Semi-Markov Processes for Power System Reliability Assessment With Application to Uninterruptible Power Supply

Antonio Pievatolo, Enrico Tironi, and Ivan Valadè

***Abstract*—We propose a state space model for electrical power systems made by independent semi-Markov components, in which restoration times can have a nonexponential distribution, thus obtaining a more realistic reliability characterization, especially regarding the outage duration distribution. We also propose a model for an energy storage unit, assuming that the storage is fully charged when it begins to deliver power. An approximate analytical evaluation based on the minimal cut sets for the outage allows to surmount the shortcomings of the Monte Carlo approach. The application of the model for an uninterruptible power supply (UPS) system shows that the autonomy of the storage plays a key role, not only for the frequency of the load point voltage failures, but also for their duration distribution.**

***Index Terms*—Energy storage system, minimal cut sets, power system reliability, semi-Markov stochastic processes, UPS.**

NOMENCLATURE

C	Number of components in the system.
$X_c(t)$	State of component c at time t.
i, j, x_c	Different ways of indicating a state taken by component c.
N_c	Number of states that component c can take.
$P_c(i, j)$	Probabilities of the transition matrix P_c of the embedded Markov chain for component c.
$F_{c,i}(t)$	Distribution function of the sojourn time of the component c in the state i.
$F_{c,ij}(t)$	Distribution function of the sojourn time of the component c in the state i, given that the next state will be j.
D_{c,x_c}	Amount of time spent by component c in state x_c from the transition to it to the next transition.
$S(t)$	State of the system at time t.
$s=(x_1, \dots x_C)$	Fixed state of the system.
k, h, u	Indexes of the generic minimal cut set of the system.
Ω	Set of the minimal cut sets of the system for a given load point.
$\bullet^{(h)}$	Quantity referring to the minimal cut set h.
$\Omega^{(h)}$	States of the failed components in the minimal cut set h.

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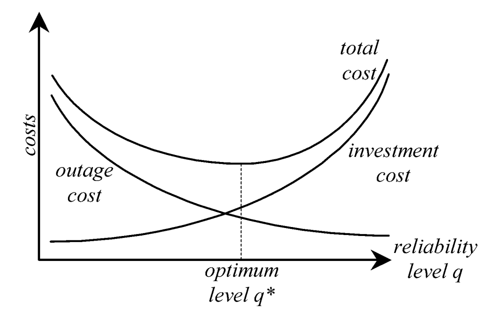

Fig. 1. Choice of the optimum reliability level minimizing the total cost.

I. INTRODUCTION

RELIABILITY is the measure that describes the ability of a system to perform its intended function. Reliability levels are interdependent with economics since increased reliability is obtained through increased investments but it also allows the consumers to reduce their outage costs. In order to carry out objective cost-benefit studies, both the previous economical aspects are important: the optimum reliability level is determined by minimizing the total cost, as reported in Fig. 1.

This paper examines some probabilistic tools that can be used to deal with what is generally known as "reliability worth," that is, the benefit derived by the investments aimed at increasing the reliability thanks to the reduction of the outage cost.

The reliability assessment is generally concerned with average indexes, such as the mean time between failure (MTBF) and the mean time to restoration (MTTR). Also, the probability distribution from which these mean values are derived is useful to more accurately estimate the outage cost of the users with very nonlinear costing function. As shown in [1] through simulations, the duration distribution of the restoration activities heavily affects the probability distribution of the outage duration, so mathematical models admitting distributions other than the negative exponential are necessary.

In [2], a state-space model was employed for the analytical calculation of the MTBF and the MTTR for a system made up of independent semi-Markov components (see [3] for the general theory about these processes): the model was applied to the reliability assessment of an uninterruptible power supply (UPS). The same model could also be useful for the electrical power distribution system when it is assumed that its behavior can be obtained from that of each component computed separately. In fact, in most cases, the complexity of the problem does not allow one to apply the analytical models embedding both dependencies and nonexponential distributions ([4], Sec. I.C), as it is necessary to solve the state transition diagram of the whole system. To the knowledge of the authors, in the field of the power system analysis, these analytical models, such as the auxiliary variable or the device of stage methods, have been applied only to a very small set of apparatuses [5]–[7]. Furthermore, some dependencies can be treated by the model used in this paper by grouping in a single component the dependent devices. This is especially true for the common-cause failures in redundant devices.

