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Robustness of the EWMA and the combined $\bar{X}$–EWMA control charts with variable sampling intervals to non-normality

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The exponentially weighted moving average (EWMA) control charts with variable sampling intervals (VSIs) have been shown to be substantially quicker than the fixed sampling intervals (FSI) EWMA control charts in detecting process mean shifts. The usual assumption for designing a control chart is that the data or measurements are normally distributed. However, this assumption may not be true for some processes. In the present paper, the performances of the EWMA and combined $\bar{X}$–EWMA control charts with VSIs are evaluated under non-normality. It is shown that adding the VSI feature to the EWMA control charts results in very substantial decreases in the expected time to detect shifts in process mean under both normality and non-normality. However, the combined $\bar{X}$–EWMA chart has its false alarm rate and its detection ability is affected if the process data are not normally distributed.

Keywords: quality control; control chart; EWMA; variable sampling intervals; non-normality

1. Introduction

Control charts are widely used in monitoring and detecting changes in process parameters. From the previous studies [7], the exponentially weighted moving average (EWMA) control chart is very effective against small process mean shifts. Traditionally, the usual practice in applying a control chart, which is called fixed sampling rate (FSR), is to take samples of equal size from the process at fixed length sampling intervals and plot this sample point on the chart with a fixed action limit coefficient. The adaptive EWMA control charts, that is, variable sampling interval (VSI), variable sample size, and variable sample size and sampling interval, have been shown to give substantially faster detection of most process parameters shifts than the FSR EWMA chart [2,9–11,13–15]. In particular, Reynolds and Stoumbos [14] showed that adding the VSI scheme to the EWMA chart results in a very substantial reduction in the expected time required

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to detect shifts in process parameters. Albin *et al*. [1] recommended that the combination of the $\bar{X}$ and EWMA charts (i.e. $\bar{X}$–EWMA chart) be plotted on the same graph and showed that this combined control chart was able to effectively detect small and large shifts in the process mean and large shifts in the process standard deviation.

Traditionally, when designing control charts, one usually assumes that the measurements or collected data in the subgroup are normally distributed. However, this assumption may not be tenable in some production processes. If the measurements are not normally distributed, the traditional way for designing the control chart may not be appropriate. Borror *et al*. [3] studied the performance of the EWMA chart under non-normality, and concluded that the EWMA control chart is relatively robust to violations of the normality assumption.

In the present paper, we study the robustness of the VSI EWMA and combined $\bar{X}$–EWMA control charts to non-normality. The non-normal distributions employed in the present paper are the Gamma, Weibull, and the t distributions. The Gamma and Weibull distributions can represent various skewed distributions, and the t distribution represents the symmetric distributions with heavier tails than the normal distribution. In the next section, the VSI EWMA chart and VSI $\bar{X}$–EWMA charts are briefly reviewed. Then, the performance measures of the VSI control charts are introduced. The performance of the control chart under non-normality is evaluated using the Gamma, Weibull, and t distributions. Finally, a comparative study of the VSI EWMA control charts is conducted and some conclusions are drawn based on the study.

2. The VSI EWMA and VSI $\bar{X}$–EWMA control charts

The VSI scheme was investigated by Reynolds *et al*. [12]. To simplify the implementation of the proposed VSI scheme, the VSI control chart generally adopts two different sampling intervals. The sampling scheme of the VSI control chart is to use the long sampling interval (h_1) as long as the sample point is close to the target so that there is no suspicion of process change. However, if the sample point is close to, but still within, the action limits so that there is some suspicion of process shift, then the short sampling interval (h_2) is used. In the VSI $\bar{X}$ control chart, the action limits (UCL_x and LCL_x) and warning limits (UWL_x and LWL_x) determine the central region C_1 (i.e. $C_1 = (\mathrm{LWL}_x, \mathrm{UWL}_x)$) and warning region C_2 (i.e. $C_2 = [\mathrm{LCL}_x, \mathrm{LW}L_x] \cup [\mathrm{UWL}_x, \mathrm{UCL}_x]$), respectively. Let $\bar{X}_t$ represent the sample average for sample t. The sampling interval function $d_1(x)$ can be defined by

$$d_1(x) = \begin{cases} h_1, & \text{if } \bar{X}_t = x \in C_1, \\ h_2, & \text{if } \bar{X}_t = x \in C_2, \end{cases} \tag{1}$$

where $(\mathrm{LWL}_x, \mathrm{UWL}_x) = \mu_0 \pm w_x \sigma_{\bar{X}}$ and $(\mathrm{LCL}_x, \mathrm{UCL}_x) = \mu_0 \pm k_x \sigma_{\bar{X}}$, μ_0 is the mean of the process when the process is in control, $\sigma_{\bar{X}}$ the standard deviation of $\bar{X}$, k_x and w_x the action limit and warning limit coefficients with the VSI $\bar{X}$ control chart, and $k_x \geq w_x$.

In the EWMA chart, the sample statistic considers the weighted average of the current and past sample points. It has been shown that the EWMA chart is very effective in detecting small mean shifts. The tth sample statistic Y_t of the EWMA chart is defined as follows:

$$Y_t = (1 - \lambda) Y_{t-1} + \lambda \bar{X}_t, \quad \text{for } t = 1, 2, 3 \ldots, \tag{2}$$

where λ is a smoothing constant and $0 < \lambda \leq 1$, and $\bar{X}_t$ is the sample average for sample t. In the VSI EWMA control chart, the action limits (UCL_e and LCL_e) and the warning limits (UWL_e and LWL_e) determine the central region D_1 (i.e. $D_1 = (\mathrm{LWL}_e, \mathrm{UWL}_e)$) and warning region D_2 (i.e.

$D_2 = [\mathrm{LCL}_e, \mathrm{LWL}_e] \cup [\mathrm{UWL}_e, \mathrm{UCL}_e]$). The sampling interval function $d_2(y)$ can be defined by

$$d_2(y) = \begin{cases} h_1, & \text{if } Y_t = y \in D_1, \\ h_2, & \text{if } Y_t = y \in D_2, \end{cases} \tag{3}$$

where

$$(\mathrm{LWL}_e, \mathrm{UWL}_e) = \mu_0 \pm w_e \sqrt{\frac{\lambda}{2-\lambda}}\, \sigma_{\bar{X}}, \tag{4}$$

$$(\mathrm{LCL}_e, \mathrm{UCL}_e) = \mu_0 \pm k_e \sqrt{\frac{\lambda}{2-\lambda}}\, \sigma_{\bar{X}}, \tag{5}$$

where k_e and w_e are the action limit and the warning limit coefficients with the VSI EWMA control chart and $k_e \geq w_e$.

