Understanding the impact of brand colour on brand image: A preference disaggregation approach☆

Mohammad Ghaderi [a,*], Francisco Ruiz [b], Núria Agell [a]

[a] *ESADE Business School-Ramon Llull University, Sant Cugat, Spain*
[b] *Automatic Control Department, BarcelonaTech, Vilanova i la Geltrú, Spain*

ARTICLE INFO

ABSTRACT

What is the role that colour plays in perception of a brand by customers? How can we explore the cognitive role that colour plays in determining brand perception? To answer these questions we propose a preference disaggregation method based on *multi-criteria decision aid*. We identify the criteria aggregation model that underlies the global preference of a brand with respect to each brand image attribute. The proposed method is inspired by the well-known UTASTAR algorithm, but unlike the original formulation, it represents preferences by means of non-monotonic value functions. The method is applied to a database of brands ranked on each brand image attribute. For each brand image attribute, non-monotonic marginal value functions from each component of the brand colour are obtained separately. These functions contain the fitness between each colour component and each brand image attribute, in an understandable manner.

© 2015 Published by Elsevier B.V.

1. Introduction

Colour is one of the key ingredients of brands which plays an important role in the purchase decisions of customers. As an aesthetic stimuli, colour can shape consumer preferences and alter perceptions by communicating meaningful messages [22]. As an essential element of a brand, colour can signal quality [16], affect perception of quality [3], contribute to brand recognition and brand image [15], and affect brand personality [13]. Colour, in addition, intrinsically communicates the desired image [2] and is considered a strategic tool for marketers and brand managers for differentiating brands from competitors, signalling product attributes, and grabbing customer attention [20].

Colour operates via two mechanisms: sensory and cognitive. In the sensory mechanism, colour helps retrieve information in blurry conditions, by distinguishing, for example, an object from its background. In the cognitive mechanism, colour helps perception by playing a diagnostic role and characterising the object that is being represented (an orange sunset and the blue of the sea have specific meaning). As brand image is characterised by the perception of the customers, brand colour influences brand image through the cognitive mechanism [21].

The importance of colour to the marketers is not limited to brand colours. Studies support a significant impact of packaging colours on customer intention to buy and perceived quality. Hoegg and Alba found that colour cues dominate taste cues. In their experiment using orange juice, participants perceived a significantly greater difference in the taste of two identical samples with different colours, than two different samples with the same colour [10]. Garber et al., in their experimental study, found that colour affects identification and flavour perception of both congruently and incongruently coloured beverages [9].

The common practice for understanding colour trends in industry is based on the opinion of field experts, whose judgements are based on past experiences and are difficult to substitute by analytic models. In this paper, we explore the relationship between brand colour and customer perception of brand image in an understandable and interpretable manner. To this end, we propose a preference disaggregation method based on a multi-criteria decision analysis (MCDA) framework. The aim of this approach is to analyse the holistic preferences of a set of alternatives in a multi-criteria setting in order to identify the criteria aggregation model that underlies global preferences, and represent the existing preferential system using a set of marginal value functions.

To address this paradigm, several methods have been proposed in the literature considering different forms of comprehensive preferences and various tasks, for instance UTA (*UTilités Additives*) [11], Pairwise comparisons UTA [6], UTADIS (*UTilités Additives DIScriminantes*) [5], fuzzy UTASTAR [19] and many others [6,12]. Most of these methods assume a monotonic relationship between preferences and

attribute levels. However, as the relationship between brand perception and colour attributes, for example colour hue, is not necessarily monotone, we introduce a new method based on UTASTAR, that is applicable in non-monotonic settings.

The paper is organised as follows. A brief introduction on colour measurement is provided in the next section. An overview of the theoretical framework of preference disaggregation is then presented. Because we are focusing on the ranking problem, the section contains a review of the most widely used UTA variant, UTASTAR, and some of the non-monotonic UTA-like methodologies for the ranking problem. In Section 4, the proposed methodology is introduced, followed by an illustrative example to make a comparison with the UTA-NM method. In Section 5 the method is applied to a comprehensive set of brand image attributes, in order to explore the impact of brand colour on brand image. Finally, we conclude the paper and present possible future directions.

2. Colour coordinates and colour spaces

Several numeric specifications for colour definition can be found in the literature. We refer the interested reader to the recent study in [8]. The most classic and internationally accepted of these are based on tristimulus values or coordinates. The most known of these is RGB, proposed by the *Commission International de l'Eclairage* (CIE) in 1931. RGB uses additive colour mixing and describes what type of light (red, green or blue) needs to be emitted to produce a given colour. The RGB colour model is implemented in different ways, depending on the capabilities of the system used. By far the most common is the 24-bit implementation. This model is thus limited to a range of 256 × 256 × 256 ≈ 16.7 million colours. It is a convenient colour model for computer graphics, but it can be unintuitive in practice. The specification of a desired colour can be difficult for untrained people (for example, selecting brown using a RGB vector can be difficult). HSV is another colour space which was developed to approximate the way humans perceive colours. For this reason, in marketing studies HSV colour space is widely used. In this single-hexcone model of colour space, hue (H) of a colour refers the pure colour it resembles and demonstrates its position on the colour wheel, where it starts from 0 for red, and continues to 60 for yellow, 120 for green, and ends up at 360 or the starting position. Saturation (S) refers to the intensity of the pure colour. In other words, it describes the purity of the colour with respect to white. The value of 100 means a very vivid colour, while 0 means the least purity, where too much white dominates the colour. Value (V) measures the brightness of the colour where 100 means a totally bright and 0 means a totally dark colour. Most colour researchers in marketing focus only on colour hue and usually do not consider the other two attributes. Geometrical representation of the two colour systems RGB and HSV is presented in Fig. 1.

