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# Inventory optimization for a customer airline in a Performance Based Contract

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A B S T R A C T

The supply of spare parts has a crucial role in the aviation sector, mainly due to the high costs of spare parts and to the strict availability requirements. In a stand-alone scenario, an airline owns the spare parts and manages the maintenance tasks by itself. A new trend consists of not owning the spare parts and delegate the maintenance tasks to an external company, taking advantage of a specific Performance Based Contract (PBC). The PBCs aim to reduce the ownership cost for the customer airline, while ensuring a target system performance. Spare parts become a variable cost for the customer airline and a business income for the maintenance supplier, which is commonly another airline.

This paper proposes an innovative model, i.e. the PBC-METRIC, which supports the customer airline manager to minimize the spare parts supply cost, in compliance with the airline availability requirements and with respect to the PBC. In detail, the PBC-METRIC models a multi-echelon, multi-item, single-indenture, multi-transportation network, by an innovative two-steps algorithm, defining the PBC specifications as modelling variables and parameters. A case study on a European airline, with the role of customer in a PBC, illustrates the outcome of the model.

## 1. Introduction

In transportation industry, defence sector, oil drilling, telecommunication and other industries, a reliable spare parts supply is a key element to provide service continuity. High inventory levels generally minimize the critical consequences of stock-outs, even in case of remote suppliers and long procurement times. However, a high inventory level generates significant stock cost. Determining the optimum inventory level for spare parts becomes a crucial strategic target for complex systems. In these cases, it is necessary to adopt the so-called system approach, which evolves the classic item approach, as prescribed by Sherbrooke's METRIC (Sherbrooke, 1968). In details, the item approach aims to define an economic order quantity and period for each item, without considering possible interactions among them in terms of global availability. On the contrary, the system approach permits to define an availability-cost function with inventory costs and required service levels for the entire system. Although it may indirectly offer measures for the supply system performance (e.g. fill rate and number of back-orders), it proposes measures in accordance with the manager or the decision-maker perspective. For example the system approach answers questions such as “What are the costs to ensure a 95% global availability? How much money do we need to spend to have an enhancement in our global availability? What does the optimal system cost-effectiveness curve look like?”

The system approach uses availability and investment targets as inputs to the decision-making process. It presents an availability-cost curve, in which the manager can easily choose the point that meets the availability constraints within budget limitations. Note that to obtain these outputs it is necessary to solve a series of item approach problems by efficient solving techniques that consider multiple conflicting goals. One of more of these features generally characterize a complex system for which it is generally fruitful to adopt a system approach:

- Commonality: different systems may have some parts in common. The manager can decide to stock these parts separately for each system or in a shared inventory;
- Service differentiation: systems may require differentiated availability levels, according the criticality levels of the sites;
- Multi-transportation modes: it is possible to transport the items in different ways from central to local warehouses;
- Multi-echelon structure: a service supply chain generally consists of central and local warehouse, where central warehouses replenish the local ones;

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- Lateral transhipment: this aspect concerns the possibility that a local warehouse provides a spare part to another local warehouse that is out of stock. In this case it is necessary a jointly optimization of the warehouses.

In particular, the aeronautical industry is one of the sectors where many of these features characterize the systems.

The International Air Transport Association (IATA)'s Maintenance Cost Task Force (IATA, 2011) shows that maintenance cost takes up about 13% of the total operating cost. While aircraft spare parts with very high price are generally not in stock (e.g. engines), and low-cost items are available in every location in a short time, stock levels for medium-range Line Replaceable Units (LRUs) require careful sizing.

As a competitive advantage to decrease these inventory costs, some airlines aims to outsource the ownership of spare parts stocks, settling contracts which regulates the performance in terms of spare parts availability. The company that manages the spare parts inventory and maintenance is the maintenance supplier. This latter is commonly another airline that benefits of risk compensation, considering the high unlikelihood of simultaneous breakdowns. Spare parts become a variable cost for the customer airline and a business income for the maintenance supplier. More formally, in these cases, the supplier proposes a Performance Based Contract (PBC) and the airline that decides to submit it, becomes the customer airline. Besides the access to the supplier inventory, generally these contracts allow the customer to stock a reduced subset of spare parts in its warehouse, paying an additional tax to the supplier. Therefore, determining the items to stock and their stock levels acquires a fundamental role in all the supply activities.

More generally, a Performance Based Contract is a results-oriented contract focusing on the outputs, quality or outcomes that may tie at least a portion of a contractor's payment, contract extensions, or contract renewals to the achievement of specific, measurable performance standards and requirements. These contracts may include both monetary and non-monetary incentives and disincentives.

What emerged from the literature review is a lack of any specific Performance Based Contract model and related solving procedure. To this extent, the contribution of the paper is to present a functional description, a mathematical modelling and an innovative two steps algorithm, based on METRIC, to fill this gap for a typical Performance Based Contract. The main role of the paper in this research field is to provide a first view on a two-party interaction, looking at the customer airline side. While the main literature on METRIC considers a multi-echelon structure of a single organization, where the optimization of the whole system depends on the complete knowledge of all the parameters, the model developed in this study supports managers to outsource the spare parts management processes, in order to reach a target availability while minimizing costs, under contractual constraints.

In summary, the structure of the paper is as follows. Firstly, the literature review in Section 2 shows the potentiality of the system approach, focusing on the METRIC research. It also highlights the importance of Performance Based Contract in the aviation industry. Section 3 describes the features of the Performance Based Contract model that will be the core of this research. Section 4 develops an innovative two steps algorithm, based on METRIC. Section 5 presents the case study, relative to a European airline that submit a Performance Based Contract as the customer airline. Lastly, Section 6 summarizes the outcomes of the study and the possibility for further research.

## 2. Literature review

From the fundamental work of Sherbrooke (Sherbrooke, 1968), the research on spare parts management considers a system-oriented inventory problem, developing many evolutions of the original Multi-Echelon Technique for Recoverable Item Control (METRIC) model. According to a general point of view, the METRIC objective is to allocate the spare parts, ensuring the parts fill rate and the minimum holding or backorder cost, subject to an availability constraint. A significant amount of METRIC-like models provide extensions to the original one, introducing multi-item, i.e. the MOD-METRIC (Muckstad, 1973), multi-indenture, i.e. the VARI-METRIC (Sherbrooke, 1986), multi-echelon network (Graves, 1985) and different demand models (Lee and Moinzadeh, 1987). Hillestad and Carrillo (1980) developed the Dyna-METRIC to take into account the systems under time-dependant operational demands and logistic decision, typical of a wartime scenarios. Some authors integrate the level of repair analysis (Alfredsson, 1997; Basten et al., 2011; Driessen et al., 2014), the capacity constraints of the maintenance centres (Selçuk, 2013; Sleptchenko et al., 2002; Zijm and Avşar, 2003) even in terms of certified skills of the Maintenance Repair and Overhaul provider (Costantino et al., 2013).

The wide application in several industries confirms the importance of these models. For example, Rustenburg et al. (2000) develop a METRIC-like model for the Royal Netherlands Navy. Sun et al. (2009) propose a METRIC model for the weapon equipment industry. Sun et al. (2010), Sun and Zuo (2013), (2010) for the civil airline context. Costantino et al. (2010) for the military aviation, Alvarez and van der Heijden (2014) for a defence systems environment. Moreover, Basten and van Houtum (2014) account several software solution based on METRIC-like models, e.g. OPUS10, MCA Solutions, Xelus, Inventri, VMetric.

Considering the demonstrated potentiality of METRIC, this study strives to customize the standard METRIC formulation, in order to develop a model that better fits the need of a customer airline in a Performance Based Contract. In the aircraft spare parts supply, as well as in other industry, the Performance Based Contract constitutes a new service model in supply management (Jin et al., 2013), with potential critical reshaping of the supply network both for commercial airlines and for defence industry (Zhang et al., 2014).