In the operation of the combined $\bar{X}$–EWMA control chart with VSI scheme, the long sampling interval h_1 is used if both the sample points falls within the central region (both C_1 and D_1), and the short sampling interval h_2 is used if either of the sample points falls within the warning region (either C_2 or D_2). The sampling interval function $\mathrm{d}(x, y)$ can be defined by

$$\mathrm{d}(x, y) = \begin{cases} h_1, & \text{if } \bar{X}_t = x \in C_1 \text{ and } Y_t = y \in D_1, \\ h_2, & \text{if } \bar{X}_t = x \in C_2 \text{ or } Y_t = y \in D_2. \end{cases} \tag{6}$$

As in the previous studies [14], the EWMA control chart is much better than the Shewhart $\bar{X}$ control chart in detecting small shifts in the mean, but not quite as fast for large shifts. When the process data are normally distributed, the combined $\bar{X}$–EWMA control chart has the advantage that the two charts are superimposed. This combined control procedure is effective against both large and small mean shifts. That is, the combined chart has better performance for large mean shifts than the EWMA chart alone, and better performance for small mean shifts than the $\bar{X}$ chart alone.

3. Performance measures of the VSI EWMA control chart

Traditionally, the average run length (ARL), which is defined as the average number of samples before the control chart signals an out-of-control condition, is applied to evaluate the performance of a control chart. However, when the sampling interval is variable, the time-to-signal is not a constant multiple of ARL, and thus the ARL is inappropriate for evaluating the effectiveness of the adaptive control charts. The widely used performance measures for VSI control charts include the in-control average time-to-signal (ATS) and adjusted average time-to-signal (AATS). In-control ATS is defined as the expected value of the time from the start of the monitoring to the time when the chart indicates a false alarm. The AATS is defined as the expected value of the time from the occurrence of an assignable cause to the time when the chart indicates an out-of-control signal. As the process is in control, the in-control ATS is a measure of the false alarm rate of the chart. A chart with a larger in-control ATS indicates a lower false alarm rate. As the process is out-of-control, the AATS is a measure of the performance of the chart. A chart with a smaller AATS indicates the better detection ability of process mean shifts.

In the previous studies on EWMA control charts, the integral equation method, the Markov chain method, and the simulation method are usually used to compute the performance measures. In the present paper, the Markov chain and the integral equation methods are applied to obtain the values of in-control ATS and AATS.

4. The Gamma, Weibull, and *t* distributions

Borror *et al.* [3], Stoumbos and Reynolds [16], Calzada and Scariano [4,5], Maravelakis *et al.* [8], and Lin and Chou [6] studied the robustness of control charts to non-normality, where the Gamma and *t* distributions were used to represent various non-normal skewed and symmetric populations. The Weibull distribution has been used extensively in reliability engineering as a model of time to failure for electrical and mechanical components and systems and is also used to represent skewed populations in general.

Therefore, in the present paper, the Gamma, Weibull, and *t* distributions are chosen to investigate the effect of various population distributions on the performance of the EWMA and the combined $\bar{X}$–EWMA charts with VSI scheme.

The Gamma distribution, denoted by Gam(α, β), has the probability density function

$$f(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1}\,\mathrm{e}^{-x/\beta}, \quad x > 0, \alpha, \beta > 0, \tag{7}$$

where α and β are the shape and scale parameters, respectively. The mean of a Gamma distribution is $\alpha\beta$ and its variance is $\alpha\beta^2$. For $\alpha = 1$, the Gamma distribution reduces to the exponential distribution with mean β. As the shape parameter α is an integer, the Gamma distribution simplifies to the Erlang distribution.

The Weibull distribution, denoted by Wbl(a, b), has the probability density function

$$f(x) = \frac{a}{b}\left(\frac{x}{b}\right)^{a-1} \mathrm{e}^{-(x/b)^{a}}, \quad x > 0, a, b > 0, \tag{8}$$

where a and b are the shape and scale parameters, respectively. The mean of a Weibull distribution is $b\Gamma(1 + (1/a))$ and its variance is $b^2(\Gamma(1 + (2/a)) - \Gamma^2(1 + (1/a)))$. The Weibull distribution is related to a number of other probability distributions; in particular, it interpolates between the exponential distribution ($a = 1$) and the Rayleigh distribution ($a = 2$).

The *t* distribution is symmetric but has more probability in the tails than the normal distribution. The *t* distribution with ν degrees of freedom, denoted by t_ν, has the probability density function

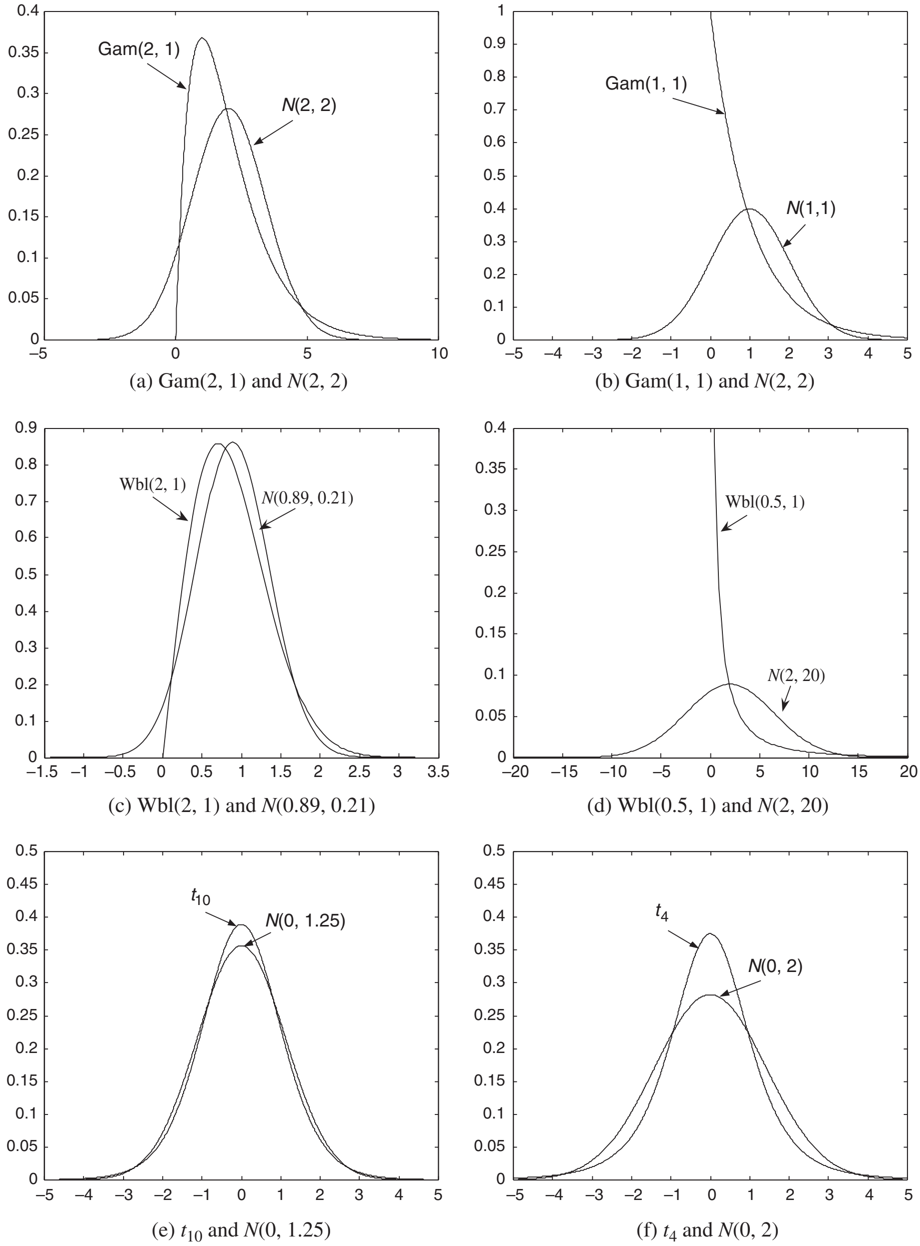
$$f(x) = \frac{\Gamma((v+1)/2)}{\Gamma(v/2)\sqrt{\pi\nu}} \left(1 + \frac{1}{\nu}x^2\right)^{-((v+1)/2)}, \quad -\infty < x < \infty. \tag{9}$$

