.0011)	0.4668 (0.9976)	0.1104 (0.0013)	0.4661	0.0930 (0.0009)	0.4752 (0.9982)	0.0143 (0.0009)	0.4745	0.5477 (0.0003)	1.0180 (0.9999)	0.1292 (0.0002)	1.0181 (1.0004)
4	0.3808	0.2224	0.5841	0.3466 (0.9778)	0.3453 (0.0206)	0.3054 (0.0016)	0.3465	0.1712 (0.9784)	0.0798 (0.0207)	0.0194 (0.0009)	0.1692	0.4941 (0.9786)	0.2313 (0.0206)	0.0634 (0.0008)	0.4884 (1.0001)
5	0.0904	0.0743	0.8213	0.0749 (0.0022)	0.0845 (0.9950)	0.0748 (0.0028)	0.0845	0.0412 (0.0017)	0.0070 (0.0247)	0.0561 (0.9736)	0.0548	0.5498 (0.0015)	0.0822 (0.0248)	0.7497 (0.8615)	0.6487 (0.8877)
6	0.8850	0.4113	0.4647	0.8612 (0.0009)	0.8838 (0.9977)	0.8435 (0.0014)	0.8837	0.4119 (0.9872)	0.0037 (0.0117)	0.0020 (0.0010)	0.4067	0.4782 (0.9621)	0.0042 (0.0117)	0.0024 (0.0010)	0.4602 (0.9748)
7	0.1230	0.0630	0.5127	0.0938 (0.0012)	0.1100 (0.9969)	0.0981 (0.0019)	0.1099	0.0531 (0.9811)	0.0020 (0.0178)	0.0301 (0.0012)	0.0522	0.5662 (0.8368)	0.0186 (0.0178)	0.3066 (0.0010)	0.4745 (0.8557)
8	0.3235	0.2988	0.9237	0.0643 (0.0009)	0.3238 (0.9982)	0.0189 (0.0008)	0.3233	0.0439 (0.0012)	0.2992 (0.9980)	0.0755 (0.0008)	0.2987	0.6825 (0.0002)	0.9239 (0.9995)	4.0014 (0.0000)	0.9239 (0.9998)
9	0.2331	0.9127	3.9153	0.1330 (0.0010)	0.2052 (0.9974)	0.1978 (0.0016)	0.2051	0.9120 (0.9851)	0.0159 (0.0140)	0.2781 (0.0009)	0.8989	6.8594 (0.6385)	0.0775 (0.0140)	1.4062 (0.0009)	4.3818 (0.6534)
10	0.4846	0.2293	0.4731	0.4283 (0.0007)	0.4285 (0.0148)	0.4522 (0.9845)	0.4519	0.2273 (0.9893)	0.0355 (0.0100)	0.0118 (0.0007)	0.2252	0.5307 (0.9377)	0.0829 (0.0095)	0.0260 (0.0007)	0.4985 (0.9478)
11	1.0000	0.5662	0.5662	1.0000 (0.9923)	0.6561 (0.0072)	1.0000 (0.0005)	0.9975	0.0633 (0.0014)	0.5511 (0.9982)	0.0004 (0.0005)	0.5502	0.0633 (0.0014)	0.8399 (0.6566)	0.0004 (0.0005)	0.5515 (0.6584)
12	0.2739	0.1648	0.6018	0.2614 (0.0020)	0.2710 (0.9949)	0.2623 (0.0031)	0.2710	0.1595 (0.9710)	0.0100 (0.0272)	0.0130 (0.0018)	0.1552	0.6103 (0.9366)	0.0367 (0.0272)	0.0496 (0.0017)	0.5727 (0.9655)
13	0.1418	0.1151	0.8117	0.0987 (0.0008)	0.1069 (0.9978)	0.0643 (0.0014)	0.1068	0.0632 (0.0054)	0.0741 (0.9938)	0.0191 (0.0007)	0.0740	0.6399 (0.0050)	0.6935 (0.9945)	0.2966 (0.0004)	0.6930 (0.9999)
14	0.0567	0.0546	0.9632	0.0466 (0.9857)	0.0115 (0.0134)	0.0086 (0.0009)	0.0461	0.0458 (0.9857)	0.0020 (0.0134)	0.0067 (0.0009)	0.0452	0.9833 (0.9965)	0.1770 (0.0034)	0.7757 (0.0002)	0.9806 (1.0000)
15	0.1998	0.1042	0.5217	0.1950 (0.9547)	0.1922 (0.0429)	0.1786 (0.0024)	0.1948	0.1044 (0.9773)	0.0024 (0.0213)	0.0020 (0.0014)	0.1021	0.5356 (0.9781)	0.0125 (0.0210)	0.0113 (0.0013)	0.5241 (1.0004)
16	0.1565	0.0884	0.5646	0.1550 (0.9699)	0.1507 (0.0226)	0.1507 (0.0076)	0.1549	0.0885 (0.9848)	0.0025 (0.0143)	0.0087 (0.0009)	0.0872	0.5708 (0.9856)	0.0167 (0.0139)	0.0576 (0.0009)	0.5628 (1.0004)
17	0.6796	0.4855	0.7144	0.4969 (0.9874)	0.1359 (0.0118)	0.4871 (0.0008)	0.4927	0.4545 (0.9874)	0.0150 (0.0118)	0.0512 (0.0008)	0.4490	0.9147 (0.9960)	0.1104 (0.0033)	0.1051 (0.0008)	0.9115 (1.0000)
18	0.1000	0.0320	0.3204	0.0334 (0.0010)	0.0963 (0.9983)	0.0110 (0.0007)	0.0962	0.0297 (0.9887)	0.0011 (0.0105)	0.0089 (0.0007)	0.0294	0.8901 (0.3432)	0.0110 (0.0106)	0.8049 (0.0001)	0.3056 (0.3538)
19	0.0949	0.1489	1.5687	0.0644 (0.9738)	0.0561 (0.0255)	0.0044 (0.0007)	0.0642	0.0669 (0.0008)	0.1110 (0.9985)	0.0128 (0.0007)	0.1109	1.0382 (0.0008)	1.9785 (0.8729)	2.9397 (0.0000)	1.7279 (0.8737)