In the commercial aviation context, Performance Based Contract arises from the possibility of sharing inventory (or part of it) among different airlines, following the principles of risk pooling: the variability of demand reduces if multiple demands across different locations aggregate. Firstly, Kilpi et al. (2009) examined the possible cooperation among different airlines. They underlined how at that time there was no evidence of formal coordination in real life, but just courtesy behaviours among airlines selling spare parts each other to face emergencies. In the commercial context, the raise of Performance Based Contract started after the collaboration pool set by the International Airlines Technical Pool association (IATP). IATP started by the pooled resources for Maintenance Repair and Overhaul of KLM, Sabena and Swissair, and now offers 11 pooled recovery kit all around the world (Harbison, 2014).

This paper focuses on a specific Performance Based Contract for spare parts supply, where the supplier owns the spare parts and the customer pays for the maintenance services. This Performance Based Contract reflects many contracts available among producers (e.g. Airbus, Boeing Company, Rolls Royce) or airlines (e.g. Air China Technic, Air France Industries, KLM Engineering & Maintenance, British Airways Engineering, Delta TechOps, Lufthansa Technik). In all these contracts, these companies provide Maintenance Repair and Overhaul services to potential customers. Therefore, the model in this paper shall be capable of managing the Performance Based Contract specifications. For this purpose, it is necessary to translate common Performance Based Contract measures such as fill rate and turnaround time into measures suitable for the METRIC model (i.e. pipeline, backorders, and availability). About fill rate and shipping time, generally set as Performance Based Contract specifications, Verrijdt et al. (1998) introduce the turnaround time variability in a

METRIC-like model. In addition, some studies provide models that guarantee a target fill rate or waiting time (Caggiano et al., 2009; Selçuk, 2013; Wong et al., 2007a). In addition, considering the standard need of a commercial airline involved in a Performance Based Contract, the model shall represent a multi-echelon structure, with the possibility of multi-transportation strategy, related to the different impact the failure of different items can have on the fleet, i.e. the items' criticality.

A literature review of these aspects clarifies the yet developed approaches. In detail, many researchers analyse multi-echelon networks by METRIC models (Luo et al., 2013; Wang and Kang, 2009; Wang et al., 2013; Wong et al., 2007b; Xu et al., 2015). The multi-echelon structure appears a milestone in METRIC theory. On the other hand, several studies aim to determine the best policy for alternative transportation mode, such as the emergency replenishment (Lee, 1987; Moinzadeh and Schmidt, 1991; Olsson, 2015). Paterson et al. (2011) propose a significant wide review on this theme. The emergency approach is similar to the activation of a lateral transhipment, i.e. the strategy capable of reducing the mean supply delay of spare parts (Tiacci and Saetta, 2011). This paper propose an innovative iterative algorithm to define the best transportation mode based on a cost/availability/criticality approach.

Although determining the spare parts criticality may represent a problem in several industrial context, in aviation, the aircraft manufacturer defines a standard classification of items, according to their criticality. This classification is usually based on the essentiality code, which indicates the criticality of an item, with respect to the effects of its failure on the aircraft to perform the intended mission. Ghobbar and Friend (2004) underline the importance of essentiality codes in airlines maintenance planning, with respect to the Minimum Equipment Lists. Papakostas et al. (2010) present the first model that takes into consideration the parameters of criticality in the scheduling of Maintenance Repair and Overhaul activities for the aircraft industry. The model developed in this paper considers the items' criticality, assigning to any essentiality code a crucial role in the iterative algorithm.

## 3. The Performance Based Contract - the customer perspective

The generic formulation of a Performance Based Contract defines the roles for two main actors:

- the supplier (Central Department – CD): a company who provides the facilities, the competences and the certifications to execute maintenance services, selling these services to other companies, i.e. the customer.
- the customer (Local Department– LD): a company who does not own the facilities, the competences or the certifications to execute maintenance services. To ensure its target operational availability, the customer acquires maintenance services from another company, i.e. the supplier.

In contrary to the theoretical discussion, which allows a cooperative discussion between customer and supplier, the empirical research by Kleemann and Essig (2013) shows that the Performance Based Contract supplier currently refrain from directly involving the customer in the offering. More specifically, the study shown as the relationships generally are not fully cooperative, mainly due to the lack of interest alignment. Generally, the supplier as a dominant role and the customer has to cope with the proposal. The model is aligned with this empirical conclusion and therefore it considers the customer cannot change the specifications. Note that the model considers only repairable items. When a failure occurs, the customer Local Maintenance Department (LMD) can only substitute the inefficient item with an efficient one and ships this repairable item to the Central Maintenance Department (CMD) of the supplier who has the facilities, the competences and the certifications to repair and manage the spare parts stock.

For each item, an (s-1, s) replenishment policy enables the shipment of a substitute efficient item anytime the supplier receives an inefficient one. The order size is always 1 with a continuous review system where an order is issued anytime the inventory lowers by 1 item. The Performance Based Contract sets some specifications in terms of shipping time of a substitute item and of on-time shipments (fill rate) in a fixed period. The customer has to pay a fee in order to have access to the main stock (pool base) of the Central Maintenance Department and a shipping cost for any shipment. The supplier determines the pool access fee based on the customer fleet size and state. This fee will not be further discussed in the model, because it is a fixed invariant cost the customer has to pay, with no possibility to change it.

The Performance Based Contract regulates two transportation modes: Standard Request (STN) and Quick Request (QCK) with different shipping time, fill rate and cost due to different shipping processes. In the Standard Request process, the customer can demand for a Local Department Kit (LDK), i.e. a subset of spare parts to keep at the Local Maintenance Department, in order to ensure a prompt response to the more critical requests. When a failure of a Local Department Kit item occurs, the customer directly substitutes the efficient item in the Local Department Kit and ships the inefficient item to the Central Maintenance Department, removed from the failed equipment. The same (s-1, s) replenishment policy enables a shipment of an efficient item from the Central Maintenance Department to the Local Maintenance Department, which stocks it for next potential requests. This process generates an additional cost to the customer, which has to pay the supplier a percentage of the entire cost of the Local Department Kit item per month, for any Local Department Kit item (tax on stock cost). In addition to the Standard Request, the supplier offers the faster Quick Request. While Quick Request does not provide any stock cost, its shipping cost is higher and, even though the shipping time is very short, it causes a degradation of customer performance in terms of availability due to the delayed substitution of the inefficient item. Furthermore, the customer can use this transportation mode only with respect to a contractual limit, in terms of Quick Request percentage of the total requests. If the customer exceeds this limit, the supplier will not accomplish the potential extra requests. This limit is mandatory to avoid the customer abuses Quick Request and to limit the level of stock at the Central Maintenance Department.

As a qualitative thought, it is possible to imagine that if the failed item is critical, it is generally preferable to manage it by Standard Request, in order not to wait the Quick Request time. Otherwise, for a less critical item, the decision-maker may a priori decide to manage it by the Quick Request transportation mode, accepting the degradation of service deriving from the shipping time from the Central Maintenance Department and saving money for its Local Department Kit value. However, for an optimum allocation, it would be necessary to consider also the amount of the item demand and balance the criticality and the costs in order to respect a budget constraint. On this path, it would be possible (e.g.) to request a Local Department Kit also for less critical items, if its demand rate would be particularly high.

Once defined the contract specifications, supplier airline offers the customer airline the possibility to decide the more appropriate management strategy for each item, i.e. Standard Request or Quick Request. Once assigned a specific strategy for an item, however, it will be managed only according to this strategy. Consequently, in case a Standard Request item is in stock-out, it is not possible to use Quick Request but it is necessary to wait the Standard Request cycle.

