IIMB Management Review (2016) 28, 31–42

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journal homepage: www.elsevier.com/locate/iimb

Sudden breaks in drift-independent volatility estimator based on multiple periods open, high, low, and close prices

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Received 17 September 2013; revised 7 April 2014; accepted 2 February 2016; available online 23 February 2016

KEYWORDS
IT-ICSS algorithm;
Sudden changes in volatility;
Monte Carlo simulation;
Yang and Zhang estimator

Abstract This paper investigates the superiority of the Yang and Zhang (YZ) estimator over the demeaned squared returns in detecting sudden breaks based on Inclan and Tiao (IT-ICSS) algorithm using Monte Carlo simulation experiments. Our findings indicate that the IT-ICSS algorithm exhibits desirable size and power properties when applied with the YZ estimator in comparison to its use with the demeaned squared returns. Empirically, we validate the superiority of the YZ estimator by relating the detected breaks with the major macroeconomic events using various US dollar exchange rates. We find that the demeaned squared returns detect many spurious breaks.

Introduction

This paper compares the performance of the Yang and Zhang (2000) (YZ) estimator and demeaned squared returns when applied with Inclan and Tiao's (1994) iterated cumulative sum of squares (IT-ICSS) algorithm to detect the sudden changes in the volatility of a random process. Analysis of market risk plays a crucial role in the financial markets literature. Volatility is known to be a popular measure in evaluating financial risks, leverage effects and to examine the impact of asymmetric shocks on markets. Volatility plays an important role in financial markets due to its application in designing investment decisions and in portfolio rebalancing and management (Aizenman & Marion, 1999), in pricing derivatives securities (Hull & White, 1987), in quantifying risk (based on value at risk and expected shortfall) (Granger, 2002) and in implementing trading strategies (Poon & Granger, 2003). It is well known in the literature that the unconditional volatility of tradable securities and portfolios may be significantly affected by infrequent structural breaks or regime shifts, which may arise due to various domestic or global macroeconomic and political events (see Aggarwal, Inclan, & Leal, 1999; Kumar & Maheswaran, 2012) including terrorist attacks, wars, sudden hike in interest rates, changes in investors' perception, crashes and crises in a market, or recession in an economy. Hence, it is important to consider the impact of sudden changes in volatility in the model for generating more accurate forecasts of volatility. This can be helpful for fund managers and investors to design investment strategies, to rebalance their portfolios and to hedge their positions based on an anticipation of future movements of the market. Regulators, policy makers and central banks also have an interest in volatility analysis to implement policy

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Peer-review under responsibility of Indian Institute of Management Bangalore.

http://dx.doi.org/10.1016/j.iimb.2016.02.001
0970-3896

measures effectively based on changes taking place in markets. This would enable them to maintain stability in financial markets and to assess the effectiveness of these policies depending on the required goals (Poon & Granger, 2003).

There exist different methods to estimate daily unconditional volatility. The most popular method involves the use of square of close to close returns. The returns based volatility measures are well established in literature and also act as inputs to generalised autoregressive conditional heteroskedasticity (GARCH) class of models. However, the squared daily return is a noisy estimate of volatility and informational inefficiency (Alizadeh, Brandt, & Diebold, 2002). Another method involves the use of high frequency intraday data. This measure of volatility is also known as realised volatility which involves summing the squares of returns sampled at shorter intervals (for example, 5 minutes or 10 minutes) for a given day. However, the high frequency data exhibit non negligible microstructure issues, which may prevent the researchers in analysing its informational contents. On the other hand, high frequency data for many assets may not be available or may be available for a shorter duration. In addition, high frequency data are generally expensive and require substantial computational resources.

The literature that started with Parkinson (1980) and Garman and Klass (1980), and extended by Rogers and Satchell (1991) and Yang and Zhang (2000), has highlighted the importance of using opening, high, low and closing prices of an asset for the efficient estimation of volatility. Alizadeh et al. (2002) highlighted that range based volatility estimates are highly efficient and are robust in terms of the non negligible market microstructure issues. Among all these range based volatility estimators, the YZ estimator proposed by Yang and Zhang (2000) is based on multi-period open, high, low, and close prices, is unbiased in the continuous limit, independent of any non-zero drift, and incorporates the impact of opening price jumps. However, the RS estimator proposed by Rogers and Satchell (1991) is also unbiased regardless of non-zero drift. The YZ estimator also makes use the of RS estimator for volatility estimation (see equation (5) in the section on *Methodology*). The other range based volatility estimators are biased in some way if the mean return (drift) is non-zero. The open, high, low, and close prices are also available for most of the traded assets, indices and commodities, and contain more information for efficient estimation of volatility.

Literature provides evidence that the volatility models, which incorporate the impact of sudden changes in unconditional volatility provide better volatility forecasts (Kumar & Maheswaran, 2012). The IT-ICSS test assumes that the zero mean returns are independent over time and normally distributed. The IT-ICSS test detects both a significant increase and decrease in the unconditional volatility and, hence, can help in identifying both the beginning and the ending of volatility regimes. The IT-ICSS test has been extensively used in detecting sudden changes in the unconditional volatility of time series based on close-to-close returns (Aggarwal et al., 1999; Fernandez & Arago, 2003; Hammoudeh & Li, 2008; Malik, 2003; Malik, Ewing, & Payne, 2005). However, Aggarwal et al. (1999), Hammoudeh and Li (2008), Malik (2003), Malik et al. (2005), and Wang and Moore (2009) use demeaned squared returns with IT-ICSS algorithm to detect sudden changes in the unconditional variance.

In this study, we compare the size and power properties of IT-ICSS test with respect to both the volatility proxies (the YZ estimator and the demeaned squared returns) for various data generating processes like the independently and identically distributed (i.i.d.) random numbers from the Gaussian, Student's t, double exponential, gamma-mixture and generalised error distributions, the GARCH model, the stochastic volatility (SV) model and the fractionally integrated GARCH (FIGARCH) model. For the GARCH, the SV and the FIGARCH models, the innovations have been taken from the normal, the Student's t and the generalised error distribution (GED) distributions. The power properties are studied by incorporating sudden breaks at 25th percentile, 50th percentile and 75th percentile of the data series from i.i.d. normal, the GARCH and the SV data generating processes. The findings of this study indicate that YZ estimator exhibits more desirable size and power characteristics when applied with IT-ICSS algorithm than the demeaned squared returns. Hence, this study proposes the use of the YZ estimator with IT-ICSS test to detect sudden changes in volatility. On the application side, this study detects sudden breaks in the YZ estimator and the demeaned squared returns of three major exchange rates (USD/Euro, USD/Japanese Yen and USD/GBP).

