



Tuning of a PID controller using improved chaotic Krill Herd algorithm



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ABSTRACT

This paper presents an improved version of the Krill Herd optimization algorithm. The proposed algorithm has been applied to determine coefficients of PID controller to achieve desired system response. For this purpose, a cost function based on weighted sum of step response characteristics is considered to be minimized. Simulation results compare performance of the ICKH algorithm with many other optimization algorithms.

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1. Introduction

Mankind has always been looking for best possible solution of problems in environment around himself. According to the situation, these solutions could be whether smallest or biggest possible solutions. Nowadays in engineering problems, achieving the solution with the lowest possible cost, is one of the designers' problems. Former methods to solve optimization problems require enormous computational efforts, which tend to fail as the problem size increases. This is the motivation for employing bio-inspired stochastic optimization algorithms as computationally efficient alternatives to deterministic approach [1]. Observation in nature for inspiration had been the old interests of scientists to achieve optimization methods. Therefore, many optimization algorithms such as ACO [2], PSO [3], GA [4] etc. have been designed with nature-inspired and animals' behavior.

Recently, a new evolutionary algorithm had been proposed by Alavi and Gandomi, inspired by herding behavior of Antarctic krill [5]. Antarctic krill is one of the best-studied species of marine animal. The Krill Herds are aggregations with no parallel orientation existing on time scales of hours to days and space scales of 10 s to 100 s of meters. One of the main characteristics of this specie is its ability to form large swarms [6,7]. Conceptual models have been proposed to explain the observed formation of the Krill Herds [8].

Krill Herd algorithm, nevertheless its powerful skill in solving optimization enigmas, has the problem of trapping in local optima. Confronting this problem using Chaos theory and its mappings would be suggested in Krill Herd algorithm. Chaos can be described as a bounded nonlinear system with deterministic dynamic behavior that has stochastic properties [9,10]. In what is called the “butterfly effect”, small variations of an initial variable will result in huge differences in the solutions after some iterations. Mathematically, chaos is random and unpredictable, yet it also possesses an element of regularity [11]. The aim of this paper is to suggest chaotic Krill Herd algorithm with logistic mapping to solve the problem of trapping in local optima. Later, the proposed algorithm is used to determine coefficients of a PID controller and then simulation results are compared with other optimization algorithms.

Proportional–Integral–Derivative controller or in extenuating words; PID is the most widely used and popular controllers in industrial means because of ease of design and low cost implementation [12,13]. But, the problem is the precise and optimal tuning of PID coefficients. In various books and papers, evolutionary algorithms such as PSO [14], DE [15], GA [16] etc. have been used to tune PID coefficients.

Surveying the operation of a closed-loop system parameters which generally are considered are maximum overshoot, settling time, steady state error, and rise time. In this paper maximum overshoot, distance of nearest pole from imaginary axis, and settling time are examined as indicators of system performance. Desirable performances are short settling time, long distance of nearest pole to the imaginary axis and low maximum overshoot. Thus, the goal is to minimize the weighted sum of maximum overshoot, settling time and distance of nearest pole from imaginary axis.

The rest of this paper is organized as follows. In section 2, the standard Krill Herd algorithm is introduced. The ICKH algorithm

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are presented in section 3. Section 4 reviews PID controller and step response characteristics. Also, cost function is introduced. In section 5, the ICKH algorithm has been applied for tuning of PID controller and results are shown. Finally, the paper is concluded in Section 6.

2. Krill Herd algorithm

Krill Herd (KH) algorithm is a new bio-inspired swarm intelligence, that is based on herding conduct of krill and formulating their swarm movement. The herding of the krill individuals is a multi-objective process including two main goals: (I) increasing krill density, and (II) reaching food. In the present study, this process is taken into account to propose a new metaheuristic algorithm for solving global optimization problems. Density-dependent attraction of krill (increasing density) and finding food (areas of high food concentration) are used as objectives which can finally lead the krill to herd around the global minima. In this process, an individual krill moves toward the best solution when it searches for the highest density and food [5].

The position of a krill in a 2D surface is determined by the following three actions [9,10]:

- (i) Movement induced by other krill individuals;
- (ii) Foraging activity; and
- (iii) Random diffusion

Therefore, the following Lagrangian model is generalized to an n dimensional decision space:

$$\frac{dX_i}{dt} = N_i + F_i + D_i \quad (1)$$

where N_i is the movement induced by other krill, F_i is the foraging movement and D_i is the physical diffusion of i th krill.