The calculation of the approximate duration distribution of a load point voltage failure via the minimal cut sets, as reported in [8], improves both [2], where this distribution is determined by simulation, and [9] in which the distribution of a single system state only is computed. The list of the minimal cut sets can be obtained through the FMEA analysis of the power distribution system, as suggested in IEEE Standard 493 [10], or from the fault tree relating to the load point being studied [11]. Today, this methodology is applied in power distribution system reliability studies [12], [13], even if for large systems, the difficulty of identifying all of the minimal cut sets compel to consider the low-order ones only.

Distributed energy storage systems are beginning to be installed in the recent power distribution systems, as it has been proposed for the Flexible, Reliable and Intelligent Electrical Energy Delivery System (FRIENDS), [14]. From one point of view, the energy storage systems allow the use of new energy resources, and especially the renewable ones; from the other, they are used to mitigate some aspects of the power quality problem. The aspect of voltage stability, which includes voltage sags, requires a small amount of energy to be stored, while the compensation of the continuity of supplying power requires a larger amount of energy stored. Therefore, an effort has been made for the explicit modeling of the storage unit, whose state of charge depends on the stochastic stories of all the semi-Markov independent components. This is different from [15] where the storage is considered as embedded within a single semi-Markov model.

After a brief summary of the results of [2] (Section II-A), special attention is paid to the calculation of the distribution of the outage time at any load point. This is done by combining the duration distribution of the minimal cut sets for the load point outage, both when an unlimited storage is available (Section II-B) and when it is not (Section III). The method is then applied to an UPS with a limited storage (Section IV).

II. Stochastic Model Made up by Independent Semi-Markov Components

In this section, the energy storage autonomy is considered unlimited and, therefore, the reliability quantities have the symbol ∞ as apex.

The load point voltage is said to be in the failure state when it is out of the rated limits, otherwise it is in the functioning state. This state results from a combination of the states of all the components that make up the electric power system: the relationship between the state of the components and of the load point can be described by a fault tree, as will be shown in Section IV.

The following hypotheses have been assumed:

- stochastic independence among the power system components;
- the repair of a component starts as soon as the fault has occurred. The repair time includes the technician's travelling time, fault identification and repair, and putting the component back into service.

The stochastic model is studied under steady-state conditions.

A. Steady-State Average Reliability Indexes of a Load Point

The stochastic behavior of the power system can be described by first modeling separately each component c through a semi-Markov process, and then using the fact that the components change states independently of each other.

For a semi-Markov process, the transitions among the states may be thought of as taking place in two stages: first of all, when the state i is entered, the next state is chosen according to the transition matrix P_c; then, given that the next state is j, the sojourn time in the state i has a distribution $F_{c,ij}(t)$, taken absolutely continuous. For the components that make up the power system, we have a positive recurrent semi-Markov process for which the steady-state probability $\Pr_c(i)$ of component c being in state i can be calculated starting from the invariant probability distribution $(\pi_{c,1},\ldots,\pi_{c,N_c})$ of P_c

$$\begin{aligned}\Pr_c(i) &= \lim_{t\to\infty} \Pr\left(X_c(t)=i|X_c(0)=j\right)\\ &= \frac{\pi_{c,i}\cdot\mu_{c,i}}{\sum\limits_{k=1}^{N_c}\pi_{c,k}\cdot\mu_{c,k}}, \quad \forall\, j\end{aligned} \tag{1}$$

where $\mu_{c,i}$ is the mean of the sojourn time of the component c in the state i.

In order to carry out the reliability analysis, it is necessary to determine the steady-state frequency of passing from a set of states, named F, to another, named B. As is shown in [2], this frequency can be calculated as

$$\begin{aligned}Fr_{F\to B} &= \lim_{t\to\infty}\lim_{\Delta\to 0}\frac{\Pr\left(S(t)\in F\cap S(t+\Delta)\in B\right)}{\Delta}\\ &= \sum_{s\in F}\Pr(s)\cdot\left(\sum_{c=1}^{C}\sum_{\substack{x'_c=1\\ s(x'_c)\in B}}^{N_c}\frac{P_c\left(x_c,x'_c\right)}{\mu_{c,x_c}}\right)\end{aligned} \tag{2}$$

where $s\left(x'_c\right)$ denotes the power system state s where only component c changes state from x_c to x'_c, because the probability of having two or more component changing states in $[t,t+\Delta]$ is negligible due to the absolute continuity of the duration distributions of the sojourn times.