20	0.1426	0.2395	1.6795	0.0927 (0.0013)	0.1232 (0.9966)	0.0677 (0.0021)	0.1231	0.0624 (0.0013)	0.0209 (0.0194)	0.2225 (0.9793)	0.2184	0.6724 (0.0010)	0.1693 (0.0194)	3.2857 (0.5389)	1.7746 (0.5593)
21	0.3810	0.1194	0.3134	0.3773 (0.9859)	0.3690 (0.0106)	0.3628 (0.0035)	0.3772	0.1131 (0.9898)	0.0094 (0.0096)	0.0041 (0.0006)	0.1120	0.2997 (0.9902)	0.0256 (0.0094)	0.0113 (0.0006)	0.2970 (1.0001)
22	1.0000	1.0000	1.0000	0.8242 (0.0027)	1.0000 (0.0132)	1.0000 (0.9841)	0.9995	0.1529 (0.0008)	1.0000 (0.9984)	0.0110 (0.0008)	0.9985	0.1855 (0.0007)	1.0000 (0.9989)	0.0110 (0.0008)	0.9990 (1.0003)
23	0.5599	0.5669	1.0125	0.2527 (0.0013)	0.5454 (0.9975)	0.0058 (0.0012)	0.5443	0.1052 (0.0018)	0.5287 (0.9970)	0.0108 (0.0012)	0.5273	0.4163 (0.0009)	0.9695 (0.9989)	1.8832 (0.0000)	0.9688 (0.9998)
24	0.0864	0.0674	0.7803	0.0731 (0.9739)	0.0651 (0.0243)	0.0472 (0.0018)	0.0728	0.0496 (0.9604)	0.0400 (0.0384)	0.0113 (0.0012)	0.0492	0.6785 (0.9636)	0.6141 (0.0343)	0.2398 (0.0008)	0.6751 (0.9987)
25	0.0427	0.0458	1.0745	0.0382 (0.9888)	0.0331 (0.0103)	0.0168 (0.0009)	0.0381	0.0375 (0.9788)	0.0292 (0.0205)	0.0279 (0.0007)	0.0373	0.9807 (0.9807)	0.8828 (0.0178)	1.6590 (0.0003)	0.9779 (0.9988)
26	0.2329	0.1054	0.4524	0.2179 (0.0014)	0.1998 (0.0189)	0.2207 (0.9797)	0.2203	0.1056 (0.9787)	0.0003 (0.0200)	0.0399 (0.0013)	0.1034	0.4844 (0.9683)	0.0016 (0.0181)	0.1807 (0.0013)	0.4693 (0.9878)
27	1.0000	0.4341	0.4341	1.0000 (0.9828)	1.0000 (0.0130)	1.0000 (0.0042)	1.0000	0.1883 (0.0011)	0.0171 (0.0130)	0.4283 (0.9859)	0.4227	0.1883 (0.0011)	0.0171 (0.0130)	0.4283 (0.9859)	0.4227 (1.0000)
28	0.4002	0.4337	1.0836	0.3169 (0.0018)	0.3869 (0.9963)	0.3077 (0.0020)	0.3866	0.0562 (0.0014)	0.4115 (0.9981)	0.0095 (0.0005)	0.4108	0.1774 (0.0011)	1.0634 (0.9988)	0.0310 (0.0004)	1.0624 (1.0004)
29	1.0000	0.5931	0.5931	1.0000 (0.9763)	1.0000 (0.0224)	1.0000 (0.0013)	1.0000	0.5938 (0.9888)	0.0037 (0.0105)	0.0054 (0.0007)	0.5872	0.5938 (0.9888)	0.0037 (0.0105)	0.0054 (0.0007)	0.5872 (1.0000)
30	0.1864	0.1133	0.6075	0.1866 (0.9785)	0.1425 (0.0192)	0.1378 (0.0023)	0.1856	0.1135 (0.9791)	0.0003 (0.0197)	0.0060 (0.0013)	0.1112	0.6086 (0.9841)	0.0022 (0.0151)	0.0435 (0.0009)	0.5990 (1.0002)