3. Preference disaggregation methodologies

UTA (Utilités Additives) is one of the most representative preference disaggregation methods. It was first introduced by Jacquet-Lagrèze and Siskos as a linear programming (LP) model to capture the preferential system of the decision maker (DM) through nonlinear (piecewise linear) monotonic additive value functions [11]. The aim of the UTA method is to reproduce, through a set of value functions, the ranking made by the DM over the set of alternatives by minimising the level of ranking errors. Ranking errors are generally defined as the distance between the global values of two consecutive alternatives that are ranked incorrectly. However, the definition of the error slightly differs in the variants of UTA. The method leads to a simple LP model where the optimal solution can be easily obtained.

Several extensions of UTA method have been introduced in the MCDA literature since then, incorporating variations on the original algorithm and considering different forms of global preference and optimality criteria. In most of the extensions of UTA method, the
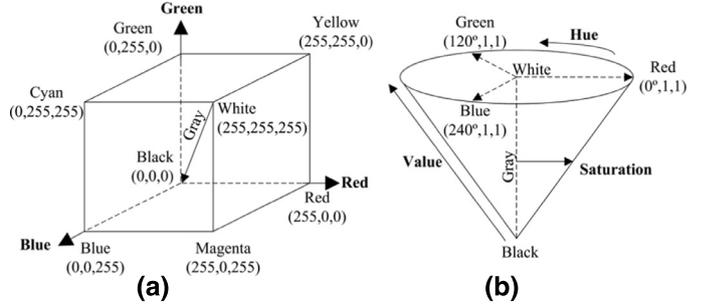

Fig. 1. RGB (a) and HSV (b) coordinates.

input attributes are normally expected to be monotone with respect to the preferences. The assumption of monotonicity is widely used, and it seems reasonable for many criteria, such as price, risk level, security, safety, comfort, required time, and effort. However, this is not the case for many other attributes, such as colour coordinates. In this paper, we propose an extension for UTA method to handle non-monotone preferences suitable for addressing the problem of understanding the impact of brand colour on brand image. In the following subsections, we present the most representative UTA method for ranking (UTASTAR) and briefly introduce some variants of the method which attempts to consider non-monotonic attributes.

3.1. UTASTAR method

Suppose that $G = \{g_1, g_2, \ldots, g_m\}$ is a set of criteria to evaluate a set of preordered alternatives $A = \{a_1, a_2, \ldots, a_N\}$ in which a_1 is the most and a_N is the least preferred alternative in the ranking list. Each criterion is defined as a function $g_i : A \to \mathbb{R}$, where $g_i(a_n) = x_i^n$. The value x_i^n is the performance of the alternative a_n over the criterion g_i. Given a weak ordering (ranking) over the set of alternatives specified by the DM, the aim of the UTASTAR algorithm is to represent the underlying preference model of the given ranking through estimating a set of monotonic additive value functions (as consistent as possible with the preferential structure of the DM). Specifically, the UTA method estimates a set of marginal value functions $v_i: g_i \to [0, 1]$ to be aggregated in an additive manner in order to estimate the comprehensive value associated with each alternative. Finally, alternatives are ranked based on the comprehensive values. The formulation of the UTASTAR method involves defining α_i characteristic points and henceforth $\alpha_i - 1$ subintervals $[g_i^0, g_i^1], [g_i^1, g_i^2], \ldots, [g_i^{\alpha_i-2}, g_i^{\alpha_i-1}]$ on the ith criterion, in which g_i^0 and $g_i^{\alpha_i-1}$ are the minimum and maximum performance levels over the ith scale, respectively. The marginal value at a characteristic point g_i^l on criterion i is expressed as in Eq. (1),

$$v_i(g_i^l) = \sum_{j=1}^{l} (v_i(g_i^j) - v_i(g_i^{j-1})) = \sum_{j=1}^{l} v_{ij} \tag{1}$$

where $v_{ij} \equiv v_i(g_i^j) - v_i(g_i^{j-1}) \geq 0$ due to the monotonicity of the criteria.

The marginal value for an alternative a_n whose performance on the ith scale is $x_i^n \in [g_i^l, g_i^{l+1}]$ is obtained by linear interpolation between $v_i(g_i^l)$ and $v_i(g_i^{l+1})$ as follows:

$$v_i(x_i^n) = \sum_{j=1}^{l} v_{ij} + \frac{x_i^n - g_i^l}{g_i^{l+1} - g_i^l} \cdot v_{i,l+1}. \tag{2}$$

The comprehensive value of an alternative a_n is obtained by the sum of all the marginal values, as in Eq. (3),

$$V(a_n) = \sum_{i=1}^{m} v_i(x_i^n). \tag{3}$$

The UTASTAR linear programming problem is provided in (4).

$$\min z = \sum_{n=1}^{N} (\sigma^+(a_n) + \sigma^-(a_n))$$

subject to

$$V'(a_n) - V'(a_{n+1}) \geq \delta \text{ iff } a_n \succ a_{n+1}, \forall n = 1, 2, \ldots, N-1$$

$$V'(a_n) - V'(a_{n+1}) = 0 \text{ iff } a_n \sim a_{n+1}, \forall n = 1, 2, \ldots, N-1$$

$$\sum_{i=1}^{m} \sum_{j=1}^{\alpha_i - 1} v_{ij} = 1$$

$$V'(a_n) = V(a_n) - \sigma^+(a_n) + \sigma^-(a_n)$$

$$v_{ij}, \sigma^+(a_n), \sigma^-(a_n) \geq 0, \forall i, j, n \qquad (4)$$

in which $\sigma^+(a_n)$ and $\sigma^-(a_n)$ are the overestimation and underestimation error terms, respectively. The term δ is a parameter (a small value), and the first two constraints represent the preorder relations provided by the DM. The third constraint ensures that the maximal shares of the criteria in the comprehensive value of the alternatives sum up to 1, and the objective function minimises the deviation of the estimated value function from the preferential model of the DM. By solving this model, the marginal value function over each criterion scale will be constructed based on the expression in (1).

3.2. Non-monotonic UTA-like algorithms

The input attributes in the UTASTAR method are normally expected to be monotone with respect to the preferences. However, this is not a reasonable requirement for colourimetric components. Obviously, no one can expect a monotonic relationship between a colour preference degree and its degree of greenness, or hue. Therefore, an improvement in the UTASTAR algorithm for handling non-monotonic preferences is of a great importance in this setting.