If F is the set of those s states of the power system that cause the examined load point voltage to be in the failure state, and B is the set of all the other states, the average load point failure rate (λ^{∞}_{LP}) can be calculated through (2). The average load point availability, which is the mean proportion of time that the load point voltage is in the rated limits, can be calculated from (1) as

$$\begin{aligned}A^{\infty}_{LP} = \sum_{s\in B}\Pr(s) &= \sum_{s\in B}\left[\prod_{c=1}^{C}\Pr_c\left(x_c\right)\right]\\ s &= (x_1,\ldots,x_C).\end{aligned} \tag{3}$$

Finally, the average load point outage duration $(\mathrm{MTTR}_{LP}^{\infty})$ can be computed as

$$\mathrm{MTTR}_{LP}^{\infty} = \frac{1}{\lambda_{LP}^{\infty}} \cdot \left(1 - A_{LP}^{\infty}\right). \tag{4}$$

Using (1)–(4), the average reliability indexes are determined from the transition matrices P_c and from the mean durations $\mu_{c,i}$ of the states of the components. Then, these average indexes do not change as long as this is true for the P_c and the mean durations, regardless of the duration distributions $F_{c,ij}$.

B. Steady-State Outage Duration Distribution

The stochastic model based on the semi-Markov components permits to determine an approximate analytic expression for the probability distribution of the outage duration, when the restoration activities have distributions other than the negative exponential.

For systems made up by binary components, a cut set is defined as a set of components which by failing causes the system to fail; the cut set is said to be minimal if it cannot be reduced without losing its status as a cut set. In the same way, for systems made up by multistate components, a cut set can be defined as a set of states of the components that causes the system to fail and the cut set is said to be minimal if the states cannot be reduced without losing its status. At any load point, the power system structure can be looked at as a series structure of minimal cut sets and the downtime distribution can be expressed starting from all of the cut sets durations.

The multistate components of the electric power system can be modeled through one state of correct operation and one or more states of incorrect operation conditions: we assume that the state following the one of correct operation is determined by the failure mode which occurs first, whereas the state following any failure mode is the one of correct operation, as shown in Fig. 2.

Given a minimal cut set k of order $n^{(k)}$ $\left(n^{(k)} \in (1, 2, \ldots, C)\right)$, the failure state $x_c^{(k)}$ is defined for each of the $n^{(k)}$ components. For the generic component c of the cut set we can extract from its life history the alternating renewal process made up by the sequence of the repair times for the failure state $x_c^{(k)}$ and the lifetime immediately before. Therefore, in order to calculate the approximate downtime distribution, we can apply the results of [8], which we are going to describe.

For highly available systems, which include the electric power systems, the steady-state downtime distribution at any load point can be approximated by the following formula [8]:

$$G_{LP}^{\infty}(t) = \sum_{h \in \Omega} r^{(h)} \cdot G^{(h)}(t) \tag{5}$$

where $r^{(h)}$ is the steady-state probability that the load point voltage failure is caused by the minimal cut set h, and $G^{(h)}(t)$ denotes the steady-state downtime distribution of the same minimal cut set.

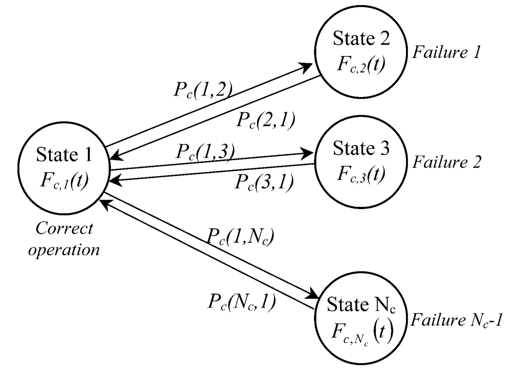

Fig. 2. Model for the generic component c of the power system.

The term $r^{(h)}$ can be computed as

$$r^{(h)} = \frac{Fr^{(h)}}{\sum\limits_{u \in \Omega} Fr^{(u)}} \tag{6}$$

where $Fr^{(h)}$ denotes the steady-state frequency of entering the minimal cut set h and can be evaluated by (2), when F is the set of the states that contain the cut set h and B is the set of the states that do not include any cut set; $Fr^{(u)}$ can be computed in the same way.

In order to calculate the term $G^{(h)}(t)$, we have to determine the downtime distribution of the parallel system comprising the components of the minimal cut set h, which is identified by some specified failure states of $n^{(h)}$ components. In [16], it is shown by Monte Carlo simulations that the dominant failure scenario for a parallel structure of highly reliable components is the one in which there is not any complete operation interval of a component during the repair of the other components. Therefore, the steady-state downtime distribution for the minimal cut set h can be computed as [8]

$$G^{(h)}(t) = \sum_{i \in \Omega^{(h)}} w_i \cdot \left\{ 1 - \left[1 - G_i(t)\right] \cdot \prod_{\substack{z \in \Omega^{(h)} \\ z \neq i}} \frac{\int_t^{\infty} \left[1 - G_z(\tau)\right] \cdot d\tau}{\mu_{G_z}} \right\} \tag{7}$$

where i is the index related to the states occupied by the components of the cut set h, G_i is the sojourn time distribution in each of these states, and μ_{G_i} is its mean, and w_i denotes the steady-state probability that entering the state i causes the cut set h. The weight w_i can be computed as [8]

$$w_i = \frac{\frac{1}{\mu_{G_i}}}{\sum\limits_{z \in \Omega^{(h)}} \frac{1}{\mu_{G_z}}}. \tag{8}$$