Therefore, the process of deciding the most appropriate strategy for each item, acquires a crucial role. This decision relies on the both on the performance ensured by the Performance Based Contract, i.e. the turnaround time and the fill rate for Quick Request and Standard Request and the maximum Quick Request movements, and on the shipment and tax on stock costs, which depends on the customer spare parts demand. For this purpose, this paper develops a METRIC formulation of a multi-echelon multi-transportation multi-item network, where the attention focuses on the customer stock level, translating the Performance Based Contract specifications into modelling variables and parameters. Fig. 1 sketches the logic features of the two transportation modes, highlighting the differences of the two processes. The warning signal in Local Maintenance Department indicates the failure of an item, which activates the substitution cycle.

Note that the item operative states can just be functioning or defect. When an item is defect, it is necessary to repair it in the Central Maintenance Department to become functioning again. This model neglects the effect of imperfection repairing activities (Zhang and Jardine, 1998). This assumption reflects a real scenario, where the Central Maintenance Department has a quite low ignorable probability of accomplishing an imperfect maintenance task.

## 4. Mathematical formulation

| **Variables** | |
|---|---|
| $A$ | Aircraft availability at LD |
| $A_i$ | Availability of item i at LD |
| $A_{SYS}$ | Availability of the fleet at LD |
| $A_{LDK}$ | Availability of all the items stocked in LDK, managed by STN |
| $A_{iLDK}$ | Availability of the item i stocked in LDK, managed by STN |
| $A_{QCK}$ | Availability of all the items managed by QCK |
| $A_{iQCK}$ | Availability of the item i managed by QCK |
| $ATI_i$ | Alternative Transportation Index of item i, to select the transportation mode |
| $C^{TOT}_{i,k}$ | Total cost for item i according to k transportation mode |
| $C^{TOT}_{k}$ | Total cost according to k transportation mode |
| $C^{TOT}$ | Total logistic cost |
| $E[PL_i]$ | Expected value of pipeline of item i at LD |
| $EBO_i$ | Expected Backorder of the item i at LD |
| $EBO_{TOT}$ | Expected Backorder of the system |
| $i_{CRF}$ | Item number in the Criticality family, ordered by decreasing values of $ATI_{iCRF}$. $i_{CRF}=1,\ldots,I_{CRF}$ |
| $i_{QCK}$ | Item number according the QCK transportation mode. $i_{QCK}=1,\ldots,I_{QCK}$ |
| $i_{LDK}$ | Item number of LDK items, managed according the STN transportation mode. $i_{LWD}=1,\ldots,I_{LDK}$ |
| $s_i$ | Level of inventory of item *i* at LDK. It is the output variable of the optimization process. According to the (s-1, s) policy, it is also the maximum level of inventory for each item |
| $\bar{s}_i$ | First approximation of the level of inventory of item *i* at LDK to support the selection of the transportation mode |
| $Var[PL_i]$ | Variance of pipeline of item i |

| **Parameters** | |
|---|---|
| $A_{target}$ | Target availability for the LD |
| $C_{i,k}$ | Shipping cost for item i, according to the k transportation mode |
| $CRI$ | Criticality family. As the *CRI* increase, the criticality level decreases |
| $CRT_i$ | Criticality family of item i |
| $F$ | Number of the criticality families (*CRI*) |
| $FR_k$ | Contractual fill rate for on-time shipments according to the k transportation mode. This parameter represents a performance ensured by the PBC |
| $g$ | Monthly fee for LDK items stocked at LD. This parameter represents a PBC specification |
| $i$ | Item number. $LRU$:$i=1\ldots I$ |
| k | Transportation mode. STN: k = 1; QCK: k = 2 |
| N | Number of aircrafts at LD |
| M | Minimum number of required aircrafts at LD |
| $m_i$ | Yearly demand mean value of item i at site LD |
| $O_{i,k}$ | Time, in years, for ordering and shipping of item i from the CD to the LD, according to the k transportation mode. This parameter represents a performance ensured by the PBC |
| $O^{std}_{i,k}$ | Standard time, in years, for ordering and shipping of item i from the CD to the LD, according to the k transportation mode. This parameter represents a performance ensured by the PBC |
| $O^{max}_{i,k}$ | Maximum time, in years, for ordering and shipping of item i from the CD to the LD, according to the k transportation mode. This parameter represents a performance ensured by the PBC |
| $O^{delay}_{i,k}$ | Delayed time, in years, for ordering and shipping of item i from the CD to the LD, according to the k transportation mode, when the supplier exceeds the contract shipping time. This parameter represents a performance ensured by the PBC |
| $P_i$ | Market value of item i |
| $P_{mod}$ | Probability of stand-by switching |
| $QCK_{target}$ | QCK maximum percentage contractual limit |

The paper models a Performance Based Contract that involves two echelons (CD-LD) and single indenture items, i.e. the spare parts are line replaceable units (LRU). This study introduces a model, the PBC-METRIC to define the optimal transportation mode and the optimal Local Department Kit stock level for a customer company that has delegated the maintenance services to a supplier company. In detail, the PBC-METRIC consists of two algorithms:

- STEP 1: application of the QCK-selection algorithm to determine the optimal transportation mode according to the Performance Based Contract specifications, costs and availability requirements of the customer;
- STEP 2: application of the LDK-allocation algorithm in order to size the Local Department Kit optimal stock level, minimizing the stock costs in full compliance with the contractual limits and reaching a target level of availability at the LD.

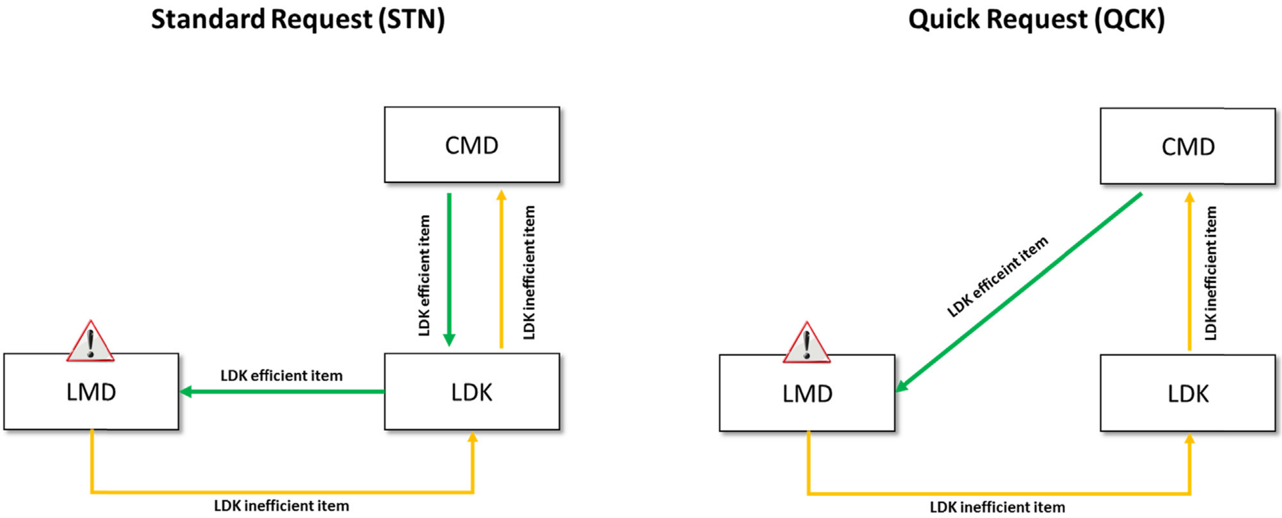

**Fig. 1.** The two transportation modes processes.

In order to assess and compare different allocations of stock, it is necessary to assign a probability distribution function to the expected backorder of an LRU at the LD. In particular, for a stock quantity $s$ and a request quantity $x$, the general formulation of the Expected Backorder $EBO(s)$ (probability that $x>s$) and the Variance of the Expected Backorder $VBO(s)$ are (1) (2):

