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# Betas and the myth of market neutrality

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ABSTRACT

Market neutral funds are commonly advertised as alternative investments that offer returns which are uncorrelated with the broad market. Utilizing recent advances in financial econometrics, we demonstrate that using standard forecasting methods to construct market (beta) neutral funds is often very inaccurate. Our findings demonstrate that the econometric methods that are commonly employed for forecasting the beta (systematic) risk typically lack sufficient accuracy to permit the successful construction of market neutral portfolios. The results in this paper also highlight the need for higher frequency returns data to be utilized more commonly. Using daily returns over the past year, we demonstrate an approach that is easy to implement and delivers a substantial improvement, relative to other methods, when attempting to construct a market neutral portfolio.


## 1. Introduction

Hedge funds are often portrayed as investments which are market (beta) neutral, in that they have little systematic exposure to market risk. However, work by Asness, Krail, and Liew (2001), Bali, Brown, and Caglayan (2011, 2012), and Patton (2009) has shown hedge funds to have substantial market exposure. The recent financial crisis provided further evidence of significant beta exposure by equity market neutral hedge fund managers, as over 70% of funds reporting to Hedge Fund Research (HFR) finished 2008 in the red. Brown, Gregoriou, and Pascalau (2012) further highlight these concerns over the risk characteristics of hedge funds, and in reference to hedge funds during the recent financial crisis, remark "we all fall down together".

The successful construction of a market neutral portfolio depends inherently on the ability of the manager to measure and forecast the beta exposure of his long and short portfolios accurately. The greater the forecast error of the betas, the more likely the fund is to have a significant residual beta exposure, and therefore, the greater the potential exposure to systematic risk factors. This, of course, is precisely what investors hope to avoid when investing in market neutral funds. They do not want to be paying alpha fees (averaging 1.5% in management fees and with 20% in incentive fees) for beta returns (which can be obtained easily through index exchange traded funds for a fee of typically less than 0.2%). In response to these concerns, this study investigates the accuracy of the methods commonly employed for constructing market neutral portfolios.

In general, daily equity returns is the highest frequency that is available reliably for the construction of equity market neutral portfolios, though it is also very common to see beta forecasts generated from monthly equity returns. The overwhelming majority of beta forecasts are generated from a constant beta model with a typical estimation period of between one and five years. This dates back to the work of Fama and MacBeth (1973), who proposed an estimation period of five years of monthly returns, and was further justified by Ghysels (1998), who showed that constant beta models have outperformed more sophisticated models of the time-varying beta. Recently

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proposed time-varying realized beta models for quarterly beta forecasting were studied by Andersen, Bollerslev, Diebold, and Wu (2005, 2006), Ghysels and Jacquier (2006), and Hooper, Ng, and Reeves (2008), among others, though Reeves and Wu (2013) showed that these time-varying realized beta models did not outperform the constant beta model estimated on daily returns over the prior year.

In this paper, we evaluate these competing beta forecasting approaches in the setting of the construction of an equity market neutral portfolio. Equity momentum portfolios are constructed, as this is a common portfolio construction technique; see Carhart (1997), Grundy and Martin (2001), Jegadeesh and Titman (1993) and Novy-Marx (2012), among others. However, we are not evaluating the return generating abilities of momentum strategies, but focus instead on evaluating the beta neutrality of portfolios by applying a commonly used and widely studied trading strategy. In addition, as a robustness check for portfolio construction based on momentum, we also construct portfolios by selecting stocks randomly and assessing the bootstrap distribution of statistics. We obtain similar results for both portfolio construction approaches, which provides an indication of the general applicability of the results beyond momentum-based strategies.

The findings in this paper show that methods that are designed to deliver beta neutrality (in the sense of low variability from a zero beta) often fail, particularly when the volatility in stock market returns is high. We define high volatility as periods when the CBOE Volatility Index (VXO) is above its median. In addition, we also find that the ex post portfolio betas of the constructed equity market neutral portfolios are generally not correlated with other well known risk premiums, such as the size, value and momentum premiums.

The results in this paper suggest that the inability of equity market neutral funds to exhibit market neutrality in their performances may be due largely to the fact that the commonly utilized approaches lack sufficient accuracy to construct an equity market neutral fund. We also find that, with a portfolio that targets a beta of zero, based on the widely used (Fama & MacBeth, 1973) beta from five years of monthly returns, the ex post beta exceeded one. Even more alarming is the fact that these non-zero ex post betas are amplified by leverage. Heavy leverage from financial institutions to invest, mistakenly, in beta (believing it to be alpha) was a leading factor in the recent financial crisis, see Acharya and Richardson (2009) and Brown (2011). Of the different beta forecasting approaches, we find that the smallest errors occur when beta neutral portfolios are constructed from realized betas computed from daily returns over the past year.

The rest of the paper is organized as follows. Section 2 provides some background on the realized beta, and Section 3 describes the methodology. Sections 4 and 5 present the data and results respectively. Our conclusion is presented in Section 6.

## 2. Realized beta

In this paper, our forecasting approaches and evaluation rely on realized beta estimates, so we begin by briefly reviewing the measurement of the realized beta. Following the work of Andersen et al. (2006) and Barndorff-Nielsen and Shephard (2004), assume that $p_t$, which represents the logarithmic $N \times 1$ vector price process, follows a multivariate continuous time stochastic volatility diffusion

$$dp_t = \mu_t dt + \theta_t dW_t \tag{1}$$

where $W_t$ is standard Brownian motion, $\omega_t = \theta_t \theta_t'$ is the instantaneous covariance matrix, and $\mu_t$ is the $N$-dimensional instantaneous drift and is jointly independent of $W(t)$. If $h$ denotes a certain period (e.g., one day, one month, etc.), then the continuously compounded return over period $h$ for stock $i$ can be defined as $r_{i,t+h,h} = p_{i,t+h} - p_{i,t}$.

Under weak regularity conditions,[1] the theory of quadratic variation leads us to the following result for all $t$ as the sampling frequency tends to infinity:

$$\sum_{j=1,\dots,[h/\Delta]} r_{t+j\cdot\Delta} \cdot r'_{t+j\cdot\Delta} - \int_0^h \omega_{t+s} ds \xrightarrow{p} 0. \tag{2}$$

The realized beta of a security is defined as the realized covariance of the security with the market divided by the realized variance of the market. Following the above discussion, the realized covariance of security $i$ and market $M$ over the period $[t, t+h]$ from the theory of quadratic variation is

$$\widehat{\nu}_{iM,t,t+h} = \sum_{j=1,\dots,[h/\Delta]} r_{i,t+j\cdot\Delta,\Delta} \cdot r_{M,t+j\cdot\Delta,\Delta}.$$

Similarly, the realized variance of the market over the period $[t, t+h]$ is

$$\widehat{\nu}_{M,t,t+h} = \sum_{j=1,\dots,[h/\Delta]} r^2_{M,t+j\cdot\Delta,\Delta}.$$

Thus, the realized beta of security $i$ is

$$\widehat{\beta}_{i,t,t+h} = \frac{\widehat{\nu}_{iM,t,t+h}}{\widehat{\nu}_{M,t,t+h}} = \frac{\sum_{j=1,\dots,[h/\Delta]} r_{i,t+j\cdot\Delta,\Delta} \cdot r_{M,t+j\cdot\Delta,\Delta}}{\sum_{j=1,\dots,[h/\Delta]} r^2_{M,t+j\cdot\Delta,\Delta}}$$
$$\xrightarrow{p} \frac{\int_0^h \omega_{(iN),t+s} ds}{\int_0^h \omega_{(NN),t+s} ds} = \beta_{i,t,t+h} \tag{3}$$

for all $t$ as $\Delta \to 0$; i.e., the realized beta measure is consistent for the true beta by sampling both the security return and the market return at an ultra high frequency.