The remainder of this paper is organised as follows: Section 2 introduces the IT-ICSS algorithm and the procedure for implementing the YZ estimator based extension of the IT-ICSS algorithm. Section 3 presents the results of the Monte Carlo simulation experiments to assess the performance of the IT-ICSS algorithm based on the YZ estimator and demeaned squared returns. Section 4 describes the application of the YZ estimator in detecting sudden breaks in USD/Euro, USD/Japanese Yen and USD/GBP exchange rates and section 5 concludes with a summary of the main findings.

Methodology

Inclan and Tiao's (1994) (IT) ICSS algorithm

The IT-ICSS algorithm is helpful in detecting multiple sudden changes in the volatility of time series. The IT-ICSS algorithm assumes stationary unconditional variance in a time series for a particular regime. This algorithm is simple to implement and is not affected by the long memory characteristics of the volatility.

Suppose $\varepsilon_t \sim$ i.i.d. $(0, \sigma^2)$, where i.i.d. means independent and identically distributed. Suppose there are T_N change points in the volatility series with change points given as $1 < k_1 < k_2 < \ldots < k_{TN} < N$, where N is the number of observations in the time series. Suppose the variance within each regime is given by τ_j^2, where $j = 0, 1, \ldots\ldots, T_N$. Then,

$$\sigma_t^2 = \tau_0^2 \quad for \quad 1 < t < \kappa_1 \tag{1a}$$

$$\sigma_t^2 = \tau_1^2 \quad for \quad \kappa_1 < t < \kappa_2 \tag{1b}$$

$$\sigma_t^2 = \tau_{TN}^2 \quad for \quad \kappa_{TN} < t < N \tag{1c}$$

Inclan and Tiao (1994) applied the cumulative sum of squares approach to detect the number of sudden changes in the variance. The cumulative sum of the squared observations from the start of the series to the k^{th} point in time is given as:

$$C_k = \sum_{t=1}^{k} \varepsilon_t^2$$

where $k = 1,\ldots, N$. The D_k statistic is given as:

$$D_k = \left(\frac{C_k}{C_N}\right) - \frac{k}{N}, \quad k = 1, \ldots, N \quad with \quad D_0 = D_T = 0 \tag{2}$$

where C_N is the sum of squared observations from the whole sample period.

The D_k statistic oscillates around zero if there are no sudden changes in the volatility. In this case, if D_k is plotted against k, it looks like a horizontal line. On the other hand, if there are sudden changes in the volatility, then the D_k statistic values drift either above or below zero. The null hypothesis of constant variance is rejected if the maximum absolute value of D_k is greater than the critical value. Hence, if $\max_k\sqrt{(N/2)}|D_k|$ is more than the predetermined boundary, then k* is taken as an estimate of the variance change point. The 95th percentile critical value for the asymptotic distribution of $\max_k\sqrt{(N/2)}|D_k|$ is 1.358 (Inclan & Tiao, 1994 and Aggarwal et al., 1999), and hence the upper and the lower boundaries can be set at ±1.358 in the D_k plot. If the value of the statistic falls outside these boundaries then a sudden change in variance is identified. Another branch in this literature makes use of regression based approach to detect the structural breaks in the time series (such as Bai & Perron, 1998; Kim & Nelson, 1999; Andreou & Ghysels, 2002; Banerjee & Urga, 2005).

Extreme value estimator of variance (Yang and Zhang (2000) (YZ) estimator)

Suppose O_t, H_t, L_t and C_t are the opening, high, low, and closing prices of an asset on day t. We define:

$$b_t = \log\left(\frac{H_t}{O_t}\right) \tag{3.1}$$

$$c_t = \log\left(\frac{L_t}{O_t}\right) \tag{3.2}$$

$$x_t = \log\left(\frac{C_t}{O_t}\right) \tag{3.3}$$

$$o_t = \log\left(\frac{O_t}{C_{t-1}}\right) \tag{3.4}$$

Suppose $varx$ denotes the usual estimator of σ^2, i.e.

$$varx = \frac{1}{N-1}\sum_{n=1}^{N}(x_n - \hat{\mu})^2$$

where

$$\hat{\mu} = \frac{1}{N}\sum_{n=1}^{N} x_n$$

Let $u_t = 2b_t - x_t$ and $v_t = 2c_t - x_t$, define the extreme value estimator $varux$ and $varvx$:

$$varux = \frac{1}{N}\sum_{n=1}^{N}\left(\frac{u_n^2 - x_n^2}{2}\right)$$

$$varvx = \frac{1}{N}\sum_{n=1}^{N}\left(\frac{v_n^2 - x_n^2}{2}\right)$$

The Rogers and Satchell (1991) estimator as studied by Maheswaran, Balasubramanian, and Yoonus (2011) is given by:

$$V_{RS} = \frac{varux + varvx}{2} \tag{4}$$

Yang and Zhang (2000) propose a volatility estimator (the YZ estimator) based on multiple period open, high, low, and close prices that are unbiased in the continuous limit, independent of the drift and account for opening price jumps. They find that the YZ estimator has the minimum variance among all estimators that have the same properties. The YZ estimator is given by:

$$V_{YZ} = V_o + kV_c + (1-k)V_{RS} \tag{5}$$

where V_{RS} is the RS estimator as given by equation (4). V_o and V_c are given as:

$$V_o = \frac{1}{N-1}\sum_{n=1}^{N}(o_n - \bar{o})^2$$

$$V_c = \frac{1}{N-1}\sum_{n=1}^{N}(c_n - \bar{c})^2$$

where $\bar{o} = (1/N)\sum_{n=1}^{N} o_n$ and $\bar{c} = (1/N)\sum_{n=1}^{N} c_n$. The constant k is chosen in such a way to minimise V_{YZ}. The value of k is given by:

$$k = \frac{0.34}{1.34 + \frac{N+1}{N-1}} \tag{6}$$

This paper suggests the use of V_{YZ} in place of ε_t^2 to detect structural breaks in the variance of the time series.

Monte Carlo simulation

This section presents the extensive Monte Carlo simulation experiments to examine the performance of the IT-ICSS (1994) test in detecting sudden changes in volatility proxies, for example, the YZ estimator and demeaned squared returns, using different data generating processes (DGP) which includes both unconditional and conditional data series. The YZ estimator represents the variance estimate of multiple period open, high, low, and close prices. As highlighted by Yang and Zhang (2000), the variance of the YZ estimator is the smallest among all the extreme value volatility estimators but possesses similar properties. In addition, the YZ estimator is unbiased regardless of the drift parameter and

Table 1 Size of the IT-ICSS test with no conditional dependence.

