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Note

Performance assessment of airlines using range-adjusted measure, strong complementary slackness condition, and discriminant analysis

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ABSTRACT

This study integrates RAM (range-adjusted measure), SCSC (strong complementary slackness condition), and DEA–DA (data envelopment analysis–discriminant analysis) to rank airlines. As conventional DEA models do not fully use all inputs and outputs, they result zero in many multipliers. These sorts of DEA models may yield many efficient decision-making units (DMUs). This decreases the discrimination power of DEA. To overcome this limitation, this study proposes a novel application of RAM–DEA/SCSC along with DA. A case study demonstrates the applicability of our proposed approach.

1. Introduction

This paper proposes a novel data envelopment analysis (DEA) model for evaluating the performance of Iran's airlines. Our proposed model not only ranks all Iranian airlines but also fully uses all inputs and outputs. To prevent alternative solutions, Sueyoshi and Sekitani (2007) integrated DEA and strong complementary slackness condition (SCSC) and proposed the DEA/SCSC model. Nonetheless, the DEA/SCSC model does not guarantee that the ties among efficient decision-making units (DMUs) are broken. To rank efficient DMUs, Sueyoshi and Sekitani (2007) used DEA–discriminant analysis (DEA–DA). Barros and Wanke (2015) used the technique for order of preference by similarity to ideal solution (TOPSIS) to assess the relative efficiency of African airlines. Merkert and Pearson (2015) developed a new approach for measuring the impact of an airline's customer service on profit. To calculate the efficiency scores of airlines (DMUs), this paper integrates one of the DEA models called range-adjusted measure (RAM) and SCSC. To rank DMUs, DEA–DA is applied.

2. Methodology

2.1. Steps of calculations

In this paper, first, the RAM model and the SCSC concept are combined. The main objective of our proposed RAM-DEA/SCSC model is to classify all DMUs into efficient and inefficient groups so that all multipliers of efficient DMUs become positive. Then, the two groups of DMUs are separated using the DEA–DA model to minimize misclassification. As a result, unique optimal solutions are calculated by adjusted efficiency score. To reduce the number of efficient DMUs, we combine RAM/SCSC and DEA–DA. Thus, our proposed approach can identify the best DMU.

2.2. Primal and dual of RAM-DEA model

Here, we review the RAM model and propose a new version of the RAM model. Suppose we have n DMUs (DMU_j; $j = 1, 2, \ldots, n$). Let $x_j = (x_{1j}, x_{2j}, \ldots, x_{ij})^T > 0$ and $y_j = (y_{1j}, y_{2j}, \ldots, y_{rj})^T > 0$ denote the input and output vectors of the jth DMU, respectively. Two vectors (d_i^x and d_r^y) represent the input and output slacks, respectively. The superscript T stands for a vector transpose. The subscripts i and r show the ith input ($i = 1, 2, \ldots, m$) and the rth output ($r = 1, 2, \ldots, s$), respectively. The subscript k shows the DMU under evaluation. The RAM model for assessing the relative efficiency of the kth DMU is as

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follows (Cooper et al., 1999):

$$\begin{aligned} & maz\ Z = \sum_{i=1}^{m} R_i^x d_i^x + \sum_{r=1}^{s} R_r^y d_r^y \\ & \sum_{j=1}^{n} x_{ij}\lambda_j + d_i^x = x_{ik} \quad i = 1, \dots, m \\ & \sum_{j=1}^{n} y_{rj}\lambda_j - d_r^x = y_{rk} \quad r = 1, \dots, s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & d_i^x, d_r^x, \lambda_j \geq 0 \quad j = 1, \dots, n \end{aligned} \tag{1}$$

where $\lambda = (\lambda_1, \dots, \lambda_n)^T$ refers to the "intensity" variable. They are used to link the input and output vectors by a convex combination. The ranges in Model (1) are calculated by upper and lower bounds on inputs and outputs. These upper and lower bounds are presented as follows:

$$\begin{aligned} R_i^x &= \frac{1}{(m+s)(max\{x_{ij}|j=1,\dots,n\} - \min\{x_{ij}|j=1,\dots,n\})} \\ R_r^y &= \frac{1}{(m+s)\left(max\left\{y_{rj}\middle|j=1,\dots,n\right\} - \min\left\{y_{rj}\middle|j=1,\dots,n\right\}\right)} \end{aligned} \tag{2}$$