### 2.1. Measurement error

The prior consistency result of the realized beta estimator provides a theoretical justification in the setting of ultra high frequency return measurement; however, in most applications, careful consideration needs to be given to the question of how high the return frequency can be taken without losing accuracy in the return measurement. For most stocks, daily returns is often

[1] See Andersen et al. (2006) and Barndorff-Nielsen and Shephard (2004) for details.

the highest frequency that will provide a reliable return measurement. However, computing realized betas from daily returns results in measurement error in the realized beta. For the purpose of ex post analysis with quarterly realized betas computed from daily returns, both Andersen et al. (2005) and Hooper et al. (2008) remove the measurement error by smoothing the realized betas using the Kalman filter. Chen and Reeves (2012) advocate the use of the Hodrick–Prescott filter (HP filter henceforth) for smoothing realized betas. This study also applies the HP filter, which we now review briefly.

Given an observed time series $y_t$ that is integrated of order zero or one, assume that $y_t = \tau_t + \epsilon_t$, with $\mathbf{y}' = (y_1, y_2, \ldots, y_N)$, $\tau' = (\tau_1, \tau_2, \ldots, \tau_N)$ and $\epsilon' = (\epsilon_1, \epsilon_2, \ldots, \epsilon_N)$, where $\tau_t$ denotes the unobserved trend component at time $t$ and $\epsilon_t$ the unobserved residual component at time $t$. Given an appropriately chosen smoothing parameter $\lambda$, the estimated trend component $\widehat{\tau_t}$ can be obtained as the solution to the following convex minimization problem:

$$\min_{[\tau_t]_{t=1}^{N}} \left[ \sum_{t=1}^{N} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{N-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 \right], \quad \lambda > 0. \tag{4}$$

The smoothing parameter $\lambda$ in the HP filter acts to balance the impacts of the two summations in the above equation. The first sum minimizes the variance in the noise component and the second minimizes the variance in the growth rate of the trend component. Chen and Reeves (2012) experimented with a wide range of $\lambda$ values, and found 100 to be suitable for filtering monthly realized betas constructed from daily returns. On average, these HP100 betas yielded close to the smallest measurement error when evaluated against the monthly realized betas constructed from 30-min returns. Their results were robust across their entire sample of Dow Jones stocks. Hence, following the same methodology, we examined the same sample of stocks at the quarterly frequency. That is, we constructed quarterly realized betas from daily returns and experimented with a wide range of $\lambda$ values, and also found 100 to yield close to the smallest measurement error on average (mean squared error) when evaluated against the quarterly realized betas constructed from 30-min returns. Thus, in this study, we measure ex post quarterly betas primarily by computing quarterly realized betas from daily returns which are smoothed using the HP100 filter in order to remove measurement error.

## 3. Methodology

The methodology involves three steps. First, we construct a dollar neutral long/short portfolio, using the price momentum of stocks listed in the S&P 100 index. We restrict ourselves to this set of stocks in order to ensure that we have reliable daily returns for beta measurement. It is common for momentum strategies to utilize stocks from a much larger set; however, given that we are evaluating beta performances, not the return performances of momentum strategies, the S&P 100 stocks are more useful for generating accurate results. In addition, as a robustness check, we also construct portfolios by selecting S&P 100 stocks at random, rather than based on the momentum. Next, we estimate the beta of the overall portfolio using four different approaches, in order to calculate the market exposure (residual beta) of the portfolio. This residual beta will be hedged using futures contracts on the S&P 500 index in order to make the portfolios beta neutral. Finally, we will calculate the ex post betas of our "market neutral" portfolios in order to ascertain which measure of beta allowed us to generate the most ex post beta-neutral portfolio.

### 3.1. *Beta forecasting models*

This paper uses four candidate models to evaluate which beta forecasting model yields the best ex post beta neutral strategy. These four models, based on different in-sample estimation periods, are selected based on their popularity and/or econometric justification.

#### 3.1.1. *Fama–MacBeth beta*

The Fama and MacBeth (1973) beta, which is the regression slope coefficient from five years of monthly stock returns regressed onto a constant and the market returns, is the beta estimate that is used most widely by both academics and practitioners. This beta forecast is regarded as the industry standard. Its widespread use could be considered an historical accident, as the choice of monthly stock returns for beta forecasting has no econometric justification when reliable daily stock returns are available.

#### 3.1.2. *Quarterly realized beta*

To calculate the quarterly realized beta, we apply the methodology described by Andersen et al. (2006), and presented in Section 2. As Andersen et al. (2006) showed, the daily return sampling frequency may be used to construct quarterly realized betas. If we refer back to Eq. (3), this means that $h$ will be quarterly and delta will be daily. Ghysels (1998) shows that constant beta models have outperformed more sophisticated models of time-varying beta. Thus, this random walk model is used as one of our beta forecasting candidates. The realized beta is calculated each month by utilizing the previous three months of daily stock and market returns.

#### 3.1.3. *Autoregressive quarterly realized beta*

Hooper et al. (2008) evaluated a number of competing forecasting models that used quarterly realized betas. The following equation is used for the autoregressive model with $p$ lags:

$$\beta_{i,t+1} = \alpha_{i,0} + \alpha_{i,1}\beta_{i,t} + \alpha_{i,2}\beta_{i,t-1} + \cdots + \alpha_{i,p}\beta_{i,t-(p-1)} + u_{i,t+1}. \tag{5}$$

Hooper et al. (2008) determine that an AR(1) model provides the most accurate forecasts if only five years of data are available. Following this result, the third beta candidate is the AR(1) quarterly realized beta forecast using the past 20 quarters of realized betas. This is calculated in two steps. Firstly, the non-overlapping three-month realized beta series is constructed from daily stock and S&P 500 index returns over the five-year period. Secondly, the AR(1) model is fitted to the constructed realized beta time series and used to produce one-step-ahead (three-month) forecasts.

*3.1.4. Annual realized beta*

Reeves and Wu (2013) evaluate the forecasting performances of constant beta models over the quarterly horizon, relative to the recently-suggested autoregressive models of quarterly realized betas, and find that a constant beta model computed from daily returns over the last 12 months generates the most accurate quarterly forecasts of beta. Therefore, we construct our fourth beta candidate by computing a realized beta from daily returns over the previous year. If we refer back to Eq. (3) again, this means that $h$ will be annual and delta will be daily.