The mean of a *t* distribution is 0 and its variance is $\nu/(\nu - 2)$, for $\nu > 2$.

In the present paper, we choose Gam(2, 1), Gam(1, 1), Wbl(2, 1), and Wbl(0.5, 1) to represent the cases of skewed distributions, and choose t_{10} and t_4 to represent symmetric distributions with heavier tails than the normal. Figure 1 depicts various Gamma, Weibull, and *t* distributions along with the corresponding normal distributions with the same mean and variance. From Figure 1, it is noted that both the Gam(1, 1) and the Wbl(0.5, 1) are much skewed to the right and far away from normality. Also, the t_4 is much more heavily tailed than the normal and the t_{10} distributions. These distributions – Gam(1, 1), Wbl(0.5, 1), and t_4 – may thus be much 'more non-normal' than is typical in industrial SPC applications, but are considered here to serve as upper bounds to the degree of departure from normality in the case of asymmetric and symmetric quality characteristics.

5. The effect of non-normality on the false alarm of EWMA charts

To study the robustness of the fixed sampling intervals (FSI) and VSI EWMA control charts to the non-normality, the action limits and warning limits are determined which give an in-control ATS approximately equal to 370 under normality. Based on this criterion, the action limit coefficients k_e for the EWMA control chart with $\lambda = 0.05$, 0.1, and 0.2 are 2.492, 2.703, and 2.860, respectively.



(a) Gam(2, 1) and $N(2, 2)$ (b) Gam(1, 1) and $N(2, 2)$

(c) Wbl(2, 1) and $N(0.89, 0.21)$ (d) Wbl(0.5, 1) and $N(2, 20)$

(e) t_{10} and $N(0, 1.25)$ (f) t_4 and $N(0, 2)$

Figure 1. The probability density function for various gamma, Weibull, t and normal distributions.

These limits are then also used in calculating all in-control ATS's for the non-normal distributions. According to Reynolds and Stoumbos [14], the short sampling interval h_2 should generally be taken as small as feasible, for example, $h_2 = 0.1$. In the present paper, the FSI control charts and VSI control charts with three sampling interval combinations, that is, $(h_1, h_2) = (1.25, 0.1)$, (1.5, 0.1), (1.9, 0.1), are studied for comparison as the process data are not normally distributed. Table 1 presents the values of the in-control ATS of the EWMA charts as the underlying distributions of the process characteristic are Gam(2, 1), Gam(1, 1), Wbl(2, 1), Wbl(0.5, 1), t_{10}, and t_4. In Table 1, the in-control ATS values of the EWMA chart with $\lambda = 0.05$ are close to 370.4 for the non-normal distributions; however, the in-control ATS values for the EWMA chart with $\lambda = 0.2$ are much less than 370 for the non-normal distributions. Thus, it may be concluded that the EWMA chart with a smaller λ value has a relatively larger in-control ATS value and consequently a lower false alarm rate. This result is expected because the EWMA statistic is a linear combination of sample means, and a small λ indicates that more previous sample means share the bulk of the total weight. Therefore, the EWMA statistic becomes ‘more normal’ and more robust to the non-normality than the individual sample mean.

For the combined $\bar{X}$–EWMA control charts, we must determine the values of λ, k_e, and k_x to obtain an in-control ATS of around 370 under the normality assumption. This is accomplished by increasing the k_e and k_x so that the chart has the same individual in-control ATS, but the joint in-control ATS is about 370. For example, the VSI $\bar{X}$ control chart with $(h_1, h_2) = (1.9, 0.1)$, $k_x = 3.193$, and $w_x = 1.025$ results in the in-control ATS of 959 and in-control ARL of 709.7. The VSI EWMA control chart with $(h_1, h_2) = (1.9, 0.1)$, $\lambda = 0.05$, $k_e = 2.756$, and $w_x = 0.983$ leads to the in-control ATS of 959 and in-control ARL of 709.7. Combining these two charts together, the VSI $\bar{X}$–EWMA control chart then produces the joint in-control ATS of 370 and in-control ARL of 370. In the present paper, this approach for determining the values of k_x and k_e to give the identical individual in-control ATS value is applied. In Table 1, it can be seen that the in-control ATS of the $\bar{X}$–EWMA control chart is not robust to non-normality. This is probably because the $\bar{X}$ control chart is relatively sensitive to non-normality. As one can see, the $\bar{X}$–EWMA control chart generally has a relatively higher false alarm rate, especially as the underlying distribution is far away from normality, for example, Gam(1, 1), Wbl(0.5, 1), or t_4. Thus, if one ignores the effect of non-normality and uses the traditional way to design the $\bar{X}$–EWMA control chart, the false alarm rate of the chart may be unreasonably high.

6. Effect of non-normality on the detection ability of the EWMA charts

For the study of detection ability, it is assumed that the process starts in control, the process mean is equal to μ_0 and the standard error is $\sigma_{\bar{X}}$. When the process is out-of-control, the occurrence of the assignable cause results in a shift in the process mean from μ_0 to μ_1. The shift in μ is measured in the standard error unit, that is, $\delta = \delta = |\mu_1 - \mu_0|/\sigma_{\bar{X}}$. Lucas and Saccucci [7] showed that the EWMA chart is substantially faster in detecting small shifts than the Shewhart control chart as the observations are from a normal distribution.

