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A new fuzzy dempster MCDM method and its application in supplier selection

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ABSTRACT

Supplier selection is a multi-criterion decision making problem under uncertain environments. Hence, it is reasonable to hand the problem in *fuzzy sets theory* (FST) and *Dempster Shafer theory* of evidence (DST). In this paper, a new MCDM methodology, using FST and DST, based on the main idea of the technique for order preference by similarity to an ideal solution (TOPSIS), is developed to deal with supplier selection problem. The *basic probability assignments* (BPA) can be determined by the distance to the ideal solution and the distance to the negative ideal solution. Dempster combination rule is used to combine all the criterion data to get the final scores of the alternatives in the systems. The final decision results can be drawn through the pignistic probability transformation. In traditional fuzzy TOPSIS method, the quantitative performance of criterion, such as crisp numbers, should be transformed into fuzzy numbers. The proposed method is more flexible due to the reason that the BPA can be determined without the transformation step in traditional fuzzy TOPSIS method. The performance of criterion can be represented as crisp number or fuzzy number according to the real situation in our proposed method. The numerical example about supplier selection is used to illustrate the efficiency of the proposed method.

1. Introduction

Many decision-making applications, such as supplier selection, within the real world inevitably include the consideration of evidence based on several criteria, rather than on a preferred single criterion. A lot of researchers have devoted themselves to solve multi-criteria decision-making (MCDM) (Bouyssou, 1986; Gal & Hanne, 2006; Narasimhan & Vickery, 1988; Shyur & Shih, 2006; Wadhwa, Madaan, & Chan, 2009). Due to the flexibility to deal with uncertain information, it is necessary to use fuzzy sets theory (FST) and Dempster Shafer theory of evidence (DST). Fruitful papers about MCDM based on FST (Ashtiani, Haghighirad, & Makui, 2009; Chu & Lin, 2009; Deng & Liu, 2005a, 2005b; Deng, 2006; Hu, 2009; Hanaoka & Kunadhamraks, 2009; Olson & Wu, 2006; Wu & Olson, 2008; Yang, Chiu, & Tzeng, 2008; Yeh & Chang, 2009; Zhang, Wu, & Olson, 2005) and DST are published (Bauer, 1997; Beynon, Curry, & Morgan, 2000, 2001; Beynon, 2002, 2005; Deng, Shi, & Liu, 2004; Mercier, Cron, & Denoeux, 2007; Srivastava & Liu, 2003; Wu, 2009; Yager, 2008; Yang & Sen, 1997; Yang & Xu, 2002).

Recently, Wu (2009) proposed a method to select international supplier using grey related analysis and Dempster–Shafer theory to deal with this fuzzy group decision making problem. Grey related analysis (Deng, 1982) is employed as a means to reflect uncertainty in multi-attribute models through interval numbers in the individual aggregation. The Dempster–Shafer combination rule is used to aggregate individual preferences into a collective preference in the group aggregation.

In this paper, however, we proposed another MCDM methodology using FST together with DST. The new method has some desired properties. First, the proposed method uses linguistic items modeled as fuzzy numbers to represents experts' subjective opinions in addition of crisp number to rank the performance of criterion. Whether using quantitative representation or qualitative representation is depending on the real situation. This property is very desired for multiple experts decision making since there are not only quantitative data but also qualitative representation in the process of decision making. Second, based on the DST, the subject fuzzy numbers can be easily combined with the crisp numbers. That is, the proposed method can efficiently fuse quantitative and qualitative data in a straightforward manner. Third, the proposed method can be easily implemented step by step to solve MCDM problems.

The remaining paper is organized as follows: Section 2 briefly introduce the preliminaries of fuzzy sets theory (FST) and DST. In Section 3, our fuzzy Dempster method to deal with MCDM is proposed. A numerical example to supplier selection is used to show the efficiency of the proposed method. Finally, some conclusions are made in Section 5.

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2. Preliminaries

In this section, we simply introduce some relative mathematics tools, such as fuzzy sets theory (FST) and Dempster Shafer theory of evidence (DST), which will be used in our new proposed method.

2.1. Fuzzy sets theory

2.1.1. Fuzzy number

Definition 2.1 (*Fuzzy set*). Let X be a universe of discourse, $\widetilde{A}$ is a fuzzy subset of X if for all $x \in X$, there is a number $\mu_{\widetilde{A}}(x) \in [0,1]$ assigned to represent the membership of x to $\widetilde{A}$, and $\mu_{\widetilde{A}}(x)$ is called the membership of $\widetilde{A}$ (Zimmermann, 1991).

Definition 2.2 (*Fuzzy number*). A fuzzy number $\widetilde{A}$ is a normal and convex fuzzy subset of X. Here, the "Normality" implies that (Zimmermann, 1991).

$$\exists x \in \mathbb{R}, \quad \bigvee_x \mu_{\widetilde{A}}(x) = 1 \tag{1}$$

and "Convex" means that

$$\begin{aligned} &\forall x_1 \in X, \quad x_2 \in X, \quad \forall \alpha \in [0,1], \\ &\mu_{\widetilde{A}}(\alpha x_1 + (1-\alpha)x_2) \geqslant \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)) \end{aligned} \tag{2}$$

Definition 2.3. A triangular fuzzy number $\widetilde{A}$ can be defined by a triplet (a,b,c) shown in Fig. 1. The membership function is defined as Zimmermann (1991).