*3.2. The momentum portfolio construction*

Price momentum is used commonly by portfolio managers as a buy or sell signal for stocks, and has been studied widely (see Carhart, 1997; Grundy & Martin, 2001; Jegadeesh & Titman, 1993 and Novy-Marx, 2012). In fact, the momentum factor is now standard in equity asset pricing models. Strategies that buy stocks that have performed well in the past and sell stocks that have performed poorly in the past generate significant positive returns over 1- to 12-month holding periods.

A strategy that selects stocks on the basis of returns over the past $J$ months and holds them for $K$ months (a $J$–$K$ strategy) is constructed as follows:

- At the beginning of each month $t$, the stocks are ranked in ascending order on the basis of their returns over the past $J$ months.
- Based on these rankings, form ten equally-weighted decile portfolios.
- In each month $t$, the strategy buys the winner portfolio and sells the loser portfolio and holds this position for $K$ months.
- The strategy closes out the position in month $t + K$.

For the purpose of our paper, we assume that the holding period $K$ is three months, and the ranking period $J$ is 11 months. Following the literature, we skip a month between the portfolio formation period and the holding period. This gap avoids the short-term reversals that might contaminate the momentum strategy. Let $r_{i,t}$ denote the return on stock $i$ in month $t$. The cumulative return for stock $i$ over the months from $t-12$ to $t-1$ is calculated as $\prod_{s=t-12}^{s=t-2}(1+r_{i,s})-1$.

S&P 100 index stock components are used as the sample, and returns on the S&P 500 index are used as the broad market return. We reconstruct the actual S&P 100 index, adjusting the index for all additions and deletions. This ensures that our portfolio construction process is realistic and that there is no survivorship bias. The portfolio construction protocol is to rank the 100 component stocks on the basis of their cumulative returns over the 11-month formation period; that is, $\prod_{s=t-12}^{s=t-2}(1+r_{i,s})-1$. We identify the top and bottom deciles, and enter into equal-weighted long positions in the 10 stocks in the top decile and equal-weighted short positions in the 10 stocks in the bottom decile.

Since our holding period $K$ is three months, we need to ensure that our results are not sensitive to the choice of the starting month. In order to overcome this possible bias, we run three portfolios with overlapping holding periods. At the end of any given month $t$, one of the three portfolios will reach the end of its holding period, and a new portfolio will need to be formed. This newly-formed portfolio, consisting of a \$100 million long position and a \$100 million short position, is dollar neutral but not necessarily beta neutral. However, our objective is to construct beta neutral portfolio; therefore, the residual market exposure of the long/short portfolio is hedged using S&P 500 E-mini futures contracts. The number of contracts purchased is calculated as $10{,}000{,}000(\sum_{i=1}^{10}\beta_{s,i,t}-\sum_{j=1}^{10}\beta_{l,j,t})/50P_t$, where $\beta_{s,i,t}$ is the beta forecast for short stock $i$ at time $t$, $\beta_{l,j,t}$ is the beta forecast for long stock $j$ at time $t$ and $P_t$ is the futures price at time $t$.

*3.3. Ex post beta analysis*

By construction, the portfolio, consisting of long and short positions and an overlaid futures hedge, is beta neutral based on beta forecasts. The key to measuring the accuracy of beta forecasts is to evaluate our ex post beta on the momentum portfolio. As has been stated, there are three portfolios in each month, and their ex post performances are treated separately. Accordingly, there are three sets of ex post betas for stocks. Specifically, the first set comprises 64 periods, including Jan–Mar 1993, Apr–Jun 1993, …, Oct–Dec 2008. The second set has 63 periods, Feb–Apr 1993, May–Jul 1993, …, Aug–Oct 2008. The third set also covers 63 periods, including Mar–May 1993, Jun–Aug 1993, …, Sep–Nov 2008. The HP100 filter is applied to each stock's realized beta series in the three different time frame sets. These smoothed betas are our ex post betas.

Ex post portfolio betas are evaluated primarily in one of two ways. Firstly, beginning-of-quarter weights are used to measure the ex post beta of the momentum portfolio at the end of the quarter. Secondly, end-of-quarter weights are used to measure the ex post beta of the momentum portfolio. End-of-quarter weights are value-weighted based on end-of-quarter prices. As a robustness check of our results, we also compute ex-post stock betas using 60-min returns for a sub-sample of the dataset. The analysis using the higher frequency data is also performed using both beginning-of-quarter stock weights and end-of-quarter stock weights.

In addition, ex post portfolio betas are also evaluated over two different volatility states in stock market returns. The high volatility state is defined as a quarter in which the CBOE Volatility Index (VXO) was initially over 18.54 (the median VXO over our sample period), while the lower volatility state is defined as a quarter in which the VXO was initially equal to or below 18.54. We also assess whether our ex post betas are correlated with other well known risk premiums, and in particular, the size, value and momentum premiums. A four-factor model is estimated, with the dependent variable being the quarterly ex post beta, and the explanatory variables being the four factors of the Carhart (1997) model.

Finally, we again conduct our initial analysis of the ex post betas, with portfolios now being constructed by selecting stocks from the S&P 100 index randomly, rather than based on momentum. We repeat the process 5000 times to construct a bootstrap distribution for the statistics and report the 90th, 50th and 10th percentiles. This is a further check of the robustness of our results, in order to ensure that our findings are applicable outside the space of momentum constructed portfolios.

## 4. Data

S&P 100 index components are considered as our stock universe. We choose the S&P 100 index because it consists of the most liquid stocks with the largest market capitalization in the US. Over the sample period, several changes were made to the composition of the S&P 100 index. In order to ensure that our analysis is unbiased and realistic, we keep a record of all of the additions to and deletions from the index, and ensure that we do not use any information for our stock selection that was not already public. The historical data on S&P 100 index components and the associated additions and deletions are collected from the website of Standard & Poor's.[2] The daily data for the S&P 100 index stock components are obtained from the Center for Research in Security Prices (CRSP). The data set covers the period from January 4, 1988, to December 31, 2008.

Although our investment universe is restricted to the S&P 100, we consider the S&P 500 as the market index. As a result, the residual betas of our long/short portfolios need to be hedged out using the S&P 500 index. We employ the front month futures on the S&P 500 E-Mini Futures index to implement this hedge. This is the most liquid futures contract, and trades almost around the clock. The S&P 500 E-Mini Futures price data from January 31, 1992 to December 31, 2008 are sourced from Datastream.

As was mentioned above, we also perform the analysis on a sub-sample of higher frequency data. We obtain hourly intraday price data for our S&P 100 stocks from Price-Data[3] and the Trade and Quote (TAQ) database. This high frequency data sample is from January 2, 2003, to December 31, 2008, and hourly intraday prices are sampled from 10:30AM to 3:30PM. In addition, as part of our further robustness checks, we also obtain the CBOE Volatility Index (VXO) from the Chicago Board Options Exchange, covering the period from December 31, 1992, to September 30, 2008. Finally, the dataset of explanatory variables used for the four factor model estimation is sourced from the Kenneth French Data Library,[4] for the period 1993Q1 to 2008Q4.