In Table 1, because the control charts under study do not have the same false alarm rates, it is difficult to compare the detection ability of these control charts. To overcome this problem, the values of k_e, w_e, k_x, and w_x are adjusted such that the in-control ATS and in-control ARL values of these charts are equal to the predetermined values (i.e. both the in-control ATS and ARL are set to be 370). Then, the AATS can be used in this section for a comparative study of the out-of-control performance of the charts. Tables 2–8 present the AATS values for the EWMA and $\bar{X}$–EWMA control charts with various parameters (e.g. λ, h_1, and h_2) and δ under normal, Gamma, Weibull, and t distributions. In the cases of greater departure from normality, for example, Gam(1, 1), Wbl(0.5, 1), or t_4, the action limit coefficients k_e or k_x need to be significantly increased to prevent an increase in the false alarm rate, and the warning limit coefficients w_e or w_x need to be decreased in order to reduce the AATS through the resulting increase in the frequency of occurrence of the short sampling interval.

From Tables 2–8, we confirm the expected result that the EWMA and $\bar{X}$–EWMA control charts with a small λ (e.g. $\lambda = 0.05$) have the better ability for detecting small mean shifts. But as the mean shift is large, to have a good detection ability, the value of λ should be increased (e.g. $\lambda = 0.2$). As expected from the previous studies on VSI control chart, adding the VSI feature to the EWMA and $\bar{X}$–EWMA control charts can substantially reduce the time required for detecting

Table 1. In-control ATS values for the EWMA and the $\bar{X}$–EWMA charts under various distributions.

Chart	FSI			VSI								
(h_1, h_2)	(1.00, 1.00)			(1.90, 0.10)			(1.50, 0.10)			(1.25, 0.1)		
λ	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2
EWMA chart												
Normal	370	370	370	370	370	370	370	370	370	370	370	370
Gam(2, 1)	372	315	207	372	317	212	372	317	213	373	318	213
Gam(1, 1)	369	274	163	370	278	168	371	279	170	372	279	171
Wbl(2, 1)	375	374	345	374	372	344	374	373	345	375	373	346
Wbl(0.5, 1)	321	199	121	340	225	149	346	229	153	347	227	144
t_{10}	361	335	280	364	339	286	363	338	285	363	337	283
t_4	343	274	188	356	291	206	354	287	202	351	283	197
k_e	2.492	2.703	2.860	2.492	2.703	2.860	2.492	2.703	2.860	2.492	2.703	2.860
w_e	–	–	–	0.626	0.648	0.662	0.852	0.884	0.903	1.137	1.177	1.206
$\bar{X}$−EWMA chart												
Normal	370	370	370	370	370	370	370	370	370	370	370	370
Gam(2, 1)	86.6	86.1	83.2	95.3	93.7	90.6	95.6	95.7	92.5	92.2	92.1	90.1
Gam(1, 1)	65.5	65	63.2	81.6	80.5	77.8	75.4	75.7	74.6	70.7	70.8	70.1
Wbl(2, 1)	211	211	206	209	208	204	210	211	207	214	214	210
Wbl(0.5, 1)	56.6	56.5	56.0	81.8	86.2	91.5	72.8	76.9	78.9	66.7	67.8	67.4
t_{10}	166	163	158	174	171	167	171	169	165	168	166	162
t_4	90.4	89.3	87.4	109	107	106	103	102	100	97.1	96.1	94.5
k_e	2.756	2.940	3.067	2.756	2.940	3.067	2.756	2.940	3.067	2.756	2.940	3.067
w_e	–	–	–	0.983	0.982	0.962	1.188	1.211	1.190	1.482	1.490	1.470
k_x	3.193	3.192	3.183	3.193	3.192	3.183	3.193	3.192	3.183	3.193	3.192	3.183
w_x	–	–	–	1.020	1.000	0.970	1.240	1.240	1.200	1.560	1.520	1.500

Table 2. Values of the AATS for the EWMA and the $\bar{X}$–EWMA charts under a normal distribution.

Chart	FSI			VSI								
(h_1, h_2)	(1.00, 1.00)			(1.90, 0.10)			(1.50, 0.10)			(1.25, 0.1)		
λ	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2
EWMA chart												
δ												
0.5	25.2	27.0	35.0	14.8[a]	15.5	21.9	15.3	16.2	23.05	16.53	17.73	25.03
1.0	10.0	9.03	9.09	5.63	4.68	4.31[a]	5.75	4.79	4.45	6.14	5.13	4.88
1.5	6.17	5.21	4.63	3.48	2.78	2.30[a]	3.55	2.81	2.30[a]	3.78	2.98	2.45
2.0	4.43	3.63	3.04	2.53	1.99	1.62	2.57	2.00	1.58[a]	2.73	2.11	1.65
3.0	2.82	2.24	1.77	1.66	1.32	1.13	1.67	1.29	1.02[a]	1.77	1.35	1.02[a]
4.0	2.07	1.62	1.25	1.25	1.08	1.00	1.26	0.97	0.83	1.32	0.99	0.77[a]
k_e	2.492	2.703	2.859	2.492	2.703	2.860	2.492	2.703	2.860	2.492	2.703	2.860
w_e	–	–	–	0.626	0.648	0.662	0.852	0.884	0.903	1.137	1.177	1.206
$\bar{X}$−EWMA chart												
δ												
0.5	29.0	32.5	44.5	16.5[a]	18.4	28.4	17.2	19.8	30.4	19.5	22.0	33.2
1.0	10.8	9.90	10.3	5.60	4.85	4.78[a]	5.70	5.00	5.00	6.38	5.48	5.56
1.5	6.17	5.36	4.92	2.95	2.55	2.31	2.95	2.54	2.27[a]	3.29	2.74	2.42
2.0	3.92	3.40	3.00	1.83	1.65	1.52	1.75	1.56	1.41[a]	1.90	1.63	1.44
3.0	1.61	1.50	1.37	1.07	1.05	1.03	0.91	0.89	0.87	0.85	0.82	0.79[a]
4.0	0.75	0.75	0.73	0.93	0.93	0.93	0.75	0.75	0.75	0.65	0.64	0.64[a]
k_e	2.756	2.940	3.067	2.756	2.940	3.067	2.756	2.940	3.067	2.756	2.940	3.067
w_e	–	–	–	0.983	0.982	0.962	1.188	1.211	1.190	1.482	1.490	1.470
k_x	3.193	3.192	3.183	3.193	3.192	3.183	3.193	3.192	3.183	3.193	3.192	3.183
w_x	–	–	–	1.020	1.000	0.970	1.240	1.240	1.200	1.560	1.520	1.500

Note: [a]The minimum value of AATS for the corresponding δ.