	YZ estimator				ε_t^2			
	N = 104	N = 208	N = 416	N = 832	N = 104	N = 208	N = 416	N = 832
Normal	0.000	0.001	0.001	0.001	0.029	0.035	0.029	0.042
Student's t	0.063	0.095	0.122	0.138	0.270	0.317	0.365	0.454
Doub_Exp	0.045	0.061	0.048	0.083	0.285	0.311	0.357	0.397
Gamma	0.009	0.010	0.003	0.003	0.126	0.151	0.166	0.152
GED	0.000	0.001	0.001	0.001	0.087	0.111	0.103	0.132

Doub_Exp and GED represent double exponential distribution and generalised error distribution.

opening jumps whereas all others are biased in one way or another if the mean return (drift) is non-zero. While the RS estimator is also unbiased, it does not account for opening jumps in the data (Kumar & Maheswaran, 2013; Yang & Zhang, 2000). According to Yang and Zhang (2000), the YZ estimator assumes that the intraday price movements follow a geometric Brownian motion.

This paper explains the Monte Carlo simulation analysis in two parts. The first part undertakes the simulation experiments to examine the size properties of the IT-ICSS algorithm for the YZ estimator and the demeaned squared returns (that is, zero mean returns). The second part of the simulation deals with the estimation of power properties of the IT-ICSS algorithm for both variance proxies. Samples of varying sizes have been taken; T = 104, T = 208, T = 416 and T = 832; assuming weekly data for 2 years, 4 years, 8 years and 16 years, respectively. The number of Monte Carlo trials is set to 10000 and the significance level[1] is set at 5%.

To examine the size of the IT-ICSS test, this study considers both unconditional and conditional data generating processes, which do not incorporate sudden change in variance. First, this study considers artificial data series, which do not have any conditional dependence and this includes the sequence of identical and independently distributed zero mean random numbers. Following Kumar and Maheswaran (2013), the unconditional data series, which have been taken for analysis include the standard normal distribution N(0,1), the Student's t distribution with 5 degrees of freedom (mixture of the normal and the chi-squared distribution), the double exponential distribution (mixture of the normal and the exponential distribution), the gamma-mixture distribution (mixture of the normal and the chi-squared distribution), and the generalised error distribution (with 1.3 degrees of freedom). To simulate the YZ estimator using unconditional data series, it is required to first simulate the RS estimator separately. This involves generating (x_i, b_i, c_i) using Gaussian random walk with initial (open) price being zero. It is also necessary to account for opening jumps in the simulation. Since, for each simulation, the initial value is taken as zero for a Gaussian random walk, the opening jumps will involve variance of natural logarithm of x_{i-1}. The value of k has been calculated using equation (6) for varying sample size as suggested by Yang and Zhang (2000). For standard normal distribution, (x_i, b_i, c_i) has been used directly to estimate the YZ estimator. Following Kumar and Maheswaran (2013), this study also makes use of mixture of distributions to generate random series for the Student's t distribution and the Gamma distribution. Suppose $Y \sim \frac{\chi^2(\nu)}{\nu}$ represents a random series generated by using the chi-squared distribution with ν degrees of freedom divided by ν. The (x_i^t, b_i^t, c_i^t) can be generated from a random walk of n steps, which is an inverse Gamma mixture of the Gaussian distribution, and this gives rise to the Student's t distribution with ν degrees of freedom, which is given as follows:

$$x_i^t = \frac{x_i}{\sqrt{Y}}, \quad b_i^t = \frac{b_i}{\sqrt{Y}}, \quad c_i^t = \frac{c_i}{\sqrt{Y}} \tag{7}$$

Similarly, the (x_i^g, b_i^g, c_i^g) can be generated for Gamma distribution from a random walk of n steps, which is a direct Gamma mixture of the Gaussian distribution and is given by:

$$x_i^g = x_i \cdot \sqrt{Y}, \quad b_i^g = b_i \cdot \sqrt{Y}, \quad c_i^g = c_i \cdot \sqrt{Y} \tag{8}$$

For the case of double exponential random walk, we first generate Z ~ Exp(1). The random walk of n steps, which is subjected to double exponential distribution, (x_i^e, b_i^e, c_i^e), can be generated from (x_i, b_i, c_i) by using the following transformation.

$$x_i^e = x_i \cdot \sqrt{Z}, \quad b_i^e = b_i \cdot \sqrt{Z}, \quad c_i^e = c_i \cdot \sqrt{Z} \tag{9}$$

In addition, in the case of the GED distribution, we generate i.i.d. random walk for the GED distribution (1.3 degree of freedom) to estimate YZ estimator.

Table 1 presents the size of the IT-ICSS test for the YZ estimator and the demeaned squared returns (ε_t^2) at 95% level of confidence. Results indicate that the IT-ICSS algorithm provides appropriate rejection ratios (size) for the YZ estimator for samples under consideration. The results for the YZ estimator are slightly oversized for a few cases; however, these over rejections are still very near to 5% (for example, for Student's t distribution, the results are slightly greater than 5% and go up to 9.5% for a sample size of 208). On the other hand, the size of the IT-ICSS test for the ε_t^2 is appropriate only for the Gaussian distribution. For all other distributions, the results are severely oversized. Overall, the results indicate that the IT-ICSS algorithm works well with the YZ estimator for unconditional data generating processes; however, it performs worse for the ε_t^2 for various data generating processes.

[1] We also perform similar analysis at 10% and 1% levels of significance. The results at 10% and 1% levels of significance provide similar inference as we obtain at 5% level of significance. Results for 10% and 1% levels of significance will be made available upon request.

To test size properties of the IT-ICSS test for data series having conditional dependence, this study makes use of the GARCH (1,1), the stochastic volatility (SV) and the FIGARCH(1,d,1) models with innovations from the Gaussian distribution, the Student's t distribution and the GED. The following models are considered to evaluate the size properties of the IT-ICSS test for ε_t^2:

Model 1: GARCH(1,1)

$$\varepsilon_t = \sqrt{h_t}u_t; \quad h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} \tag{10}$$

Model 2: Stochastic volatility

$$\varepsilon_t = \exp(0.5h_t)u_t; \quad h_t = \delta h_{t-1} + \epsilon_t; \quad \epsilon_t \sim N(0, 0.1) \tag{11}$$

Model 3: FIGARCH(1,d,1)

$$\varepsilon_t = \sqrt{h_t}u_t; \quad \phi(L)(1-L)^d \varepsilon_t^2 = \omega + [1-\beta(L)]v_t, \tag{12}$$

where $v_t = \varepsilon_t^2 - h_t$ and $\phi(L) = [1-\alpha(L)-\beta(L)]$.