Although several attempts have been made in the literature to overcome the mentioned shortcoming [4,6,7,12], all are computationally intensive, or require extra information from the DM. One way to address non-monotone preferences is to divide the range of the criteria into intervals so that the preferences are monotonic in each interval, and then treat each interval separately. Following this idea, in the approach of Despotis and Zopounidis, it is assumed that each marginal value function is non-decreasing from the starting point of the range to a middle point, and it is non-increasing from this middle point to the end of the range [4]. This middle point corresponds to the most preferable value of the criterion. The main drawback of this method is that the exact value function shape and the most preferable value need to be known beforehand. Kliegr proposed another non-monotone methodology called UTA-NM, which relaxes the monotonicity condition of the UTASTAR algorithm, that in theory allows any shape for the marginal value function [12]. To avoid the over-fitting problem, UTA-NM simultaneously minimise the sum of the errors and the complexity of the model expressed by the number of changes in the sign of the marginal value functions. The method suffers from severe performance issues. Even for very small toy problems, tens of binary variables were involved, causing the method to be computationally infeasible for real-world problems. In another paper, Eckhardt and Kliegr propose *local preferences transformation*, a heuristic attribute preprocessing algorithm that transforms arbitrary input attributes into a space approximately monotone with respect to user preferences, thus making it suitable for UTA [7]. Finally, non-monotonic additive value functions, introduced by Doumpos in 2012 [6], consider a broader class of non-monotonic value functions that leads to a nonlinear integer programming problem, which is difficult to solve with data sets of realistic size. Thus, an evolutionary approach is employed, based on the differential evolution algorithm.

4. Proposed methodology

The method we introduce here, inspired by the UTA methodology, is fast and tractable. The general idea is to relax sign constraint in the decision variables that represent difference of value levels between two consecutive breakpoints. Therefore, marginal value function can change the monotonicity at any breakpoint.

This may lead to two problems: the first is the over-fitting problem in the case that monotonicity changes arbitrarily many times. This potential problem is handled simultaneously in two ways. First, we defined a small, but reasonable, number of breakpoints. The breakpoints are constructed so that each sub-interval contains the same number of data points and hence the same amount of information. Second, the slope of the marginal value function in each sub-interval is controlled by defining upper and lower bounds for the associated decision variables. The bounds for each decision variable is defined with respect to the length of the corresponding sub-interval. The longer the sub-interval, the wider the bound. This constraint not only controls the over-fitting problem, but also increase interpretability of the extracted value functions.

The second problem is about normalisation. By normalisation, we mean that the minimum and maximum global values must be equal to zero and one, respectively. Fixing the minimum and maximum global values is essential for obtaining the relative importance of the criteria. The challenge is that we cannot predict where the maximum value will be achieved on each criterion scale in order to force the sum to be one. Furthermore, we do not know the attribute level corresponding to the minimum marginal value on each criterion to set them equal to zero. To solve this problem, an iterative approach is followed. Whenever the maximum global value is less than one, its value is forced to be increased in the next iteration, by adding a new constraint considering the performance level corresponding to the highest marginal value in the current stage. The added constraint is applied in the next iteration, and will be removed from the LP model in the following iterations, because it does not have to be necessarily satisfied in the final solution. Whenever the maximum global utility is greater than one, a restrictive constraint is imposed to ensure that the global utility of the attribute levels corresponding to the highest marginal utility in the current stage will not have a value greater than one in all the following iterations. Furthermore, to satisfy another condition of normalisation (namely, minimum global utility being zero), a penalisation term is added to the objective function to penalise any deviation.

4.1. Characteristic points definition

Defining the breakpoints is an important step in all the UTA-like methodologies as it directly affects the number of decision variables. We define the breakpoints based on the idea of equal frequency intervals. This means that we expect equal numbers of distinct performance values in each sub-interval of the criterion, except for the two ends of the criterion scale. Let us denote by h_i the number of distinct performance levels of alternatives over the ith criterion, and by c_i the desired frequency in each sub-interval of the ith scale. It is easy to show that the number of decision variables corresponding to the ith criterion is equal to $[h_i/c_i] + 1$, in which $[x]$ is the largest integer number less than or equal to x. Considering that the number of distinct performance levels might be much higher for some criteria than others, defining the same value for all the c_i variables leads to associating many decision variables with the former, and few with the latter. This leads to over-fitting on the former criteria and inaccurate results on the latter, resulting in dramatically different degrees of freedom for the different estimated value functions. To overcome this issue, we suggest that c_i be a function of h_i, and we propose $c_i = [\sqrt{h_i}]$. Following this method, we expect that all the criteria will have almost the same degree of freedom.

4.2. Initial solution

Following UTASTAR notation, the marginal utilities for each criterion $i \in \{1, 2, \ldots, m\}$ are represented as in Eq. (1), except for a new type of decision variable that has been added, $v_{i,0}$, which allows any level of value within the range [0, 1] for the lowest possible performance over the criterion scale. The marginal value at a breakpoint g_i^l on criterion i is expressed as:

$$v_i(g_i^l) = v_{i,0} + \sum_{j=1}^{l} (v_i(g_i^j) - v_i(g_i^{j-1})) = \sum_{j=0}^{l} v_{ij} \tag{5}$$

and the marginal value for an alternative a_n whose performance on the ith scale is $x_i^n \in [g_i^l, g_i^{l+1}]$ is obtained by linear interpolation between $v_i(g_i^l)$ and $v_i(g_i^{l+1})$, as follows:

$$v_i(x_i^n) = \sum_{j=0}^{l} v_{ij} + \frac{x_i^n - g_i^l}{g_i^{l+1} - g_i^l} \cdot v_{i,l+1}. \tag{6}$$