Table 3. Values of the AATS for the EWMA and the $\bar{X}$–EWMA charts under the Gam(2, 1) distribution.

Chart	FSI			VSI								
(h_1, h_2)	(1.00, 1.00)			(1.90, 0.10)			(1.50, 0.10)			(1.25, 0.1)		
λ	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2
EWMA chart												
δ												
0.5	26.9	30.6	48.2	15.4[a]	17.0	30.8	15.9	18.0	32.2	17.4	19.9	34.7
1.0	10.6	10.2	13.1	5.83	4.90	4.66[a]	5.97	5.00	4.82	6.37	5.35	5.36
1.5	6.44	5.70	6.11	3.57	2.87	2.44[a]	3.67	2.93	2.46	3.92	3.11	2.60
2.0	4.59	3.90	3.77	2.59	2.04	1.67[a]	2.65	2.08	1.67[a]	2.83	2.20	1.75
3.0	2.92	2.39	2.09	1.70	1.32	1.10	1.72	1.34	1.03[a]	1.83	1.40	1.07
4.0	2.14	1.73	1.45	1.26	1.05	1.00	1.30	0.98	0.83	1.36	1.04	0.77[a]
k_e	2.490	2.792	3.234	2.490	2.792	3.234	2.490	2.792	3.234	2.490	2.792	3.234
w_e	–	–	–	0.626	0.646	0.654	0.851	0.877	0.885	1.130	1.165	1.170
$\bar{X}$–EWMA chart												
δ												
0.5	31.6	39.0	65.9	20.2[a]	25.0	51.5	20.2[a]	25.5	50.1	21.2	26.9	50.6
1.0	11.9	11.8	16.3	6.47	5.63	5.82	6.46	5.62[a]	5.98	6.72	5.90	6.62
1.5	7.13	6.43	7.12	3.18	2.73	2.51	3.25	2.75	2.49[a]	3.47	2.91	2.60
2.0	5.02	4.33	4.26	1.66	1.51	1.43	1.71	1.50	1.38[a]	1.88	1.61	1.43
3.0	3.07	2.56	2.27	1.16	1.11	1.08	0.98	0.93	0.91	0.87	0.82	0.79[a]
4.0	2.01	1.70	1.47	1.05	1.03	1.00	0.88	0.85	0.82	0.76	0.74	0.71[a]
k_e	2.760	3.087	3.521	2.760	3.087	3.521	2.760	3.087	3.521	2.760	3.087	3.521
w_e	–	–	–	0.979	0.975	0.949	1.200	1.202	1.160	1.452	1.466	1.420
k_x	4.785	4.752	4.698	4.785	4.752	4.698	4.785	4.752	4.698	4.785	4.752	4.698
w_x	–	–	–	0.930	0.925	0.900	1.100	1.090	1.060	1.280	1.270	1.250

Note: [a]The minimum value of AATS for the corresponding δ.

Table 4. Values of the AATS for the EWMA and the $\bar{X}$–EWMA charts under the Gam(1, 1) distribution.

Chart	FSI			VSI								
(h_1, h_2)	(1.00, 1.00)			(1.90, 0.10)			(1.50, 0.10)			(1.25, 0.1)		
λ	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2
EWMA chart												
δ												
0.5	27.8	34.6	60.5	15.6[a]	17.7	37.0	16.1	19.0	39.1	17.6	21.3	42.2
1.0	10.9	11.1	16.5	5.88	4.98	4.88[a]	6.04	5.09	5.02	6.43	5.43	5.49
1.5	6.52	6.09	7.27	3.59	2.90	2.52[a]	3.71	2.97	2.55	3.96	3.15	2.68
2.0	4.64	4.11	4.29	2.60	2.06	1.71[a]	2.68	2.10	1.71[a]	2.86	2.23	1.79
3.0	2.96	2.51	2.30	1.72	1.32	1.11	1.74	1.36	1.03[a]	1.85	1.41	1.09
4.0	2.17	1.81	1.59	1.26	1.06	1.01	1.32	0.98	0.84	1.37	1.05	0.76[a]
k_e	2.494	2.910	3.489	2.494	2.910	3.489	2.494	2.910	3.489	2.494	2.910	3.489
w_e	–	–	–	0.624	0.643	0.646	0.849	0.870	0.870	1.124	1.151	1.140
$\bar{X}$−EWMA chart												
δ												
0.5	33.2	45.5	82.9	20.5	26.9	63.6	20.2[a]	27.3	61.3	21.3	29.2	61.5
1.0	12.3	13.2	20.9	6.54	5.70	6.04	6.36	5.57[a]	6.07	6.64	5.81	6.69
1.5	7.32	6.98	8.60	2.97	2.57	2.50	2.97	2.57	2.44[a]	3.29	2.74	2.51
2.0	5.17	4.63	4.90	1.37	1.32	1.35	1.19	1.15	1.18	1.39	1.20	1.11[a]
3.0	3.23	2.77	2.53	1.18	1.13	1.11	1.00	0.96	0.93	0.89	0.85	0.82[a]
4.0	2.27	1.93	1.70	1.08	1.05	1.03	0.90	0.87	0.85	0.79	0.76	0.74[a]
k_e	2.784	3.239	3.792	2.784	3.239	3.792	2.784	3.239	3.792	2.784	3.239	3.792
w_e	–	–	–	0.973	0.960	0.929	1.185	1.170	1.130	1.432	1.430	1.381
k_x	5.449	5.404	5.328	5.449	5.404	5.328	5.449	5.404	5.328	5.449	5.404	5.328
w_x	–	–	–	0.844	0.840	0.820	0.940	0.930	0.920	1.100	1.070	1.020

Note: [a]The minimum value of AATS for the corresponding δ.

Table 5. Values of the AATS for various EWMA charts and the $\bar{X}$–EWMA charts under the Wbl(2, 1) distribution.

