Journal of Air Transport Management 54 (2016) 104–110

Contents lists available at ScienceDirect

Journal of Air Transport Management

journal homepage: www.elsevier.com/locate/jairtraman

Optimization of assigning passengers to seats on airplanes based on their carry-on luggage

R. John Milne [a, *], Mostafa Salari [b]

[a] Neil '64 and Karen Bonke Associate Professor in Engineering Management, Clarkson University School of Business, 333 B. H. Snell Hall, P.O. Box 5790, Potsdam, NY 13699-5790, USA

[b] Department of Civil Engineering, Schulich School of Engineering, University of Calgary, 2500 University Drive Northwest, Calgary, Alberta, T2N 1N4, Canada

ARTICLE INFO

Article history:
Received 7 August 2015
Received in revised form
17 March 2016
Accepted 28 March 2016

Keywords:
Airplane boarding
Passenger luggage
Sequencing
Mixed integer program

ABSTRACT

We build upon previous work that assigns passengers to a specific numerical position in line that depends on their seat location. The assignment of seat locations to passengers depends on the number of luggage they carry aboard the plane. In particular, we propose a mixed integer programming model that determines the number of luggage to be carried by passengers assigned to each seat. Numerical results indicate that the proposed approach results in a reduction of the time to complete the boarding of the plane. The improvement is greatest when many luggage are carried onto the plane. The optimal distribution of luggage assigns passengers with few carry-on bags to the rows of the plane closest to the entrance.

1. Introduction

The total annual cost of airplane delays in 2007 in the United States alone was $29 billion (Ball et al., 2010). Jaehn and Neumann (2015) cite cost estimates of airplane delays ranging from $30 to $250 per minute. While some delays result from bad weather, mechanical issues, and congested airspace, as noted in Ball et al. (2010), other delays are due to the time to board passengers. To reduce the time it takes passengers to board their airplanes, Delta Airlines offered valet services on some flights to pre-load passengers' luggage (i.e. bags) for them (Koeing, 2015). Clearly, methods that reduce the time to board airplanes would be advantageous for the airlines and their passengers.

Skorupski and Wierzbinska (2015) determine the optimal time to wait for a late passenger to arrive at the gate. Many publications assume passengers are called to board in blocks or groups (e.g., Kuo, 2015; Bachmat et al., 2013; Bachmat and Elkin, 2008; Bazargan, 2007; Soolaki et al., 2012; Van den Briel et al., 2005) and that passengers board in a random sequence within a group. In an invited literature review, Jaehn and Neumann (2015) provide a broad overview of boarding methods and describe the 12 most relevant papers in detail. Of the methods they studied, the Steffen (2008) boarding sequence results in the fastest time to complete the boarding of all passengers.

Boarding starts when the first passenger begins entering the aisle of the airplane in row 1 and concludes when all passengers have been seated. We assume a fully loaded airplane with 20 rows and three seats on each side of a single aisle. Fig. 1 illustrates the Steffen (2008) boarding sequence. If we assume that all passengers walk down the aisle at the same speed and there is always an empty row between them, then with Steffen (2008), the first set of 10 passengers to board the plane all begin storing their carry-on luggage, if any, at the same time and occupy a window seat in every other row. For instance, as indicated in Fig. 1, the 10th passenger to board the plane sits adjacent to the window in row 2 and begins storing his or her luggage in an overhead bin at the same time that the first passenger to board begins to store his or her luggage in row 20. The first group of 10 passengers is followed by a second group of 10 passengers sitting on the opposite side of the plane. As implied by Fig. 1, the process continues until the final 10 passengers to board (passengers 111–120) take their aisle seats in the 10 odd-numbered rows of the plane.

Milne and Kelly (2014) and Qiang et al. (2014) build upon the work of Steffen (2008) by considering the amount of carry-on

* Corresponding author.
E-mail addresses: jmilne@clarkson.edu (R.J. Milne), Mostafa.salari2@ucalgary.ca (M. Salari).

http://dx.doi.org/10.1016/j.jairtraman.2016.03.022
0969-6997/

Row	Window	Middle	Aisle	Entrance	Aisle	Middle	Window
1	40		120				30
2	20						10
3	39		119				29
4	19						9
5	38		118				28
6	18						8
7	37		117				27
8	17						7
9	36		116				26
10	16						6
11	35		115				25
12	15						5
13	34		114				24
14	14						4
15	33		113				23
16	13					43	3
17	32		112				22
18	12					42	2
19	31		111				21
20	11					41	1

Fig. 1. Passengers boarding a plane in the sequence of the Steffen (2008) method.