It is to be noted that for all the three models, this study examines of size of the IT-ICSS test for three cases of u_t. The u_t ~ iid N(0,1) for innovations from the standard normal distribution, u_t ~ iid t(ν) for innovations from the standardised Student's t distribution with $\nu = 5$ degrees of freedom (to account for finite fourth moment in the data series) and u_t ~ iid GED(η) for innovations from the standardised GED distribution with $\eta = 1.3$ degrees of freedom[2] (to account for thicker tail of the distribution).

On the other hand, to evaluate the size properties of the YZ estimator for conditional data series, this study generates random walk from respective distributions, that is, the normal distribution, the Student's t distribution and the GED distribution. This can help us to take out (x_t, b_t, c_t).

*Model 1**: GARCH(1,1)

$$x_t^* = \sqrt{h_t}x_t; \quad h_t = \omega + \alpha\left(x_t^*\right)^2 + \beta h_{t-1} \tag{13}$$

To preserve the joint distribution of maximum and minimum of the random walk, b_t^* and c_t^* are computed as follows:

$$b_t^* = \sqrt{h_t}b_t$$

$$c_t^* = \sqrt{h_t}c_t$$

*Model 2**: Stochastic volatility

$$x_t^* = \exp(0.5h_t)x_t; \quad h_t = \delta h_{t-1} + \epsilon_t; \quad \epsilon_t \sim N(0, 0.1) \tag{14}$$

Here, to preserve the joint distribution of maximum and minimum of the random walk, b_t^* and c_t^* are computed as follows:

$$b_t^* = e^{h_t/2}b_t$$

$$c_t^* = e^{h_t/2}c_t$$

*Model 3**: FIGARCH(1,d,1)

$$x_t^* = \sqrt{h_t}x_t; \quad \phi(L)(1-L)^d(x_t^*)^2 = \omega + [1-\beta(L)]v_t, \tag{15}$$

Here, $v_t = (x_t^*)^2 - h_t$. In order to preserve the joint distribution of maximum and minimum of the random walk, b_t^* and c_t^* are computed as follows:

$$b_t^* = \sqrt{h_t}b_t$$

$$c_t^* = \sqrt{h_t}c_t$$

Table 2 reports the size of the IT-ICSS test for the YZ estimator and the ε_t^2 for different sample at 5% level of significance for the GARCH(1,1) conditional data generating process with the normal distribution, the Student's t distribution and the GED distribution.

It can be seen that when the underlying distribution is normal, the results indicate desirable size properties of the IT-ICSS test for both volatility proxies. However, when the underlying distribution is Student's t, the ε_t^2 is severely oversized for all possible values of β. On the other hand, the YZ estimator exhibits desirable size properties up to $\beta = 0.4$ for all samples under consideration. For $\beta = 0.5$, the results are oversized for sample of size 832. For $\beta \geq 0.6$, the rejection frequency is greater than conventional level of significance (5%) for all samples. However, the size of the IT-ICSS test is still near the conventional level of significance for the YZ estimator than the ε_t^2. Overall for Student's t distribution, the YZ estimator provides a more desirable size than ε_t^2 for all values of β. For the GED distribution, the YZ estimator exhibits desirable size properties for all cases. On the flipside, the ε_t^2 exhibit desirable size up to $\beta = 0.6$ (except for sample size of 832, the size value is 0.08 for $\beta = 0.6$). For $\beta \geq 0.6$, the results are substantially oversized. Overall, for the GARCH(1,1) model, the rejection frequency related to the IT-ICSS test is much better for the YZ estimator for different underlying distributions than the rejection frequencies obtained using ε_t^2.

Table 3 presents the size of the IT-ICSS test for the stochastic volatility model with the innovations from the Gaussian, the Student's t and the GED distributions for both the variance estimators for varying sample sizes at 95% level of confidence. It can be seen that the rejection frequencies of the IT-ICSS test for the YZ estimator is properly sized for all cases. However, the results for the IT-ICSS test for ε_t^2 exhibit severely oversized behaviour for all the cases under consideration. Here also, the results indicate that the YZ estimator exhibits desirable size characteristics for the SV model with different underlying distributions.

Table 4 reports the rejection frequencies of the IT-ICSS test for the FIGARCH(1,d,1) model for both the volatility proxies. It can be seen that the results are severely oversized for all the cases for both the volatility estimators. However, if the results of the YZ estimator are compared with the results of the ε_t^2, it can be seen that the YZ estimator exhibits a less oversized behaviour than the ε_t^2. This indicates that the IT-ICSS test does not account for long memory in the volatility. However, for most of the data generating processes considered in this study, the size of the IT-ICSS test

[2] We also perform analysis for different degrees of freedom less than 2 and the results provide a similar inference to what we have obtained here.

Table 2 Size of the IT-ICSS test for the GARCH(1,1) model.

$\omega = 0.1$ and $\alpha = 0.1$								
Β	YZ estimator				ε_t^2			
	N = 104	N = 208	N = 416	N = 832	N = 104	N = 208	N = 416	N = 832
Normal								
0.1	0.004	0.002	0.000	0.001	0.002	0.001	0.001	0.001
0.2	0.001	0.002	0.002	0.000	0.001	0.001	0.001	0.000
0.3	0.005	0.003	0.008	0.002	0.001	0.004	0.000	0.000
0.4	0.003	0.003	0.013	0.005	0.002	0.003	0.003	0.001
0.5	0.007	0.009	0.016	0.012	0.005	0.005	0.001	0.002
0.6	0.014	0.016	0.020	0.027	0.012	0.009	0.013	0.011
0.7	0.026	0.061	0.053	0.059	0.032	0.029	0.038	0.043
Student's t								
0.1	0.007	0.009	0.013	0.019	0.177	0.232	0.296	0.369
0.2	0.015	0.016	0.013	0.019	0.166	0.261	0.328	0.418
0.3	0.020	0.025	0.019	0.027	0.177	0.258	0.348	0.449
0.4	0.030	0.038	0.050	0.058	0.242	0.325	0.384	0.479
0.5	0.040	0.044	0.061	0.092	0.239	0.356	0.454	0.574
0.6	0.087	0.124	0.171	0.214	0.298	0.436	0.550	0.681
0.7	0.183	0.289	0.410	0.499	0.408	0.592	0.731	0.806
GED								
0.1	0.004	0.000	0.000	0.002	0.018	0.014	0.010	0.006
0.2	0.002	0.004	0.004	0.003	0.013	0.017	0.009	0.008
0.3	0.003	0.002	0.001	0.006	0.015	0.014	0.013	0.019
0.4	0.006	0.007	0.005	0.014	0.023	0.027	0.027	0.022
0.5	0.009	0.014	0.006	0.014	0.041	0.039	0.034	0.040
0.6	0.021	0.018	0.035	0.025	0.038	0.063	0.064	0.080
0.7	0.039	0.046	0.051	0.057	0.083	0.119	0.133	0.162

Table 3 Size of the IT-ICSS test for the stochastic volatility (SV) model.