2 http://www.standardandpoors.com/.

3 http://www.price-data.com/.

4 http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.

## 5. Results

In this section we focus primarily on the ex post beta analysis of our portfolios, paying particular attention to their variability from the zero target. The momentum portfolios are dollar neutral, meaning that the absolute values of long and short positions are equal, and their residual beta exposure is hedged based on forecasted betas. The four different beta forecasting techniques result in different hedging dynamics of the three momentum portfolios. Although the general pattern of movement of the time series of hedging is similar for all four beta forecasts across all three momentum portfolios, the number of futures contracts needed to hedge the residual beta exposure can vary substantially depending on the beta forecast. Fig. 1 displays the hedging dynamics of portfolio 1 as the number of S&P 500 E-mini futures contracts.

As was discussed in Section 3, two sets of portfolio weights are used in our ex post analysis of the portfolio beta. The first set is the beginning-of-quarter weights in the portfolio. If we are able to forecast stock betas with perfect foresight, then these portfolio weights will generate an ex post portfolio beta of zero. The second set is the end-of-quarter weights in the portfolio. These weights are the associated proportional weight of each asset in the overall portfolio based on end-of-quarter prices.

Six statistics are computed on the ex post portfolio beta: mean, standard deviation (STD), minimum (MIN), maximum (MAX), mean squared error from zero (MSE), and mean absolute error from zero (MAE). MSE and MAE are calculated as $\frac{1}{m}\sum_{t=1}^{m}\hat{\beta}_t^{\,2}$ and $\frac{1}{m}\sum_{t=1}^{m}|\hat{\beta}_t|$, respectively, with $m$ being the number of quarters over the forecast evaluation period. Table 1 displays the results when beginning-of-quarter stock weights are used to calculate the ex post betas of the three momentum portfolios. The case of perfect foresight, where the beginning-of-quarter stock beta forecast is the actual HP100 filtered stock beta of the quarter, is displayed in the last column. For the four beta forecasting approaches, the portfolios' mean ex post betas are in the range −0.0161 to 0.0745, demonstrating that, on average, the portfolios are close to being beta neutral. However, the main criterion for evaluating whether a portfolio is beta neutral is whether the ex post beta varies from zero substantially over time. We measure this variability in the ex post beta using STD, MIN, MAX, MSE and MAE.

These statistics show a substantial degree of variability in the ex post beta. The worst performing beta forecasting method is the Fama–MacBeth, with the range (MAX–MIN) being in excess of 1.6 for all three portfolios. The quarterly realized beta method also has a relatively large range, being in excess of 1.47 for all three portfolios. The ex post beta variability in these methods is also seen in the relatively large values of STD, MSE and MAE. The range of the ex post beta is narrower for the annual realized beta and the AR(1) quarterly realized beta forecasts: while the range for the AR(1) quarterly realized beta is still in excess of one for all three portfolios, the largest range for the annual realized beta is 0.7711. These statistics show that the annual realized beta forecast generates the portfolio with the greatest beta neutrality. The AR(1) quarterly realized beta forecast is the second best performer,

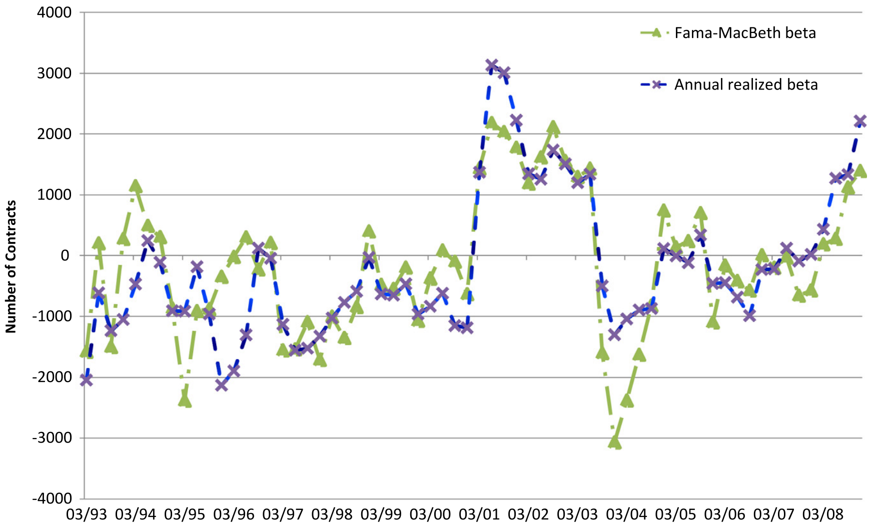


**Fig. 1.** Dynamics of the hedge for momentum portfolio 1. The Fama–MacBeth beta is computed from monthly returns over the previous five years and the annual realized beta is computed from daily returns over the previous year.

**Table 1**
Ex post beta exposure using daily returns and beginning-of-quarter weights.

| | | Fama–MacBeth beta | Quarterly realized beta | AR(1) quarterly realized beta | Annual realized beta | Perfect foresight |
|---|---|---|---|---|---|---|
| Portfolio 1 | mean | 0.0510 | −0.0161 | 0.0630 | 0.0606 | 0.0000 |
| | std | 0.3615 | 0.3190 | 0.2136 | 0.1398 | 0.0000 |
| | min | −0.8532 | −0.8629 | −0.5087 | −0.2341 | 0.0000 |
| | max | 0.9512 | 1.1213 | 0.5953 | 0.4663 | 0.0000 |
| | MSE | 0.1312 | 0.1004 | 0.0489 | 0.0229 | 0.0000 |
| | MAE | 0.2837 | 0.2443 | 0.2494 | 0.1170 | 0.0000 |
| | MSE (1st subperiod) | 0.1355 | 0.0768 | 0.0321 | 0.0162 | 0.0000 |
| | MSE (2nd subperiod) | 0.1269 | 0.1240 | 0.0657 | 0.0296 | 0.0000 |
| Portfolio 2 | mean | 0.0745 | −0.0026 | 0.0677 | 0.0655 | 0.0000 |
| | std | 0.3679 | 0.2884 | 0.2179 | 0.1459 | 0.0000 |
| | min | −0.7272 | −0.4880 | −0.5582 | −0.2461 | 0.0000 |
| | max | 0.9940 | 0.9886 | 0.5701 | 0.5250 | 0.0000 |
| | MSE | 0.1387 | 0.0819 | 0.0513 | 0.0252 | 0.0000 |
| | MAE | 0.2876 | 0.2297 | 0.1756 | 0.1201 | 0.0000 |
| | MSE (1st subperiod) | 0.1484 | 0.0530 | 0.0527 | 0.0204 | 0.0000 |
| | MSE (2nd subperiod) | 0.1287 | 0.1117 | 0.0498 | 0.0302 | 0.0000 |
| Portfolio 3 | mean | 0.0745 | −0.0053 | 0.0698 | 0.0742 | 0.0000 |
| | std | 0.3477 | 0.2620 | 0.2255 | 0.1487 | 0.0000 |
| | min | −0.8138 | −0.8013 | −0.4418 | −0.2682 | 0.0000 |
| | max | 0.8312 | 0.6922 | 0.5917 | 0.5024 | 0.0000 |
| | MSE | 0.1246 | 0.0676 | 0.0549 | 0.0273 | 0.0000 |
| | MAE | 0.2896 | 0.1918 | 0.1856 | 0.1277 | 0.0000 |
| | MSE (1st subperiod) | 0.1166 | 0.0592 | 0.0433 | 0.0279 | 0.0000 |
| | MSE (2nd subperiod) | 0.1328 | 0.0762 | 0.0670 | 0.0267 | 0.0000 |