luggage that passengers bring aboard the plane. Both of these papers utilize the Steffen (2008) sequence of boarding in which passengers board in a specified numeric sequence determined by their seat assignments. However, Steffen (2008) ignores the volume of carry-on luggage in the seat assignments. Milne and Kelly (2014) assign passengers to seats so that the carry-on luggage is distributed evenly throughout the plane and so that passengers with the most bags sit nearest to the windows. Qiang et al. (2014) assign passengers with the most bags to seats in the rear of the plane. Qiang et al. (2014) achieve a reduction in the boarding time (versus Steffen, 2008) that they point out is "much consistent with works done by Milne and Kelly (2014)." They say that their approach is easier to understand and implement than Milne and Kelly (2014). However, a limitation of Qiang et al. (2014) is the potential for overcrowding of luggage near the rear of the plane. Suppose, for instance, that there are 30 passengers carrying two bags and they are all assigned to sit in the final five rows of the airplane. Would there be room in the overhead bins for six bags on both sides of the aisle for five consecutive rows? We suspect that applying the Qiang et al. (2014) method consistently may lead to some situations of overcrowding near the rear of the plane. This could lead to blockage in earlier rows and result in an increase in total boarding time. Milne and Kelly (2014) avoid such overcrowding. Consequently, we use Milne and Kelly (2014) as a benchmark to test against the proposed method. In our method, we propose using a mixed integer program (MIP) to determine the number of luggage to be carried by passengers assigned to each seat assignment. Similarly to Milne and Kelly (2014) and Qiang et al. (2014), we propose first assigning luggage to seats and after the luggage assignment has been completed, assign passengers carrying those amounts of luggage to those seats, and have them board the plane in the Steffen (2008) sequence. Our objective is to minimize the time to complete the boarding of the airplane.

In Section 2, we describe the assumptions we make regarding passenger flow, the storing of luggage, and sitting down. In Section 3, we describe the MIP model we propose. In Section 4, we describe numerical results comparing the proposed method with the method of Milne and Kelly (2014). Section 5 concludes our paper by highlighting insights, discussing practical considerations for implementation, and suggesting future research directions.

2. Passenger movement assumptions

We make the following assumptions on passengers flow, the storing of luggage, and the time to sit down. In the absence of interference from other passengers, we assume that the time it takes a passenger to move down the aisle from one row to the next, *Trow*, is 2.4 seconds and that the time for a passenger to sit down after storing any carry-on bags, *Tsit*, is 8 s. These times are the same as the average times used by Milne and Kelly (2014) and are based upon Van Landeghem and Beuselinck (2002)—who gathered data at Brussels National Airport—and Audenaert et al. (2009). We use average times in our assignment of luggage to seat locations because we assume we do not know the speed of individual passengers.

At time zero, the first passenger begins walking down the aisle. We assume a passenger walking or standing in the aisle consumes the aisle space of an entire row. This includes some personal space for passenger comfort. Because we assume a Steffen (2008) sequence of boarding, there will be at least one row separating the seat of a passenger from the seat of the next passenger that follows in the boarding sequence; consequently, there will not be two passengers storing their luggage at the same time in adjacent rows; this allows for further personal comfort and safety. We assume a passenger begins storing his or her carry-on bags in the overhead bin after completely entering the row in which he or she will be sitting. For instance, referring again to Fig. 1, the first passenger will begin storing any bags in the overhead bin above his or her window seat in row 20 after completing entering row 20 at time 48 s (calculated via $20 * Trow = 20 \times 2.4 = 48$). Consistent with Milne and Kelly (2014), we assume that a passenger does not begin entering a row until the row has been completely cleared of other passengers. For instance, the second passenger will wait until time 4.8 s before he or she begins to enter row 1. That is because it takes the first passenger 2.4 s to enter the first row and another 2.4 s to clear (exit) it. Time 4.8 s is the instant at which the first passenger has immediately left row 1, is standing in row 2, and is about to enter row 3. At time 7.2 s, the first passenger is standing in row 3 and the second passenger is standing in row 1. Until the final passenger begins walking down the aisle, we assume there is always a passenger waiting at the aisle's entrance for aisle space to become available to enter the first row. That first row aisle space becomes available when the previous passenger has cleared row 1 (either by completing a move into row 2 or by sitting down in a row 1 seat).