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leqslant x \leqslant b \\ \frac{c-x}{c-b}, & b \leqslant x \leqslant c \\ 0, & x > c \end{cases} \tag{3}$$

2.1.2. Linguistic variable

The concept of linguistic variable is very useful in dealing with situations which are too complex or ill-defined to be reasonably described in conventional quantitative expressions. Linguistic variables are represented in words or sentences or artificial languages, where each linguistic value can modeled by a fuzzy set (Kauffman & Gupta, 1985). In this paper, the importance weights of various criteria and the ratings of qualitative criteria are considered as linguistic variables. For example, these linguistic variables can be expressed in positive triangular fuzzy numbers as Table 1. It should be noticed that there are many different methods to represent linguistic items. Which kind of represent method is used is depend on the real application systems and the domain experts' opinions.

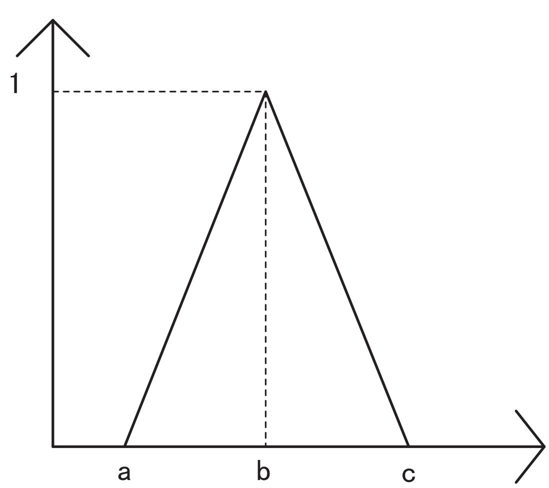

Fig. 1. A triangular fuzzy number.

Table 1
Linguistic variables for the importance weight and ratings.

Very low (VL)	(0,0.1,0.3)
Low (L)	(0.1,0.3,0.5)
Medium (M)	(0.3,0.5,0.7)
High (H)	(0.5,0.7,0.9)
Very high (VH)	(0.7,0.9,1.0)

2.1.3. Defuzzification

Defuzzification is an important step in fuzzy modeling and fuzzy multi-criteria decision-making. The defuzzification entails converting the fuzzy value into a crisp value, and determining the ordinal positions of n-fuzzy input parameters vector. Many defuzzification techniques are available (Zimmermann, 1991), but the common defuzzification methods include centre of area, first of maximums, last of maximums, and middle of maximums (MoM).

Different defuzzification techniques extract different levels of information. In this paper, the canonical representation of operation on triangular fuzzy numbers (Chou, 2003), which is based on the graded mean integration representation method is used in defuzziness process. For detailed information, please refer (Chou, 2003).

Definition 2.4. Given a triangular fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$, the graded mean integration representation of triangular fuzzy number $\widetilde{A}$ is defined as

$$P\left(\widetilde{A}\right) = \frac{1}{6}(a_1 + 4 \times a_2 + a_3) \tag{4}$$

By applying Eq. (4), the graded mean integration representation for importance weight of each criterion and ratings are shown in Table 2

2.2. Dempster shafer theory of evidence

DST (Dempster, 1967; Shafer, 1976) can be regarded as a general extension of Bayesian theory that can robustly deal with incomplete data. In addition to this, DST offers a number of advantages, including the opportunity to assign measures of probability to focal elements, and allowing for the attachment of probability to the frame of discernment. In this section, we briefly review the basic concepts of evidence theory.

Evidence theory first supposes the definition of a set of hypotheses Θ called the frame of discernment, defined as follows:

$$\Theta = \{H_1, H_2, \ldots, H_N\}$$

It is composed of N exhaustive and exclusive hypotheses. Form the frame of discernment Θ, let us denote $P(\Theta)$, the power set composed with the 2^N propositions A of Θ:

$$P(\Theta) = \{\emptyset, \{H_1\}, \{H_2\}, \ldots, \{H_N\}, \{H_1 \cup H_2\}, \{H_1 \cup H_3\} \ldots, \Theta\}$$

where Ø denotes the empty set. The N subsets containing only one element are called singletons. A key point of evidence theory is the basic probability assignment (BPA). The mass of belief in an element of Θ is quite similar to a probability distribution, but differs by the fact that the unit mass is distributed among the elements of $P(\Theta)$,

Table 2
Graded mean integration representation for the importance weight of each criterion.