The optimal efficiency score of DMU under evaluation can be determined as follows:

$$\theta^* = 1 - \left[\sum_{i=1}^{m} R_i^x d_i^x + \sum_{r=1}^{s} R_r^y d_r^y\right] \tag{3}$$

where d_i^x and d_r^y are slack variables and represent the level of inefficiency. The optimal efficiency score is calculated by subtracting the level of inefficiency from unity. The dual formulation of Model (1) is as follows:

$$\begin{aligned} & \min P = \sum_{i=1}^{m} v_i x_{ik} - \sum_{r=1}^{s} u_r y_{rk} + \sigma \\ & s.t \\ & \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + \sigma \geq 0, \quad j = 1, \dots, n \\ & v_i \geq R_i^x, \quad i = 1, \dots, m \\ & u_r \geq R_r^y, \quad r = 1, \dots, s \\ & \sigma, v_i, u_r : URS \end{aligned} \tag{4}$$

where v_i and u_r represent all dual variables related to the first and second set of constraints in Model (1). The dual variable σ is obtained from the third constraint of Model (1).

2.3. RAM−DEA/SCSC

Given the complementary slackness condition (CSC), correlations between the optimal solution of Model (1) (σ^*, λ^*, d_i^x, d_r^y) and the optimal solution of Model (4) (Z^*,v^*,u^*) are shown as follows (Bazaraa et al., 2010):

$$\lambda_j^*\left(\sigma^* + v^* x_j + u^* y_j\right) = 0 \quad (j = 1, \dots, n) \tag{5}$$

$$d_i^{x*} v_i^* = 0 \quad (i = 1, \dots, m) \tag{6}$$

$$d_r^{y*} u_r^* = 0 \quad (r = 1, \dots, s) \tag{7}$$

Both the optimal solutions of Model (1) and optimal solutions of Model (4) are satisfied in the following conditions:

$$\lambda_j^* + \left(\sigma^* + v^* x_j + u^* y_j\right) > 0 \quad (j = 1, \dots, n) \tag{8}$$

$$d_i^{x*} + v_i^* > 0 \quad (i = 1, \dots, m) \tag{9}$$

$$d_r^{y*} + u_r^* > 0 \quad (r = 1, \dots, s) \tag{10}$$

Here, we combine Model (1) and Model (4) as follows:

$$\begin{aligned} & \text{Max } \eta \\ & \text{s.t.} \\ & \sum_{j=1}^{n} x_{ij}\lambda_j + d_i^x = x_{ik}, \\ & \sum_{j=1}^{n} y_{rj}\lambda_j - d_r^y = y_{rk}, \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + \sigma \geq 0 \\ & \sum_{i=1}^{m} R_i^x d_{ik}^x - \sum_{r=1}^{s} R_r^y d_{rk}^y = \sum_{i=1}^{m} v_i x_{ik} - \sum_{r=1}^{s} u_r y_{rk} + \sigma, \quad j = 1, \dots, n \\ & \lambda_j + \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + \sigma \geq \eta e^T, \quad j = 1, \dots, n \\ & v_i \geq R_i^x, \quad i = 1, \dots m \\ & u_r \geq R_r^y, \quad r = 1, \dots s \\ & v_i + d_i^x \geq \eta e^T, \quad i = 1, \dots m \\ & u_r + d_r^y \geq \eta e^T, \quad i = 1, \dots s \\ & d_i^x, d_r^x, \lambda_j, x_i, y_r, \eta \geq 0; \quad \sigma, v_i, u_r : URS; \quad j = 1, \dots, n \end{aligned} \tag{11}$$

The fifth constraint of Model (11) ensures that the objective function of Model (1) is equivalent to the objective function of Model (4). The last constraints of Model (11) are related to SCSC (5)–(10). The unit vector is represented by $e = (1, 1, \dots, 1)$. A new decision variable (η) is added to Model (11) to keep SCSC optimal.