The Fama–MacBeth beta is computed from monthly returns over the previous five years. The quarterly realized beta is computed from daily returns over the previous quarter. The AR(1) quarterly realized beta is estimated on five years of quarterly realized betas computed from daily returns. The annual realized beta is computed from daily returns over the previous year. Perfect foresight is when stock betas are forecast without error. The forecast evaluation period is from January 2, 1993, to December 31, 2008.

followed by the quarterly realized beta forecast. The widely used Fama–MacBeth beta is the worst predictor, having the highest variability: the Fama–MacBeth MSE is typically more than four times that of the annual realized beta MSE. The annual realized beta forecast has MAEs for our portfolios of 0.1170, 0.1201 and 0.1277, whereas the other beta forecasting methods typically have MAEs in excess of 0.2. Fig. 2 displays the ex post portfolio betas of portfolio 1 for the Fama–MacBeth beta and the annual realized beta. Most notable are the large deviations from zero that sometimes occur for the Fama–MacBeth beta. The large deviations of around one are typically associated with crisis periods in the market, such as the bursting of the technology bubble in the early 2000s and the recent financial crisis.

Table 2 presents the summary statistics on the ex post portfolio betas from end-of-quarter weights for the three momentum portfolios. The results are similar to those in

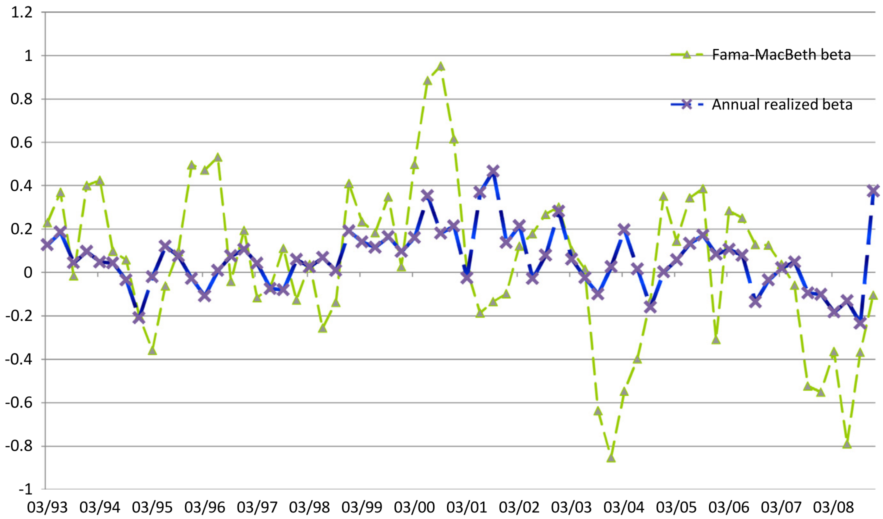


**Fig. 2.** Ex post beta exposure for momentum portfolio 1 using beginning-of-quarter weights. The Fama–MacBeth beta is computed from monthly returns over the previous five years and the annual realized beta is computed from daily returns over the previous year.

**Table 2**
Ex post beta exposure using daily returns and end-of-quarter weights.

| | | Fama–MacBeth beta | Quarterly realized beta | AR(1) quarterly realized beta | Annual realized beta | Perfect foresight |
|---|---|---|---|---|---|---|
| Portfolio 1 | mean | 0.0607 | −0.0065 | 0.0726 | 0.0703 | 0.0097 |
| | std | 0.3538 | 0.3204 | 0.1951 | 0.1377 | 0.0525 |
| | min | −0.8517 | −0.8968 | −0.3968 | −0.2257 | −0.1448 |
| | max | 0.9377 | 0.9945 | 0.5820 | 0.4640 | 0.1760 |
| | MSE | 0.1269 | 0.1011 | 0.0428 | 0.0236 | 0.0028 |
| | MAE | 0.2841 | 0.2502 | 0.1587 | 0.1187 | 0.0367 |
| | MSE (1st subperiod) | 0.1369 | 0.0824 | 0.0335 | 0.0186 | 0.0012 |
| | MSE (2nd subperiod) | 0.1169 | 0.1198 | 0.0520 | 0.0286 | 0.0044 |
| Portfolio 2 | mean | 0.0947 | 0.0177 | 0.0880 | 0.0858 | 0.0203 |
| | std | 0.3586 | 0.3109 | 0.2055 | 0.1504 | 0.0541 |
| | min | −0.7048 | −0.4833 | −0.3820 | −0.2011 | −0.0567 |
| | max | 1.0005 | 1.1164 | 0.5766 | 0.5467 | 0.2886 |
| | MSE | 0.1355 | 0.0954 | 0.0493 | 0.0296 | 0.0033 |
| | MAE | 0.2865 | 0.2439 | 0.1774 | 0.1316 | 0.0347 |
| | MSE (1st subperiod) | 0.1533 | 0.0617 | 0.0583 | 0.0254 | 0.0014 |
| | MSE (2nd subperiod) | 0.1171 | 0.1302 | 0.0401 | 0.0340 | 0.0053 |
| Portfolio 3 | mean | 0.0380 | −0.0497 | 0.0395 | 0.0401 | −0.0155 |
| | std | 0.4058 | 0.3657 | 0.2914 | 0.2883 | 0.2413 |
| | min | −0.8602 | −0.7900 | −0.7473 | −0.7964 | −1.0101 |
| | max | 1.1811 | 0.8286 | 0.6603 | 0.7260 | 0.5548 |
| | MSE | 0.1635 | 0.1341 | 0.0851 | 0.0834 | 0.0575 |
| | MAE | 0.3136 | 0.2896 | 0.2277 | 0.2229 | 0.1633 |
| | MSE (1st subperiod) | 0.1526 | 0.1277 | 0.0739 | 0.0791 | 0.0064 |
| | MSE (2nd subperiod) | 0.1747 | 0.1406 | 0.0967 | 0.0878 | 0.0510 |

The Fama–MacBeth beta is computed from monthly returns over the previous five years. The quarterly realized beta is computed from daily returns over the previous quarter. The AR(1) quarterly realized beta is estimated on five years of quarterly realized betas computed from daily returns. The annual realized beta is computed from daily returns over the previous year. Perfect foresight is when stock betas are forecast without error. The forecast evaluation period is from January 2, 1993, to December 31, 2008.