The comprehensive value is obtained by the formula in (3). No normalisation constraint is imposed in the initial solution, and sign constraint over decision variables are relaxed. However, some constraints are imposed to obtain a solution as close as possible to the feasible solution. The first issue to be considered here is having a non-negative estimated marginal value over any characteristic point. Suppose that vector $\boldsymbol{V_i} = (v_{i,0}, v_{i1}, \ldots, v_{i,\alpha_i-1})$ demonstrates the decision variables corresponding to the marginal value of the ith criterion. The following set of constraints then guarantees that the estimated marginal value at any point on a criterion scale is non-negative:

$$\left. \begin{array}{l} v_{i,0} \geq 0 \\ v_{i,0} + v_{i1} \geq 0 \\ v_{i,0} + v_{i1} + v_{i2} \geq 0 \\ \ldots \\ \ldots \\ \ldots \\ v_{i,0} + v_{i1} + v_{i2} + \cdots + v_{i,\alpha_i-1} \geq 0 \end{array} \right\} \text{ for } i = 1, 2, .., m. \tag{7}$$

The following set of constraints also guarantees that the estimated marginal value at any point on a criterion scale is less than 1:

$$\left. \begin{array}{l} v_{i,0} \leq 1 \\ v_{i,0} + v_{i1} \leq 1 \\ v_{i,0} + v_{i1} + v_{i2} \leq 1 \\ \ldots \\ \ldots \\ \ldots \\ v_{i,0} + v_{i1} + v_{i2} + \cdots + v_{i,\alpha_i-1} \leq 1 \end{array} \right\} \text{ for } i = 1, 2, .., m. \tag{8}$$

Note that (7) and (8) can be written in a more compact way using the $\alpha_i \times \alpha_i$ lower triangular matrix A_i with $a_{kp} = 1$ for elements where $k \geq p$. Then (7) and (8) can be written as:

$$A_i V_i \geq 0, \forall i \tag{9}$$

$$A_i V_i \leq 1, \forall i. \tag{10}$$

It is important to bear in mind that the normalisation condition is not guaranteed in the initial solution because the maximum of the estimated comprehensive value is not necessarily equal to 1.

Finally, the following set of constraint limits the slope of the value function at any interval

$$\left| \frac{v_{ij}}{g_i^j - g_i^{j-1}} \right| \leq \frac{1}{g_i^0 - g_i^{\alpha_i-1}}, \forall i = 1, 2, .., m, \forall j = 1, 2, .., \alpha_{i-1}. \tag{11}$$

The linear format of the above constraints is presented as follows:

$$\left. \begin{array}{l} v_{ij} \leq \dfrac{g_i^j - g_i^{j-1}}{g_i^0 - g_i^{\alpha_i-1}} \\[2mm] -v_{ij} \leq \dfrac{g_i^j - g_i^{j-1}}{g_i^0 - g_i^{\alpha_i-1}} \end{array} \right\} \forall i = 1, 2, \ldots, m \quad \forall j = 1, 2, \ldots, \alpha_{i-1}. \tag{12}$$

The LP model of the initial solution is presented in (13).

$$\min z = \sum_{n=1}^{N} (\sigma^+(a_n) + \sigma^-(a_n))$$

subject to

$$V'(a_n) - V'(a_{n+1}) \geq \delta \text{ iff } a_n \succ a_{n+1}, \forall n = 1, 2, \ldots, N-1$$

$$V'(a_n) - V'(a_{n+1}) = 0 \text{ iff } a_n \sim a_{n+1}, \forall n = 1, 2, \ldots, N-1$$

set of constraints in (7)

set of constraints in (8)

set of constraints in (12)

$$V'(a_n) = V(a_n) - \sigma^+(a_n) + \sigma^-(a_n)$$

$$v_{ij} \text{ URS}, i = 1, 2, \ldots, m, j = 0, 1, \ldots, \alpha_i - 1$$

$$\sigma^+(a_n), \sigma^-(a_n) \geq 0, n = 1, 2, \ldots, N \tag{13}$$

in which URS means UnRestricted in Sign variable.

In the outcome achieved by solving the model in (13), let the breakpoints with maximum and minimum marginal value on the ith criterion scale be g_i^* and g_{i*}, respectively,

$$g_i^* = \arg \max_j v_i(g_i^j) \tag{14}$$

$$g_{i*} = \arg \min_j v_i(g_i^j). \tag{15}$$

Furthermore, let us assume that f^* denotes the sum of the overestimation and underestimation errors in the optimal solution of the model in (13). By storing this information, the iterative part of the algorithm can be started as explained in the following section.

4.3. Iterative part

The missing piece in the aforementioned model is the normalisation to ensure that the maximum achievable comprehensive value is equal to 1. Because the comprehensive value is the sum of m marginal values and the maximum marginal value of each criterion might occur at any breakpoint of the criterion scale, $\prod_i \alpha_i$ possible combinations of decision variables exist to constitute the maximum comprehensive value. The general idea is to detect the combinations of decision variables that have the potential to cause the maximum comprehensive value to exceed 1 and restrict them by adding a new constraint. Another possibility is that the maximum affordable comprehensive value is less than 1. In this case, we impose a new constraint to enforce an increase in the maximum comprehensive value by a small number, ϵ, in the next iteration and subsequently remove this constraint. The contribution in the objective function is to introduce two types of penalisation for deviating from the normalised solution. Suppose that we already have the solution from iteration t and we want to move to the next iteration, $t + 1$. The idea constitutes the following three aspects:

1. If the maximum comprehensive value achieved in the iteration t, V_t^* is greater than 1, a new constraint will be added to the model that considers the position of the maximum value in the marginal value functions. The added constraint will be kept in all the subsequent iterations. Let us denote by v_i^t the extracted marginal value function in the tth iteration and g_i^{t*} the breakpoint on the ith criterion with the highest marginal value. The constraint that has to

be added and kept in all of the subsequent iterations is as follows:

$$\sum_{i=1}^{m} v_i^t(g_i^{t*}) \leq 1. \tag{16}$$

Going forward, we call these types of constraints 'restrictive constraints'.