2.4. Review of characteristics of supporting hyperplane in DEA

Sueyoshi and Goto (2011) characterized the supporting hyperplane mathematically by the following proposition:

Proposition 1. A supporting hyperplane of DMU_k is as follows:

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \sigma \geq 0, \quad j = 1, \dots n \tag{12}$$

Here, v_i ($i = 1, \dots, m$) and u_r ($r = 1, \dots, s$) are parameters for indicating the direction of a supporting hyperplane, and σ indicates the intercept of the supporting hyperplane. The parameters are unknown and should be measured by the following equations:

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \sigma = 0, \quad j \in R_k \tag{13}$$

where R_k stands for a reference set of the kth DMU, for which operational performance is measured by Expression (13). Sueyoshi and Goto (2011) indicated that Proposition 1 characterizes a supporting hyperplane in data space. The proposition shows how the reference set of the kth DMU characterizes the position of a supporting hyperplane(s). Therefore, the proposition indicates the significance of uniqueness of a reference set. To discriminate between efficient and inefficient DMUs, we can adjust the position of a supporting hyperplane.

3. DEA–DA

3.1. DEA–DA model

Determining a unique reference set and a unique projection are important issues in DEA. One of the main objectives of this paper is to reduce the number of efficient DMUs. To address this, the paper extends a previous version of the DEA–DA model proposed by Farzipoor Saen (2013). The proposed DEA–DA model can be formulated as follows:

Stage 1: After running Model (11), all DMUs can be separated into efficient (E) and inefficient (IE) groups.
Stage 2: To reduce the number of efficient DMUs, the following DEA–DA model is applied to the two groups obtained in stage 1:

$$\begin{aligned}
& \min M \sum_{j\in E} y_j + \sum_{j\in IE} y_j \\
& \text{s.t.} \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \sigma + My \geq 0 \quad , j\in E \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \sigma - My \leq -\eta, \quad j\in IE \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} = 1, \quad j = 1,\ldots,n \\
& v_i > \varepsilon\zeta_i, \quad i = 1,\ldots,m \\
& u_r > \varepsilon\zeta_r, \quad r = 1,\ldots,s \\
& \sum_{i=1}^{m} \zeta_i = m \\
& \sum_{r=1}^{s} \zeta_r = s \\
& \sigma : unrestricted \\
& y_j : \text{binary} \\
& \zeta_i : \text{binary} \\
& \zeta_r : \text{binary}
\end{aligned} \tag{14}$$

where M is a given large number and η is a given small number. The η value is used to avoid DMU(s) existing on an estimated discriminant function. The objective function is designed to minimize the total number of incorrectly classified DMUs by counting a binary variable (y_j). The discriminant scores ($-\sigma$) and ($-\sigma-\eta$) are used to separate the efficient group and inefficient group, respectively. In Model (14), as efficient DMUs are of greater priority than the inefficient group, M is incorporated into the objective function of the efficient group. Note that all inputs and outputs of DMUs are incorporated into the discriminant function, which can be regarded as a supporting hyperplane in DEA.

Stage 3: After running Model (14), the optimal solution can be obtained for efficient DMUs by the following equation:

$$\rho_j = \sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} + \sigma^* \quad j = (1,\ldots,n) \tag{15}$$

Using ρ_j, we compute their adjusted efficiency scores by the following formulation:

$$\text{Efficiency of DMU}_k = \left[\rho_j - min_j\rho_j\right] \Big/ \text{Range} \ \rho_j \tag{16}$$

Note that Expression (16) ranges between zero and one. Each DMU, which has a higher level of efficiency score, is selected as the best DMU. Therefore, we can rank efficient DMUs by (16). To find the range, we first need to find the lowest and highest values of ρ_j. The range is determined by subtracting the highest value from the lowest value as follows:

$$\begin{aligned}
& \overline{\rho}_j = \max\left(\rho_j\right) \\
& \underline{\rho_j} = \min\left(\rho_j\right) \\
& Range \ \ \rho_j = \overline{\rho}_j - \underline{\rho_j}
\end{aligned}$$

3.2. Supporting hyperplane and discriminant line

3.2.1. Definition of discriminant line

Discriminant analysis (DA) is a statistical technique to predict acategoricaldependent variable by one or more continuous orbinary independent variables. Original dichotomous DA was developed by Fisher (1936). DA is used to determine whether a set of variables are effective in predicting category membership. Furthermore, DA is used when groups are known in advance. DA is a technique used to predict DMUs' membership in predetermined groups (Sueyoshi, 1999).

4. Case study

4.1. Data set

One of the main objectives of this paper is to measure the performance of Iranian airlines. In conventional DEA models, the multipliers are positive or zero. Zero multipliers imply that the DEA does not utilize all inputs and outputs. To overcome this limitation, this paper proposes a new model for evaluating airlines using all inputs and outputs. Table 1 summarizes the data set of Iranian airlines (DMUs) from 2007 to 2011. There are two outputs and three inputs. Passenger plane (km) and cargo plane (km) are selected as outputs, whereas the number of airplanes, number of employees, and number of flights are selected as the inputs.