Very low (VL)	0.1167
Low (L)	0.3000
Medium (M)	0.5000
High (H)	0.7000
Very high (VH)	0.8333

that is to say not only on the singletons in H_N in Θ but on composite hypotheses too. A BPA is a function from $P(\Theta)$ to [0,1] defined by:

$$m : P(\Theta) \to [0,1]$$

and which satisfies the following conditions:

$$\sum_{A \in P(\Theta)} m(A) = 1$$

$$m(\varnothing) = 0$$

In the case of imperfect data (uncertain, imprecise and incomplete), fusion is an interesting solution to obtain more relevant information. Evidence theory offers appropriate aggregation tools. From the basic belief assignment denoted m_i obtained for each information source S_i, it is possible to use a combination rule in order to provide combined masses synthesizing the knowledge of the different sources. Dempsteris rule of combination (also called orthogonal sum), noted by $m = m_1 \oplus m_2$, is the first one within the framework of evidence theory which can combine two BPAs m_1 and m_2 to yield a new BPA:

$$m(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - k} \tag{7}$$

with

$$k = \sum_{B \cap C = \varnothing} m_1(B) m_2(C) \tag{8}$$

where k is a normalization constant, called conflict because it measures the degree f conflict between m_1 and m_2, $k = 0$ corresponds to the absence of conflict between m_1 and m_2, whereas $k = 1$ implies complete contradiction between m_1 and m_2. The belief function resulting from the combination of J information sources $S_j$defined as

$$m = m_1 \oplus m_2 \cdots \oplus m_j \cdots \oplus m_J \tag{9}$$

Given reliability coefficients, the next step is to incorporate them into the fusion process.To handle conflict between information sources, a discounting rule has been introduced in DSET given by the following theorems.

Theorem 1. *Let BEL: $2^\Theta \to [0,1]$ be a belief function and discounting coefficient $\alpha (0 \leqslant \alpha \leqslant 1)$ qualify the strength of the reliability of the evidence. The discounting function, BEL^α: $2^\Theta \to [0,1]$, is defined as* (Shafer, 1976)

$$BEL^\alpha(\Theta) = 1, \tag{10}$$

$$BEL^\alpha(A) = (1 - \alpha) \cdot BEL(A), \quad \forall A \subset \Theta \quad \text{and} \quad A \neq \phi \tag{11}$$

The function BEL^α is also a belief function.

Theorem 2. *From the definition of discounted belief function BEL^α given by* Theorem 1, *the BPA m^αcorresponding to BEL^α are further modified in the following manner* (Shafer, 1976):

$$m^\alpha(\Theta) = (1 - \alpha) m(\Theta) + \alpha, \tag{12}$$

$$m^\alpha(A) = (1 - \alpha) m(A), \quad \forall A \subset \Theta \quad \text{and} \quad A \neq \phi \tag{13}$$

The intrinsic meaning of the transformation is that the reliability of any hypothesis is reflected in its own BPA by redistributing the degree of support among the hypotheses based on the reliability coefficients. So the weight of any evidence holds the value of 1. These discounted BPAs can be combined to obtain the fused result, using the Dempster's rule of combination.

Beliefs manifest themselves at two levels - the *credal* level (from credibility) where *belief* is entertained, and the *pignistic* level where beliefs are used to make decisions. The term "pignistic" was proposed by Smets (2000) and originates from the word pignus, meaning '*bet*' in Latin. Pignistic probability is used for decision-making and uses *Principle of Insufficient Reason*to derive from *basic probability assignment*. It is a point (crisp) estimate in a *belief interval* and can be determined as

$$bet(A_i) = \sum_{A_i \subseteq A_k} \frac{m(A_k)}{|A_k|} \tag{14}$$

Eq. (14) is also called as Pignistic Probability Transformation (PPT).

3. Proposed method

In this section, a new fuzzy evidential approach to deal with MCDM is proposed. Assume that a committee of k decision-makers $(D_1, D_2, \ldots, D_k)$. In general, a multiple criteria decision-making (MCDM) problem can be concisely expressed in matrix format as Hwang and Yoon (1981).

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \end{array}$$

where $A_1, A_2, \ldots, A_m$ are possible alternative, $C_1, C_2, \ldots, C_n$ are criteria with which performance of alternatives are measured, x_{ij} is the rating of alternative A_i with respect to criteria C_j. In this paper, the rating r_{ij} of alternative A_i and the weights of criteria are assessed in linguistic terms represented as triangular fuzzy numbers.

In many MCDM, we do not care the final scores of each alternative. What we need usually is the ranking order of all alternatives to choose the best alternative. Hence, an ideal based on TOPSIS (Hwang & Yoon, 1981) is used to develop our method. For each criteria in MCDM, it can easily determine the ideal solution and negative ideal solution. The distance of an alternative between ideal solution and negative ideal solution can also be determined. The distance functions can be used to generate BPA to describe how close the alternative to ideal solution and to negative ideal solution.

For example, the frame of discernment is $\{IS, NS\}$, where IS means that ideal solution and NS means negative ideal solution. For one alternative, the BPA is shown as follows:

$$m_1\{IS\} = 0.8; \quad m_1\{NS\} = 0.1; \quad m_1\{IS, NS\} = 0.1 \tag{BPA1}$$

It means that:

(1) The BPA supports the hypothesis "the alternative is an ideal solution with belief degree 0.8".
(2) The BPA supports the hypothesis "the alternative is a negative ideal solution to with belief degree 0.1".
(3) The BPA supports the hypothesis "We do not known the alternative is an ideal solution or a negative ideal solution. We know nothing about the alternative with belief degree 0.1".