Δ	YZ estimator				ε_t^2			
	N = 104	N = 208	N = 416	N = 832	N = 104	N = 208	N = 416	N = 832
Normal								
0.1	0.001	0.000	0.000	0.000	0.065	0.065	0.089	0.061
0.2	0.000	0.000	0.000	0.003	0.060	0.078	0.091	0.077
0.3	0.001	0.000	0.000	0.001	0.066	0.075	0.074	0.082
0.4	0.000	0.004	0.004	0.001	0.064	0.077	0.098	0.105
0.5	0.002	0.000	0.005	0.003	0.096	0.091	0.108	0.111
0.6	0.006	0.008	0.013	0.014	0.099	0.107	0.116	0.151
0.7	0.029	0.027	0.034	0.043	0.140	0.168	0.183	0.229
Student's t								
0.1	0.002	0.001	0.002	0.000	0.266	0.364	0.425	0.467
0.2	0.001	0.001	0.003	0.002	0.273	0.382	0.425	0.480
0.3	0.000	0.002	0.000	0.003	0.255	0.401	0.425	0.523
0.4	0.001	0.002	0.000	0.001	0.293	0.383	0.447	0.508
0.5	0.004	0.003	0.005	0.007	0.303	0.397	0.439	0.495
0.6	0.007	0.008	0.011	0.014	0.302	0.405	0.467	0.525
0.7	0.025	0.027	0.047	0.049	0.366	0.474	0.549	0.587
GED								
0.1	0.001	0.000	0.002	0.002	0.169	0.200	0.238	0.242
0.2	0.001	0.001	0.002	0.002	0.182	0.202	0.237	0.267
0.3	0.000	0.001	0.003	0.002	0.160	0.219	0.230	0.272
0.4	0.000	0.002	0.003	0.001	0.201	0.223	0.254	0.282
0.5	0.001	0.005	0.002	0.007	0.223	0.265	0.284	0.297
0.6	0.007	0.014	0.019	0.014	0.226	0.277	0.324	0.324
0.7	0.022	0.031	0.042	0.034	0.270	0.322	0.375	0.405

Table 4 Size of the IT-ICSS test for the FIGARCH(1,d,1) model.

$\omega = 0.1$, $\phi = 0.05$ and $d = 0.75$

B	YZ estimator				ε_t^2			
	N = 104	N = 208	N = 416	N = 832	N = 104	N = 208	N = 416	N = 832
Normal								
0.1	0.622	0.756	0.857	0.895	0.766	0.908	0.989	0.997
0.2	0.606	0.755	0.859	0.892	0.749	0.921	0.970	0.997
0.3	0.596	0.751	0.862	0.894	0.760	0.908	0.984	0.999
0.4	0.566	0.704	0.847	0.896	0.740	0.912	0.979	0.999
0.5	0.545	0.746	0.870	0.890	0.694	0.895	0.968	0.998
0.6	0.489	0.677	0.847	0.893	0.635	0.856	0.977	0.997
0.7	0.380	0.655	0.820	0.889	0.539	0.813	0.956	0.999
Student's t								
0.1	0.612	0.742	0.843	0.890	0.803	0.935	0.982	0.996
0.2	0.593	0.745	0.844	0.892	0.791	0.926	0.985	0.998
0.3	0.584	0.734	0.847	0.890	0.787	0.915	0.979	0.999
0.4	0.591	0.727	0.848	0.891	0.797	0.910	0.985	0.998
0.5	0.535	0.711	0.833	0.889	0.752	0.909	0.987	0.999
0.6	0.483	0.713	0.838	0.891	0.729	0.910	0.981	0.999
0.7	0.475	0.670	0.835	0.887	0.673	0.856	0.978	0.995
GED								
0.1	0.601	0.764	0.850	0.888	0.787	0.928	0.988	1.000
0.2	0.593	0.768	0.851	0.892	0.808	0.906	0.984	0.996
0.3	0.595	0.733	0.856	0.891	0.776	0.923	0.979	0.996
0.4	0.591	0.730	0.854	0.890	0.758	0.929	0.984	0.999
0.5	0.549	0.757	0.849	0.896	0.719	0.908	0.985	0.996
0.6	0.494	0.705	0.851	0.895	0.736	0.898	0.976	0.999
0.7	0.470	0.719	0.841	0.886	0.642	0.878	0.967	0.994

for the YZ estimator is much better than the size of the test associated with the ε_t^2.

Tables 5 to 7 report the power of the IT-ICSS test, for both the volatility estimators with data generating processes from the i.i.d. standard normal, the GARCH(1,1) model and the SV model, when breaks are incorporated at 50th percentile, 25th percentile and 75th percentile of the series for varying sample sizes at 95% confidence level. To generate ε_t^2, we first generate the sample for a given data generating process (i.i.d. standard normal, GARCH(1,1) (equation (10)) and SV (equation (11))). Based on the xth percentile (50th, 25th and 75th) analysis, we keep the first half of sample as it is and change the standard deviation of the second half of the sample by multiplying by a factor $(1 + \lambda)$, where λ indicates the percentage change in the volatility of the series. On the other hand, to generate the YZ estimator, we first generate the pure Gaussian random walk to take out (x, b, c). This (x, b, c) is directly used to find YZ estimator for the case of i.i.d. standard normal distribution. Moreover, we use equations (13) and (14) to generate the YZ estimator for the GARCH(1,1) and the stochastic volatility (SV) data generating process. Here also, we change the standard deviation of the second half of the sample by multiplying it by $(1 + \lambda)$.

Table 5 reports the power of the IT-ICSS test when a sudden change in volatility is incorporated at 50th percentile of the series from the i.i.d. standard normal, the GARCH(1,1) model (with $\omega = 0.1$, $\alpha = 0.1$ and $\beta = 0.6$) and the stochastic volatility model (with $\delta = 0.6$) at 95% level of confidence for both the variance estimators. The results indicate desirable power properties of the IT-ICSS test for both volatility estimators. However, the power properties are better for YZ estimator when a sudden change of more than 100% is incorporated in volatility for all data generating processes under consideration.