4.2. Results of RAM/SCSC

Table 2 shows the results of our proposed models. The third

Table 1
Summary of data set from 2007 to 2011.

	Inputs			Outputs	
	Number of airplanes	Number of employees	Number of flights	Passenger plane (km)	Cargo plane (km)
Max	63	8166	58,551	6,507,092	1,491,545
Min	6	200	1594	93,403	791
Average	20	1800	18,440	2,081,307	203,813
Standard deviation	18	1983	16,353	1,960,865	474,086

Table 2
Results.

Airline (DMU)	Year	Efficiency score by Model (1) or (4)	Efficiency score by Model (11)	V_1	V_2	V_3	U_1	U_2
Iran Aseman	2007	0.5187	0.5187	0.44E-02	0.30E-04	0.45E-05	0.30E-07	0.1E-06
	2008	0.5349	0.5349	0.46E-02	0.40E-04	0.30E-05	0.30E-07	0.1E-06
	2009	0.5411	0.5411	0.36E-02	0.40E-04	0.40E-05	0.23E-07	0.1E-06
	2010	0.5095	0.5095	0.33E-02	0.30E-04	0.36E-05	0.10E-07	0.1E-06
	2011	0.4430	0.4430	0.40E-02	0.20E-04	0.40E-05	0.30E-07	0.1E-06
Iran Air Tour	2007	0.3032	0.3032	0.40E-02	0.30E-04	0.42E-05	0.22E-07	0.9E-06
	2008	0.2531	0.2531	0	0.39E-04	0.63E-05	0.55E-07	0.6E-06
	2009	0.2304	0.2304	0.33E-02	0.35E-04	0.40E-05	0.30E-07	0.1E-06
	2010	0.2996	0.2996	0	0.40E-04	0.30E-05	0.40E-07	0.3E-06
	2011	0.4009	0.4009	0.40E-02	0.13E-04	0.26E-05	0.40E-07	0.1E-06
Kish Air	2007	0.9936	0.9936	0	0.25E-04	0.38E-05	0.29E-07	0.5E-06
	2008	0.8366	0.8366	0.48E-02	0.40E-04	0.30E-05	0.30E-07	0
	2009	0.8339	0.8339	0.36E-02	0.82E-04	0.40E-05	0.10E-07	0.1E-06
	2010	1	1	0.33E-02	0.40E-04	0.40E-05	0.20E-07	0.1E-06
	2011	0.8794	0.8794	0.63E-02	0	0.56E-05	0.33E-07	0.2E-06
Mahan	2007	0.9482	0.9482	0.44E-02	0.20E-04	0.36E-05	0.65E-07	0.2E-06
	2008	0.7475	0.7475	0.46E-02	0.50E-04	0.31E-05	0.30E-07	0.1E-06
	2009	0.7175	0.7175	0.30E-02	0.40E-04	0.33E-05	0.10E-07	0.1E-06
	2010	1	1	0.33E-02	0.40E-04	0.40E-05	0.20E-07	0.1E-06
	2011	0.5851	0.5851	0.40E-02	0.40E-04	0.33E-05	0.20E-07	0.2E-06
Taban	2007	1	1	0.44E-02	0.31E-04	0.45E-05	0.30E-07	0.1E-06
	2008	0.9631	0.9631	0	0.40E-04	0.31E-05	0.30E-07	0.1E-06
	2009	0.9350	0.9350	0.36E-02	0.65E-04	0.23E-05	035E-07	0.2E-06
	2010	1	1	0.88E-02	0.40E-04	0.30E-05	0.10E-07	0.1E-06
	2011	1	1	0.40E-02	0.20E-04	0.40E-05	0.82E-07	0.1E-06
Naft Iran	2007	1	1	0.44E-02	0.30E-04	0.45E-05	0.30E-07	0.1E-06
	2008	1	1	0.46E-02	0.40E-04	0.83E-05	0.30E-07	0.1E-06
	2009	1	1	0.33E-02	0.42E-04	0.32E-05	0.22E-07	0.1E-06
	2010	1	1	0.33E-02	0.40E-04	0.18E-05	0.30E-07	0.1E-06
	2011	1	1	0.35E-02	0.53E-04	0.33E-05	0.30E-07	0.2E-06
Caspian	2007	1	1	0.19E-02	0.37E-04	0.45E-05	0.30E-07	0.1E-06
	2008	1	1	0.46E-02	0.40E-04	0.31E-05	0.30E-07	0.1E-06
	2009	0.9857	0.9857	0.69E-02	0.52E-04	0.36E-05	0	0.6E-06
	2010	1	1	0.33E-02	0.40E-04	0.23E-05	0.65E-07	0.1E-06
	2011	0.84	0.84	0.40E-02	0.20E-02	0.30E-05	0	0.1E-06