It is shown in [17] that the steady-state formula gives very accurate results also for the first system failure, provided that the number of component failures before the occurrence of the system failure is relatively large for each component, as in the case of the electric power systems.

III. Storage Modeling

In this section, a model for the storage, appropriate for reliability assessment, will be presented.

The basic hypothesis of this model is that the storage unit is assumed to be fully charged when it begins to deliver power. This is a reasonable assumption because the mean interval between two consecutive events requiring energy from the storage is very long with respect to the duration of the same events.

When an energy storage unit is present, the key point for the reliability assessment at any load point is to determine the distribution $G_{ST}(t)$ of the duration of the required drawing from the unit. This distribution can be calculated as reported in Section II-B considering the minimal cut sets k_{ST} made up by the states of the components that cause the power system to draw energy from the storage. Some of these states are related to the failure modes of the corresponding components but others can require the correct operation of the component, in such a way to have a path from the storage unit to the load point. Due to the high availability of the components, only the failure states have been taken into consideration in the cut sets, thus overestimating the energy required.

Γ_{ST} denotes the set of the states in which the load power is drawn from the storage unit and $Y_{\Gamma_{ST}}$ denotes the sojourn time in Γ_{ST}, which is distributed according to $G_{ST}(t)$. The load point availability can be computed as

$$\begin{aligned} A_{LP} &= 1 - \Pr\left(S(t) \in F \cup (S(t) \in \Gamma_{ST} \cap E_{ST}(t) = 0)\right) \\ &= A_{LP}^{\infty} - \Pr\left(S(t) \in \Gamma_{ST} \cap E_{ST}(t) = 0\right) \end{aligned} \tag{9}$$

in which $E_{ST}(t)$ denotes the energy stored at time t. The equality is valid because the two events $\{S(t) \in F\}$ and $\{S(t) \in \Gamma_{ST} \cap E_{ST}(t) = 0\}$ are obviously mutually exclusive. The second term can be evaluated via the conditional probability factorization

$$\begin{aligned} &\Pr\left(S(t) \in \Gamma_{ST} \cap E_{ST}(t) = 0\right) \\ &\quad = \Pr\left(E_{ST}(t) = 0 | S(t) \in \Gamma_{ST}\right) \cdot \\ &\quad \cdot \Pr\left(S(t) \in \Gamma_{ST}\right) \\ &\quad = \frac{\int_{D_{ST}}^{+\infty} (\tau - D_{ST}) \cdot g_{ST}(\tau) \cdot d\tau}{\int_0^{+\infty} \tau \cdot g_{ST}(\tau) \cdot d\tau} \\ &\quad \cdot \Pr\left(S(t) \in \Gamma_{ST}\right) \end{aligned} \tag{10}$$

where $g_{ST}(\tau)$ is the probability density corresponding to the distribution $G_{ST}(t)$, and D_{ST} is the autonomy (in time units) of the energy storage unit, assuming that the load draws the rated power.

After computing the steady-state frequency Fr_{ST} of visits to Γ_{ST} with (2), the load point failure rate has been calculated taking into account that only the fraction $[1 - G_{ST}(D_{ST})]$ of visits to Γ_{ST} causes a load point outage

$$\lambda_{LP} = \lambda_{LP}^{\infty} + Fr_{ST} \cdot [1 - G_{ST}(D_{ST})]. \tag{11}$$