$$EBO(s) = \sum_{x=s+1}^{\infty} (x-s)\Pr\{x\} \tag{1}$$

$$VBO(s) = \sum_{x=s+1}^{\infty} (x-s)^2 Pr\{x\} - [EBO(s)]^2 \tag{2}$$

Under the assumption of $m_i$ following a Poisson distribution and according to Palm's theorem, the expected value of the pipeline of the item $i$ at the LD ($E[PL_i]$) equals to its variance ($Var[PL_i]$), which represents the parameter of a Poisson distribution (3):

$$E[PL_i] = Var[PL_i] = m_i O_{i,k} \tag{3}$$

The choice of the Poisson distribution does not affect the validity of the model, since alternative distributions can be used to represent specific demand patterns. In particular, what is to verify is only the specific formulation of $E[PL_i]$ and $Var[PL_i]$ deriving from the Palm's theorem which, e.g. in case of a Negative Binomial distribution, allows developing the VARI-METRIC (Sherbrooke, 1986).

Differently from standard METRIC formulation, this model does not consider the supplier stock level. This concept reflects the real case scenario where the customer airline has to optimize its stock level without having information on the supplier stock level. More in detail, the customer has to define its inventory level based on the Performance Based Contract specifications. This paper shows how it is possible to combine these specifications in the standard METRIC formulation, i.e. by the PBC-METRIC. For this purpose, it is necessary to define the Performance Based Contract specifications in terms of time constraints. In details, the supplier defines a standard time for ordering and shipping of item $i$ from the Central Department to the Local Department $O^{std}_{i,k}$ and ensures respecting it according to the $FR_k$. In the other situations, i.e. $(1{-}FR_k)$, the supplier defines in the contract a delayed time $O^{delay}_{i,k}$, which follows a triangular distribution, with $mode = minimum = O^{std}_{i,k}$ and $maximum = O^{MAX}_{i,k}$. Here follows the formulation (4) (5) (6) to describe these aspects.

Thus, the distribution function of the shipping time variability is (4):

$$\begin{cases} O_{i,k} = O^{std}_{i,k} & if \quad OTP_{i,k} = 1 \\ O_{i,k} = O^{delay}_{i,k} & if \quad OTP_{i,k} = 0 \end{cases} \tag{4}$$

Where $OTP_{i,k}$ is a random number generated from a Bernoulli distribution with number of success for each trial, $FR_k$, as described in (5):

$$\begin{cases} Pr(OTP_{i,k} = 1) = FR_k \\ Pr(OTP_{i,k} = 0) = 1 - FR_k \end{cases} \tag{5}$$

And where, $O^{delay}_{i,k}$ follows a triangular distribution (6) with $mode = minimum = O^{std}_{i,k}$ and $maximum = O^{MAX}_{i,k}$, as shown in Fig. 2.

$$O^{delay}_{i,k} \text{such that } Pr\left(O^{delay}_{i,k}\right) = \frac{2(1-FR_k)\left(O^{MAX}_{i,k} - O^{delay}_{i,k}\right)}{\left(O^{MAX}_{i,k} - O^{std}_{i,k}\right)^2} \tag{6}$$

The total value of the Expected Backorder of LRU can be expressed (7) by:

$$EBO_{TOT} = \sum_{i=1}^{I} EBO(s_i|E[PL_i]) \tag{7}$$

Given that any site has a minimum required number of available aircrafts to satisfy its flight plan, a failure on an aircraft reduces the site availability only if the number of available aircrafts is lower than the minimum required.

The system redundancy can be active or passive. More specifically, in case of passive redundancy, an imperfection switching

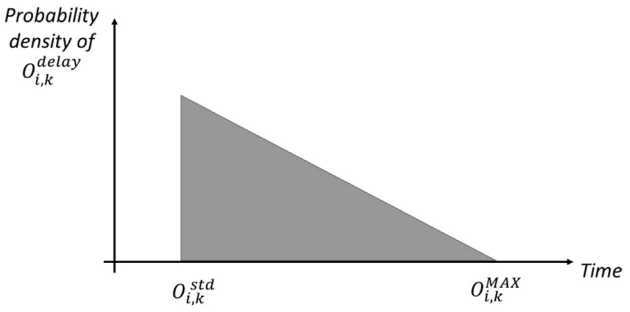

**Fig. 2.** $O^{delay}_{i,k}$ Probability density function.

action represents the possibility to use one of the *cold aircrafts* as a stand-by aircraft. A cold aircraft is an aircraft not scheduled in the flight plan and therefore redundant to allow a prompt substitution in case of unavailability of a scheduled one. $P_{mod}$ is the probability of successfully reassign the cold aircraft in a specific flight plan. Taking into account the possibility of ongoing extraordinary maintenance operations on it, it is generally slightly minor than 1. In this condition, a redundant system with (*N-M*) stand-by units, in which *M* of *N* must be operating (8) models the availability of the equipment at Local Department:

$$A_{SYS} = A^M \left( 1 + P_{mod} \sum_{n=0}^{N-M} \frac{\left[ -\ln A^M \right]^n}{n!} \right) \tag{8}$$

If the redundancy is active, it is possible to model the system as a redundant system with $N_j$ parallel elements where at least $M_j$ elements are available (9):

$$A_{SYS} = \sum_{n_j=M_j}^{N_j} \left( \binom{N_j}{n_j} (A_j)^{n_j} (1 - A_j)^{N_j - n_j} \right) \tag{9}$$

The aircraft availability *A* is the series system of the availability of the LRUs (10), which are functions of their relative $EBO_{i,1}$ (11) (12) under the hypothesis that each system needs all its LRUs to be available:

$$A = \prod_{i=1}^{I} A_i \tag{10}$$

$$A_i = \left( 1 - \frac{EBO_i}{N} \right) \tag{11}$$

$$EBO_i \le N \tag{12}$$

### 4.1. The QCK-selection algorithm: determining the transportation mode

As the percentage of Quick Request is a contract requirement ($QCK_{target}$ ), the customer has to preliminarily associate of each item with each transportation mode. The two transportation modes differ in terms of cost, shipping time, and therefore, contribution to the global availability of the fleet at Local Department. The customer has to decide if it is better to stock an item and adopt the Standard Request transportation mode, or do not stock it and adopt the Quick Request transportation mode, accepting a certain level of unavailability due to the expected backorder of that item. If an item is in stock, it is immediately ready for the substitution and does not generate backorder. In this case, the LDK-allocation algorithm will define its stock level. On the other hand, if the item has a big Local Department Kit stock cost and a low demand, it might be worthwhile to choose the Quick Request; the item is not immediately ready for the substitution but it is available after $O_{i,2}$, because its stock level is set to 0 (i.e. $s_{iQCK}$=0). The evaluation of $EBO(s_{iQCK})$ consequently reduces the global availability. The availability of an equipment depends on the transportation mode for each one of its LRU, specifying (10), as prescribed by (13) (14) (15) (16) (17):

$$A = \prod_{i=1}^{I} A_i = A_{QCK}\ A_{LDK} = \prod_{i_{QCK}=1}^{I_{QCK}} A_{i_{QCK}} \prod_{i_{LDK}=1}^{I_{LDK}} A_{i_{LDK}} \tag{13}$$

$$A_{i_{QCK}} = \left( 1 - \frac{EBO_{i_{QCK}}}{N} \right) \tag{14}$$

$$E\left[ PL_{i_{QCK}} \right] = Var\left[ PL_{i_{QCK},1} \right] = m_{i_{QCK}} O_{i,2} \tag{15}$$

$$A_{i_{LDK}} = \left( 1 - \frac{EBO_{i_{LDK}}}{N} \right) \tag{16}$$

$$E\left[ PL_{i_{LDK}} \right] = Var\left[ PL_{i_{LDK}} \right] = m_{i_{LDK}} O_{i,1} \tag{17}$$