31	0.1032	0.0655	0.6344	0.0979 (0.9838)	0.0671 (0.0145)	0.0695 (0.0017)	0.0974	0.0633 (0.9836)	0.0047 (0.0154)	0.0158 (0.0010)	0.0623	0.6462 (0.9886)	0.0707 (0.0106)	0.2268 (0.0007)	0.6398 (0.9999)
32	1.0000	0.8158	0.8158	0.6238 (0.0008)	0.6181 (0.0113)	1.0000 (0.9878)	0.9954	0.3010 (0.0008)	0.0193 (0.0116)	0.7454 (0.9876)	0.7366	0.4825 (0.0005)	0.0312 (0.0072)	0.7454 (0.9922)	0.7401 (0.9999)
33	0.2750	0.1499	0.5451	0.2660 (0.9890)	0.2266 (0.0099)	0.2225 (0.0012)	0.2656	0.1448 (0.9899)	0.0120 (0.0095)	0.0617 (0.0006)	0.1435	0.5445 (0.9916)	0.0528 (0.0081)	0.2771 (0.0005)	0.5405 (1.0002)
34	0.3432	0.1180	0.3439	0.3369 (0.9865)	0.3310 (0.0101)	0.3252 (0.0034)	0.3368	0.1006 (0.9902)	0.0179 (0.0092)	0.0064 (0.0006)	0.0998	0.2986 (0.9905)	0.0540 (0.0091)	0.0198 (0.0006)	0.2963 (1.0001)
35	0.1551	0.0884	0.5703	0.1312 (0.0020)	0.1302 (0.0096)	0.1383 (0.9884)	0.1382	0.0755 (0.9898)	0.0164 (0.0096)	0.0433 (0.0006)	0.0749	0.5753 (0.9399)	0.1257 (0.0090)	0.3133 (0.0006)	0.5420 (0.9495)
36	0.1462	0.0842	0.5758	0.1284 (0.9891)	0.0741 (0.0097)	0.1017 (0.0012)	0.1278	0.0824 (0.9890)	0.0044 (0.0104)	0.0701 (0.0007)	0.0816	0.6419 (0.9933)	0.0594 (0.0060)	0.6889 (0.0005)	0.6383 (0.9998)
37	0.0853	0.0653	0.7652	0.0000 (0.9833)	0.0511 (0.0149)	0.0507 (0.0018)	0.0841	0.0652 (0.9848)	0.0020 (0.0143)	0.0052 (0.0009)	0.0643	0.7700 (0.9913)	0.0385 (0.0087)	0.1018 (0.0006)	0.7637 (1.0006)
38	0.0731	0.0585	0.8000	0.0730 (0.9850)	0.0348 (0.0134)	0.0455 (0.0016)	0.0725	0.0577 (0.9856)	0.0035 (0.0135)	0.0230 (0.0009)	0.0570	0.7905 (0.9932)	0.1000 (0.0065)	0.5049 (0.0006)	0.7861 (1.0002)
39	0.1508	0.0796	0.5278	0.1510 (0.9732)	0.1311 (0.0238)	0.1225 (0.0030)	0.1504	0.0792 (0.9869)	0.0074 (0.0122)	0.0052 (0.0009)	0.0783	0.5248 (0.9906)	0.0564 (0.0106)	0.0423 (0.0007)	0.5205 (1.0019)
Ave.	0.3487	0.2558	0.7950	0.2901 (0.5038)	0.2967 (0.3431)	0.2829 (0.1530)	0.3343	0.1498 (0.6817)	0.0986 (0.2169)	0.0622 (0.1015)	0.2416	0.7318 (0.6488)	0.2997 (0.2034)	0.5811 (0.0872)	0.7955 (0.9395)