If we get another alternative with the following BPA

$$m_2\{IS\} = 0.1; \quad m_1\{NS\} = 0.7; \quad m_1\{IS, NS\} = 0.2 \tag{BPA2}$$

It means that:

(1) The BPA supports the hypothesis "the alternative is an ideal solution with belief degree 0.1".
(2) The BPA supports the hypothesis "the alternative is a negative ideal solution to with belief degree 0.7".
(3) The BPA supports the hypothesis "We do not known the alternative is an ideal solution or a negative ideal solution. We know nothing about the alternative with belief degree 0.2".

According to the BPA shown in Eqs. (BPA1) and (BPA2), it can easily to say that alternative 1 is better than alternative 2 due to the fact that the BPA support alternative 1 is more like an ideal solution while alternative 2 is more like a negative ideal solution. Based on the idea mentioned above, the proposed method can be listed step by step as follows:

Step 1. Selects the ideal solution and negative ideal solution and determine the BPA of each performance.
Step 2. Discounts the BPA of performance using the criteria weights as discounting coefficient. Combined the BPA of each criterion to get one comprehensive evaluation of an alternative.
Step 3. Discounts the BPA of combined performance (obtained in Step 2) using the DMs' weights as discounting coefficient. Combined the BPA of all DMs' to get the final performance of each alternative.
Step 4. Determine the final ranking order based on pignistic probability transformation (PPT).

4. Numerical example

In this section, the numerical example used in Wu (2009) is adopted to illustrate the efficiency of our proposed method. Supplier selection is a typical MCDM problem. The initial condition, such as the performance and the weighs of each criterion as well as the weights of experts are shown in Table 3.

There are four criteria, namely product late delivery, cost, risk factor and suppliers' service performance detailed as follows:

C1 Product late delivery – late delivery in percentage is to be minimized.
C2 Cost – overall cost of the product including procurement cost, transportation cost, tariff and custom duties is to be minimized.
C3 Risk factor – the risk of supplier located (political risk, economic risk, terrorism, etc.) is to be minimized.
C4 Supplier's service performance – the ongoing improvement of the product and service (e.g., product quality acceptance level, technological and R&D support, information process) is to be maximized.

Costs and product late delivery rate are crisp values as outlined in Table 3, but risk factors and supplier's service performance have fuzzy data for each source supplier. Now we implement the method from the prior section to this data.

Step 1. Selects the ideal solution and negative ideal solution and determine the BPA of each performance.

For the sake of simplicity, we give following assumptions: We used the crisp number to represent the fuzzy number in Table 3. Also, we transfer the interval weight into crisp number. For example, the weights of four criteria are [0.20, 0.35], [0.30, 0.55], [0.05, 0.30] and [0.25, 0.50], respectively. The crisp weights can be determined as follows:

$$W_1 = \frac{(0.20 + 0.35)}{(0.20 + 0.35) + (0.30 + 0.55) + (0.05 + 0.30) + (0.25 + 0.50)} = \frac{11}{50}$$

$$W_2 = \frac{(0.30 + 0.55)}{(0.20 + 0.35) + (0.30 + 0.55) + (0.05 + 0.30) + (0.25 + 0.50)} = \frac{17}{50}$$

$$W_3 = \frac{(0.05 + 0.30)}{(0.20 + 0.35) + (0.30 + 0.55) + (0.05 + 0.30) + (0.25 + 0.50)} = \frac{7}{50}$$

Table 3
Data for international supplier selection (Wu, 2009).

	Performance	C1 (%)	C2	C3	C4
DM1 [0.20 0.45]	Weights	[0.20 0.35]	[0.30 0.55]	[0.05 0.30]	[0.25 0.50]
	Supplier1	60	40	Low	High
	Supplier2	60	40	Medium	Medium
	Supplier3	70	80	Low	Very high
	Supplier4	50	30	Medium	Medium
	Supplier5	90	130	Very high	Very low
	Supplier6	80	120	Very low	Very low
DM2 [0.35 0.55]	Weights	[0.25 0.45]	[0.2 0.55]	[0.05 0.3]	[0.2 0.6]
	Supplier1	60	40	Medium	High
	Supplier2	60	40	High	Medium
	Supplier3	70	80	Low	Very high
	Supplier4	50	30	Medium	Medium
	Supplier5	90	130	High	Low
	Supplier6	80	120	Low	Very low
DM3 [0.70 0.95]	Weights	[0.20 0.55]	[0.20 0.70]	[0.10 0.40]	[0.20 0.60]
	Supplier1	60	40	Medium	High
	Supplier2	60	40	Low	Low
	Supplier3	70	80	Low	High
	Supplier4	50	30	Medium	High
	Supplier5	90	130	Very high	Low
	Supplier6	80	120	Low	Very low

Table 4
Crisp number of Linguistic items.

Linguistic item	VL	L	M	H	VH
Crisp value	0.1	0.3	0.5	0.7	0.9

$$W_4 = \frac{(0.25+0.50)}{(0.20+0.35)+(0.30+0.55)+(0.05+0.30)+(0.25+0.50)} = \frac{15}{50}$$

Since the discounting coefficient is used in next step, the weights can be transformed into discounting coefficient as follows

$$\alpha_{C1} = \frac{11}{50} / \frac{17}{50} = 0.6471$$

$$\alpha_{C2} = \frac{17}{50} / \frac{17}{50} = 1$$

$$\alpha_{C1} = \frac{7}{50} / \frac{17}{50} = 0.4118$$

$$\alpha_{C1} = \frac{15}{50} / \frac{17}{50} = 0.8824$$

Using the same method, the interval weights of DMs' importance can be transformed into crisp number as follows

$$\alpha_{DM1} = 0.3939$$
$$\alpha_{DM2} = 0.5455$$
$$\alpha_{DM3} = 1.0000$$

The data after transformation mentioned above can be shown in Tables 4 and 5, where all weights and performance are crisp numbers now. It can easily to choose the ideal solution and negative ideal solution. In addition, the distance of an alternative between the ideal solution and negative ideal solution can be easily calculated.