According to the (s-1, s) inventory policy, the Standard Request cost (18) is the sum of the annual stock cost (based on the monthly fee *g*, the market value $P_i$ and the number of items in stock $s_i$), and the Central Maintenance Department — Local Maintenance Department round trip shipping, with the outbound equals the return cost. In the Quick Request, there is no Local Department Kit fee contribution to costs (19), but only the round trip cost, which is $C_{i,2}$ for the outbound and $C_{i,1}$ for the return, because the return trip is accomplished with normal priority, costing the same as the Standard Request trip:

$$C_{i,1}^{TOT} = \left( 12 P_i\, s_i g + 2 C_{i,1}\ \ m_i \right) \tag{18}$$

$$C_{i,2}^{TOT} = \left( C_{i,1} + C_{i,2} \right) m_i \tag{19}$$

Determining the association of each item with the most appropriate transportation mode, respecting the Performance Based Contract requirements, is a complex problem, which needs to balance the effects of each transportation mode in terms of total costs and total availability. To reduce this complexity it is possible to implement an item-by-item procedure, ordering them in terms of criticality and cost saving, evaluating the progressive contribution to the availability. The procedure consists of four steps:

a. Group the items according to their *CRI* code. This step creates *F* criticality family (*CRI*).
b. Evaluate for each item a specific Alternative Transport Indicator (*ATI*), which offers a comparison of the two alternative transportation modes in terms of economic value. The bigger the $ATI_i$, the more economically advantageous is the Quick Request transportation mode. If the $ATI_i$< 0, it means that stocking that item in Local Department Kit is preferable, even in economic terms. Its expression is (20):

$$ATI_i = \left( 12 P_i\, \bar{s}_i g + 2 C_{i,1}\ m_i \right) - \sum_{k=1}^{2} C_{i,k} m_i \tag{20}$$

where $\bar{s}_i$ is a first approximation of the stock level (21):

$$\bar{s}_i = m_i O_{i,1}^{MAX} \tag{21}$$

Its value derives from the hypothesis that the demand of the item *i* has a constant frequency and the time for ordering and shipping is constant and fixed a priori equal to the maximum time for ordering and shipping in the *STN* transportation mode $O_{i,1}^{MAX}$, following a conservative approach.

c. For each *CRI*, assign an increasing $i_{CRF}$ rank to the items, according to the decreasing value of the *ATI*. The output of this step is an organized dataset, as described in a generic formulation in Table 1.

**Table 1**
Generic example of organized dataset.

| CRI = F | | | CRI = F − 1 | | | (F − 1) < CRI < 1 | | | CRI = 1 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{item^F_{i_F}}$ | $\boldsymbol{ATI^F_{i_F}}$ | $\boldsymbol{m^F_{i_F}}$ | $\boldsymbol{item^{(F-1)}_{i_2}}$ | $\boldsymbol{ATI^{(F-1)}_{i_{(F-1)}}}$ | $\boldsymbol{m^{(F-1)}_{i_{(F-1)}}}$ | … | … | … | $\boldsymbol{item^1_{i_1}}$ | $\boldsymbol{ATI^1_{i_1}}$ | $\boldsymbol{m^1_{i_1}}$ |
| $\boldsymbol{item^F_1}$ | $ATI^F_1$ | $m^F_1$ | $item^{(F-1)}_1$ | $ATI^{(F-1)}_1$ | $m^{(F-1)}_1$ | … | … | … | $item^1_1$ | $ATI^1_1$ | $m^1_1$ |
| $\boldsymbol{item^F_2}$ | $ATI^F_2$ | $m^F_2$ | $item^{(F-1)}_2$ | $ATI^{(F-1)}_2$ | $m^{(F-1)}_2$ | … | … | … | $item^1_2$ | $ATI^1_2$ | $m^1_2$ |
| **…** | … | … | … | … | … | … | … | … | … | … | … |
| $\boldsymbol{item^F_{I_F}}$ | $ATI^F_{I_F}$ | $m^F_{I_3}$ | $item^{(F-1)}_{I_{(F-1)}}$ | $ATI^{(F-1)}_{I_{(F-1)}}$ | $m^{(F-1)}_{I_{(F-1)}}$ | … | … | … | $item^1_{I_1}$ | $ATI^1_{I_1}$ | $m^1_{I_1}$ |

d. Select the spare parts to manage in Standard Request and the ones in Quick Request. The logic beyond this algorithm consists in assigning the Quick Request transportation mode firstly to the less critical items, whose $CRI = F$, which offer the highest economic contribute in terms of ATI value ($ATI^F_1$). The calculation is performed respecting the contractual limit of percentage maximum Quick Request ($QCK_{target}$) and evaluating the contribution of $EBO(s_{iQCK})$ in terms of global availability. The structure of the algorithm follows these sub-steps:
   1. Start considering less critical items, the ones with the higher Criticality family ($CRI = F$).
   2. Check if there are any items in this category($I_F$>0). If it is false, the algorithm proceeds in evaluating the family of more critical items, whose $CRI = (F\text{-}1)$, proceeding in this way until the most critical ones ($CRI = 1$).
   3. If there are items of the specified Criticality family, fix $i_{CRI}$=1.
   4. Check if the associated $ATI^{CRF}_{i_{CRF}}$ value verifies $ATI^{CRI}_{i_{CRI}} > 0$. If it is true, it means that for this item, the Quick Request could be worthwhile. Since the items are organized according the decreasing value of $ATI^{CRI}_{i_{CRI}}$, the first item which verifies $ATI^{CRI}_{i_{CRI}} < 0$ implies that it is worthwhile to adopt Standard Request transportation mode for all the successive items of that Criticality family.
   5. Check if the hypothetic Quick Request assignment of the item verifies the contractual constraint, in terms of $QCK_{target}$. If an item does not verify the constraint, it would be possible that a successive item ($i_{CRI}$+1), less favourable in terms of $ATI^{CRI}_{i_{CRI+1}} < ACI^{CRI}_{i_{CRI}}$, may verify the constraint (e.g.) because $m^{CRI}_{i_{CRI+1}} < m^{CRI}_{i_{CRI}}$.
   6. In case the hypothesis have been verified, assign the stock level of the item equal to zero ($s^{CRI}_{i_{CRI}} = 0$) and update the $QCK_{target}$, decreasing its value in order to take into account the demand yet assigned to Quick Request $QCK_{target} = QCK_{target} - \frac{m^{CRI}_{i_{CRI}}}{m_{TOT}}$.
   7. Iterate this process until $i_{CRI}$<$I_{CRI}$.
   8. Iterate the entire process for each Criticality family, till $CRI = 1$.

Once accomplished these steps, the items are classified in two groups, the ones to manage by the Quick Request transportation mode, whom stock level is set to zero, and the ones to manage by the Standard Request transportation mode, whom stock level in the Local Department Kit will be determined by the application of the LDK-allocation algorithm. The selection of Quick Request items allows the calculation of $A_{QCK}$ in (13).