column of Table 2 describes the efficiency score of airlines from 2007 to 2011, which are measured by Model (1) or (4). Further, Table 2 represents efficiency scores and associated multipliers measured by Model (11). Note that the results of Model (1) or (4) and Model (11) are similar. As shown in Table 2, Naft Iran is efficient in all periods and is selected as the best DMU.

4.3. Results of DEA–DA and adjusted efficiency score

Tables 3 and 4 provide the results of the DEA–DA (Expression 13) and adjusted efficiency scores calculated by Expression (15). Note that the range of efficiency scores of Model (15) is between zero and one ($0 \leq \rho^* \leq 1$). Using the proposed approach, each DMU has a unique efficiency score indicating the high discrimination power of the proposed approach. If $\rho^* = 1$, then the DMU under evaluation has the best performance and can be selected as the best DMU. If $\rho^* = 0$, then the DMU under evaluation has the worst performance. The results of the proposed approach reveal that Naft Iran in 2007, Taban in 2008, Kish Air in 2009, and Taban in 2010 and 2011 had the best performance (adjusted efficiency score of 1 and rank 1). Moreover, Iran Air Tour had the worst performance

Table 3
Results of DEA–DA (Expression 13).

Year	Optimal multipliers of inputs			Optimal multipliers of outputs	
	V_1	V_2	V_3	U_1	U_2
2007	0.001	0.8973	0.0996	0.001	0.001
2008	0.423	0.3772	0.1964	0.002	0.001
2009	0.001	0.5404	0.4565	0.001	0.001
2010	0.724	0.1462	0.1277	0.001	0.001
2011	0.001	0.2234	0.7735	0.001	0.001

Table 4
Results of adjusted efficiency score (Expression 15).

Firm	Year	Adjusted efficiency score	Rank
Iran Aseman	2007	0.2710	4
	2008	0.4710	4
	2009	0.1299	6
	2010	0.0900	6
	2011	0.1326	6
Iran Air Tour	2007	0	7
	2008	0	7
	2009	0	7
	2010	0	7
	2011	0	7
Kish Air	2007	0.2455	5
	2008	0.3669	5
	2009	1	1
	2010	0.9088	3
	2011	0.8640	4
Mahan	2007	0.2055	6
	2008	0.2626	6
	2009	0.3055	5
	2010	0.5249	5
	2011	0.6022	5
Taban	2007	0.5217	2
	2008	1	1
	2009	0.7175	2
	2010	1	1
	2011	1	1
Naft Iran	2007	1	1
	2008	0.5812	3
	2009	0.5413	4
	2010	0.8685	4
	2011	0.9296	2
Caspian	2007	0.2708	3
	2008	0.8333	2
	2009	0.6300	3
	2010	0.9238	2
	2011	0.8932	3

(adjusted efficiency score 0 and rank 7) in all periods. Therefore, the results show that only one efficient DMU exists every year.

5. Conclusions

Due to the significance of the transportation industry in every economy, the performance evaluation of transportation systems has become an integral part of their management. Performance measurement of airlines is mostly based on the application of conventional DEA models. However, the conventional use of DEA models may yield zero in many multipliers. This implies that these models do not use all inputs and outputs and thus produce many efficient DMUs. To overcome this limitation, this paper proposed a novel application of RAM/SCSC along with DEA–DA. The proposed model, in this study, was applied to evaluate the performance of seven airlines. Three inputs and two outputs were selected. We used RAM/SCSC to evaluate the performance of DMUs and to separate them into two efficient and inefficient groups. To rank DMUs, this study applied DEA–DA. The results indicated that Naft Iran has the maximum efficiency score and is selected as the best DMU with an efficiency score of 1 in all periods. Further, Iran Air Tour was the most inefficient DMU. To improve the performance of airlines, the sources of poor performance were identified.

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