As in Section II, the average load point outage duration (MTTR_{LP}) can be calculated as

$$\mathrm{MTTR}_{LP} = \frac{1}{\lambda_{LP}} \cdot (1 - A_{LP}). \tag{12}$$

The outage duration distribution is the mixture of the downtime distribution $G_{LP}^{\infty}(t)$ and the one due to the discharge of the storage unit, named $G_{LP}^{ST}(t)$

$$G_{LP}(t) = p_{\infty} \cdot G_{LP}^{\infty}(t) + p_{ST} \cdot G_{LP}^{ST}(t) \tag{13}$$

where p_{∞} and p_{ST} are the weights of the two terms. These weights denote the steady-state probability that a load point voltage failure is due to the discharge of the storage unit or to another event and can be computed from the frequencies of these events

$$p_{\infty} = \frac{\lambda_{LP}^{\infty}}{\lambda_{LP}} \tag{14}$$

$$p_{ST} = 1 - \frac{\lambda_{LP}^{\infty}}{\lambda_{LP}}. \tag{15}$$

The downtime distribution $G_{LP}^{ST}(t)$ can be calculated from the distribution of the duration of the drawing from the storage conditioning on the fact that this duration is greater than the autonomy of the storage

$$\begin{aligned} G_{LP}^{ST}(t) &= \Pr\left(Y_{ST} \le t | \Gamma_{ST}, Y_{\Gamma_{ST}} > D_{ST}\right) \\ &= \frac{G_{ST}(t + D_{ST}) - G_{ST}(D_{ST})}{1 - G_{ST}(D_{ST})} \end{aligned} \tag{16}$$

in which T denotes the duration of the load point outage.

By (13)–(16), the approximate outage duration distribution was computed analytically considering also the autonomy of the storage system.

IV. Application: The UPS Reliability Assessment

Our method has been tested for a simple electrical power system composed by the mains, an active compensation device, and its critical load.

The following procedure has been followed:

- identification of the components of the power system and of their failure modes;
- study of the power system behavior under various fault conditions of one or more components, resulting in the fault-tree construction;
- identification of the minimal cut sets in the fault tree;
- probabilistic analysis, as reported in the Sections II and III.

A. Electric Model of the Power System

The mains is modeled as a single component whose output voltage reliability characterization has been obtained from some studies on the public electric network [18], [19].

There are a lot of topologies that improve the quality of the power supplied to a sensitive load, which can be well-tried solutions such as the double-conversion UPS, or innovative ones, such as the unified power quality conditioner (UPQC), the shunt, or the series compensators. The double conversion UPS, that consists of a rectifier, an inverter, an energy storage unit, and a static transfer switch is taken into consideration. This electrical power system has been chosen for the following reasons.

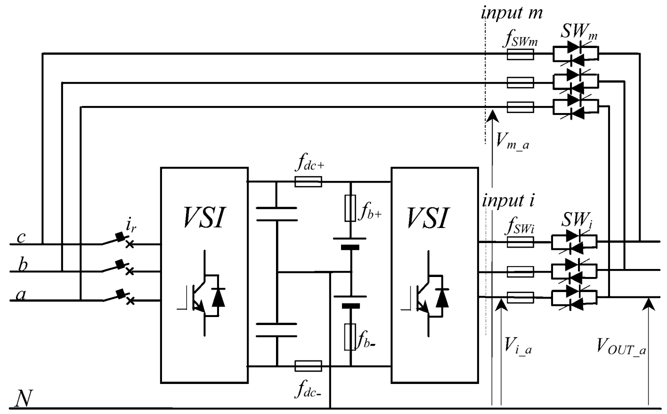

Fig. 3. Double conversion UPS with its protection devices. Voltages V_m, V_i, and V_{out} pertaining to phase "a" of the system are also shown.

- This kind of application requires a careful reliability assessment of the load voltage, owing to the high outage cost of the critical load.
- Even if simple, it has some generality because there are multistate components and also an energy storage unit.

In Fig. 3, the double-conversion UPS is shown with its protection devices. In order to obtain a conversion with a unity power factor and very low current harmonic distortion, a rectifier consisting of a voltage source inverter (VSI) has been considered.

The following failure modes have been identified for the components of the UPS:

1) in the semiconductor devices: short circuit or open circuit due to faults in the power or firing circuits;
2) in the control logics of the components, which result in the generation of "ON" or "OFF" signals at the wrong times;
3) short circuits in either the ac or dc section;
4) anomalous behavior of the energy storage unit, made up by batteries.

Table I shows the operational states of the UPS components.

For the UPS components, Weibull distributions, with a shape parameter equal to 10, have been assumed for the restoration times; the failure times are exponentially distributed. In [2], it is shown how to compute the scale parameters of all the distributions and the transition matrices for each component, when the frequencies of visits to the different failure states and their mean durations are available. These are reported in Table II, obtained from [18] and [19]. The mains voltage has been classified according to four operational modes: between 0.9 and 1.1 of the rated voltage V_n (mode Ma_1), voltage sags or short interruptions with residual voltage between 0.7 and $0.9V_n$ (mode Ma_2) and below $0.7V_n$ (mode Ma_3), long interruptions (mode Ma_4).