Direction of N_i which is called α_i , is affected by local density and position of the best krill. N_i is calculated with following formula:

$$N_i^{\text{new}} = N^{\text{max}} \alpha_i + \omega_n N_i^{\text{old}} \quad (2)$$

where

$$\alpha_i = \alpha_i^{\text{local}} + \alpha_i^{\text{target}} \quad (3)$$

and N^{max} is the maximum induced speed, ω_n is the inertia weight of the motion induced in the range [0,1], N_i^{old} is the last motion induced, α_i^{local} is the local effect provided by the neighbors and α_i^{target} is the target direction effect provided by the best krill individual. According to the measurements of the maximum induced speed [9], it is taken 0.01 (ms^{-1}).

In KH algorithm, the effect of neighbors (α_i^{local}) is formulated as follows:

$$\alpha_i^{\text{local}} = \sum_{j=1}^{NN} \hat{K}_{i,j} \hat{X}_{i,j} \quad (4)$$

$$\hat{X}_{i,j} = \frac{X_i - X_j}{\|X_i - X_j\| + \varepsilon} \quad (5)$$

$$\hat{K}_{i,j} = \frac{K_j - K_i}{K_{\text{worst}} - K_{\text{best}}} \quad (6)$$

where K_{best} and K_{worst} are the best and the worst fitness values of the krill individuals so far; K_i represents the fitness or the objective function value of the i th krill individual; K_j is the fitness of j th ($j = 1, 2, \dots, NN$) neighbor; X represents the related positions; and NN is the number of the neighbors.

For choosing neighbors at the first hand, the sensing distance of each krill are calculated with following equation:

$$d_{s,i} = \frac{1}{5N} \sum_{j=1}^N \|X_i - X_j\| \quad (7)$$

If the distance of two krill individuals is less than the defined sensing distance, then they are neighbors [5].

The best krill affects on others by α_i^{target} which is given by:

$$\alpha_i^{\text{target}} = C^{\text{best}} \hat{K}_{i,\text{best}} \hat{X}_{i,\text{best}} \quad (8)$$

and

$$C^{\text{best}} = 2 \left(\text{rand} + \frac{I}{I_{\text{max}}} \right) \quad (9)$$

The foraging motion (F_i) is estimated by the two main components. One is the food location and the other is the prior knowledge about the food location. For the i th krill individual, this motion can be approximately formulated as follows: [17]

$$F_i = V_f \beta_i + \omega_f F_i^{\text{old}} \quad (10)$$

where

$$\beta_i = \beta_i^{\text{food}} + \beta_i^{\text{best}} \quad (11)$$

V_f is the foraging speed, ω_f is the inertia weight of the foraging motion and it is a number in the range [0,1], F_i^{old} is the last foraging motion. In this paper, we set V_f to 0.02 [18].

The physical diffusion of the krill individuals is considered to be a random process. It can be formulated as follows:

$$D_i = D^{\text{max}} \delta \quad (12)$$

where D^{max} is the maximum diffusion speed, and δ is the random directional vector, and its arrays are random values in range of [-1,1]. The better position of krill leads to the less random motion. Furthermore, another term is added to the physical diffusion formula to consider this effect. This term linearly decreases the random speed with the time (iterations):

$$D_i = D^{\text{max}} \left(1 - \frac{I}{I_{\text{max}}} \right) \delta \quad (13)$$

According to three main actions mentioned above, velocity of each krill can be calculated. The new position of each krill from t to $t + \Delta t$ is formulated as below:

$$X_i(t + \Delta t) = X_i(t) + \Delta t \frac{dX_i}{dt} \quad (14)$$

Δt is a very important parameter which determines the effect of velocity on the new position of krill. This parameter is extremely affected by search space, so that it can be as the following equation:

$$\Delta t = C_t \sum_{j=1}^{NV} (UB_j - LB_j) \quad (15)$$

where NV is the number of variables and LB_j and UB_j are the lower and the upper limits of the j th variable. C_t is varied in the range [0,2]. It is obvious that small values of C_t results precisely search in the search space.