We use the same luggage storage assumptions as Milne and Kelly (2014). A passenger carries zero, one, or two bags onto the plane. Each row has an overhead bin on each side of the aisle. We assume each bin has unlimited storage space but account for the fact that a passenger takes longer to store luggage when the passenger has more luggage to store and when there is already more luggage in the bin. In particular, a passenger takes *Tstore* seconds to store his or her luggage using Eq. (1) derived by Audenaert et al. (2009).

$$Tstore = ((Nbin + Npassenger) * Npassenger/2) * Trow \qquad (1)$$

The terms in Eq. (1) are defined as follows:

Tstore Time to store the luggage (calculated)
Nbin The number of luggage in the bin prior to the passenger's arrival
Npassenger The number of luggage the passenger has
Trow Time for a passenger to walk from one row to the next

Table 1 shows the result of applying Eq. (1) to calculate the time it will take a passenger to store luggage as a function of the passenger's seat position and the amount of luggage being stored in the overhead bin above the seat. In consistence with Milne and Kelly (2014), we assume each passenger can carry 0, 1 or 2 carry-on bags. Each row in the table indicates luggage storage time when a particular combination of luggage is assigned to the overhead bin of a particular row and particular side of the plane. For example, the combination $c = 17$ reflects a decision that assigns to the window, middle, and aisle seats three passengers who are carrying 1, 2, and 1 bags respectively, which results in these passengers taking 1.2, 7.2, and 4.8 s respectively to store their luggage as indicated in the table.

3. Mixed integer programming model

The purpose of the proposed MIP is to determine the number of luggage to be carried by passengers assigned to each seat assignment. We refer to this as the luggage assignment. At first glance, a natural modeling approach would be to assign a decision variable that represents the number of bags carried by a passenger in a specified seat on the plane. However, this approach would result in a nonlinear model due to the calculation of the time to store luggage. Nonlinearities are best avoided within a mathematical programming model because of computational run time issues and the possibility of being trapped in a local optimum. In contrast, the solution to a MIP is guaranteed to result in a global optimal solution (Powell and Baker, 2014). Consequently, to keep our model as a MIP, we introduce the decision variable $C_{r,s,c,}$ to indicate whether the assigned combination of bags of passengers in row r on side s of the plane corresponds to combination c in Table 1.

We describe our mathematical model as follows:

Subscripts:

r = row of the plane

Table 1
Time to store luggage as a function of luggage carried for each passenger on one side of a row.

Combination (C)	Luggage carried			Time to store		
	Window	Middle	Aisle	Window	Middle	Aisle
1	0	0	0	0	0	0
2	0	0	1	0	0	1.2
3	0	0	2	0	0	4.8
4	0	1	0	0	1.2	0
5	0	1	1	0	1.2	2.4
6	0	1	2	0	1.2	7.2
7	0	2	0	0	4.8	0
8	0	2	1	0	4.8	3.6
9	0	2	2	0	4.8	9.6
10	1	0	0	1.2	0	0
11	1	0	1	1.2	0	2.4
12	1	0	2	1.2	0	7.2
13	1	1	0	1.2	2.4	0
14	1	1	1	1.2	2.4	3.6
15	1	1	2	1.2	2.4	9.6
16	1	2	0	1.2	7.2	0
17	1	2	1	1.2	7.2	4.8
18	1	2	2	1.2	7.2	12
19	2	0	0	4.8	0	0
20	2	0	1	4.8	0	3.6
21	2	0	2	4.8	0	9.6
22	2	1	0	4.8	3.6	0
23	2	1	1	4.8	3.6	4.8
24	2	1	2	4.8	3.6	12
25	2	2	0	4.8	9.6	0
26	2	2	1	4.8	9.6	6
27	2	2	2	4.8	9.6	14.4

s = side of the plane
c = combination of luggage assignment for a particular row and particular side of the plane
p = passenger to board the plane (e.g. $p = 43$ corresponds to the 43rd passenger to board the plane)
b = number of bags carried by a passenger

Sets:

R = set of rows on the plane = {1, …20}
S = set of sides on the plane = {left, right}
C = set of possible luggage combinations for a single side of a single row = {1, …, 27}
P = set of passengers to board the plane = {1, …, 120}
B = set of the possible numbers of bags carried by a passenger = {0, 1, 2}

Parameters:

row_p = row in which passenger p will be seated (this is predetermined according to the Steffen (2008) sequence of boarding the plane)
$side_p$ = side of the plane in which passenger p will be seated (this is predetermined according to the Steffen (2008) sequence of boarding the plane)
$Y_{c,b}$ = the number of passengers carrying b bags who will sit in one row's side of the plane if combination c is chosen for that side of the row
$Tstore_{p,c}$ = time for passenger p to store luggage in the event that combination c is chosen for the row and side of the plane in which this passenger is seated according to the Steffen (2008) boarding sequence (these values are shown in Table 1)
$NumPassengersWithBbags_b$ = the number of passengers boarding the plane who are carrying b bags where $b \in B$
$Trow$ = time it takes a passenger to walk from one row to the next row (2.4 s)
$Tsit$ = time it takes a passenger to sit after storing luggage (8 s)