2. If V_t^* is less than 1, a new constraint will be added to the model in the next iteration, $t + 1$, considering the location of the breakpoints corresponding to the maximum value in each criterion. The constraint that has to be added in iteration $t + 1$ is as follows:

$$\sum_{i=1}^{m} v_i^t(g_i^{t*}) \geq V_t^* + \epsilon \tag{17}$$

in which ϵ is a very small real number so that $\epsilon \in (0, 1 - V_t^*)$. The added constraint will be imposed only in the next iteration, and will be removed later. We refer to this type of constraint as an 'incremental constraint'.

3. Two types of penalties are defined and considered in the objective function, one for the case that the maximum comprehensive value deviates from 1 and another for the case that the minimum comprehensive value deviates from 0, all based on the solution obtained in the last iteration. For the first type, the penalty is proportional to the distance between the maximum comprehensive value and 1, $| \sum_{i=1}^{m} v_i^t(g_i^{t*}) - 1 |$. Based on the imposed constraints, explained above, we know that in the case that $\sum_{i=1}^{m} v_i^t(g_i^{t*})$ exceeds 1 in iteration t, a new constraint will be imposed in the iteration $t + 1$ that forces this term to have a value less than 1. Therefore, this penalisation term can be rewritten as $1 - \sum_{i=1}^{m} v_i^t(g_i^{t*})$. The second penalisation factor is proportional to the distance of the lowest comprehensive value and 0, $| \sum_{i=1}^{m} v_i^t(g_{i*}^t) - 0 | = \sum_{i=1}^{m} v_i^t(g_{i*}^t)$, in which g_{i*}^t denotes the breakpoints on the ith criterion with the lowest marginal value. To prevent penalties from dominating the two error terms in the objective function, the coefficients of the penalty terms are defined as a certain percentage of the sum of the error values in the optimal solution of the last iteration. Therefore, penalty terms in the objective function of iteration $(t + 1)$ are multiplied by the coefficients $p_{\max} \cdot f^t$ and $p_{\min} \cdot f^t$ in which $p_{\max}$ and $p_{\min}$ are real positive numbers, and f^t is the sum of the error terms in the optimal solution of the last iteration. The underlying logic behind these two penalisation terms is that the position of the maximum and minimum marginal values over criterion breakpoints will change only if it leads to a significant decrease in the error term values.

The LP model of iteration $(t + 1)$ is as follows in (18):

$$\min z = \sum_{n=1}^{N} (\sigma^+(a_n) + \sigma^-(a_n)) + p_{\max} f^t \left(1 - \sum_{i=1}^{m} v_i^t(g_i^{t*}) \right)$$

$$+ p_{\min} f^t \sum_{i=1}^{m} v_i^t(g_{i*}^t)$$

Subject to

$$V'(a_n) - V'(a_{n+1}) \geq \delta \text{ iff } a_n \succ a_{n+1}, \forall n = 1, 2, \ldots, N - 1$$
$$V'(a_n) - V'(a_{n+1}) = 0 \text{ iff } a_n \sim a_{n+1}, \forall n = 1, 2, \ldots, N - 1$$
$$A_i V_i \geq 0, \forall i = 1, 2, \ldots, m$$
$$A_i V_i \leq 1, \forall i = 1, 2, \ldots, m$$

for all the iterations $k \leq t$ with V_k^* greater than 1

$$\sum_{i=1}^{m} v_i^k(g_i^{k*}) \leq 1$$
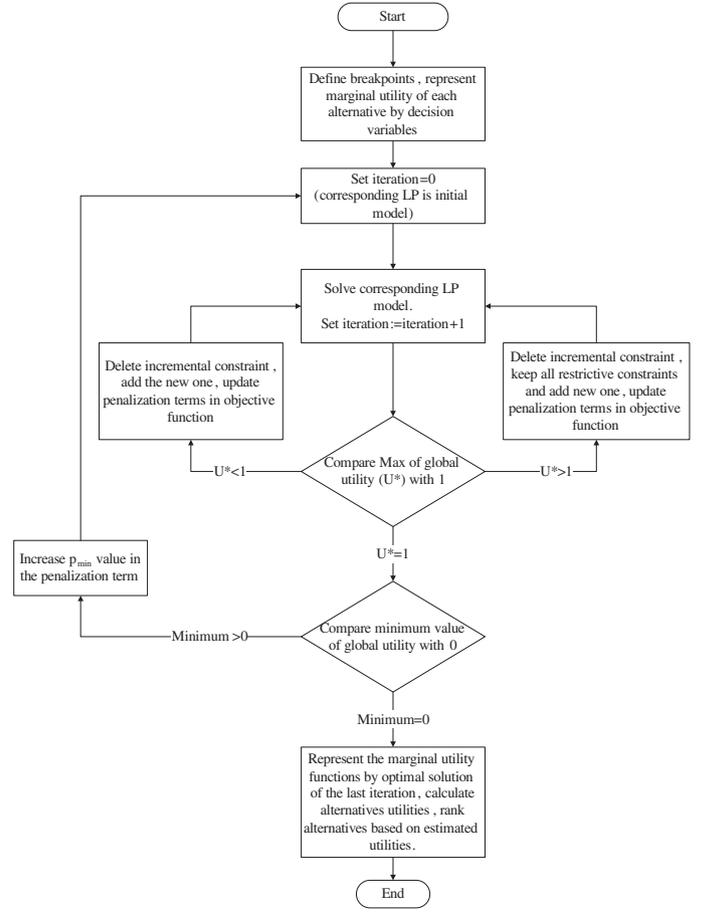
only if V_t^* less than 1

Fig. 2. Proposed method flowchart.

$$\sum_{i=1}^{m} v_i^t(g_i^{t*}) \geq V_t^* + \epsilon$$
$$V'(a_n) = V(a_n) - \sigma^+(a_n) + \sigma^-(a_n)$$
$$v_{ij} \text{ URS}, i = 1, 2, \ldots, m, j = 0, 1, \ldots, \alpha_i - 1$$
$$\sigma^+(a_n), \sigma^-(a_n) \geq 0, n = 1, 2, \ldots, N. \tag{18}$$

The algorithm can be summarised by the following steps (Fig. 2):

Step 0: Define the appropriate breakpoints on each criterion scale and represent the marginal value of each alternative in terms of the decision variables v_{ij}.