For example, as can be seen from Tables 4 and 5, the ideal solution of the performance according to criterion 1, given by DM1 is 50, while the negative ideal solution of the performance of DM1 is 90. The performance of the supplier 1 is 60. The distance can be calculated as follows:

$$d_{11}(IS) = |60 - 50| = 10$$

$$d_{11}(NS) = |60 - 90| = 30$$

$$d_{11}(IS, NS) = \left|60 - \frac{(50+90)}{2}\right| = 10$$

Hence, the BPA for the first DM1 about supplier 1 according to criterion 1 is obtained as follows:

$$m_{11}(IS) = \frac{d_{11}(NS)}{d_{11}(IS) + d_{11}(NS) + d_{11}(IS, NI)} = \frac{30}{10+30+10} = 0.6$$

$$m_{11}(NS) = \frac{d_{11}(IS)}{d_{11}(IS) + d_{11}(NS) + d_{11}(IS, NS)} = \frac{10}{10+30+10} = 0.2$$

$$m_{11}(IS, NS) = \frac{d_{11}(IS, NS)}{d_{11}(IS) + d_{11}(NS) + d_{11}(IS, NI)} = \frac{10}{10+30+10} = 0.2$$

All BPA can be calculated and shown as in Table 6.

Step 2. Discounts the BPA of performance using the criteria weights as discounting coefficient. Combined the BPA of each criterion to get one comprehensive evaluation of an alternative.

For example, for the first supplier, the BPA of the four criteria can be listed in Table 7.

Using the weighs as discounting coefficient, then the first discounted BPA of supplier 1 according to C1 can be calculated as follows:

$$m_{11}^{\alpha}\{IS\} = \alpha \times m_{11} = 0.6471 \times 0.6 = 0.3883$$

$$m_{11}^{\alpha}\{NS\} = \alpha \times m_{12} = 0.6471 \times 0.2 = 0.1294$$

$$m_{11}^{\alpha}\{IS, NS\} = \alpha \times m_{11}\{IS, NS\} + (1-\alpha) = 0.4823$$

Hence, the performance represented by discounted BPA of the supplier 1 given by the first DM1 is listed in Table 8.

Using the classical Dempster combination rule to combine the four criterion discounted BPA to get the comprehensive opinions of the supplier 1. The results can be shown as follows:

Table 5
Data for international supplier selection after transformation.

	Performance	C1 (%)	C2	C3	C4
DM1 0.3939	Weights	0.6471	1.0000	0.4118	0.8824
	Supplier1	60	40	0.3	0.7
	Supplier2	60	40	0.5	0.5
	Supplier3	70	80	0.3	0.9
	Supplier4	50	30	0.5	0.5
	Supplier5	90	130	0.9	0.1
	Supplier6	80	120	0.1	0.1
DM2 0.5455	Weights	0.8750	0.9375	0.4375	1.0000
	Supplier1	60	40	0.5	0.7
	Supplier2	60	40	0.7	0.5
	Supplier3	70	80	0.3	0.9
	Supplier4	50	30	0.5	0.5
	Supplier5	90	130	0.7	0.3
	Supplier6	80	120	0.3	0.1
DM3 1.0000	Weights	0.8333	1.0000	0.5556	0.8889
	Supplier1	60	40	0.5	0.7
	Supplier2	60	40	0.3	0.3
	Supplier3	70	80	0.3	0.7
	Supplier4	50	30	0.5	0.7
	Supplier5	90	130	0.9	0.3
	Supplier6	80	120	0.3	0.1

Table 6
Generating BPA according to the distance functions.