Afterwards, the fault tree has been built for the load point voltage supply, whose top event is the out-of-limit condition of this voltage. For clarity, the section of this fault tree for the UPS compensator and the other section, for the static transfer switch, are represented separately in Fig. 4.

The UPS device with a VSI rectifier should be capable of drawing from the mains the active power required by the load even when the residual voltage is very low, increasing the absorbed current. It has been assumed that the current rating of the rectifier makes it possible to draw the full load power as long as the mains voltage exceeds 70% of the nominal; below this point, the storage system must come into play.

TABLE I
OPERATIONAL STATES OF THE UPS COMPONENTS

Section	Operational modes	Typical cause
Inverter	In_1: correct operation	
	In_2: short circuit	-power or firing circuit failure -output filter short circuit
	In_3: open circuit	-power or firing circuit failure
	In_4: control failure	-control logic failure
Rectifier	R_1: correct operation	
	R_2: short circuit	-power or firing circuit failure -input filter short circuit -dc capacitor short circuit
	R_3: open circuit	-power or firing circuit failure
	R_4: control failure	-control logic failure
Energy storage unit	E_1: correct operation	
	E_2: short circuit	-cell short circuit
	E_3: high impedance	-positive grid corrosion -dry-out -plate sulphation
Static transfer switch	SW_1: correct operation	
	SW_2: SW_m thyristor short circuit	-thyristor power circuit failure -thyristor firing circuit failure
	SW_3: SW_i thyristor short circuit	
	SW_4: SW_m thyristor open	
	SW_5: SW_i thyristor open	
	SW_6: control failure	control logic failure

TABLE II
RELIABILITY INFORMATION ABOUT THE COMPONENTS

Component	Failure		Mean restoration time
Mains	$Fr_{Ma}(2)=6.772\cdot10^{-3}h^{-1}$ $Fr_{Ma}(3)=5.016\cdot10^{-3}h^{-1}$ $Fr_{Ma}(4)=0.480\cdot10^{-3}h^{-1}$		$\mu_{Ma,2}=0.22s$ $\mu_{Ma,3}=1.54s$ $\mu_{Ma,4}=1.0h$
Static transfer switch	$MTBF_{power}=1250kh$	$Fr_{SW}(2)=Fr_{SW}(3)=0.2/(1250\cdot10^3)h^{-1}$ $Fr_{SW}(4)=Fr_{SW}(5)=0.8/(1250\cdot10^3)h^{-1}$	$\mu_{SW,2}=\mu_{SW,3}=\mu_{SW,4}=\mu_{SW,5}=\mu_{SW,6}=10h$
	$MTBF_{control}=600kh$	$Fr_{SW}(6)=1/(600\cdot10^3)h^{-1}$	
Converters (rectifier, inverter)	$MTBF_{power}=130kh$	$Fr_R(2)=Fr_{In}(2)==0.2/(130\cdot10^3)h^{-1}$ $Fr_R(3)=Fr_{In}(3)==0.8/(130\cdot10^3)h^{-1}$	$\mu_{R,2}=\mu_{R,3}=\mu_{R,4}=\mu_{In,2}=\mu_{In,3}=\mu_{In,4}=10h$
	$MTBF_{control}=600kh$	$Fr_R(4)=Fr_{In}(4)==1/(600\cdot10^3)h^{-1}$	
Energy storage unit	$MTBF=100kh$	$Fr_E(2)=Fr_E(3)==0.5/(100\cdot10^3)h^{-1}$	$\mu_{E,2}=\mu_{E,3}=10h$

B. Reliability Analysis of the Power System

The reliability of the voltage supplied to the critical load has been assessed varying the characteristics of the energy storage system. In order to check the analytical results, Monte Carlo simulations have also been carried out. We generated ten independent life histories, with 3000 V_{OUT} failures each, for a few