Tables 6 and 7 report the power of the IT-ICSS test when a sudden change in volatility estimators is incorporated at the 25th and 75th percentile of the series from various data generating processes. For the i.i.d. standard normal DGP (for both 25th and 75th percentile cases), the YZ estimator exhibits better power for a sample of size of 832 when there is a sudden change in volatility by 30% (or more). Here also, the power properties are better for the YZ estimator when a sudden break of greater than 100% is incorporated in volatility for samples of different sizes. For the GARCH and the SV model, the results find higher power for the YZ estimator for sample sizes 832, 416, 208 and 104 when there is a sudden change in volatility by 50%, 100%, 150% and 200%, respectively. This indicates that as sample size increases, the YZ estimator provides better power for the IT-ICSS algorithm. Overall, the power properties of the YZ estimator and the ε_t^2 are comparable; however, the YZ estimator provides more desirable power properties than the ε_t^2.

The findings from the Monte Carlo simulation experiments indicate that the YZ estimator provides desirable size and power characteristics for the IT-ICSS test. The ε_t^2 exhibits substantial size distortion and lower power in capturing sudden changes in the volatility. The daily open, high, low,

Table 5 Power of the test when there is a change in a variance at 50th percentile of the series.

Λ	YZ estimator				ε_t^2			
	N = 104	N = 208	N = 416	N = 832	N = 104	N = 208	N = 416	N = 832
i.i.d. Normal								
0.2	0.018	0.101	0.507	0.969	0.155	0.358	0.618	0.929
0.3	0.088	0.525	0.971	1.000	0.315	0.669	0.928	0.999
0.4	0.308	0.885	1.000	1.000	0.534	0.864	0.990	1.000
0.5	0.594	0.990	1.000	1.000	0.685	0.952	1.000	1.000
1	0.999	1.000	1.000	1.000	0.991	1.000	1.000	1.000
1.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
GARCH (1,1) with $\omega = 0.1$, $\alpha = 0.1$ and $\beta = 0.6$								
0.2	0.038	0.066	0.132	0.297	0.180	0.242	0.341	0.495
0.3	0.047	0.115	0.278	0.616	0.184	0.298	0.478	0.704
0.4	0.060	0.233	0.508	0.865	0.222	0.417	0.614	0.849
0.5	0.113	0.314	0.699	0.954	0.283	0.476	0.742	0.933
1	0.420	0.847	0.994	1.000	0.535	0.826	0.987	1.000
1.5	0.767	0.990	1.000	1.000	0.762	0.954	1.000	1.000
2	0.915	1.000	1.000	1.000	0.887	0.990	1.000	1.000
SV with $\delta = 0.6$								
0.2	0.017	0.040	0.115	0.272	0.149	0.195	0.281	0.445
0.3	0.033	0.104	0.296	0.602	0.165	0.281	0.446	0.683
0.4	0.072	0.195	0.497	0.847	0.239	0.357	0.588	0.848
0.5	0.101	0.317	0.699	0.976	0.286	0.477	0.737	0.946
1	0.447	0.883	0.999	1.000	0.560	0.839	0.981	1.000
1.5	0.754	0.987	1.000	1.000	0.771	0.976	1.000	1.000
2	0.924	1.000	1.000	1.000	0.906	1.000	1.000	1.000

Table 6 Power of the test when there is a change in a variance at 25th percentile of the series.

Λ	YZ estimator				ε_t^2			
	N = 104	N = 208	N = 416	N = 832	N = 104	N = 208	N = 416	N = 832
i.i.d. Normal								
0.2	0.001	0.004	0.066	0.488	0.063	0.128	0.305	0.657
0.3	0.003	0.034	0.431	0.991	0.087	0.268	0.630	0.948
0.4	0.008	0.121	0.902	1.000	0.178	0.442	0.875	1.000
0.5	0.011	0.396	0.994	1.000	0.230	0.602	0.972	1.000
1	0.361	1.000	1.000	1.000	0.633	0.998	1.000	1.000
1.5	0.935	1.000	1.000	1.000	0.930	1.000	1.000	1.000
2	1.000	1.000	1.000	1.000	0.998	1.000	1.000	1.000
GARCH (1,1) with $\omega = 0.1$, $\alpha = 0.1$ and $\beta = 0.6$								
0.2	0.013	0.033	0.060	0.146	0.125	0.181	0.247	0.384
0.3	0.023	0.046	0.108	0.295	0.140	0.228	0.341	0.503
0.4	0.019	0.078	0.172	0.519	0.146	0.272	0.408	0.660
0.5	0.032	0.092	0.322	0.781	0.181	0.254	0.505	0.767
1	0.186	0.554	0.896	0.998	0.269	0.565	0.881	0.998
1.5	0.265	0.782	0.998	1.000	0.388	0.756	0.977	1.000
2	0.544	0.913	1.000	1.000	0.496	0.885	0.997	1.000
SV with $\delta = 0.6$								
0.2	0.007	0.016	0.032	0.108	0.123	0.150	0.202	0.296
0.3	0.010	0.033	0.089	0.240	0.116	0.177	0.271	0.472
0.4	0.014	0.046	0.171	0.470	0.144	0.202	0.383	0.646
0.5	0.020	0.081	0.280	0.798	0.169	0.276	0.505	0.790
1	0.078	0.571	0.882	1.000	0.250	0.531	0.882	0.995
1.5	0.143	0.791	0.996	1.000	0.395	0.776	0.987	1.000
2	0.545	0.897	1.000	1.000	0.493	0.896	0.999	1.000

Table 7 Power of the test when there is a change in a variance at 75th percentile of the series.

Λ	YZ estimator				ε_t^2			
	N = 104	N = 208	N = 416	N = 832	N = 104	N = 208	N = 416	N = 832
i.i.d. Normal								
0.2	0.015	0.051	0.261	0.762	0.109	0.242	0.420	0.777
0.3	0.053	0.298	0.820	0.999	0.223	0.486	0.755	0.977
0.4	0.221	0.685	0.994	1.000	0.404	0.712	0.961	1.000
0.5	0.456	0.932	1.000	1.000	0.526	0.866	0.987	1.000
1	0.995	1.000	1.000	1.000	0.956	1.000	1.000	1.000
1.5	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
GARCH (1,1) with $\omega = 0.1$, $\alpha = 0.1$ and $\beta = 0.6$								
0.2	0.020	0.056	0.106	0.201	0.148	0.212	0.266	0.415
0.3	0.047	0.094	0.191	0.410	0.167	0.258	0.391	0.553
0.4	0.047	0.159	0.292	0.666	0.224	0.343	0.481	0.699
0.5	0.085	0.236	0.499	0.850	0.256	0.360	0.618	0.835
1	0.343	0.700	0.961	1.000	0.469	0.710	0.934	0.997
1.5	0.630	0.933	1.000	1.000	0.656	0.890	0.994	1.000
2	0.839	0.995	1.000	1.000	0.777	0.968	0.999	1.000
SV with $\delta = 0.6$								
0.2	0.019	0.029	0.067	0.141	0.143	0.181	0.252	0.331
0.3	0.034	0.066	0.148	0.375	0.161	0.230	0.344	0.521
0.4	0.063	0.113	0.304	0.650	0.198	0.311	0.479	0.711
0.5	0.074	0.213	0.486	0.876	0.254	0.369	0.589	0.850
1	0.333	0.707	0.981	1.000	0.466	0.735	0.937	0.996
1.5	0.635	0.949	0.998	1.000	0.663	0.915	0.996	1.000
2	0.842	0.994	1.000	1.000	0.786	0.971	1.000	1.000