Step 1: Set iteration: $= 0$, solve the LP model (13) and find g_i^*, g_{i*} and f^0 (iteration $= 0$).

Step 2: Set iteration: $=$ iteration $+1$. Delete the incremental constraint, if any. Keeping all of the restrictive constraints that were previously added to the model, add the new restrictive constraint (16) to the model if the maximum comprehensive value of the previous iteration exceeds 1. If the maximum comprehensive value of the previous iteration is less than 1, add the incremental constraint (17) to the next iteration.

Step 3: Check if the normalisation condition is satisfied (i.e. if the maximum marginal values add up to 1 and the minimum comprehensive value is 0. If both conditions are satisfied, go to step 4. If not, go back to step 2.

Step 4: Represent the marginal value function of each criterion by the v_{ij} variables achieved in the last iteration. Calculate the value of each alternatives by (6). Rank the alternatives based on the estimated values.
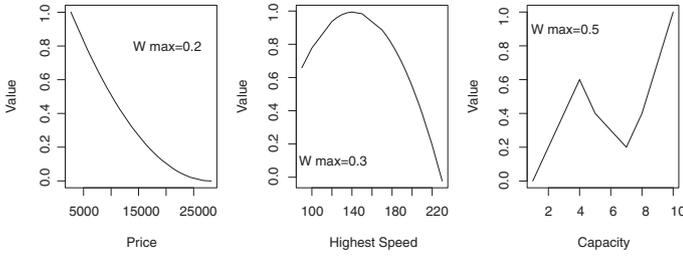
Fig. 3. Assumed value functions for the illustrative example of car characteristics.
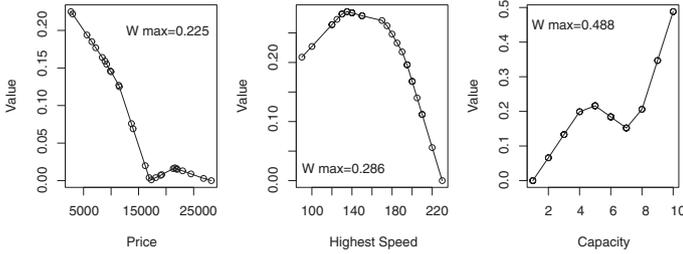

Fig. 4. Extracted value functions for the proposed method in the illustrative example of car characteristics.

Fig. 5. Extracted value functions by UTA-MN method in the illustrative example.

Table 1
Brand colour components and rankings with respect to brand image items.

Brand name	Colour components			Rankings in image items		
	H	S	V	Daring	Fun	Trendy
Always	214	82	66	4	34	22
Caress	45	50	100	18	21	18
Charmin	203	78	86	28	27	29
Clairol	208	98	53	3	8	4
Colgate	358	88	93	8	14	27
CoverGirl	324	10	20	2	1	2
Crest	199	99	76	23	3	21
Dial Soap	205	100	71	7	23	25
...	...	...	...	...	...	...

4.4. An illustrative example to compare with UTA-NM

In order to illustrate the method, a typical example based on car characteristics is employed. In this example, we assume a set of marginal value functions over a set of three criteria as DM tacit knowledge and we calculate the rank of alternatives based on them. Without prior knowledge over the marginal values and considering only the ranking, we then analyse the extent to which the captured set of marginal value functions are really aligned with the ones previously assumed.

The three criteria are *price*, *maximum speed* and *personal capacity* and the considered alternatives are 28 different cars. The assumed marginal value functions over each of the three criteria and its maximal shares in the comprehensive values that the DM tacitly assigns to each of the criteria are depicted in Fig. 3.

The model parameters are set such that both proposed method and UTA-NM have the same number of decision variables for each specific criterion, and thus the same degree of freedom for the associated value function. The extracted marginal values and the maximal shares over the criteria for the proposed method and UTA-NM are provided in Figs. 4 and 5, respectively. It is important to highlight that using the proposed method, the final solution was obtained after 54 iterations in less than 1 s (using a 64-bit OS on a 2.53 GHz Intel Core2Duo using MATLAB R2012b), while using UTA-NM exceeded 15 s.

From these results, it can be deduced that by using the proposed method both the marginal value function shapes and criteria weights accord with the preferential system of the DM. However, it can be seen that UTA-NM was not successful in estimating the assumed value functions, also the maximal shares are far from the expected values.

To assess the strength of the methods in reproducing the ranking given by the DM, the Kendall τ measure has been used. In the proposed method a value of 90.0% is obtained, while, in UTA-NM the obtained value was 83.1%.

5. Brand colour and brand image

Our focus in this study is to explore the contribution of brand colour in brand image in an understandable way. Several studies examined impacts of brand colour on v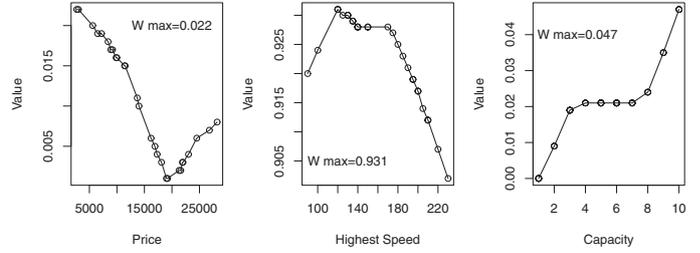arious aspects of the brand. In a scenario-based experiment, Babin et al. found that effects of colour on behavioural intentions are mediated by the cognitive reactions they create [1]. As by definition, the concept of brand image is based on the perception of consumers, we expect a strong association between brand image and brand colour.