Decision Variables:

$C_{r,s,c}$ = binary variable indicating whether the combination of bags chosen for side s of the plane in row r corresponds to combination c
= 1 if combination c is chosen for row r and side s; 0 otherwise
$ClearRow_{p,r}$ = time at which passenger p has cleared (exited) the aisle of row r (defined for all $p \in P$ and $r \le row_p$; and assumed for notational convenience to be to zero when $r > row_p$)
$TimeToCompleteBoarding$ = time at which the final passenger to sit has been seated

Objective Function:

$$\text{Minimize } TimeToCompleteBoarding \tag{2}$$

Constraints:

$$TimeToCompleteBoarding \ge ClearRow_{p,r} \;\; \forall p \in P, r \in R : r = row_p \tag{3}$$

$$\sum_{c \in C} C_{r,s,c} = 1 \;\; \forall r \in R, s \in S \tag{4}$$

$$\sum_{r\in R}\sum_{s\in S}\sum_{c\in C} Y_{c,b}\times C_{r,s,c} = NumPassengerWithBbags_b \quad \forall b\in B \tag{5}$$

$$ClearRow_{p,r} \geq \; ClearRow_{p,r-1} + Trow \quad \forall p\in P, r < row_p \tag{6}$$

$$ClearRow_{p,r} \geq ClearRow_{p',r} + 2\times Trow \quad \forall p' < p, p\in P, r < row_p \tag{7}$$

$$ClearRow_{p,r} \geq \; ClearRow_{p',r+1} + Trow \; \forall p' < p, p\in P, r < row_p \tag{8}$$

$$ClearRow_{p,row_p} \geq ClearRow_{p,row_{p-1}} + \sum_{c\in C} Tstore_{p,c}\times C_{row_p,side_p} + Tsit \;\forall p\in P \tag{9}$$

$$ClearRow_{p,row_p} \geq ClearRow_{p',row_p} + Trow + \sum_{c\in C} Tstore_{p,c} \times C_{row_p,side_p} + Tsit \quad \forall p' < p, p\in P \tag{10}$$

The objective function (2) minimizes the time to complete boarding of the airplane. Constraints (3) ensure that the plane has not completed boarding until all passengers have been seated. These are the times at which the passengers have cleared the aisle in the rows where they are sitting. Constraints (4) ensure that exactly one combination of luggage assignment is chosen for each side of each row of the airplane. Constraints (5) ensure that the total number of passengers boarding the plane with zero, one, and two bags respectively equals the total number of passengers carrying zero, one, and two bags who are assigned to seats on the plane.

Constraints (6) through (8) model passengers flow for the situation in which passenger p will walk from row r to row $r+1$, and constraints (9) and (10) model the situation in which passenger p will sit in row_p. Passenger p will be standing in row r at time $ClearRow_{p,r-1}$. This is the instant when he or she has completed exiting the previous row, $r-1$. Because it takes a passenger *Trow* seconds to walk from row r to row $r+1$, constraints (6) ensure that passenger p cannot clear row r until *Trow* seconds after clearing the previous row, $r-1$. Passenger p cannot begin to enter row r prior to it being clear of all passengers p' who boarded the plane prior to passenger p. Because it takes passenger p a total time of *Trow* seconds to completely enter row r, the passenger will have completely entered (filled the aisle of) row r no earlier than $ClearRow_{p',r} + Trow$. It will take passenger p an additional *Trow* seconds after entering row r to completely clear row r as ensured in constraints (7). Passenger p cannot begin to enter row $r+1$ until after this row has been cleared of all previous passengers p'. Consequently, constraints (8) ensure that passenger p cannot clear row r any earlier than *Trow* seconds after row $r+1$ has become clear of previous passengers. That is because it takes passenger p a total of *Trow* seconds to walk from row r to row $r+1$ after which time passenger p will have cleared row r by having completely entered (filled the aisle in) row $r+1$. Constraints (9) ensure that passenger p, who will sit in row_p, has first cleared the previous row and will clear row_p after storing any carry-on bags and then sitting. Constraints (10) have the same rationale as constraints (7), except with passenger p sitting in row_p. In constraints (10), first row_p must become clear of previous passengers (which happens at time $ClearRow_{p',row_p}$); second, passenger p enters row_p (which takes *Trow* seconds); third, the passenger stores any carry-on bags, and finally the passenger sits (which takes *Tsit* seconds).