### *4.2. The LDK-allocation algorithm: determining the optimum stock level*

The total cost $C^{TOT}$ (22) is due to the Standard Request cost (23) and the Quick Request cost (24). Once decided the best transportation mode for each item, $C^{TOT}_2$ and the shipping cost of the Standard Request transportation mode are independent from the Local Department Kit stock level. Therefore the cost $C^*_1$(25) represents the objective function, to be minimized in the optimization process. $A_{SYS}$, whatever type of redundancy it models, represents a fundamental constraint, because it shows the availability of the fleet at Local Department, which has to satisfy the limit imposed by the customer policy $A_{target}$ (26). Furthermore, it is necessary to consider the constraint in (12), which in this case becomes $EBO_{i_{LDK}} < N$, considering only the Local Department Kit items. Lastly, the optimization solution shall respect a fixed budget (27).

$$C^{TOT} = \sum_{k=1}^{2} C^{TOT}_k \tag{22}$$

$$C^{TOT}_1 = \sum_{i_{LDK}=1}^{I_{LDK}} \left(C^{TOT}_{i_{LDK},1}\right) = \sum_{i_{LDK}=1}^{I_{LDK}} \left(12\, P_{i_{LDK}} s_{i_{LDK},1}\ \ g + 2\, C_{i_{LDK},1}\ \ m_{i_{LDK}}\right) \tag{23}$$

$$C^{TOT}_2 = \sum_{i_{QCK}=1}^{I_{QCK}} \left(C^{TOT}_{i_{QCK},2}\right) = \sum_{i_{QCK}=1}^{I_{QCK}} \left(C_{i_{QCK},1} +\ C_{QCK,2}\right) m_{i_{QCK}} \tag{24}$$

$$C^*_1 = C^{TOT}_1 - \sum_{i_{LDK}=1}^{I_{LDK}} 2\, C_{i_{LDK},1}\ \ m_{i_{LDK}} = \sum_{i_{LDK}=1}^{I_{LDK}} 12\ \ P_{i_{LDK}} s_{i_{LDK},1} tax \tag{25}$$

$$A_{SYS} \geq A_{target} \tag{26}$$

$$C^{TOT} \leq C_{budget} \tag{27}$$

## 5. Numerical example

A case study of a European airline with an Airbus 320 fleet illustrates the features of the model. The airline, which assumes the role of the Performance Based Contract customer, has to decide which items are to stock in Local Department Kit, managed by Standard Request, and which items to manage by Quick Request. For each Local Department Kit item, the airline has to define the optimal stock level, according to the supplier Performance Based Contract specifications, and respecting the contractual limits, the availability performance and the cost budget. In this application, the model analyses 20 LRUs, which all belong to the hydraulic power plant, i.e. 29 ATA chapter of the Federal Aviation Administration (2008). Table 2 summarizes the input data of each item. In this case, the cold redundancy best models $A_{SYS}$, because the airline uses some aircrafts as cold aircrafts to safeguard its flight plan, and thus enhance its availability.

Table 3 shows the customer fleet data, in terms of number of total aircraft ($N$), minimum required aircraft ($M$), switching probability ($P_{mod}$) budget constraint ($C_{budget}$) and fleet availability target

**Table 2**
Item input data.

| i | Code | Item | $m_i$ | $CRT_i$ | $P_i$ | $C_{i,1}$ | $C_{i,2}$ |
|---|---|---|---|---|---|---|---|
| 1 | $LRU_1$ | Ground service manifold | 11.60 | 1 | € 8075.00 | € 50.00 | € 175.00 |
| 2 | $LRU_2$ | Manifold assy | 3.25 | 1 | € 49,865.00 | € 50.00 | € 175.00 |
| 3 | $LRU_3$ | HP PTU-MANIFOLD | 1.70 | 1 | € 62,780.00 | € 45.00 | € 190.00 |
| 4 | $LRU_4$ | MANIFOLD PTU | 2.00 | 1 | € 33,950.00 | € 40.00 | € 170.00 |
| 5 | $LRU_5$ | Electrical pump | 35.20 | 2 | € 44,765.00 | € 65.00 | € 240.00 |
| 6 | $LRU_6$ | Hydraulic electrical pump | 2.25 | 2 | € 47,185.00 | € 75.00 | € 255.00 |
| 7 | $LRU_7$ | Engine drive pump | 47.50 | 1 | € 44,860.00 | € 45.00 | € 240.00 |
| 8 | $LRU_8$ | Detector balance | 1.85 | 2 | € 5715.00 | € 50.00 | € 170.00 |
| 9 | $LRU_9$ | Power transfer unit | 1.30 | 1 | € 89,525.00 | € 50.00 | € 220.00 |
| 10 | $LRU_{10}$ | Xformer | 8.00 | 2 | € 3125.00 | € 30.00 | € 105.00 |
| 11 | $LRU_{11}$ | Priority valve | 3.15 | 1 | € 7620.00 | € 35.00 | € 115.00 |
| 12 | $LRU_{12}$ | Manifold-HP | 8.00 | 1 | € 18,085.00 | € 50.00 | € 140.00 |
| 13 | $LRU_{13}$ | Switch | 5.00 | 1 | € 18,120.00 | € 35.00 | € 195.00 |
| 14 | $LRU_{14}$ | Hydraulic hand pump | 15.16 | 2 | € 3710.00 | € 40.00 | € 160.00 |
| 15 | $LRU_{15}$ | Ram air turbine | 2.00 | 1 | € 109,340.00 | € 85.00 | € 200.00 |
| 16 | $LRU_{16}$ | Valve assy | 10.20 | 2 | € 4610.00 | € 70.00 | € 200.00 |
| 17 | $LRU_{17}$ | Fire shut-off valve | 6.00 | 1 | € 2135.00 | € 40.00 | € 200.00 |
| 18 | $LRU_{18}$ | Valve | 3.40 | 1 | € 7690.00 | € 25.00 | € 100.00 |
| 19 | $LRU_{19}$ | Quantity indicator | 5.75 | 3 | € 3155.00 | € 80.00 | € 240.00 |
| 20 | $LRU_{20}$ | Drain valve | 10.15 | 1 | € 2265.00 | € 45.00 | € 100.00 |

**Table 3**
Customer data.

| | |
|---|---|
| $\boldsymbol{N}$ | 96 |
| $M$ | 94 |
| $P_{mod}$ | 95% |
| $C_{budget}$ | € 150,000.00 |
| $A_{target}$ | 99% |

($A_{target}$). Table 4 shows the supplier contract specifications. The supplier ensures the same time for ordering and shipping regardless of the item. $FR_2 > FR_1$, implies that Quick Request ensures a higher on-time performance rate. Monthly fee ($tax = 0.8\%$) shows that, each year, the customer has to pay the 9.6% of the item value to the supplier. The supplier unilaterally fixes these contract specs according to the structure of its spare parts pool, i.e. total maintenance costs, size of inventory and flight hours of all the parts.

The first step of the PBC-METRIC consists in the application of the QCK-selection algorithm. Note that, in the aviation context, the items are classified, according to their criticality, in 3 families. These three families categorize the effect of an item failure on the intended mission, where the higher the Criticality family the less critical the value. Table 5 summarizes the results, where the highlighted elements correspond to the seven items in Quick Request ($LRU_6$, $LRU_8$, $LRU_{15}$, $LRU_9$, $LRU_3$, $LRU_2$, $LRU_4$).

The LDK-allocation algorithm has to determine the optimal stock level of the remaining 13 items. Due to the characteristics of the problem (complexity, non-linearity, probability distribution of the variables), it is necessary to adopt a solving process that does not require the gradient or higher derivatives of the target function. A genetic algorithm (GA) can therefore solve the problem, considering the complexity of the optimization function and of the constraints. GA is only one of the possible adoptable algorithms to solve this non-linear constrained optimization problem, but it would be possible to adopt other methods, e.g. pattern search, or specific algorithms, e.g. interior-point algorithm, to reach a pseudo-optimal solution.