Although studies show that all three colour components influence brand personality [13], most colour research in marketing focuses only on colour hue, and usually ignores the other two attributes: saturation and value [14]. However, in this study, we consider all three attributes together. Furthermore, as studies show that the influence of colour differs across product categories [16], we analyse only brands from a particular sector, namely beauty products.

5.1. Experiment description and dataset

Our data comes from a survey conducted by Young and Rubicam's BrandAsset Valuator consulting group. The dataset contained many measurements of several aspects of brand and was published recently [18]. In their quarterly survey (ten quarters from 2008 to the second quarter of 2010) a representative sample of the U.S. population, 17,000 individuals, were asked about 250 brands. The survey measures a broad range of perceptions and attitudes of brands. In their survey, 40 different attributes of brand image (arrogant, energetic, chic, etc.) are included, and each respondent is asked to check whether (s)he can associate the brand with each of these attributes. For each attribute, the dataset contains the percentage of respondents who associated this attribute with the brand. In our experiment, we converted all the percentages into a ranking. Hence we considered the relative position of brands with respect to each brand image attribute. The brands at the top of the ranking with respect to each brand image attribute are deemed to have a strong association with that particular attribute in customer minds. Furthermore, we only considered single-coloured brands, as the interaction among colours is not the topic of this study. We used 34 single-coloured beauty brands. Finally, we measured the HSV colour component for each brand logo and added these to the dataset. A small portion of the dataset is presented in Table 1 for illustration, where only 8 brands and 3 brand image attributes together with the brand colour components are presented.
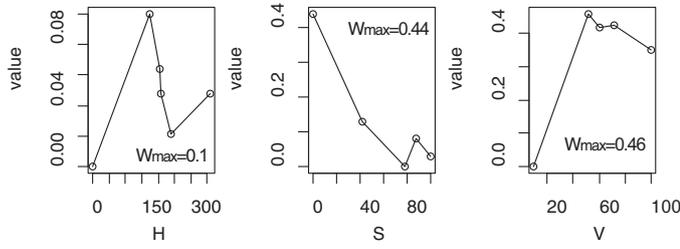
Fig. 6. Value functions for the item *fun*.

Table 2
Extracted weights of the colour components and Kendall τ of the reproduced ranking (single star means significant at 0.01 and double star means significant at 0.001).

Brain image attributes	weights			Kendall τ
	H	S	V	
Arrogant	0.192	0.425	0.373	0.383**
Authentic	0.161	0.416	0.414	0.318*
Best brand	0.120	0.455	0.424	0.540**
Care-free	0.184	0.407	0.408	0.451**
Cares for customers	0.155	0.400	0.438	0.487**
Charming	0.111	0.441	0.438	0.344*
Daring	0.290	0.357	0.353	0.380**
Down to earth	0.156	0.397	0.441	0.455**
Energetic	0.188	0.373	0.434	0.333*
Friendly	0.147	0.410	0.443	0.430**
Fun	0.097	0.441	0.462	0.629**
Gaining in popularity	0.149	0.395	0.455	0.365*
Glamorous	0.120	0.442	0.430	0.526**
Good value	0.081	0.469	0.443	0.480**
Healthy	0.106	0.401	0.492	0.316*
Helpful	0.163	0.419	0.414	0.469**
Healthy	0.106	0.401	0.492	0.316*
High performance	0.195	0.401	0.401	0.390**
Independent	0.207	0.366	0.425	0.419**
Intelligent	0.156	0.400	0.436	0.258
Kind	0.323	0.440	0.228	0.590**
Obliging	0.181	0.386	0.426	0.415**
Original	0.155	0.383	0.458	0.383**
Prestigious	0.246	0.429	0.323	0.458**
Progressive	0.148	0.378	0.467	0.533**
Restrained	0.187	0.418	0.394	0.451**
Rugged	0.172	0.468	0.353	0.619**
Sensuous	0.120	0.430	0.443	0.601**
Simple	0.232	0.427	0.331	0.554**
Social	0.108	0.446	0.446	0.358*
Socially responsible	0.111	0.431	0.451	0.487**
Straightforward	0.168	0.407	0.427	0.554**
Stylish	0.176	0.385	0.433	0.501**
Traditional	0.158	0.407	0.425	0.458**
Trendy	0.097	0.430	0.463	0.326*
Trustworthy	0.105	0.435	0.453	0.601**
Unapproachable	0.208	0.389	0.399	0.376*
Up to date	0.137	0.425	0.429	0.465**
Upper class	0.174	0.409	0.417	0.298*
Visionary	0.121	0.435	0.435	0.326*
Worth more	0.111	0.442	0.442	0.412**