	Performance	C1 ({IS},{NS},{IS,NS})	C2 ({IS},{NS},{IS,NS})	C3 ({IS},{NS},{IS,NS})	C4 ({IS},{NS},{IS,NS})
DM1	Weights	0.6471	1	0.4118	0.8824
	Supplier1	(0.60,0.20,0.20)	(0.6429,0.0714,0.2857)	(0.60,0.20,0.20)	(0.60,0.20,0.20)
	Supplier2	(0.60,0.20,0.20)	(0.6429,0.0714,0.2857)	(0.50,0.50,0)	(0.50,0.50,0)
	Supplier3	(0.50,0.50,0)	(0.50,0.50,0)	(0.60,0.20,0.20)	(0.6667,0,0.3333)
	Supplier4	(0.6667,0,0.3333)	(0.6667,0,0.3333)	(0.50,0.50,0)	(0.50,0.50,0)
	Supplier5	(0,0.6667,0.3333)	(0,0.6667,0.3333)	(0,0.6667,0.3333)	(0,0.6667,0.3333)
	Supplier6	(0.20,0.60,0.20)	(0.0714,0.6429,0.2857)	(0.6667,0,0.3333)	(0,0.6667,0.3333)
DM2	Weights	0.8750	0.9375	0.4375	1
	Supplier1	(0.60,0.20,0.20)	(0.6429,0.0714,0.2857)	(0.50,0.50,0)	(0.60,0.20,0.20)
	Supplier2	(0.60,0.20,0.20)	(0.6429,0.0714,0.2857)	(0,0.6667,0.3333)	(0.50,0.50,0)
	Supplier3	(0.50,0.50,0)	(0.50,0.50,0)	(0.6667,0,0.3333)	(0.6667,0,0.3333)
	Supplier4	(0.6667,0,0.3333)	(0.6667,0,0.3333)	(0.50,0.50,0)	(0.50,0.50,0)
	Supplier5	(0,0.6667,0.3333)	(0,0.6667,0.3333)	(0,0.6667,0.3333)	(0.20,0.60,0.20)
	Supplier6	(0.20,0.60,0.20)	(0.0714,0.6429,0.2857)	(0.6667,0,0.3333)	(0,0.6667,0.3333)
DM3	Weights	0.8333	1	0.5556	0.8889
	Supplier1	(0.60,0.20,0.20)	(0.6429,0.0714,0.2857)	(0.5714,0.2857,0.1429)	(0.6667,0,0.3333)
	Supplier2	(0.60,0.20,0.20)	(0.6429,0.0714,0.2857)	(0.6667,0,0.3333)	(0.2857,0.5714,0.1429)
	Supplier3	(0.50,0.50,0)	(0.50,0.50,0)	(0.6667,0,0.3333)	(0.6667,0,0.3333)
	Supplier4	(0.6667,0,0.3333)	(0.6667,0,0.3333)	(0.5714,0.2857,0.1429)	(0.6667,0,0.3333)
	Supplier5	(0,0.6667,0.3333)	(0,0.6667,0.3333)	(0,0.6667,0.3333)	(0.2857,0.5714,0.1429)
	Supplier6	(0.20,0.60,0.20)	(0.0714,0.6429,0.2857)	(0.6667,0,0.3333)	(0,0.6667,0.3333)

Table 7
BPA of the four criteria for the first supplier.

	Performance	C1 ({IS},{NS},{IS,NS})	C2 ({IS},{NS},{IS,NS})	C3 ({IS},{NS},{IS,NS})	C4 ({IS},{NS},{IS,NS})
	Weights	0.6471	1	0.4118	0.8824
DM1	Supplier1	(0.60,0.20,0.20)	(0.6429,0.0714,0.2857)	(0.60,0.20,0.20)	(0.60,0.20,0.20)

$$\begin{aligned} m^1_{DM1} &= BPA_{C1}^{\alpha_{C1}} \oplus BPA_{C2}^{\alpha_{C2}} \oplus BPA_{C3}^{\alpha_{C3}} \oplus BPA_{C4}^{\alpha_{C4}} \\ &= (m^1_{DM1}\{IS\} = 0.8829, m^1_{DM1}\{NS\} = 0.0760, m^1_{DM1}\{IS,NS\} = 0.0411) \end{aligned}$$

In above equation, m^1_{DM1} means the BPA given by DM1 about the supplier 1. All the results of the experts of each alternative, taking consideration of combination of four criteria can be listed in Table 9.

Step 3. Discounts the BPA of combined performance (obtained in Step 2) using the DMs' weights as discounting coefficient. Combined the BPA of all DMs' to get the final performance of each alternative.

For example, all the performance about the supplier 1 given by the three DMs can be listed as follows:

$$m^1_{DM1}\{IS\} = 0.8829, m^1_{DM1}\{NS\} = 0.0760, m^1_{DM1}\{IS,NS\} = 0.0411$$

$$m^1_{DM2}\{IS\} = 0.8885, m^1_{DM2}\{NS\} = 0.0904, m^1_{DM2}\{IS,NS\} = 0.0211$$

$$m^1_{DM3}\{IS\} = 0.9270, m^1_{DM3}\{NS\} = 0.0177, m^1_{DM3}\{IS,NS\} = 0.0097$$

The relative weights of each DM are 0.3939, 0.5455 and 1, respectively. Using weights of DMs' as discounting coefficient, then the discounted BPA of three DMs about supplier 1 can be calculated. The discounted BPA of DM1, taking consideration of DM's weights, is listed as follows

$$m^1_{DM1}{}^{\alpha}\{IS\} = 0.3939 \times 0.8829 = 0.3478$$

$$m^1_{DM1}{}^{\alpha}\{NS\} = 0.3939 \times 0.0760 = 0.0299$$

$$m^1_{DM1}{}^{\alpha}\{IS,NS\} = 1 - m^1_{DM1}{}^{\alpha}\{IS\} - m^1_{DM1}{}^{\alpha}\{NS\} = 0.6223$$

All the three DMs' BPAs about supplier 1 can be combined using Dempster rule. The results are shown in Table 10.

Step 4. Determine the final ranking order based on pignistic probability transformation (PPT).