**Table 4**
PBC specifications.

| | |
|---|---|
| $g$ | 0.8% |
| $QCK_{target}$ | 15% |
| $O_{i,1}^{std}$ | 0.0274 |
| $O_{i,2}^{std}$ | 0.0014 |
| $O_{i,1}^{MAX}$ | 0.0548 |
| $O_{i,2}^{MAX}$ | 0.0055 |
| $FR_1$ | 93% |
| $FR_2$ | 97% |

GAs evolved a starting population of potential solutions, i.e. individuals, over successive generations using a set of genetic operators called selection, crossover and mutation. The algorithm follows three steps:

a. Creation of a random initial population.

The initial population contains a number of individuals equal to the number of items about which the determination of the optimum stock level is required. The initial range of the population is set in [0; 20].

b. Creation of a sequence of new populations, or generations. At each step, the algorithm uses the individuals in the current generation to create the next generation. To create the new generation, the algorithm performs the following sub-steps:
   1. Scores each member of the current population by computing its fitness value.
   2. Scales the raw fitness scores to convert them into a more usable range of values.
   3. Selects parents basing on their fitness.
   4. Produces children from the parents. The algorithm produces three different children for the next generation. In detail: Elite children (individuals in the current generation with the best fitness values, which automatically survive to the next generation); Crossover children (created by combining pairs of parents in the current population); Mutation children (created by randomly changing the genes of individual parents, adding random vector from a Gaussian distribution). A crossover fraction of 0 means that all children are mutation children, while a crossover fraction of 1 means that all the children are mutation children.
   5. Replaces the current population with the children to form the next generation.

c. Stop when reaching one of the stopping criteria: generations' number, time limit, stall generations' number, stall time limit.

**Table 5**
Organized dataset once applied the QCK-selection algorithm.

| CRI = 3 | | | | CRI = 2 | | | | CRI = 1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **$item^3_{i_3}$** | Code | **$ATI^3_{i_3}$** | **$m^3_{i_3}$** | **$item^2_{i_2}$** | Code | **$ATI^2_{i_2}$** | **$m^2_{i_2}$** | **$item^1_{i_1}$** | Code | **$ATI^1_{i_1}$** | **$m^1_{i_1}$** |
| **$item^3_1$** | $LRU_{19}$ | −617.12 | 5.75 | $item^2_1$ | $LRU_6$ | 4124.76 | 2.25 | $item^1_1$ | $LRU_{15}$ | 10,266.64 | 2.00 |
| | | | | $item^2_2$ | $LRU_8$ | 326.64 | 1.85 | $item^1_2$ | $LRU_9$ | 8373.4 | 1.30 |
| | | | | $item^2_3$ | $LRU_{10}$ | −300 | 8.00 | $item^1_3$ | $LRU_3$ | 5780.38 | 1.70 |
| | | | | $item^2_4$ | $LRU_{16}$ | −883.44 | 10.20 | $item^1_4$ | $LRU_2$ | 4380.79 | 3.25 |
| | | | | $item^2_5$ | $LRU_{14}$ | −1463.04 | 15.16 | $item^1_5$ | $LRU_4$ | 2999.2 | 2.00 |
| | | | | $item^2_6$ | $LRU_5$ | −1862.56 | 35.20 | $item^1_6$ | $LRU_{12}$ | 1016.16 | 8.00 |
| | | | | | | | | $item^1_7$ | $LRU_{13}$ | 939.52 | 5.00 |
| | | | | | | | | $item^1_8$ | $LRU_{18}$ | 483.24 | 3.40 |
| | | | | | | | | $item^1_9$ | $LRU_{11}$ | 479.52 | 3.15 |
| | | | | | | | | $item^1_{10}$ | $LRU_{20}$ | −340.81 | 10.15 |
| | | | | | | | | $item^1_{11}$ | $LRU_1$ | −674.8 | 11.60 |
| | | | | | | | | $item^1_{12}$ | $LRU_{17}$ | −755.04 | 6.00 |
| | | | | | | | | $item^1_{13}$ | $LRU_7$ | −4955.94 | 47.50 |

> The stall represents the situation in which there is no improvement in the objective function for a sequence of consecutive generations or during an interval of time in seconds.

For the purpose of the analysis, it is possible to adopt a Real Coded Genetic Algorithm (RCGA), in detail the MI-LXPM. MI-LXPM is efficient in solving integer optimization problems as it utilizes the self-adaptive Laplace Crossover operator (LX) and the tuneable Power Mutation operator (PM) to increase randomness and consequently the possibility of achieving a global optimum. Note that in a general formulation, LX should consider a different scaling parameter for integer and continuous variables. In this model, however it is not necessary to differentiate it, because all the variables are integer (Deep and Thakur, 2007). The algorithm adopts binary tournament selection operator as reproduction operator. Then in order to ensure the satisfaction of integer restrictions, the algorithm applies the following procedure

- If $s_i$ is integer then $s_i^* = s_i$ otherwise
- $s_i^*$ is equal to either $[s_i]$ or $([s_i]+1)$ each with probability 0.5

:

This choice ensures greater randomness, avoiding the generation of the same integer values (Deep et al., 2009).

Lastly, the algorithm follows (Deb, 2000) to handle the constraints: the fitness of an infeasible solution not only depends on the amount of constraint violation, but also on the population of solution at hand. Therefore, the genetic algorithm attempts to minimize a penalty function, not the fitness function. The penalty function includes a term for infeasibility and combines with binary tournament selection to select individuals for subsequent generations. If a member of a population is feasible, its penalty function value is the fitness function. Otherwise, if the member is infeasible, the penalty function is the maximum fitness function among feasible members of the population, plus a sum of the constraints violations of the (infeasible) point.

More specifically, in this numerical example, the fitness function considered is $C_1^*$. The algorithm was implemented in MATLAB, taking advantage of its powerful integer GA solver (less than 3 min of computational time on an Intel® Core™ i5-6200U–3 MB cache, 2,8 GHz – and 8 Gb of RAM – LPDDR3 at 1.866 MHz). On this path, it has been fixed a tolerance in terms of function's evaluation, i.e. 1e-10 (two points are equal if the difference in terms of function value is lower than the tolerance) and another one, in terms of constraints' evaluation, i.e. 1e-7 (the constraints are respected if the difference between the desired values is lower than the tolerance). The GA solver adopts real values for the variables and creates the initial population following a uniform distribution in the range constituted by the assigned lower and upper bounds for each item, respectively 0 and $m_i$. A parametric study on a subpart of 5 items ($i$=1,…,5) allows tuning the algorithm, defining the relevant parameters (e.g.) the elite ratio (set to 0.20), a mutation ratio (set to 0.30), and a crossover ratio (set to 0.40). The solver sets the maximum number of iteration steps at 100, as a compromise between execution time and stability of results. Fig. 3 shows the parameter testing for the crossover ratio, with separate curves. Note that $C_1^*$ in the graph reflects only the first five items considered here. The results show a quite high deviation, possibly due to the low size of population and to a not optimal choice of the initial parameters.

Table 6 defines the optimal spare parts allocation according to the PBC-METRIC, taking into account the QCK-selection and the LDK-allocation algorithms. Table 6 also shows the stock level without considering the Quick Request transportation mode, thus applying the LDK-allocation algorithm for all the 20 items.