4. Numerical results

We implemented the MIP using GAMS for the algebraic model formulation and GURBOI as the mixed integer programming solver on a personal computer with a 2.5 GHz Intel® Core™ i7-4710 HQ processor and 12 GB of memory. We used GUROBI "out of the box" with its default parameter settings (except with zero tolerance gap). Each MIP instance was solved in a fraction of a second.

4.1. Test case 1: base case

For Test Case 1, we assume that 43, 52, and 25 passengers board the plane with zero, one, and two bags respectively. This is the same distribution of luggage as contained in an example of Milne and Kelly (2014). Fig. 2 shows the luggage assignment as determined by Milne and Kelly (2014), in which the luggage is distributed approximately evenly throughout the plane with passengers carrying two bags sitting adjacent to the windows.

Fig. 3 shows the luggage assignment as determined by the proposed MIP. Observe that the MIP assigned no luggage to the first two rows of the airplane. To understand the reason, consider the Steffen (2008) boarding sequence under the assumption used by the MIP that each passenger walks at the same rate. The first 10 passengers proceed down the aisle and begin storing their bags at the same time in the even numbered rows. This group of 10 passengers is followed by another group of 10 passengers and so on, with each group of 10 passengers beginning to store their bags at the same time. Once the first passenger of a group begins walking down the aisle, if this passenger's progress is not impeded waiting for passengers of the previous group to move out of the way, then no passenger within the group will have to wait for any passengers. Consequently, optimal solutions tend to have the first passengers of the groups incur no (or limited) waiting times. These passengers directly follow the last passengers of the previous group. That is, passengers 11, 21, 31, …111 (the first passengers of their groups) directly follow the previous groups' passengers 10, 20, 30, …110. The latter passengers sit in rows 1 and 2 according to the Steffen

				Entrance			
Row	Window	Middle	Aisle		Aisle	Middle	Window
1	1	1	0		0	1	2
2	2	1	0		0	1	1
3	1	1	0		0	1	2
4	2	1	0		0	0	2
5	1	1	0		0	1	2
6	2	1	0		0	1	1
7	2	1	0		0	1	2
8	1	1	0		0	1	2
9	2	1	0		0	1	1
10	1	1	0		0	1	2
11	2	1	0		0	0	2
12	1	1	0		0	1	2
13	2	1	0		0	1	1
14	2	1	0		0	1	2
15	1	1	0		0	1	2
16	2	1	0		0	1	1
17	1	1	0		0	1	2
18	2	1	0		0	0	2
19	1	1	0		0	1	2
20	2	1	0		0	1	1

Fig. 2. Each cell shows the number of bags carried by passengers in each seat according to the Milne/Kelly algorithm for Test Case 1.

Row	Window	Middle	Aisle	Entrance	Aisle	Middle	Window
1	0	0	0		0	0	0
2	0	0	0		0	0	0
3	0	0	0		0	2	0
4	0	0	0		1	1	1
5	2	2	0		1	2	2
6	0	0	2		1	1	1
7	0	0	0		0	1	1
8	0	2	2		1	1	1
9	1	2	0		1	2	1
10	0	0	0		1	1	1
11	0	1	0		1	1	1
12	1	1	1		1	1	1
13	2	0	0		1	1	1
14	1	1	1		1	1	1
15	1	2	0		0	2	2
16	2	2	1		1	1	1
17	1	0	0		1	1	1
18	2	2	2		1	1	1
19	0	2	0		2	2	2
20	1	1	1		0	2	2

Fig. 3. Each cell shows the number of bags carried by passenger in each seat according to the proposed MIP for Test Case 1.

(2008) boarding sequence. As indicated in Fig. 3, the MIP assigned zero bags to these passengers sitting in the first two rows of the plane. Because these final passengers of each group do not spend time storing luggage, this reduces waiting times for the first passengers of the subsequent groups who follow them.

The final group of passengers to board (111–120) sit in odd-numbered aisle seats as indicated in Fig. 1. In the optimal solution of Fig. 3, these final 10 passengers carry zero luggage. In this solution, all 10 of these passengers have finished sitting down at 643.2 s. This equals the total time to complete boarding of the plane in the event that no passengers bring luggage aboard the plane. Observe that if even one of these final 10 passengers carried a single bag aboard the plane, the time to complete the plane's boarding would increase. Conversely, passengers 91–100 (who are sitting in the even numbered rows in the same column as passengers 111–120) can store their luggage without impacting the time at which passengers 101–110 arrive at their seats. Consequently, passengers 91–100 can and often do have a positive number of carry-on bags in the optimal solution of Fig. 3.