Chart	FSI			VSI								
(h_1, h_2)	(1.00, 1.00)			(1.90, 0.10)			(1.50, 0.10)			(1.25, 0.1)		
λ	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2
EWMA chart												
δ												
0.5	25.8	27.2	33.7	15.1[a]	16.0	22.9	15.6	16.8	23.8	17.0	18.5	25.4
1.0	10.3	9.29	9.57	5.74	4.80	4.43[a]	5.86	4.89	4.58	6.26	5.26	5.06
1.5	6.29	5.34	4.87	3.54	2.84	2.36[a]	3.62	2.87	2.36[a]	3.85	3.06	2.51
2.0	4.50	3.70	3.16	2.56	2.03	1.65	2.61	2.04	1.62[a]	2.78	2.16	1.70
3.0	2.86	2.28	1.82	1.68	1.33	1.11	1.70	1.31	1.03[a]	1.80	1.38	1.05
4.0	2.10	1.65	1.28	1.26	1.06	0.99	1.28	0.98	0.82	1.34	1.02	0.77[a]
k_e	2.487	2.700	2.889	2.487	2.700	2.889	2.487	2.700	2.889	2.487	2.700	2.889
w_e	–	–	–	0.627	0.650	0.666	0.853	0.885	0.905	1.136	1.180	1.203
$\bar{X}$–EWMA chart												
δ												
0.5	29.5	32.4	42.9	18.6[a]	21.2	32.8	19.1	22.3	33.6	20.5	23.7	34.9
1.0	11.2	10.3	11.1	6.31	5.49	5.48[a]	6.26	5.47	5.55	6.66	5.80	6.11
1.5	6.59	5.68	5.36	3.26	2.80	2.49	3.27	2.77	2.43[a]	3.52	2.93	2.58
2.0	4.45	3.77	3.34	1.92	1.71	1.55	1.91	1.66	1.47[a]	2.06	1.74	1.52
3.0	2.23	1.94	1.69	1.08	1.05	1.03	0.91	0.88	0.85	0.86	0.81	0.77[a]
4.0	1.05	1.00	0.92	0.96	0.96	0.95	0.78	0.78	0.77	0.67	0.66	0.66[a]
k_e	2.747	2.940	3.122	2.747	2.940	3.122	2.748	2.940	3.122	2.747	2.940	3.122
w_e	–	–	–	0.988	0.991	0.973	1.190	1.216	1.190	1.467	1.491	1.485
k_x	3.610	3.601	3.582	3.610	3.601	3.582	3.605	3.601	3.582	3.610	3.601	3.582
w_x	–	–	–	1.040	1.030	1.000	1.260	1.230	1.200	1.500	1.480	1.460

Note: [a]The minimum value of AATS for the corresponding δ.

Table 6. Values of the AATS for various EWMA charts and the $\bar{X}$–EWMA charts under the Wbl(0.5, 1) distribution.

Chart	FSI			VSI								
(h_1, h_2)	(1.00, 1.00)			(1.90, 0.10)			(1.50, 0.10)			(1.25, 0.1)		
λ	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2
EWMA chart												
δ												
0.5	36.8	70.3	144.3	15.1[a]	16.9	37.8	15.4	17.8	53.1	16.5	20.9	66.7
1.0	12.2	17.8	51.7	5.68	5.15[a]	7.64	5.89	5.25	7.68	6.18	5.45	7.76
1.5	7.12	7.91	18.2	3.53	2.87[a]	3.35	3.64	2.92	3.29	3.81	3.03	3.34
2.0	5.07	5.18	7.47	2.56	2.07	1.72[a]	2.63	2.04	1.83	2.78	2.15	1.88
3.0	3.23	3.09	3.35	1.70	1.21	1.19	1.72	1.31	1.01	1.80	1.39	0.92[a]
4.0	2.37	2.21	2.23	1.16	1.09	1.08	1.29	0.92	0.91	1.36	0.94	0.79[a]
k_e	2.650	3.492	4.592	2.650	3.492	4.592	2.650	3.492	4.592	2.650	3.492	4.592
w_e	–	–	–	0.584	0.573	0.549	0.770	0.750	0.698	0.991	0.960	0.868
$\bar{X}$–EWMA chart												
δ												
0.5	45.6	90.0	174.3	18.8[a]	23.3	85.4	21.0	33.7	97.6	19.0	30.4	96.0
1.0	14.0	21.6	63.3	2.26	3.04	7.21	2.09[a]	2.87	7.04	2.51	3.06	7.02
1.5	8.10	9.06	22.3	1.67	1.77	3.10	1.49	1.60	2.93	1.38[a]	1.49	2.84
2.0	5.72	5.79	8.87	1.43	1.44	1.75	1.25	1.27	1.58	1.14[a]	1.16	1.47
3.0	3.61	3.43	3.65	1.21	1.20	1.23	1.04	1.03	1.05	0.93	0.92[a]	0.94
4.0	2.64	2.42	2.35	1.12	1.10	1.09	0.94	0.92	0.91	0.83	0.81	0.80[a]
k_e	2.976	3.813	4.878	2.976	3.813	4.878	2.975	3.813	4.878	2.976	3.813	4.890
w_e	–	–	–	0.852	0.820	0.757	1.231	1.131	0.936	1.255	1.140	1.013
k_x	8.211	8.038	7.850	8.211	8.038	7.850	8.215	8.038	7.833	8.211	8.038	7.840
w_x	–	–	–	0.440	0.440	0.440	0.450	0.450	0.450	0.520	0.510	0.500

Note: [a]The minimum value of AATS for the corresponding δ.

Table 7. Values of the AATS for various EWMA charts and the $\bar{X}$–EWMA charts under the t_{10} distribution.

Chart	FSI			VSI								
(h_1, h_2)	(1.00, 1.00)			(1.90, 0.10)			(1.50, 0.10)			(1.25, 0.1)		
λ	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2
EWMA chart												
δ												
0.5	25.6	28.6	42.5	14.7[a]	15.6	24.4	15.3	16.4	26.0	16.5	18.1	28.7
1.0	10.1	9.29	10.1	5.60	4.65	4.33[a]	5.73	4.77	4.48	6.12	5.13	4.93
1.5	6.19	5.31	4.96	3.46	2.76	2.29[a]	3.54	2.80	2.29	3.77	2.98	2.44
2.0	4.45	3.69	3.21	2.52	1.98	1.61	2.56	1.99	1.57[a]	2.73	2.11	1.65
3.0	2.84	2.28	1.86	1.65	1.31	1.13	1.67	1.28	1.01[a]	1.76	1.34	1.02
4.0	2.08	1.65	1.31	1.25	1.08	1.01	1.25	0.97	0.84	1.31	0.99	0.77[a]
k_e	2.502	2.745	2.978	2.502	2.745	2.978	2.502	2.745	2.978	2.502	2.745	2.978
w_e	–	–	–	0.623	0.641	0.648	0.849	0.876	0.885	1.133	1.173	1.188
$\bar{X}$−EWMA chart												
δ												
0.5	29.9	35.2	56.5	16.4[a]	18.6	32.9	17.5	20.2	35.9	19.6	22.9	40.3
1.0	11.3	10.5	11.9	5.44	4.70	4.69[a]	5.72	4.92	4.98	6.42	5.50	5.71
1.5	6.77	5.83	5.53	2.85	2.46	2.23	2.94	2.48	2.21[a]	3.33	2.74	2.41
2.0	4.72	3.95	3.47	1.82	1.63	1.50	1.78	1.55	1.39[a]	1.95	1.64	1.43
3.0	2.50	2.13	1.81	1.17	1.12	1.07	1.01	0.95	0.91	0.94	0.88	0.82[a]
4.0	1.15	1.08	0.98	0.98	0.97	0.96	0.80	0.79	0.78	0.69	0.68	0.67[a]
k_e	2.763	2.984	3.195	2.763	2.984	3.195	2.763	2.984	3.195	2.763	2.984	3.195
w_e	–	–	–	0.982	0.975	0.946	1.201	1.198	1.169	1.476	1.481	1.464
k_x	3.885	3.872	3.836	3.885	3.872	3.836	3.885	3.872	3.836	3.885	3.872	3.836
w_x	–	–	–	0.970	0.950	0.910	1.210	1.180	1.140	1.520	1.490	1.450

Note: [a]The minimum value of AATS for the corresponding δ.