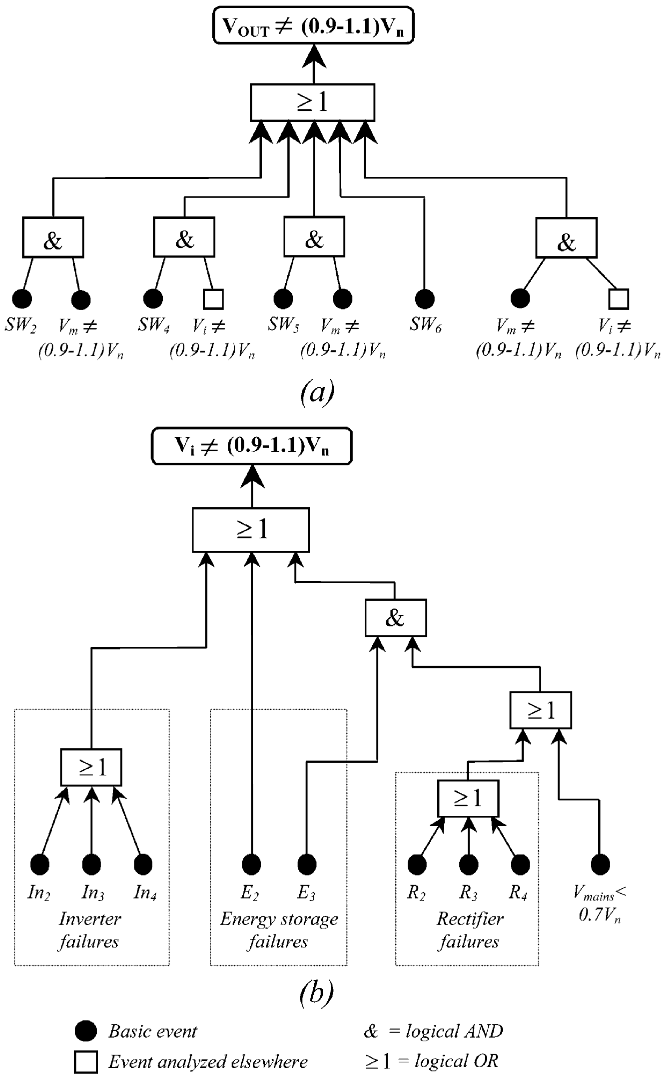

Fig. 4. (a) Static transfer switch fault tree. (b) UPS compensator fault tree.

TABLE III
RELIABILITY INDEXES OF THE CRITICAL LOAD VOLTAGE

Scenario	Analytical results	Simulation results
Unlimited autonomy	MTBF=2.6250·10^5h MTTR=4.4006h A=0.99998324	Simulation has not been carried out because the analytical results are exact
D_{ST}=8h	MTBF=2.5187·10^5h MTTR=4.2617h A=0.99998308	Storage unit fully charged immediately after the usage MTBF=2.5355·10^5h (σ=0.0544·10^5h) MTTR=4.2418h (σ=0.0848h) A=0.99998327 β=0.1 MTBF=2.5237·10^5h (σ=0.0376·10^5h) MTTR=4.2775h (σ=0.0996h) A=0.99998305
D_{ST}=4h	MTBF=7.9460·10^4h MTTR=2.0295h A=0.99997446	β=0.1 MTBF=7.8646·10^4h (σ=0.1273·10^4h) MTTR=2.0202h (σ=0.0619h) A=0.99997431
D_{ST}=2h	MTBF=1.4566·10^4h MTTR=1.1888h A=0.99991839	β=0.1 MTBF=1.4474·10^4h (σ=0.0213·10^4h) MTTR=1.1747h (σ=0.0406h) A=0.99991884

β = rate of recharge of the energy storage system, expressed in per unit value of the rated load power.

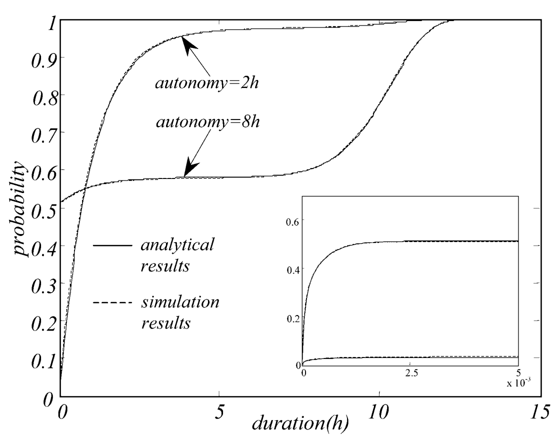

Fig. 5. Comparison between the analytical and simulation results for the outage duration distributions.

different autonomies, with the β recharge rate of the energy storage unit equal to 10% of the rated load power. With an autonomy of 8 h, the simulations have been also carried out assuming the storage unit to be fully charged when it begins to deliver power.

The mean indexes are reported in Table III. The table exhibits a good accordance between the analytical and simulation results, thus confirming the validity of the analytical model proposed. In particular, the simulations show that the reliability indexes are very slightly influenced by the rate of recharge of the energy storage unit and, thus, the hypothesis assumed for the storage modeling is valid. On the contrary, the autonomy of the storage plays a very important role: the greater the autonomy, the longer the load point MTBF, and MTTR.

The steady-state outage duration distributions are reported in Figs. 5 and 6. The comparison between the analytical and simulation results, carried out in Fig. 5, shows that they are in good accordance when the energy storage autonomy is equal both to 2 and 8 h.