and close prices for most traded assets are readily available. Hence, this study recommends the use of the YZ estimator with the IT-ICSS test in detecting sudden changes in the volatility associated with the tradable asset.

Empirical application

It is clear from the Monte Carlo simulation experiments that the YZ estimator performs better than the demeaned squared returns in detecting changes in unconditional volatility when applied with the IT-ICSS test. Now, to examine how the YZ estimator performs on data against the demeaned squared returns in detecting sudden changes in volatility, daily data of three widely traded exchange rates—USD/Euro, USD/Japanese Yen and USD/GBP—is used. All data have been obtained from the Bloomberg database. The period of study for USD/Euro is from April 1999 to February 2013 (3631 observations), and for USD/Japanese Yen and USD/GBP it is from January 1989 to February 2013 (6301 observations each). The sample period for each exchange rate starts from the date when the open, high, low, and close prices are available. Fig. 1 reports the volatility regimes associated with the YZ estimator (left column) and the demeaned squared returns (right column) based on the IT-ICSS algorithm.

Table 8 reports the break points detected in the YZ estimator using the IT-ICSS algorithm (also see left column of Fig. 1). Six break points are detected in the USD/Euro and the USD/GBP exchange rates and eight break points are detected in the USD/Japanese yen exchange rates. This indicates the presence of $(n + 1)$ distinct volatility regimes in the time series of the YZ estimator, where n represents the number of break points in the unconditional variance. The time points of the sudden breaks in the YZ estimator series of the exchange rates are associated with various macroeconomic events. In September 1992, under the impact of speculative currency trades related to pounds, the British government withdrew the pound from the European Exchange Rate Mechanism as the British government was not able to keep the value of a pound above the agreed upon lower limit. This incident, known as Black Wednesday, impacted trade in pounds till the end of 1993. The impact of the Mexican peso crisis in 1994–1995 was not limited to Mexico alone, but also impacted the US dollar exchange rates relative to various other exchange rates, which can be seen in USD/Japanese yen exchange rates. The impact of the Asian financial crisis, which hit many Asian markets such as Thailand, Indonesia, South Korea, Philippines, and other Asian countries, also adversely impacted the USD/Japanese yen exchange rates. In 1997–98, the devaluation of the Russian ruble relative to the US dollar also impacted other US dollar exchange rates. Under the impact of the dot-com bubble by the end of 1999, many markets around the globe experienced sudden changes in volatility, which also affected the volatility of various exchange rates during the period of 2000. In addition, the dot-com bubble burst adversely impacted the major Asian markets which also affected the Japanese yen exchange rates. The terrorist attack of 11 September 2001 in the US affected the global markets severely and resulted in the collapse of major global markets, thereby affecting the US dollar exchange rates.