Table 8. Values of the AATS for various EWMA charts and the $\bar{X}$–EWMA charts under the t_4 distribution.

Chart	FSI			VSI								
(h_1, h_2)	(1.00, 1.00)			(1.90, 0.10)			(1.50, 0.10)			(1.25, 0.1)		
λ	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2	0.05	0.1	0.2
EWMA chart												
δ												
0.5	26.7	34.6	77.8	14.2[a]	15.6	34.5	14.7	16.6	37.9	16.0	18.6	43.8
1.0	10.3	10.1	14.3	5.40	4.47	4.45[a]	5.54	4.60	4.62	5.95	4.96	5.11
1.5	6.26	5.63	6.07	3.34	2.65	2.23[a]	3.42	2.70	2.25	3.66	2.88	2.41
2.0	4.49	3.89	3.75	2.44	1.90	1.56	2.48	1.92	1.53[a]	2.65	2.04	1.61
3.0	2.87	2.40	2.12	1.60	1.27	1.14	1.63	1.24	1.00	1.72	1.31	0.99[a]
4.0	2.10	1.73	1.49	1.21	1.07	1.02	1.22	0.94	0.85	1.28	0.96	0.76[a]
k_e	2.531	2.874	3.325	2.531	2.874	3.325	2.531	2.874	3.325	2.531	2.874	3.325
w_e	–	–	–	0.600	0.607	0.599	0.819	0.833	0.825	1.099	1.121	1.119
$\bar{X}$–EWMA chart												
δ												
0.5	31.2	43.4	104.1	15.0[a]	18.2	46.8	16.4	20.4	54.0	18.5	24.0	64.1
1.0	11.5	11.5	17.6	4.72	4.13[a]	4.59	5.15	4.44	4.96	5.97	5.14	5.84
1.5	6.95	6.26	6.92	2.41	2.13	2.06	2.52	2.16	2.03[a]	2.95	2.45	2.25
2.0	4.96	4.28	4.16	1.64	1.50	1.44	1.56	1.39	1.31[a]	1.69	1.45	1.33
3.0	3.13	2.60	2.30	1.21	1.15	1.11	1.04	0.98	0.94	0.96	0.89	0.84[a]
4.0	2.20	1.81	1.55	1.08	1.04	1.02	0.91	0.87	0.84	0.80	0.76	0.73[a]
k_e	2.784	3.117	3.561	2.784	3.117	3.561	2.784	3.117	3.561	2.784	3.117	3.561
w_e	–	–	–	0.936	0.926	0.870	1.150	1.145	1.094	1.410	1.420	1.372
k_x	5.341	5.239	5.096	5.341	5.239	5.096	5.341	5.239	5.096	5.341	5.239	5.096
w_x	–	–	–	0.830	0.810	0.780	1.060	1.030	1.000	1.370	1.340	1.310

Note: [a]The minimum value of AATS for the corresponding δ.

shifts in the mean. The control charts with $(h_1, h_2) = (1.9, 0.1)$ have a relatively low AATS for detecting small mean shifts, and the charts with $(h_1, h_2) = (1.25, 0.1)$ or $(1.5, 0.1)$ have a relatively low AATS for detecting large mean shifts, which indicates that a large h_1 (e.g. $h_1 = 1.9$) is generally good for small mean shift detection. The control scheme with a large h_1 results in a narrow central region (i.e. the area between warning limits), and consequently a relatively large probability of applying the short sampling interval when the process is out-of-control, which definitely leads to a reduction of the time for detecting mean shifts.

The results from Tables 3 to 8 show that the AATS values for the EWMA chart with $\lambda = 0.1$ and 0.2 under Gamma, Weibull, and t distributions are generally larger than the corresponding AATS for the normal distribution, which indicates that it usually takes more time to detect the mean shifts if the underlying distribution is not normal. However, the AATS values for the EWMA chart with $\lambda = 0.05$ under normality are quite close to the values of AATS under non-normal distributions. Here, we confirm the expected result that for the EWMA chart with a small value of λ (e.g. $\lambda = 0.05$), the EWMA statistic is 'more normal' and the corresponding chart would be quite robust to non-normality.

For the normally distributed case, the combined $\bar{X}$–EWMA control chart depends on its $\bar{X}$ chart for detecting the large shifts in the mean, and depends on the EWMA chart for detecting the small shifts in the mean. Thus, this combined $\bar{X}$–EWMA control procedure is generally effective against both large and small mean shifts for normal underlying distributions. In order to determine the effectiveness of the $\bar{X}$–EWMA control chart in detecting the shifts in the mean under various non-normal distributions, a computer simulation with 100,000 replicates is run to estimate the conditional probability of a signal by the EWMA or the $\bar{X}$ chart, given that at least one chart signals. The results are shown in Table 9, where $P(\bar{X}$ chart), P(EWMA), and P(both) denote the conditional probabilities that the $\bar{X}$ chart alone signals, the EWMA chart alone signals, and both charts signal, respectively, given that at least one chart signals.

From Table 9, for the case of normality, it may be seen that P(EWMA) is large and $P(\bar{X}$ chart) is small when the mean shifts is small, for example, $P(\text{EWMA}) = 0.891$ and $P(\bar{X}\text{ chart}) = 0.076$ as $\lambda = 0.05$ and $\delta = 0.5$; P(EWMA) is small and $P(\bar{X}$ chart) is large when the mean shifts is large, for example, $P(\text{EWMA}) = 0.007$ and $P(\bar{X}\text{ chart}) = 0.905$ as $\lambda = 0.05$ and $\delta = 4$. Thus, the EWMA chart alone is likely to signal when a small shift in the mean occurs in the process, and the $\bar{X}$ chart alone is likely to signal when a large shift in the mean occurs in the process. This result is consistent with the general conclusion in the literature [14]. However, in the cases of a large departure from normality, for example, Gam(1, 1), Wbl(0.5, 1), or t_4, P(EWMA) is always large and $P(\bar{X}$ chart) is always small no matter the mean shift is small or large. For example, for the distribution of Gam(1, 1), $P(\text{EWMA}) = 0.914$ and $P(\bar{X}\text{ chart}) = 0.026$ as $\lambda = 0.05$ and $\delta = 0.5$, and $P(\text{EWMA}) = 0.767$ and $P(\bar{X}\text{ chart}) = 0.103$ as $\lambda = 0.05$ and $\delta = 4$. The EWMA chart alone is likely to signal when the small or large shift in the mean occurs in the process under extreme non-normal situations. This result should also be expected since the $\bar{X}$ chart is quite sensitive to non-normality, therefore it would require wide action limits in order not to increase the false alarm rate, and this situation definitely hampers its performance; however, the EWMA chart is less sensitive to non-normality, therefore its control limits are less affected and its performance is preserved under non-normality. So, based on this observation, it may be concluded that the $\bar{X}$–EWMA control chart is not as effective as the EWMA chart in detecting mean shifts under non-normality.