Table 1 for beginning-of-quarter weights. Again, the MSE is systematically lower when the hedge is implemented using the annual realized beta and quarterly AR(1) forecast models, and this result is robust across all three portfolios and all sub-periods. These results indicate the robustness of our findings to the starting month of our momentum portfolios and also to the use of beginning- and end-of-quarter portfolio weights. In addition to reporting the MSEs over our forecast evaluation periods, we also report the MSEs separately for the first and second halves of our forecast evaluation periods. Similar patterns across the forecast errors from the different approaches are seen over these different subsamples, and thus, we conclude that our results cannot be attributed to sampling variability in the data generating processes of stock returns. Demonstrating the robustness of our results over subsamples of data in the current setting is adopted as an alternative to measuring the statistical significance between approaches directly.

Next, we conduct an additional robustness check of our results by computing ex post stock betas on 60 min

**Table 3**
Ex post beta exposure using hourly returns and beginning-of-quarter weights.

| | | Fama–MacBeth beta | Quarterly realized beta | AR(1) quarterly realized beta | Annual realized beta | Perfect foresight |
|---|---|---|---|---|---|---|
| Portfolio 1 | mean | −0.0759 | 0.1055 | 0.0539 | 0.0754 | 0.0000 |
| | std | 0.4871 | 0.4105 | 0.3050 | 0.2781 | 0.0000 |
| | min | −0.9022 | −0.3331 | −0.4872 | −0.4241 | 0.0000 |
| | max | 0.8697 | 1.7391 | 0.6215 | 0.6978 | 0.0000 |
| | MSE | 0.2331 | 0.1726 | 0.0920 | 0.0798 | 0.0000 |
| | MAE | 0.4055 | 0.2368 | 0.2437 | 0.2206 | 0.0000 |
| Portfolio 2 | mean | −0.0423 | 0.0701 | 0.0612 | 0.0805 | 0.0000 |
| | std | 0.5283 | 0.5390 | 0.3821 | 0.3533 | 0.0000 |
| | min | −1.0177 | −0.5763 | −0.9029 | −0.6173 | 0.0000 |
| | max | 0.8389 | 2.0519 | 0.8127 | 0.9392 | 0.0000 |
| | MSE | 0.2688 | 0.2828 | 0.1434 | 0.1259 | 0.0000 |
| | MAE | 0.4501 | 0.3373 | 0.2793 | 0.2692 | 0.0000 |
| Portfolio 3 | mean | −0.0405 | 0.0609 | 0.0398 | 0.0820 | 0.0000 |
| | std | 0.5056 | 0.4185 | 0.3342 | 0.3203 | 0.0000 |
| | min | −1.2190 | −0.6027 | −0.7420 | −0.7048 | 0.0000 |
| | max | 0.7251 | 1.5777 | 0.6847 | 0.7455 | 0.0000 |
| | MSE | 0.2462 | 0.1712 | 0.1084 | 0.1048 | 0.0000 |
| | MAE | 0.3803 | 0.2423 | 0.2413 | 0.2305 | 0.0000 |

The Fama–MacBeth beta is computed from monthly returns over the previous five years. The quarterly realized beta is computed from daily returns over the previous quarter. The AR(1) quarterly realized beta is estimated on five years of quarterly realized betas computed from daily returns. The annual realized beta is computed from daily returns over the previous year. Perfect foresight is when stock betas are forecast without error. The forecast evaluation period is from January 2, 2003, to December 31, 2008.

**Table 4**
Ex post beta exposure using hourly returns and end-of-quarter weights.

| | | Fama–MacBeth beta | Quarterly realized beta | AR(1) quarterly realized beta | Annual realized beta | Perfect foresight |
|---|---|---|---|---|---|---|
| Portfolio 1 | mean | −0.0672 | 0.1140 | 0.0620 | 0.0842 | 0.0689 |
| | std | 0.4755 | 0.4070 | 0.3003 | 0.2807 | 0.2149 |
| | min | −0.7864 | −0.3260 | −0.5246 | −0.4262 | −0.2413 |
| | max | 0.8857 | 1.7277 | 0.6144 | 0.7137 | 0.6190 |
| | MSE | 0.2212 | 0.1718 | 0.0903 | 0.0826 | 0.0490 |
| | MAE | 0.3885 | 0.2447 | 0.2345 | 0.2264 | 0.1561 |
| Portfolio 2 | mean | −0.0255 | 0.0869 | 0.0780 | 0.0973 | 0.0778 |
| | std | 0.4928 | 0.4996 | 0.3297 | 0.3040 | 0.3027 |
| | min | −0.7985 | −0.6185 | −0.5704 | −0.3683 | −0.3446 |
| | max | 0.8708 | 1.8428 | 0.7984 | 0.7301 | 1.0633 |
| | MSE | 0.2329 | 0.2463 | 0.1101 | 0.0979 | 0.0937 |
| | MAE | 0.4217 | 0.3225 | 0.2525 | 0.2483 | 0.2060 |
| Portfolio 3 | mean | −0.0782 | 0.0465 | 0.0366 | 0.0724 | 0.0587 |
| | std | 0.5897 | 0.4558 | 0.4178 | 0.3807 | 0.2649 |
| | min | −1.0777 | −0.8747 | −0.7962 | −0.5758 | −0.4052 |
| | max | 0.9476 | 0.7638 | 0.6613 | 0.7344 | 0.8855 |
| | MSE | 0.3388 | 0.2009 | 0.1683 | 0.1439 | 0.0706 |
| | MAE | 0.4730 | 0.3464 | 0.3217 | 0.2894 | 0.1597 |

The Fama–MacBeth beta is computed from monthly returns over the previous five years. The quarterly realized beta is computed from daily returns over the previous quarter. The AR(1) quarterly realized beta is estimated on five years of quarterly realized betas computed from daily returns. The annual realized beta is computed from daily returns over the previous year. Perfect foresight is when stock betas are forecast without error. The forecast evaluation period is from January 2, 2003, to December 31, 2008.

returns, using Eq. (3). These results for our portfolios are displayed for beginning-of-quarter stock weights in Table 3 and for end-of-quarter stock weights in Table 4. The forecast evaluation period when using this high frequency dataset is from January 2, 2003, to December 31, 2008. The patterns observed in Tables 1 and 2 are also present in Tables 3 and 4. Substantial deviations from beta neutrality are observed, with MAEs in excess of 0.22 for all forecasting methods. Due to the greater precision in estimating the ex post beta exposure for 60 min returns, it is found that the Fama–MacBeth MAE is over 0.4 for some portfolios. Across our three portfolios, for beginning- and end-of-quarter weights, the annual realized beta forecasting method delivers the lowest STD, MSE and MAE values.