3. Improved chaotic Krill Herd optimization (ICKH)

3.1. Applying limits for velocity and position

To control and improve the Krill Herd algorithm, velocity and range of movement are limited. In order to apply these limits, velocity of each krill is limited as the following equation:

$$-\left(\frac{dX_i}{dt}\right)_{\max} < \left|\frac{dX_i}{dt}\right| < \left(\frac{dX_i}{dt}\right)_{\max} \quad (16)$$

$$\left(\frac{dX_i}{dt}\right)_{\max} = \alpha (UB_j - LB_j) \quad (17)$$

$$\frac{dX_i}{dt} = \min \left\{ \max \left(\frac{dX_i}{dt}, -\left(\frac{dX_i}{dt}\right)_{\max} \right), \left(\frac{dX_i}{dt}\right)_{\max} \right\} \quad (18)$$

where the value of α has been empirically estimated about 0.1.

Sometimes according to the calculated velocity, new position of the krill is somewhere out of the search space. Therefore, the position is limited with the following equation:

$$X_i = \min \left\{ \max (X_i, LB_j), UB_j \right\} \quad (19)$$

3.2. Chaotic Krill Herd

Chaos theory studies the systems that follow deterministic law but exhibit random and unpredictable behavior. In the other words, chaotic systems are sensitive to initial conditions and small changes in initial conditions may lead to quite different outcomes [19–20]. Due to these characteristics, chaos theory is applied in optimization. In KH algorithm, D_i and N_i are parameters contained random numbers which can be modified with chaos mappings. In this paper, the successions are made by the logistic mapping instead of above-mentioned random numbers. Sequences generated by the logistic mapping are formulated as below:

$$x(t + 1) = 4 \times x(t) \times (1 - x(t)) \quad (20)$$

In Eq. (20), $x(0)$ is created randomly between 0 and 1 for each iteration. Notice that $x(0)$ should not be 0, 0.25, 0.5, 0.75 or 1. Fig. 1 shows the chaotic $x(t)$ using a logistic map for 50 iterations where $x(0)=0.2$. Also, Fig. 2 illustrates the flowchart of ICKH algorithm.

4. PID controller

PID is an acronym for *Proportional–Integral–Derivative*, referring to the three terms operating on the error signal to produce a control signal. PID control is one of the earlier control strategies [21]. Its early implementation was in pneumatic devices, followed by vacuum and solid state analog electronics, before arriving at today's digital implementation of microprocessors. Since many process plants controlled by PID controllers have similar dynamics, it has been found possible to set satisfactory controller parameters from less plant information than a complete mathematical model [21].

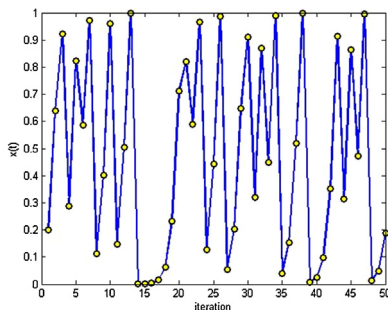


Fig. 1. Chaotic $x(t)$ using logistic mapping.

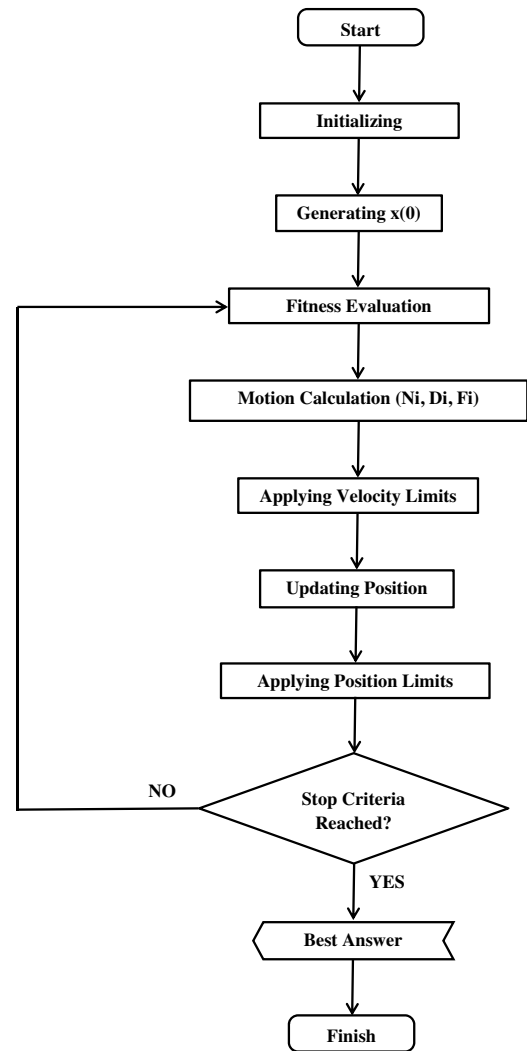


Fig. 2. Flowchart of Improved Chaotic Krill Herd algorithm.