7. Conclusions

Non-normal data commonly exist in many industrial processes. In the present paper, we use the Gamma, Weibull, and t distributions to evaluate the robustness of the VSI EWMA and the combined $\bar{X}$–EWMA control charts to non-normality. Based on our observations, the charts

Table 9. The conditional signal probabilities for the $\bar{X}$–EWMA charts under various distributions.

	$\lambda = 0.05$			$\lambda = 0.1$			$\lambda = 0.2$		
	$P(\bar{X}$ chart)	P(both)	P(EWMA)	$P(\bar{X}$ chart)	P(both)	P(EWMA)	$P(\bar{X}$ chart)	P(both)	P(EWMA)
Normal									
δ									
0.5	0.076	0.034	0.891	0.073	0.047	0.879	0.090	0.080	0.830
1.0	0.104	0.058	0.839	0.076	0.072	0.852	0.056	0.101	0.843
1.5	0.200	0.104	0.696	0.143	0.122	0.734	0.095	0.155	0.749
2.0	0.361	0.158	0.481	0.264	0.191	0.545	0.177	0.239	0.584
3.0	0.736	0.165	0.100	0.590	0.259	0.151	0.423	0.379	0.199
4.0	0.905	0.089	0.007	0.779	0.206	0.015	0.577	0.396	0.027
Gam(2, 1)									
δ									
0.5	0.036	0.055	0.909	0.031	0.086	0.883	0.041	0.170	0.789
1.0	0.026	0.039	0.935	0.017	0.052	0.932	0.010	0.089	0.901
1.5	0.031	0.043	0.926	0.020	0.051	0.929	0.012	0.071	0.917
2.0	0.043	0.055	0.902	0.029	0.062	0.909	0.018	0.078	0.905
3.0	0.101	0.108	0.791	0.069	0.118	0.813	0.048	0.136	0.816
4.0	0.242	0.210	0.549	0.174	0.238	0.589	0.125	0.267	0.608
Gam(1, 1)									
δ									
0.5	0.026	0.061	0.914	0.023	0.102	0.875	0.030	0.215	0.756
1.0	0.016	0.037	0.947	0.011	0.051	0.939	0.006	0.096	0.897
1.5	0.018	0.036	0.946	0.012	0.043	0.946	0.007	0.065	0.928
2.0	0.023	0.041	0.936	0.015	0.047	0.938	0.009	0.061	0.930
3.0	0.046	0.068	0.886	0.032	0.075	0.893	0.021	0.086	0.893
4.0	0.103	0.131	0.767	0.074	0.143	0.783	0.055	0.158	0.787
Wbl(2, 1)									
δ									
0.5	0.084	0.053	0.863	0.079	0.073	0.848	0.087	0.121	0.792
1.0	0.084	0.064	0.853	0.059	0.078	0.863	0.041	0.114	0.846
1.5	0.131	0.091	0.778	0.092	0.109	0.800	0.057	0.136	0.808
2.0	0.212	0.133	0.655	0.149	0.155	0.696	0.094	0.189	0.717
3.0	0.500	0.210	0.290	0.372	0.269	0.360	0.248	0.335	0.417
4.0	0.830	0.129	0.041	0.661	0.280	0.059	0.493	0.387	0.120

Wbl(0.5, 1)									
δ									
0.5	0.003	0.106	0.891	0.002	0.228	0.771	0.000	0.474	0.525
1.0	0.002	0.039	0.960	0.001	0.067	0.932	0.000	0.211	0.789
1.5	0.002	0.029	0.969	0.001	0.034	0.965	0.000	0.092	0.907
2.0	0.002	0.023	0.975	0.001	0.027	0.972	0.000	0.046	0.954
3.0	0.003	0.022	0.976	0.002	0.025	0.973	0.001	0.031	0.968
4.0	0.005	0.026	0.969	0.003	0.029	0.968	0.001	0.033	0.965
t_{10}									
δ									
0.5	0.039	0.025	0.936	0.038	0.038	0.923	0.052	0.080	0.869
1.0	0.030	0.026	0.944	0.021	0.031	0.948	0.015	0.049	0.936
1.5	0.048	0.040	0.912	0.032	0.045	0.923	0.021	0.059	0.921
2.0	0.092	0.069	0.839	0.062	0.077	0.861	0.040	0.093	0.867
3.0	0.336	0.187	0.477	0.244	0.221	0.535	0.165	0.265	0.569
4.0	0.721	0.197	0.082	0.571	0.306	0.123	0.410	0.435	0.155
t_4									
δ									
0.5	0.028	0.029	0.943	0.032	0.053	0.916	0.064	0.159	0.777
1.0	0.011	0.016	0.973	0.008	0.021	0.971	0.009	0.040	0.951
1.5	0.009	0.014	0.977	0.006	0.017	0.977	0.004	0.024	0.972
2.0	0.010	0.016	0.974	0.007	0.019	0.974	0.005	0.024	0.971
3.0	0.022	0.032	0.946	0.016	0.037	0.947	0.012	0.045	0.942
4.0	0.083	0.095	0.822	0.066	0.112	0.822	0.058	0.142	0.800

adjusted to maintain the specified in-control ATS under non-normality exhibit almost the same behavior as the charts under normal data in what concerns the effects of the VSI feature of the value of λ, h_1 (the larger sampling interval), and warning limits. Therefore, we confirm the expected results that applying the VSI feature in a EWMA chart brings substantial improvements in the ability to detect shifts in the process mean for both normality and non-normality, and that small values of λ and w and large values of h_1 are more effective against small shifts. In addition, we also confirm that the EWMA control chart with a small λ is quite robust to non-normality. However, the combined $\bar{X}$–EWMA chart is less effective than the EWMA chart in detecting mean shifts for non-normal distributions. Thus, it seems that the $\bar{X}$–EWMA control chart is not a good choice for detecting the shifts in mean under non-normality.

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