For example, for the supplier 1, the final performance is listed as follows

$$m^1\{IS\} = 0.9727$$

$$m^1\{NS\} = 0.0177$$

$$m^1\{IS,NS\} = 0.0096$$

Then, the results using PPT is shown as follows

$$Bet^1\{IS\} = 0.9727 + \frac{0.0096}{2} = 0.9775$$

The *Bet* and the final ranking order are shown in Table 10.

It can be easily seen that the rank order is supplier 4 > supplier 1 > supplier 2 > supplier 3 > supplier 6 > supplier 5. It is coincided with the results of that presented in Wu (2009).

Table 8
The performance represented by discounted BPA of the supplier 1 given by the first DM1.

	Performance	C1 ({IS},{NS},{IS,NS})	C2 ({IS},{NS},{IS,NS})	C3 ({IS},{NS},{IS,NS})	C4 ({IS},{NS},{IS,NS})
DM1	Weights	0.6471	1	0.4118	0.8824
	Supplier1	(0.3883, 0.1294, 0.4823)	(0.6429, 0.0714, 0.2857)	(0.2471, 0.0824, 0.6705)	(0.5294, 0.1765, 0.2941)

Table 9
Fuse multi-criteria data using discounting coefficient of each criterion.

	Performance	DM1	DM2	DM3	Combined results
	Weights	0.3939	0.5455	1	
({IS},{NS},{IS,NS})	Supplier1	(0.8829, 0.0760, 0.0411)	(0.8885, 0.0904, 0.0211)	(0.9270, 0.0431, 0.0299)	(0.9727, 0.0177, 0.0097)
	Supplier2	(0.7827, 0.1959, 0.0214)	(0.7249, 0.2571, 0)	(0.8138, 0.1545, 0.0317)	(0.8947, 0.0930, 0.0122)
	Supplier3	(0.7475, 0.2525, 0)	(0.8080, 0.1870, 0.0050)	(0.7959, 0.2041, 0)	(0.8888, 0.1111, 0.0001)
	Supplier4	(0.8366, 0.1379, 0.0255)	(0.8649, 0.1351, 0)	(0.9516, 0.0113, 0.0372)	(0.9804, 0.0077, 0.0119)
	Supplier5	(0, 0.9343, 0.0566)	(0.0269, 0.9462, 0.0269)	(0.0308, 0.9404, 0.0289)	(0.0100, 0.9840, 0.0061)
	Supplier6	(0.0768, 0.8584, 0.0648)	(0.0676, 0.8924, 0.0400)	(0.0902, 0.8642, 0.0456)	(0.0367, 0.9477, 0.0156)

Table 10
Fuse multi-persons data using discounting coefficient of each alternative.

Performance	Combined results	$bet(IS)$	Final ranking order
Supplier1	(0.9727, 0.0177, 0.0096)	0.9775	2
Supplier2	(0.8947, 0.0930, 0.0123)	0.9008	3
Supplier3	(0.8888, 0.1111, 0.0001)	0.8888	4
Supplier4	(0.9804, 0.0077, 0.0119)	0.9864	1
Supplier5	(0.0100, 0.9840, 0.0060)	0.0130	6
Supplier6	(0.0367, 0.9477, 0.0156)	0.0445	5

5. Conclusions

In this paper, a new MCDM method based on DST is proposed. It is shown that the new method can deal with MCDM in an efficient manner. We use a supplier selection example to illustate the use of the proposed method. It can easily be applied to other MCDM. In the future, the conflict data fusion alogrithm will be taken into consideration since that the DM may conflict with each other DM and the criterion in MCDM may conflict with each other criterion. If highly conflicting evidence are collected, the classical DS combination rule will get uncorrect results. Hence, it is necessary to efficiently handle conflict evidence in the decision making process (Guo, Shi, & Deng, 2006; Lefevre, 2002; Murphy, 2000).