Fig. 3 shows a closed loop system controlled by PID. In a PID controller, the error signal is used to generate the proportional, integral and derivative actions, with the resulting signal weighted and summed to form the control signal applied to the plant model. A mathematical description of the PID controller is:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{d}{dt} e(t) \quad (21)$$

where $u(t)$ is the control signal, $e(t)$ is the error signal defined as $e(t) = R(t) - y(t)$, $R(t)$ is the reference input signal, $y(t)$ is the output signal, k_p , k_i and k_d denote the coefficients for the proportional, integral and derivative terms, respectively.

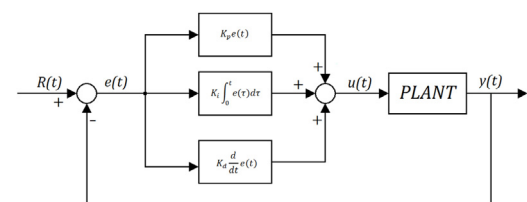


Fig. 3. A closed loop system controlled by PID.

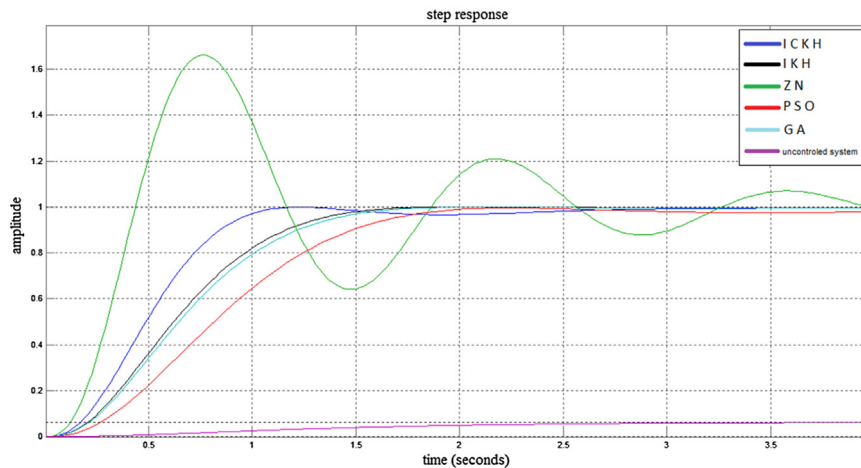


Fig. 4. Step responses for the system of Example 1.

Table 1
Effect of each PID coefficient on system performance indicators.

| | Rise time | Overshoot | Settling time |
|-------|--------------|-----------|---------------|
| K_p | Decrease | Increase | Small change |
| K_i | Decrease | Increase | Increase |
| K_d | Small change | Decrease | Decrease |

The effects of PID Coefficients on step response of system are shown in Table 1.

To optimize system behavior, the following cost function is considered to be minimized:

$$f = T_s + 2MP + SI \tag{22}$$

where T_s is settling time of step response, MP is maximum overshoot, and SI is stability index defined as:

$$SI = \frac{-1}{\min(\max(\text{real}(\text{poles}(G))), 0)} \tag{23}$$

where G is the transfer function of closed loop system. It is clear that big real parts of poles lead to small SI index.

5. Illustrative examples

In this section, two examples are given to illustrate the proposed algorithm. Later, we are going to compare this algorithm with other algorithms such as KH, GA, PSO, CPSO and Ziegler–Nichols tuning

method [22]. The lower and upper bounds of PID coefficients are considered as: K_p : [0,100], K_i : [0,100], K_d : [0,10].

Example 1. Consider the following third order system:

$$G(s) = \frac{1}{s^3 + 9s^2 + 23s + 15} \tag{24}$$

Utilizing the optimization method presented in this paper, the three coefficients of PID controller are tuned as follows:

$$K_p = 39.709, \quad K_i = 27.03, \quad K_d = 10$$

The results of the step response characteristics are shown in Table 2. In all simulations, the population size and number of iterations are 20 and 30, respectively.