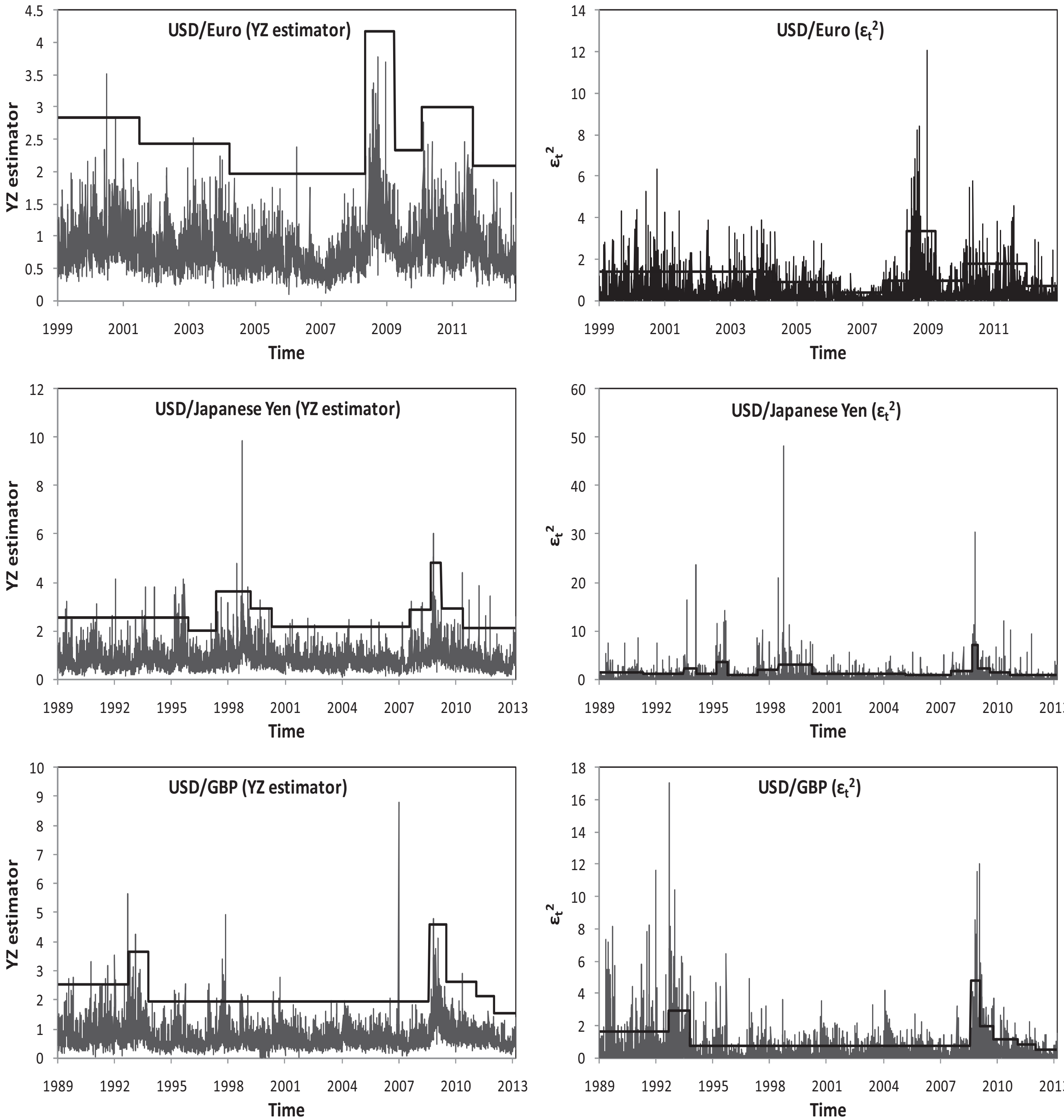

Figure 1 Time plots for YZ estimator (left column) and demeaned squared returns (right column) with a band of three times the average volatility in the given regime.

The higher volatility experienced by all US dollar exchanges rates during the period of 2007–09 can be related to the impact of the global financial crisis, which initiated a period of recession in all major markets. In addition, the sudden changes in the volatility of exchange rates during the period 2009–12 are affected by the impact of crises in various European economies.

Table 9 presents the sudden changes detected in the demeaned square return based on the IT-ICSS algorithm (also see right column of Fig. 1). It can be seen that many breaks detected in the demeaned square return by the IT-ICSS algorithm are spurious in nature. In addition, the IT-ICSS test detects more number of breaks in demeaned squared returns than the YZ estimator. Seven break points are detected in the USD/Euro exchange rates, fifteen break points in the USD/Japanese Yen exchange rates and eight break points are detected in the USD/GBP exchange rates. It can be seen that many breaks cannot be related to macroeconomic events and

Table 8 Change points detected based on the YZ estimator with the IT-ICSS algorithm.

Index	Number of breaks	Break date detected	Reason
USD/Euro	6	24-09-2001	Impact of September 11 terrorist attack
		17-06-2004	–
		06-08-2008	Impact of global financial crisis
		25-06-2009	Bull rally after global financial crisis
		22-04-2010	Impact of European debt crisis
		15-11-2011	Recession due to problems in Greece
USD/Yen	8	14-11-1995	Impact of crisis in Mexico
		08-05-1997	Impact of Asian financial crisis
		01-03-1999	Impact of Russian debt crisis
		17-04-2000	Impact of dot-com bubble burst
		26-07-2007	Impact of global financial crisis
		05-09-2008	Impact of global financial crisis
		02-04-2009	Bull rally after global financial crisis
		24-05-2010	Impact of European debt crisis
USD/GBP	6	09-09-1992	Impact of Black Wednesday
		05-10-1993	Impact of speculative currency trades against the European Exchange Rate Mechanism
		08-08-2008	Impact of global financial crisis
		14-07-2009	Bull rally after global financial crisis
		03-02-2011	Impact of crisis in Europe
		17-01-2012	Impact of crises in Greece and Italy

Table 9 Changes detected with the demeaned squared returns with IT-ICSS algorithm.

Index	Number of breaks	Break date detected	Reason
USD/Euro	7	01-07-2004	–
		26-07-2006	–
		09-11-2007	Impact of global financial crisis
		06-08-2008	Impact of global financial crisis
		24-06-2009	Bull rally after global financial crisis
		26-04-2010	Impact of European debt crisis
		15-11-2011	Recession due to problems in Greece
USD/Yen	15	22-04-1991	–
		10-06-1993	–
		21-02-1994	–
		01-03-1995	–
		05-10-1995	Impact of crisis in Mexico
		07-05-1997	Impact of Asian financial crisis
		11-06-1998	-
		17-02-1999	Impact of Russian debt crisis
		03-04-2000	Impact of dot-com bubble burst
		24-02-2005	–
		26-07-2007	Global financial crisis
		15-09-2008	Global financial crisis
		19-12-2008	Global financial crisis
		12-08-2009	Bull rally after global financial crisis
		16-09-2010	Impact of European debt crisis
USD/GBP	8	02-09-1992	Impact of Black Wednesday
		03-02-1993	–
		18-10-1993	Impact of speculative currency trades against the European Exchange Rate Mechanism
		08-08-2008	Global financial crisis
		11-02-2009	–
		26-10-2009	Impact of crisis in Dubai
		02-02-2011	Impact of crisis in Europe
		12-01-2012	Impact of crises in Greece and Italy

hence are deemed spurious. These findings are in line with the findings from Monte Carlo simulation experiments where it has been shown that the IT-ICSS test exhibit superior size and power properties with the YZ estimator in comparison to the demeaned squared returns.

These findings are also supported by the findings of Schwert (1989) and Kumar and Maheswaran (2013), which say that volatility breaks detected in volatility based on close to close returns may be difficult to relate to macroeconomic events. Hence, in this context, this study proposes the use the YZ estimator based on multi-period open, high, low, and close prices for volatility estimation.

Conclusion

This paper examines the performance of the multiple period drift-independent Yang and Zhang (2000) volatility estimator, the YZ estimator and the demeaned squared returns in detecting sudden breaks in volatility using the IT-ICSS algorithm by means of Monte Carlo simulation experiments. Using data generating processes from sequence of i.i.d. random numbers (the Gaussian, the Student's t, the double exponential, the gamma-mixture and the generalised error distributions), the generalised autoregressive conditional heteroskedasticity model, the stochastic volatility model and the fractionally integrated GARCH model, this study assesses the size and power properties of the YZ estimator and the demeaned squared returns. The findings from Monte Carlo simulation experiments indicate that the YZ estimator exhibits outstanding size and power characteristics when used with the IT-ICSS algorithm. However, the demeaned squared return exhibits oversized behaviour and severe size distortion for most of the data generating processes taken for simulation experiments. This indicates that the IT-ICSS algorithm can detect appropriate sudden breaks in the YZ estimator; however, the sudden breaks detected in the demeaned squared returns may be spurious. To confirm the findings of simulation experiments, this study applies the IT-ICSS algorithm on the YZ estimator and the demeaned squared returns of the USD/Euro, the USD/Japanese yen and the USD/GBP exchange rates to detect sudden changes in the respective volatility proxies. The empirical findings indicate that most of the sudden breaks detected in the YZ estimator can be related to major macroeconomic events. On the other hand, the IT-ICSS algorithm detects too many breaks in the demeaned squared returns, and most of the detected breaks cannot be related to any macroeconomic events and are probably spurious.

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