In Fig. 6, the outage duration distributions, given a load voltage failure, are represented when the autonomy is unlimited or equal to 8, 4, or 2 h. Since the long interruptions have a mean duration equal to 1 h, there is only a small difference between the unlimited autonomy or the 8-h autonomy. In these circumstances, the probability is concentrated mainly on two areas: the first one represents short interruptions and is due to the voltage sags that reach the load for some failures in the UPS; the second represents longer interruptions and is due to the output voltage failures generated directly by a UPS component, such as the failure of the control of the static transfer switch, which has Weibull distributed restoration times with mean equal to 10 h.

The frequencies of these kinds of failures do not depend on the autonomy. On the contrary, for the lower autonomies, the frequency of the failures due to the discharge of the storage becomes larger relative to the frequency of those due to voltage

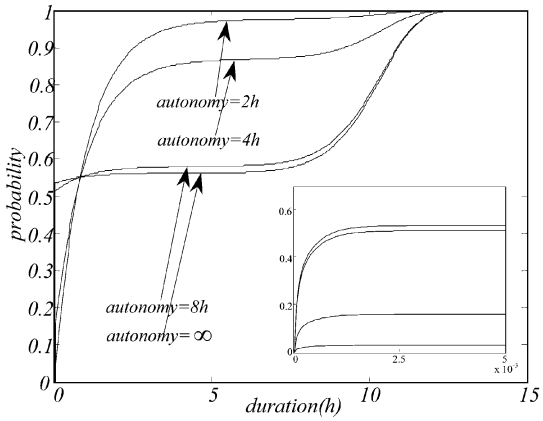

Fig. 6. Effect of the autonomy on the outage duration distribution.

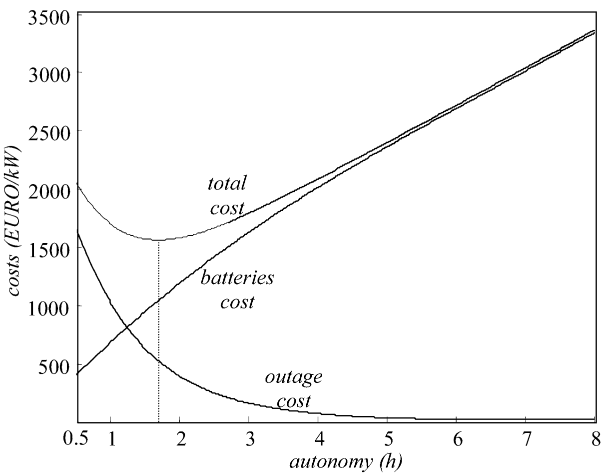

Fig. 7. Optimization of the storage autonomy minimizing the total cost.

sags, and this is reflected by the change of shape of the distribution which concentrates some more mass on a new area between the previous two.

C. Optimization of the Storage Autonomy

The optimization of the storage autonomy requires the minimization of the total cost, as explained in the introduction.

This application does not refer to any specific real situation and so the outage cost, shown in Fig. 7, has been calculated referring to the industrial customer sector of a survey performed in Great Britain in 1993 [20]. This survey provides the industrial sector cost function $cf(d)$ which gives the interruption cost as a function of its duration d. The mean cost $E(C_{\text{int}})$ of an interruption, given a certain autonomy, can be calculated as

$$E(C_{\text{int}}) = \int_0^{\infty} cf(\tau) \cdot g(\tau) \cdot d\tau \quad (17)$$

where $g(\tau)$ is the probability density function corresponding to the downtime distribution. The expected number of interruptions in the UPS lifetime, assumed equal to 30 years, was obtained from the MTBF.

Only the cost of the batteries, furnished by a UPS manufacturer, has been considered as investment cost because the other costs do not vary with the storage autonomy.

From Fig. 7, it comes out that the lowest total cost is obtained with an autonomy close to 2 h.

V. Conclusion

The state space model made by independent semi-Markov components allows a complete assessment of the electrical power system reliability, also when the restoration times have distributions other than the exponential. A realistic outage duration distribution can be calculated analytically, thus surmounting the shortcomings of the Monte Carlo approach due mainly to the computational burden it implies.

In the case of highly available components, such as that of power systems, the energy storage can be modeled assuming it to be fully charged when it begins to deliver power.

The application to a double conversion UPS shows that the autonomy of the energy storage plays a key role both for the reliability mean indexes and for the outage duration distribution. On the contrary, the storage recharge rate does not modify considerably the reliability of the critical load supply.

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