We also examine ex post betas over two volatility states (high and low). Tables 5 and 6 display statistics on ex post portfolio betas with beginning-of-quarter stock weights for the high and low volatility states, respectively. In the high volatility state (quarters in which the CBOE Volatility Index is above its median), we find the ex post betas to vary more from the zero target. For example, the annual realized beta forecast in the high volatility state has MSEs for our portfolios of 0.0299, 0.0403 and 0.0332, whereas in the low volatility state the MSEs are 0.0172, 0.0124 and 0.0231. These results imply that the successful construction of a

**Table 5**
Ex post beta exposure under high volatility using daily returns and beginning-of-quarter weights.

| | | Fama–MacBeth beta | Quarterly realized beta | AR(1) quarterly realized beta | Annual realized beta | Perfect foresight |
|---|---|---|---|---|---|---|
| Portfolio 1 | mean | 0.0273 | 0.0639 | 0.0325 | 0.0855 | 0.0000 |
| | std | 0.3670 | 0.3186 | 0.2577 | 0.1528 | 0.0000 |
| | min | −0.8532 | −0.5352 | −0.5087 | −0.2341 | 0.0000 |
| | max | 0.8844 | 1.1213 | 0.5953 | 0.3752 | 0.0000 |
| | MSE | 0.1308 | 0.1021 | 0.0652 | 0.0299 | 0.0000 |
| | MAE | 0.2669 | 0.2367 | 0.1945 | 0.1373 | 0.0000 |
| Portfolio 2 | mean | 0.0758 | 0.0878 | 0.0342 | 0.0896 | 0.0000 |
| | std | 0.4076 | 0.2985 | 0.2445 | 0.1827 | 0.0000 |
| | min | −0.7272 | −0.4880 | −0.5582 | −0.2461 | 0.0000 |
| | max | 0.9940 | 0.9886 | 0.5701 | 0.5250 | 0.0000 |
| | MSE | 0.1662 | 0.0937 | 0.0589 | 0.0403 | 0.0000 |
| | MAE | 0.2895 | 0.2289 | 0.1818 | 0.1552 | 0.0000 |
| Portfolio 3 | mean | 0.0542 | 0.0811 | 0.0307 | 0.0908 | 0.0000 |
| | std | 0.3655 | 0.2437 | 0.2747 | 0.1611 | 0.0000 |
| | min | −0.7545 | −0.3076 | −0.4418 | −0.1701 | 0.0000 |
| | max | 0.8312 | 0.6922 | 0.5917 | 0.5024 | 0.0000 |
| | MSE | 0.1314 | 0.0637 | 0.0735 | 0.0332 | 0.0000 |
| | MAE | 0.2931 | 0.1892 | 0.2201 | 0.1384 | 0.0000 |

The Fama–MacBeth beta is computed from monthly returns over the previous five years. The quarterly realized beta is computed from daily returns over the previous quarter. The AR(1) quarterly realized beta is estimated on five years of quarterly realized betas computed from daily returns. The annual realized beta is computed from daily returns over the previous year. Perfect foresight is when stock betas are forecast without error. The forecast evaluation period is from January 2, 1993, to December 31, 2008.

**Table 6**
Ex post beta exposure under low volatility using daily returns and beginning-of-quarter weights.

| | | Fama–MacBeth beta | Quarterly realized beta | AR(1) quarterly realized beta | Annual realized beta | Perfect foresight |
|---|---|---|---|---|---|---|
| Portfolio 1 | mean | 0.0707 | −0.0824 | 0.0883 | 0.0400 | 0.0000 |
| | std | 0.3610 | 0.3190 | 0.1685 | 0.1266 | 0.0000 |
| | min | −0.6375 | −0.8629 | −0.3529 | −0.2084 | 0.0000 |
| | max | 0.9512 | 0.7560 | 0.4290 | 0.4663 | 0.0000 |
| | MSE | 0.1316 | 0.0991 | 0.0354 | 0.0172 | 0.0000 |
| | MAE | 0.2977 | 0.2506 | 0.1511 | 0.1002 | 0.0000 |
| Portfolio 2 | mean | 0.0734 | −0.0796 | 0.0963 | 0.0450 | 0.0000 |
| | std | 0.3366 | 0.2596 | 0.1914 | 0.1034 | 0.0000 |
| | min | −0.6296 | −0.4390 | −0.5060 | −0.1667 | 0.0000 |
| | max | 0.6322 | 0.5356 | 0.5207 | 0.2029 | 0.0000 |
| | MSE | 0.1153 | 0.0717 | 0.0448 | 0.0124 | 0.0000 |
| | MAE | 0.2859 | 0.2303 | 0.1703 | 0.0902 | 0.0000 |
| Portfolio 3 | mean | 0.0887 | −0.0659 | 0.0973 | 0.0625 | 0.0000 |
| | std | 0.3391 | 0.2604 | 0.1825 | 0.1405 | 0.0000 |
| | min | −0.8138 | −0.8013 | −0.2394 | −0.2682 | 0.0000 |
| | max | 0.5906 | 0.5477 | 0.5231 | 0.3809 | 0.0000 |
| | MSE | 0.1198 | 0.0703 | 0.0419 | 0.0231 | 0.0000 |
| | MAE | 0.2871 | 0.1936 | 0.1614 | 0.1202 | 0.0000 |

The Fama–MacBeth beta is computed from monthly returns over the previous five years. The quarterly realized beta is computed from daily returns over the previous quarter. The AR(1) quarterly realized beta is estimated on five years of quarterly realized beta computed from daily returns. The annual realized beta is computed from daily returns over the previous year. Perfect foresight is when stock betas are forecast without error. The forecast evaluation period is from January 2, 1993, to December 31, 2008.

beta neutral portfolio is easier to achieve during periods in which the overall stock market volatility is lower.

In a further examination of the properties of our portfolio ex post betas, we assess whether these betas are correlated with well-known risk premiums in the stock market, and in particular the size, value and momentum premiums. Ex post betas are regressed on a constant and the four factors of Carhart's (1997) model. The results are displayed in Table 7 for our three portfolios with beginning-of-quarter weights. For all four beta forecasting approaches, the size and value premiums are not significant in explaining our portfolio ex post betas, at conventional significant levels. In addition, the momentum premium is found to be significant only for the AR(1) quarterly realized beta with portfolio 2. We therefore conclude that, overall, these other risk premiums are not correlated with our portfolio ex post betas.

As a final robustness check, we also assess the variability of our statistics on the ex post betas. The bootstrap procedure described in Section 3.3 is applied over the three subsamples, corresponding to the sample periods for the three momentum portfolios. Table 8 reports the 90th, 50th and 10th percentiles of the bootstrap distribution of our statistics for subsample 1 (corresponding to the same sample period as momentum portfolio 1). An examination of the differences between the 90th and