The luggage assignment of Fig. 3 is one of many optimal solutions. We get the same total boarding time of 64.2 s from using the proposed MIP, except with an added limitation that rows 3–20 are each permitted to have a total of only two, three, or four bags on each side of the aisle. This excludes combinations 1, 2, 4, 10, 18, 24, 26, and 27 that have 0, 1, 1, 1, 5, 5, 5, and 6 bags respectively as shown in Table 1. As with the Fig. 3 solution, the optimal solution of the MIP with these limitations results in zero bags assigned to the first two rows of the plane and to the final 10 passengers to board it.

To determine the total boarding time resulting from the Milne and Kelly (2014) luggage assignment of Fig. 2, we used a special version of the proposed MIP in which the luggage assignment decision variables ($C_{r,s,c,}$) have their values fixed to correspond to the luggage assignment of Fig. 2. This approach ensures a valid comparison between Milne and Kelly (2014) and the proposed approach using the MIP. The time to board for Test Case 1 using Milne and Kelly (2014) is 673.2 s in contrast to the 643.2 s resulting from using the proposed MIP. In other words, using the proposed MIP-based approach results in a 4.5% reduction in boarding time from that attained using Milne and Kelly (2014).

4.2. Test case 2: high volume of luggage

For Test Case 2, we investigate the impact of the proposed method when passengers bring more luggage aboard the plane than in Test Case 1. In particular, for Test Case 2, there are 12, 72, and 36 passengers carrying zero, one, and two bags respectively. For this data, the proposed MIP results in a time to board of 646.8 s that is 6.4% less than the 691.2 s to board resulting from the Milne and Kelly (2014) method.

Fig. 4 shows the luggage assignment resulting from using a version of the proposed MIP with an added limitation that rows 3–20 are each permitted to have a total of only three, four, or five bags on each side of the aisle. This results in the same time to board (646.8 s) as the proposed MIP without this limitation on valid luggage configurations. The Fig. 4 luggage assignment has only one bag in rows 1 and 2. In rows 3–20, each passenger is carrying at least one bag and nine of the final ten passengers to board each carries exactly one bag.

When the proposed MIP is used, the boarding time of Test Case 2 (646.8 s) is 3.6 s higher than the boarding time of Test Case 1 (643.2). The value of 3.6 s occurs multiple times in the *Time to store* columns of Table 1. In Fig. 4, passenger 111 stores a bag in an overhead bin in row 19 that takes him or her 3.6 s to store according to the assigned combination 14. Conversely, in Fig. 3 (containing MIP results for Test Case 1), passenger 111 is not carrying any luggage.

4.3. Varying luggage volumes

Each row of Table 2 shows the time to board using the proposed MIP and using the Milne and Kelly (2014) method for a specified percentage of zero, one, and two carry-on bags. For each row in the

Row	Window	Middle	Aisle	Entrance	Aisle	Middle	Window
1	0	0	1		0	0	0
2	0	0	0		0	0	0
3	2	1	0		1	1	1
4	1	1	1		1	1	1
5	1	1	1		2	2	1
6	1	2	2		2	2	1
7	1	1	1		1	2	2
8	1	2	1		2	1	2
9	1	1	1		1	1	1
10	2	2	1		1	1	1
11	1	1	1		1	1	2
12	1	2	2		2	2	1
13	1	1	1		2	1	2
14	2	2	1		1	1	1
15	1	1	1		2	2	1
16	1	1	2		1	1	1
17	1	1	1		2	1	2
18	1	2	2		2	2	1
19	1	1	1		1	2	2
20	2	2	1		1	1	1

Fig. 4. Each cell shows the number of bags carried by passengers in each seat according to the proposed MIP when limited combinations per row/side are permitted in rows 3–20 for Test Case 2.

table, the proposed MIP performs better than Milne and Kelly (2014), except of course for the case when no luggage is carried onto the plane. The relative improvement of the proposed MIP (versus Milne and Kelly, 2014) tends to increase as more luggage is brought aboard the plane, with a 9.7% improvement for the case with the most luggage (180 bags). Though not shown in the table, we ran the 180 bags data through a version of the proposed MIP that contained an added limitation that rows 3–20 are each permitted to have a total of only three, four, or five bags on each side of the aisle. The boarding time resulting from this run (649.2) was higher than that resulting from the proposed MIP without the limitation on permitted combinations.