In terms of costs, there is a relevant increase, i.e. in case of application of the QCK-selection algorithm $C_1^* = €57558$, $C^{TOT}$=€96782 and $A_{SYS}$=99,20%, while $C_1^* = €\ 84691$ and $C^{TOT}$=€121924 and $A_{SYS}$=99.45%, that confirms the importance of the QCK-selection algorithm, which allows a greater savings and however does not downgrade the availability. Fig. 4 shows the differences in terms of total logistic cost ($C^{TOT}$), dividing this latter in Standard Request logistic cost ($C_1^{TOT}$) and Quick Request logistic cost ($C_2^{TOT}$). This graph highlights the importance of the developed model, which motivates the low increment in Quick Request logistic costs by a significant reduction in Standard Request logistic cost. Note that the application in two separate steps of the PBC-METRIC does not ensure that the obtained stock level is a global optimum. Although it would be necessary to combine the two steps for each generation of the LDK-allocation algorithm and jointly optimize all the spare parts, the achieved results represent a fast and significant solution of the problem, even if compared to the effort required for a jointly optimization of a large set of spare parts, in line with the managerial perspective.

## 6. Conclusions

The paper supports decision-makers of the maintenance function to decrease capital investment and improve availability through submitting a PBC. This is a novel contribution to take as a

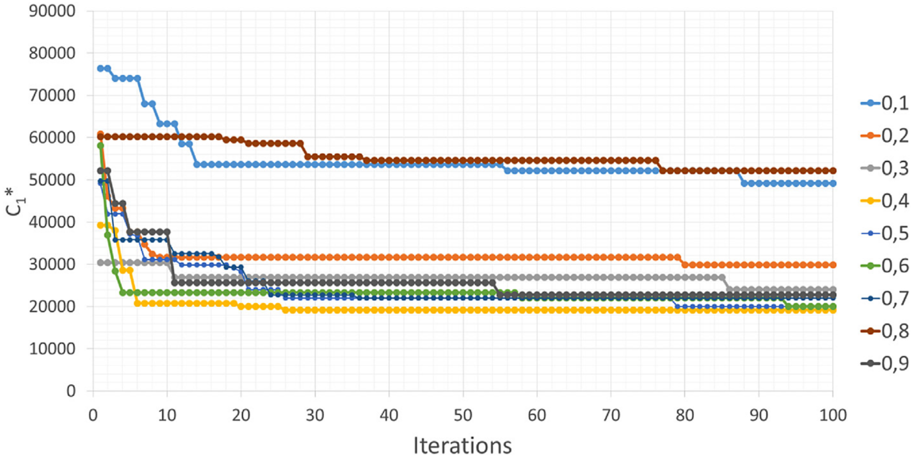

**Fig. 3.** Results of first test on the crossover ratio for a subpart of items.

**Table 6**
Optimum stock level and associated stock cost.

| Item | PBC-METRIC | | Stock without QCK | |
|---|---|---|---|---|
| | Stock level | Stock cost [€] | Stock level | Stock cost [€] |
| LRU$_1$ | 2 | 1550.40 | 2 | 1550.40 |
| LRU$_2$ | 0 | 0.00 | 1 | 4787.04 |
| LRU$_3$ | 0 | 0.00 | 1 | 6026.88 |
| LRU$_4$ | 0 | 0.00 | 1 | 3259.20 |
| LRU$_5$ | 4 | 17,189.76 | 4 | 17,189.76 |
| LRU$_6$ | 0 | 0.00 | 1 | 4529.76 |
| LRU$_7$ | 6 | 25,839.36 | 5 | 21,532.80 |
| LRU$_8$ | 0 | 0.00 | 5 | 2743.20 |
| LRU$_9$ | 0 | 0.00 | 0 | 0.00 |
| LRU$_{10}$ | 2 | 600.00 | 5 | 1500.00 |
| LRU$_{11}$ | 2 | 1463.04 | 5 | 3657.60 |
| LRU$_{12}$ | 2 | 3472.32 | 6 | 10,416.96 |
| LRU$_{13}$ | 1 | 1739.52 | 1 | 1739.52 |
| LRU$_{14}$ | 3 | 1068.48 | 3 | 1068.48 |
| LRU$_{15}$ | 0 | 0.00 | 0 | 0.00 |
| LRU$_{16}$ | 2 | 885.12 | 3 | 1327.68 |
| LRU$_{17}$ | 2 | 409.92 | 2 | 409.92 |
| LRU$_{18}$ | 2 | 1476.48 | 2 | 1476.48 |
| LRU$_{19}$ | 4 | 1211.52 | 2 | 605.76 |
| LRU$_{20}$ | 2 | 1550.40 | 4 | 869.76 |

first example of modelling contractual requirements to fix performance and service level of both parties. In this case, the PBC-METRIC helps the customer airline to select among multiple transportation options, by identifying the more worthwhile items to keep in stock. This first result is a novel introduction in the METRIC literature as all the main contributions refers to a single organization where the multi-echelon structure and all its parameters are known. The paper shows how the METRIC approach can remain valid for a customer airline company of a PBC where the information on the maintenance network are partial and some details are hidden by contract specs.

Moreover, the methodology provides an algorithm for optimizing the stock levels. The case study of a European airline, solved with a genetic algorithm, presents the achievable results in terms of availability target level while containing the budget. This showed how also other specific requirements (e.g. percentage of no fault found items to ship at the supplier's central department) could be included in the first stage of modelling that associates the items with the services in scope. Furthermore, the optimization function of the second stage could include other performance of the service (e.g. imperfect maintenance). The value of the methodology is to draw a path for managers and practitioners to set the best allocation of technical and economic resources on maintenance services.

New directions in research can expand the view on the modelling context, considering the Maintenance Repair and Overhaul perspective. A new model could integrate the different requirements and service levels of the Maintenance Repair and Overhaul portfolio to manage its total inventory of spare parts, selecting the best strategy for multiple customers, analysing the

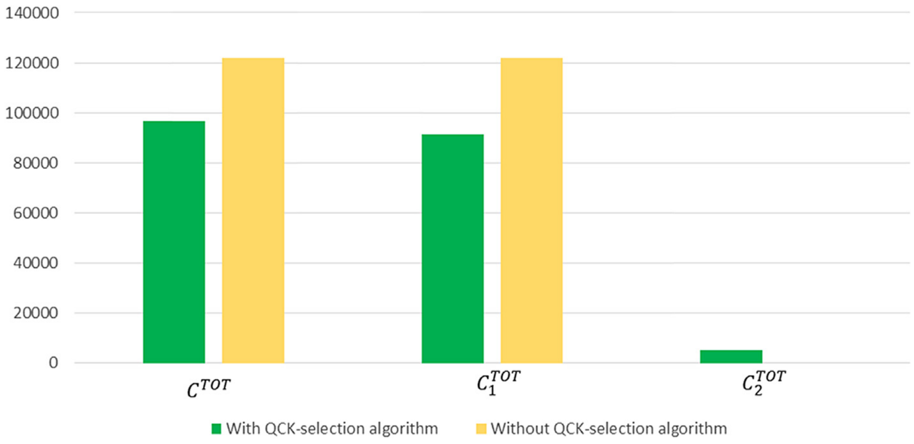

**Fig. 4.** Cost differences with or without the QCK-selection algorithm.

total cost of a multi-base scenario and the effects of the contract specifications. Furthermore, the single-indenture of items can evolve to multi-indenture, considering the maintainability at both line replaceable unit and shop-replaceable unit level. In this model, each stock-out generates a backorder, and each backorder has a downgrading effect in terms of the system availability. It would be possible to evolve this approach, considering the effect of backorders also in the operation process of the customer company, with respect to the deriving costs of stock out. It would be also possible to perform a sensitivity analysis on the optimum stock level, also considering other potential optimization methods and algorithms. This analysis could help the decision-makers to evaluate the effects of a change (e.g.) in the market value of the items, or even in the demand rate, due (e.g.) to the fleet ageing.

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