**Table 7**
Factors and ex post beta exposure using daily returns and beginning-of-quarter weights.

| | | Fama–MacBeth beta | Quarterly realized beta | AR(1) quarterly realized beta | Annual realized beta |
|---|---|---|---|---|---|
| Portfolio 1 | Constant | 0.0410 | −0.0173 | 0.0820 | 0.0574 |
| | | (0.534) | (0.725) | (0.025) | (0.026) |
| | Market | 0.0025 | −0.0161 | −0.0017 | −0.0013 |
| | | (0.685) | (0.038) | (0.653) | (0.560) |
| | Size | −0.0026 | 0.0098 | −0.0066 | −0.0002 |
| | | (0.768) | (0.202) | (0.275) | (0.960) |
| | Value | 0.0085 | −0.0013 | −0.0045 | −0.0003 |
| | | (0.344) | (0.855) | (0.441) | (0.920) |
| | Momentum | −0.0006 | 0.0056 | −0.0032 | 0.0019 |
| | | (0.875) | (0.262) | (0.414) | (0.357) |
| Portfolio 2 | Constant | 0.0714 | −0.0150 | 0.0815 | 0.0607 |
| | | (0.349) | (0.649) | (0.048) | (0.039) |
| | Market | 0.0046 | −0.0182 | 0.0037 | −0.0024 |
| | | (0.603) | (0.002) | (0.219) | (0.534) |
| | Size | 0.0021 | 0.0088 | −0.0055 | 0.0016 |
| | | (0.781) | (0.080) | (0.171) | (0.587) |
| | Value | 0.0050 | 0.0000 | −0.0043 | 0.0007 |
| | | (0.608) | (0.999) | (0.395) | (0.800) |
| | Momentum | −0.0033 | 0.0108 | −0.0040 | 0.0023 |
| | | (0.572) | (0.004) | (0.362) | (0.274) |
| Portfolio 3 | Constant | 0.0407 | −0.0169 | 0.0620 | 0.0715 |
| | | (0.519) | (0.715) | (0.125) | (0.020) |
| | Market | 0.0086 | −0.0093 | 0.0062 | −0.0001 |
| | | (0.152) | (0.116) | (0.212) | (0.974) |
| | Size | 0.0015 | 0.0073 | −0.0028 | 0.0014 |
| | | (0.797) | (0.067) | (0.416) | (0.516) |
| | Value | 0.0101 | 0.0058 | 0.0025 | 0.0024 |
| | | (0.256) | (0.373) | (0.714) | (0.513) |
| | Momentum | 0.0048 | 0.0039 | 0.0002 | −0.0002 |
| | | (0.319) | (0.392) | (0.962) | (0.943) |

The Fama–MacBeth beta is computed from monthly returns over the previous five years. The quarterly realized beta is computed from daily returns over the previous quarter. The AR(1) quarterly realized beta is estimated on five years of quarterly realized betas computed from daily returns. The annual realized beta is computed from daily returns over the previous year. The four factor regressions are over the period from January 2, 1993, to December 31, 2008. Newey–West $p$-values are in brackets below the coefficient estimate.

**Table 8**
Bootstrap percentiles of ex post beta exposure using daily returns and beginning-of-quarter weights (subsample 1).

| | | Fama–MacBeth beta | Quarterly realized beta | AR(1) quarterly realized beta | Annual realized beta | Perfect foresight |
|---|---|---|---|---|---|---|
| 10th percentile | mean | −0.0320 | −0.0231 | −0.0164 | −0.0121 | 0.0000 |
| | std | 0.1800 | 0.1252 | 0.0871 | 0.0650 | 0.0000 |
| | min | −0.6453 | −0.5242 | −0.4380 | −0.2599 | 0.0000 |
| | max | 0.3762 | 0.2660 | 0.1804 | 0.1352 | 0.0000 |
| | MSE | 0.0319 | 0.0154 | 0.0075 | 0.0042 | 0.0000 |
| | MAE | 0.1400 | 0.0965 | 0.0675 | 0.0505 | 0.0000 |
| 50th percentile | mean | 0.0012 | 0.0006 | 0.0002 | −0.0001 | 0.0000 |
| | std | 0.2039 | 0.1440 | 0.1018 | 0.0743 | 0.0000 |
| | min | −0.4900 | −0.3599 | −0.2543 | −0.1833 | 0.0000 |
| | max | 0.4951 | 0.3617 | 0.2553 | 0.1821 | 0.0000 |
| | MSE | 0.0409 | 0.0204 | 0.0102 | 0.0054 | 0.0000 |
| | MAE | 0.1592 | 0.1102 | 0.0775 | 0.0574 | 0.0000 |
| 90th percentile | mean | 0.0343 | 0.0232 | 0.0166 | 0.0121 | 0.0000 |
| | std | 0.2299 | 0.1643 | 0.1201 | 0.0844 | 0.0000 |
| | min | −0.3747 | −0.2638 | −0.1824 | −0.1362 | 0.0000 |
| | max | 0.6444 | 0.5227 | 0.4347 | 0.2589 | 0.0000 |
| | MSE | 0.0520 | 0.0266 | 0.0142 | 0.0070 | 0.0000 |
| | MAE | 0.1796 | 0.1250 | 0.0883 | 0.0651 | 0.0000 |

The Fama–MacBeth beta is computed from monthly returns over the previous five years. The quarterly realized beta is computed from daily returns over the previous quarter. The AR(1) quarterly realized beta is estimated on five years of quarterly realized betas computed from daily returns. The annual realized beta is computed from daily returns over the previous year. Perfect foresight is when stock betas are forecast without error. The forecast evaluation period is from January 2, 1993, to December 31, 2008.

10th percentiles indicates that none of the statistics show any substantial variability. For example, the 90th, 50th and 10th percentiles of the MAE for the annual realized beta are 0.0651, 0.0574 and 0.0505, respectively, with the difference between the 90th and 10th percentiles being just 0.0146. The 90th, 50th and 10th percentiles of the MAE for the Fama–MacBeth beta are 0.1796, 0.1592 and 0.1400, respectively, with the difference between the 90th and 10th percentiles being 0.0396. The relatively small differences between the 90th and 10th percentiles increases our confidence in the reliability of our results. Patterns similar to that with the momentum portfolios again occur in the performances of the different beta forecasting methods. The annual realized beta is the most accurate in targeting a beta of zero, while the Fama–MacBeth beta is the least accurate. The results for subsamples 2 and 3 are very similar to those for subsample 1, and are available upon request.

## 6. Conclusion

This paper has demonstrated that ex post betas can vary from zero substantially for portfolios constructed with the intention of being beta neutral using standard beta forecasting approaches. Portfolios were constructed both based on momentum and by selecting stocks randomly, in order to increase the general applicability of the results. This leads us to conclude that the inability of equity market neutral funds to exhibit market neutrality in their performance may be attributable largely to the fact that the methods employed in the construction of equity market neutral funds are often unreliable. Beta neutral portfolios may be more achievable in certain specific cases, such as with highly liquid assets and reliable intra-day return measurement, but this is left for future research. In the more general case where daily return measurements are the highest return frequency that is available reliably, we find that the annual realized beta (computed from daily returns over the past year) provides a greater accuracy in the construction of market neutral portfolios at the quarterly frequency than other methods.

An important finding in this paper is that it is particularly difficult to achieve beta neutrality in equity portfolios during periods of high volatility in financial markets. These high volatility periods are often when beta neutrality is most desirable, due to the risk of substantial drawdowns in equities over these periods. In the portfolios that were constructed to target beta neutrality with respect to the overall broad market, it was also found that the ex post portfolio betas were generally not correlated with other well-known risk premiums, such as size, value and momentum. However, we did not study the specific construction of portfolios that target neutrality to risk premiums other than the broad market, but left this for future research.

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