4.4. Impact of randomness

As noted above, the proposed MIP assigns luggage to seats under the assumption that all passengers walk at the same rate and take the same time to sit. We continue with this assumption in this section for the purposes of luggage assignment to seats. However, once those luggage allocation decisions have been made, we investigate the impact on boarding time that results from passengers walking and sitting at various speeds. In particular, we generate ten sets of random data. For each set, the row to row time (*Trow*) and time to sit (*Tsit*) for each of the 120 passengers is generated at random in the same manner used by Milne and Kelly (2014). *Trow* follows a triangular distribution with a min, mode, and max of 1.8, 2.4, and 3 s respectively, and *Tsit* follows a triangular distribution with a min, mode, and max of 6, 8, and 10 s respectively. Because passengers who walk quickly are likely to be quick in taking their seat, the value of a single uniform random variable is generated between 0 and 1 for each passenger. This value is used with the inverse of the two cumulative probability density functions to determine *Trow* and *Tsit* for each passenger. See Milne and Kelly (2014) for further details.

With ten random sets and 120 passengers in each set, we generated random walking and sitting times for a total of 1200 passengers. Table 3 shows the time to board for each of these random sets when using the methods of Milne and Kelly (2014) and the proposed MIP for Test Case 1 (with a total of 102 bags from 43, 52, and 25 passengers boarding the plane with zero, one, and two bags respectively). As indicated in Table 3, the proposed MIP results in an improvement over Milne and Kelly (2014) that varies between 3.5 and 4.2% for the ten sets of random data. This relative improvement is not as high as the 4.5% improvement of Test Case 1 when each passenger walks and sits at the same speed. We performed similar calculations for the ten sets of random data for the case with a total of 168 bags (from 12, 48, and 60 passengers carrying zero, one, and two bags respectively.) A similar pattern emerged where the proposed MIP results in improvements over Milne and Kelly (2014) that vary between 7.5% and 7.9% and is less than the relative improvement of 8.3% of the 168 bags case in Table 2 when passengers have the same speed. While the relative improvement resulting from use of the proposed MIP decreased in the presence of randomness, in all of these cases, the proposed method outperforms Milne and Kelly (2014). The proposed method optimizes using deterministic values of row movement, storage, and sitting times. Consequently, the proposed method appears slightly more vulnerable to the randomization of these times than the Milne and Kelly (2014) method that ignores these times when determining seat assignments.

5. Conclusions

Using the proposed mixed integer programming (MIP) model—to determine the number of bags to be carried by passengers assigned to each seat—results in a meaningful reduction in the time to complete the boarding of an airplane when compared with the method of Milne and Kelly (2014). As noted in Milne and Kelly (2014), their method results in a two to three percent improvement versus Steffen (2008)—the best previous method for minimizing boarding time. Both the proposed method and that of Milne and Kelly (2014) use the Steffen (2008) approach of assigning passengers to a specific numerical position in the boarding line that depends on their seat location. The proposed method and the Milne and Kelly (2014) method utilize information on the number of bags passengers carry aboard the plane. This raises two practical concerns: 1) How can an airline know in advance the number of carry-on bags a passenger will bring on board? 2) How to line up the passengers so that they board the plane in the proper sequence? We address these two concerns in turn.

Spirit Airlines (2015) charges lower fees to those passengers who specify—prior to check-in—that they are bringing bags aboard the plane. Similarly, Allegiant Air (2015) charges a lower fee for carry-on luggage when specified at the time of purchasing the ticket. For those airlines not wishing to charge carry-on fees, the volume of carry-on luggage may be estimated from factors such as whether the passenger has checked luggage and the duration of time between the first and final legs of a passenger's round trip ticket.

Lining up passengers in the proper boarding sequence can be accomplished in several ways. Milne and Kelly (2014) discuss some of these ways including the Southwest Airlines approach of "having passengers line up next to columns which are labeled with relative boarding sequence numbers."

Practical aspects for future research include families traveling together and passenger seating preferences. Other future research

Table 2
Summary statistics for Milne/Kelly vs proposed MIP when varying luggage volumes.

Probability passenger carries			Total	Time to board		
0 bag	1 bag	2 bags	#Bags	Milne/Kelly	Proposed MIP	%Improvement
10%	30%	60%	180	716.4	646.8	9.7
10%	40%	50%	168	705.6	646.8	8.3
10%	50%	40%	156	697.2	646.8	7.2
10%	60%	30%	144	691.2	646.8	6.4
20%	50%	30%	132	690.0	643.2	6.8
30%	50%	20%	108	673.2	643.2	4.5
40%	40%	20%	96	667.2	643.2	3.6
50%	40%	10%	72	656.4	643.2	2.0
60%	30%	10%	60	654.0	643.2	1.7
70%	20%	10%	48	651.6	643.2	1.3
80%	10%	10%	36	649.2	643.2	0.9
100%	0%	0%	0	643.2	643.2	0.0

Table 3
Summary statistics for random sets of times when passengers carry 102 bags on board.