The last four columns show the MPIs of the three processes and the system calculated using Equation (7). There are only five branches, Numbers 3, 9, 19, 20, and 28, whose performances have improved. The average of 0.7955 indicates that the performances of the thirty-nine branches have in general declined. This finding is consistent with that obtained from the black-box model. However, the black-box model misjudges two branches, Numbers 23 and 25, as having improved performance. The average MPIs shown in the last row indicate that process two is in general the one whose performance declines the most. The coefficients associated with the process MPIs in expressing the system MPIs, as obtained from Equation (8), are shown in parentheses. The sum of the MPIs of the three processes multiplied by their respective coefficients produce the system MPI, as is clear, for example, from branch Number 1: $0.9619 \times 0.4583+0.0130 \times 0.0198+0.0008 \times 0.0926=0.4412$. Most of the total coefficients associated with the three process MPIs have values close to one (except those for branches Numbers 9, 11, 18, and 20), indicating that the system MPI is approximately equal to the weighted average of the process MPIs.

4. Conclusion

The conventional DEA models for measuring efficiencies treat the production system as a black box, ignoring the operations of the component processes. As a result, the measured efficiencies are misleading, and the subsequent measures of MPI are incorrect. This paper thus develops a relational model to measure the MPIs for parallel production systems in a more meaningful manner.

A merit of the relational model for parallel systems is that the efficiency of the system can be decomposed into a weighted average of those of the component processes. The MPI used in this paper is the biennial MPI (which is the global MPI involving only two periods), based on which a number of properties are obtained. The changes in performance between 2000 and 2001 for thirty-nine branches of an Iranian commercial bank are presented to illustrate the proposed method, and this work has several findings, as follows.

First, the MPIs measured from the conventional black-box model, without taking the operations of the component processes into account, may misjudge the changes in performance of a DMU in two periods. Second, the system MPI measured from the parallel model is a linear combination of the process MPIs. The empirical results show that this linear combination is quite close to a convex combination, implying that the system MPI is approximately a weighted average of the process MPIs. Third, the decomposition of the system MPI into a linear combination of the process MPIs helps identify the processes that cause the decline in the performance of the system. Based on these findings, a general conclusion is that the parallel model should be used whenever the data is available.

In this paper the returns to scale are assumed to be constant. Whether the system MPI can be decomposed into a linear combination of the process MPIs under variable returns to scale or not requires further study. This is a topic for future research.

While real world systems are more complicated than the parallel one discussed in this paper, they can usually be expressed as a combination of the two basic network structures, series and parallel [24]. It is thus believed that the MPI of a general network system can be expressed as a function of the MPIs of its component processes based on the decompositions for series systems discussed in Kao and Hwang [11] and for parallel systems in this paper. This is a direction for future studies.

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Highlights

- This paper measures the MPI of a parallel system.
- The system MPI is a linear combination of the process MPIs.
- The parallel model is more discriminative than the black-box one in judging changes in performance.
- The changes in performance of 39 branches of a commercial bank are investigated.