Fig. 4 shows the step response of the uncontrolled system and system controlled by the designed PIDs using various optimization algorithms.

Example 2. As the second experiment, consider a second order system with transfer function as below:

$$G(s) = \frac{0.01}{(0.01s + 0.1)(0.5s + 1)(0.01)^2} \tag{25}$$

Using the ICKH optimization algorithm, the three parameters of PID controller are calculated as follows:

$$K_p = 47.48, \quad K_i = 81.86, \quad K_d = 4.06$$

Table 2
Step response characteristics for the system of Example 1.

| | MP (%) | Rise time (s) | Settling time (s) | Peak time (s) | SI | Cost function |
|---------------------|--------|---------------|-------------------|---------------|------|---------------|
| ICKH | 0.15 | 0.68 | 1.01 | 1.39 | 0.84 | 2.15 |
| IKH | 0.06 | 0.9 | 1.32 | 2.02 | 1 | 2.44 |
| GA | 0 | 0.95 | 1.39 | 2.1 | 1.05 | 3.49 |
| PSO | 0 | 1.14 | 1.67 | 11.59 | 1.52 | 2.66 |
| ZN | 66.2 | 0.26 | 5.1 | 0.75 | 1.86 | 139.36 |
| Uncontrolled system | 0 | 2.24 | 4.12 | 7.34 | 1.62 | 5.74 |

Table 3
Step response characteristics for the system of Example 2.

| | MP (%) | Rise time (s) | Settling time (s) | Peak time (s) | SI | Cost function |
|---------------------|------------------------------|---------------|-------------------|---------------|------|---------------|
| ICKH | $8 \times 10^{-4} \approx 0$ | 0.15 | 0.21 | 0.28 | 0.36 | 0.57 |
| IKH | 0.01 | 0.206 | 0.28 | 0.39 | 0.55 | 0.85 |
| GA | 0 | 0.24 | 0.34 | 1.19 | 0.47 | 0.81 |
| PSO | 0 | 0.15 | 0.53 | 0.28 | 0.53 | 1.06 |
| ZN | 62 | 0.06 | 2.02 | 0.15 | 0.72 | 126.74 |
| Uncontrolled system | 0 | 1.01 | 1.84 | 3.34 | 0.48 | 2.32 |

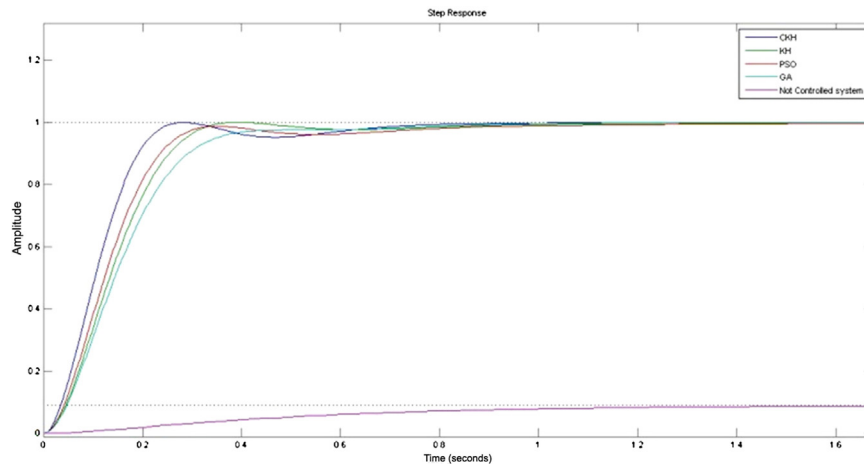


Fig. 5. Step responses for the system of Example 2.

The results of the step response characteristics are shown in Table 3. In all simulations, the population size and number of iterations are 20 and 30, respectively.

Fig. 5 shows the step response of the uncontrolled system and system controlled by the designed PIDs using various optimization algorithms.

6. Conclusions

In this paper, a recently bio-inspired optimization algorithm called Krill Herd has been modified using chaos theory and some other improvements. The proposed algorithm (ICKH) has been applied for tuning of PID coefficients. Simulation results show that the ICKH algorithm has better performance in terms of step response characteristics compared to some optimization algorithms.

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