Random set	Time to board		%Improvement from using
	Milne/Kelly	Proposed MIP	Proposed MIP
1	672	646	3.8
2	677	652	3.7
3	668	640	4.2
4	675	649	3.9
5	668	641	4.0
6	679	655	3.6
7	680	656	3.5
8	674	650	3.6
9	677	652	3.8
10	678	653	3.8
Average:	675	649	3.8

could exploit individual passenger walking speeds that may be estimated from information such as the passenger's age (often specified at the time of ticket purchase) or information attained from a passenger's purchases of other products (e.g. running shoes, weight loss pills). Because the proposed MIP uses deterministic parameters, this suggests opportunities for stochastic optimization or other methods for generating solutions that are robust to variations from averages.

In the meantime, this manuscript provides a mixed integer programming model that results in airplane boarding times that are faster than those resulting from other known methods. The improvement is particularly helpful when a lot of luggage is carried on board. Furthermore, this manuscript suggests insights into the assignment of passengers to seats in the event the Steffen (2008) boarding sequence is used, for example, to have the first two rows of the plane occupied (or mostly occupied) with passengers without carry-on bags.

References

Allegiant Air, 2015. Accessed 2 August 2015. http://www.allegiantair.com/popup/optional-services-fees.

Audenaert, J., Verbeeck, K., Berghe, G.V., 2009. Multi-agent based simulation for boarding. In: Proceedings of the 21st Belgian-Netherlands Conference on Artificial Intelligence, pp. 3–10.

Bachmat, E., Elkin, M., 2008. Bounds on the performance of back-to-front airplane boarding policies. Operations Res. Lett. 36, 597–601.

Bachmat, E., Khachaturov, V., Kuperman, R., 2013. Optimal back-to-front airplane boarding. Phys. Rev. E 87 (6), 062805.

Ball, M., Barnhart, C., Dresner, M., Hansen, M., Neels, K., Odoni, A., Peterson, E., Sherry, L., Trani, A., Zou, B., 2010. Total delay impact study. In: NEXTOR Draft Final Report Prepared for the Federal Aviation Administration.

Bazargan, M., 2007. A linear programming approach for aircraft boarding strategy. Eur. J. Operational Res. 183, 394–411.

Jaehn, F., Neumann, S., 2015. Airplane boarding. Eur. J. Operational Res. 244, 339–359.

Koeing, D., 2015. Speedier boarding? Delta to try preloading carry-on bags. U. S. A. Today. June 2, 2015.

Kuo, C.-C., 2015. A review of heuristic and optimal aircraft boarding strategies in the US airline industry. In: Trends and Research in the Decision Sciences: Best Papers from the 2014 Annual Conference. Pearson Education, pp. 313–330.

Milne, R.J., Kelly, A.R., 2014. A new method for boarding passengers onto an airplane. J. Air Transp. Manag. 34, 93–100.

Powell, S.G., Baker, K.R., 2014. Management Science: the Art of Modeling with Spreadsheets, fourth ed. Wiley, Hoboken, New Jersey.

Qiang, S.-J., Jia, B., Xie, D.-F., Gao, Z.-Y., 2014. Reducing airplane boarding time by accounting for passengers' individual properties: a simulation based on cellular automaton. J. Air Transp. Manag. 40, 42–47.

Spirit Airlines, 2015. Accessed 2 August 2015. http://www.spirit.com/OptionalFees.aspx.

Skorupski, J., Wierzbinska, M., 2015. A method to evaluate the time of waiting for a late passenger. J. Air Transp. Manag. 47, 79–89.

Soolaki, M., Mahdavi, I., Mahdavi-Amiri, N., Hassanzadeh, R., Aghajani, A., 2012. A new linear programming approach and genetic algorithm for solving airline boarding problem. Appl. Math. Model. 36, 4060–4072.

Steffen, J.H., 2008. Optimal boarding method for airline passengers. J. Air Transp. Manag. 14, 146–150.

Van den Briel, M.H.L., Villalobos, J.R., Hogg, G.L., Lindemann, T., Mule, A.V., 2005. America west airlines develops efficient boarding strategies. Interfaces 35, 191–201.

Van Landeghem, H., Beuselinck, A., 2002. Reducing passenger boarding time in airplanes: a simulation based approach. Eur. J. Operational Res. 142, 294–308.