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References

Ashtiani, B., Haghighirad, F., Makui, A., et al. (2009). Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets. *Applied Soft Computing, 9*(2), 457–461.
Bauer, M. (1997). Approximation algorithms and decision making in the Dempster–Shafer theory of evidence – An empirical study. *International Journal of Approximate Reasoning, 17*(2–3), 217–237.
Beynon, M. (2002). DS/AHP method: A mathematical analysis, including an understanding of uncertainty. *European Journal of Operational Research, 140*(1), 148–164.
Beynon, M. (2005). A method of aggregation in DS/AHP for group decision-making with the non-equivalent importance of individuals in the group. *Computers and Operations Research, 32*(7), 1881–1896.
Beynon, M., Cosker, D., & Marshall, D. (2001). An expert system for multi-criteria decision making using Dempster Shafer theory. *Expert System with Applications, 20*(4), 357–367.
Beynon, M., Curry, B., & Morgan, P. (2000). The Dempster–Shafer theory of evidence: an alternative approach to multicriteria decision modeling. *Omega-International Journal of Management, 28*(1), 37–50.
Bouyssou, D. (1986). Some remarks on the notion of compensation in MCDM. *European Journal of Operational Research, 26*(1), 150–160.
Chou, C. C. (2003). The canonical representation of multiplication operation on triangular fuzzy numbers. *Computers and Mathematics with Applications: An International Journal, 45*, 1601–1610.
Chu, T. C., & Lin, Y.-C. (2009). An extension to fuzzy MCDM. *Computers and Mathematics with Applications, 57*(3), 445–454.
Dempster, A. P. (1967). Upper and lower probabilities induced by a multivalued mapping. *Annals of Mathematical Statistics, 38*, 325–328.
Deng, J. L. (1982). Control problems of grey systems. *Systems and Controls Letters, 5*, 288–294.
Deng, Y. (2006). Plant location selection based on fuzzy TOPSIS. *International Journal of Advanced Manufacturing Technology, 28*, 839–844.
Deng, Y., & Liu, Q. (2005a). An improved optimal fuzzy information fusion method and its application in group decision. *Journal of Computer and Systems Sciences International, 44*, 531–541.
Deng, Y., & Liu, Q. (2005b). A TOPSIS-based centroid-index ranking method of fuzzy numbers and its application in decision-making. *Cybernetics and Systems, 36*, 581–595.
Deng, Y., Shi, W. K., & Liu, Q. (2004). Combining belief function based on distance function. *Decision Support Systems, 38*, 489–493.
Gal, T., & Hanne, T. (2006). Nonessential objectives within network approaches for MCDM. *European Journal of Operational Research, 168*(2), 584–592.
Guo, H. W., Shi, W. K., & Deng, Y. (2006). Evaluating sensor reliability in classification problems based on evidence theory. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics, 36*, 970–981.
Hanaoka, S., & Kunadhamraks, P. (2009). Multiple criteria and fuzzy based evaluation of logistics performance for intermodal transportation. *Journal of Advanced Transportation, 43*(2), 123–153.
Hu, Y. C. (2009). Fuzzy multiple-criteria decision making in the determination of critical criteria for assessing service quality of travel websites. *Expert System with Applications, 26*(3), 6439–6445.
Hwang, C. L., & Yoon, K. (1981). *Multiple attribute decision making: Methods and applications*. New York: Springer.
Kauffman, A., & Gupta, M. M. (1985). *Introduction of fuzzy arithmetic: Theory and applications*. New York: Van Nostrand Reinhold.
Lefevre, E. (2002). Belief function combination and conflict management. *Information fusion, 3*, 149–162.
Mercier, D., Cron, G., Denoeux, T., et al. (2007). Postal decision fusion based on the transferable belief model. *Traitement Du Signal, 24*(2), 133–151.
Murphy, C. K. (2000). Combining belief functions when evidence conflicts. *Decision support systems, 29*, 1–9.
Narasimhan, R., & Vickery, S. K. (1988). An experimental evaluation of articulation of preferences in multiple criterion decision-making (MCDM) methods. *Decision Sciences, 19*(4), 880–888.
Olson, D. L., & Wu, D. (2006). Simulation of fuzzy multiattribute models for grey relationships. *European Journal of Operational Research, 175*(11), 111–120.
Shafer, G. (1976). *A mathematical theory of evidence*. Princeton, NJ: Princeton University Press.

Shyur, H. J., & Shih, H. S. (2006). A hybrid MCDM model for strategic vendor selection. *Mathematical and Computer Modelling, 44*(7–8), 749–761.

Smets, P. (2000). Data fusion in the transferable belief model. In *Proceedings of 3rd international conference on information fusion, fusion 2000. PS21-PS33, Paris, France, July 10–13*.

Srivastava, R. P., & Liu, L. P. (2003). Applications of belief functions in business decisions: A review. Information systems frontiers. *Information Systems, 5*(4), 359–378.

Wadhwa, S., Madaan, J., & Chan, F. T. S. (2009). Flexible decision modeling of reverse logistics system: A value adding MCDM approach for alternative selection. *Robotics and Computer-Integrated Manufacturing, 25*(2), 460–469.

Wu, D. (2009). Supplier selection in a fuzzy group setting: A method using grey related analysis and Dempster–Shafer theory. *Experts Systems with Applications, 36*, 8892–8899.

Wu, D., & Olson, D. L. (2008). Comparison of stochastic dominance and stochastic DEA for vendor evaluation. *International Journal of Production Research, 46*(8), 2313–2327.

Yager, R. R. (2008). A knowledge-based approach to a diversarial decision Making. *International Journal of Intelligent System, 23*(1), 1–21.

Yang, J. L., Chiu, H. N., Tzeng, G.-H., et al. (2008). Vendor selection by integrated fuzzy MCDM techniques with independent and interdependent relationships. *Information Sciences, 178*(21), 4166–4183.

Yang, J. B., & Sen, P. (1997). Multiple attribute design evaluation of complex engineering products using the evidential reasoning approach. *Journal of Engineering Design, 8*(3), 211–230.

Yang, J. B., & Xu, D. L. (2002). On the evidential reasoning algorithm for multiple attribute decision analysis under uncertainty. *IEEE Transactions on Systems Man and Cybernetics Part A-Systems and Humans, 32*(3), 289–304.

Yeh, C. H., & Chang, Y. H. (2009). Modeling subjective evaluation for fuzzy. *European Journal of Operational Research, 194*(2), 464–473.

Zhang, J., Wu, D., & Olson, D. L. (2005). The method of grey related analysis to multiple attribute decision making problems with interval numbers. *Mathematical and Computer Modelling, 42*(9–10), 991–998.

Zimmermann, H. J. (1991). *Fuzzy set theory and its applications*. Boston: Kluwer Academic Publishers..