5.2. Experimental results

For each of the brand image items, the association with the brand colour is analysed using the proposed methodology. The set of extracted value functions represents the colour patterns with respect to that brand image item. The value functions are used to calculate the utility of each brand from the perspective of that particular brand image item. The greater the utility of a brand, the greater is the likelihood of a strong connection between the brand and the brand image attribute. Finally, the brands were ranked on their utilities. The obtained ranking was compared with the initial ranking from the data
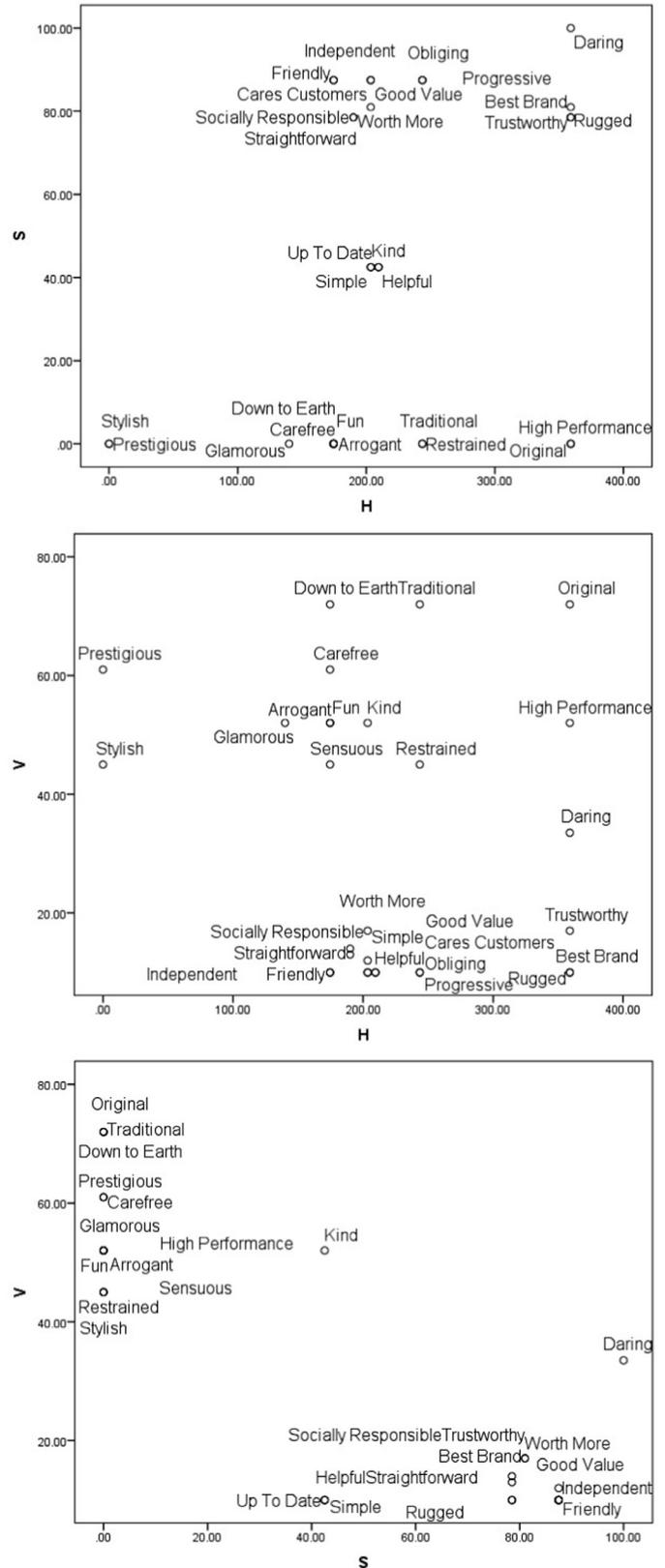

Fig. 7. Colour map of the brand image attributes.

in order to measure the accountability of the extracted value functions. The Kendall τ measure was used for this purpose.

The extracted value functions for the brand image attribute *fun* is presented in Fig. 6. By setting $P_{min} = 0.6$ and $P_{max} = 0.01$, the results are obtained in 1252 iterations. The Kendall τ measure between

the initial ranking and the ranking from the extracted value functions is equal to 0.63. This indicates a strong association between the two rankings. So we can conclude that brand colour significantly explains the perception of the brand to be *fun*. Under the null hypothesis, when there is no dependency between the two rankings, and when the number of elements in the ranking list is sufficiently large, namely larger than 10, the τ measure follows a normal distribution with the mean equal to zero and variance equal to $\frac{2(2n+5)}{9n(n-1)}$ in which n is the number of elements in the ranking list [17]. In our experiment n is equal to 34 which is the number of brands in the list. The statistical test shows that τ is significantly positive with p-value < 0.001.

The maximal contribution of each H, S, and V colour component into the comprehensive utility of the brand with respect to the item *fun* is 0.10, 0.44, and 0.46, respectively. This indicates that the colour hue plays the least role in the perception of a brand as fun. The value functions demonstrate that brands with the colour hue green, less saturated, and moderately bright tend to be perceived as fun.

The same analysis has been conducted for all the other 39 brand image attributes. The Kendall τ measure of each analysis and colour component weights are given in Table 2.

As it can be seen from Table 2, for 29 of the 40 brand image attributes the Kendall τ measure is significant at the 0.001 level of significance. This reveals a significant contribution of brand colour in the perception of brand by customers. More interestingly, comparison among the weights of colour components shows that colour hue is usually the least important component. Colour hue has the least weight in 39 of the 40 brand image attributes, and only for the image attribute *kind* does it have the second highest weight at the top of component V. Statistical tests show that colour hue (H) weight is significantly less than colour saturation (S) weight ($t(39) = -27.66$ and $p - value < 0.001$), and colour value (V) weight ($t(39) = -19.90$ and $p - value < 0.001$), while there is no significant difference between colour saturation and colour value weights ($t(39) = -1.06$ and $p - value = 0.29$). This indicates that the two colour components S (how pure or whitened is the colour) and V (how dark or bright is the colour) play much more of a role in determining the customer perception of brand image than colour hue. Finally, it is important to highlight that customer perception of a brand as *intelligent* is not influenced by the brand colour.

From each set of the extracted marginal value functions, it is possible to determine the H, S, and V values which lead to the highest utility with respect to each particular brand image attribute. For example, from the extracted value functions for the brand image *fun* in Fig. 6, it can be seen that $(V, H, S) = (174.5, 0, 52)$ leads to the highest possible utility. Therefore, a brand manager can choose the corresponding colour to be perceived as *fun* by customers. We did the same analysis for all the attributes which are significant at 0.001 level, and obtained their position in the HSV colour space. The resulted brand image-colour map is presented in Fig. 7.

The map clearly describes the colour space by the brand image attributes. It also demonstrates the interrelation of brand image items from the brand colour perspective.

6. Conclusion and future work

This paper presents a disaggregation methodology based on the UTA method that enables the use of non-monotonic additive models in ranking and other multi-criteria decision problems. The main difference between the proposed methodology and existing non-monotonic methods is that our method is capable of obtaining marginal value functions and the relative importance of attributes (maximal shares in the comprehensive values) following an LP approach. Marginal value functions obtained by the proposed method are free in shape. Over-fitting is prevented by appropriate breakpoints definition and value functions slope restriction.

The proposed method does not require further information regarding the shape of the value functions, nor the most desirable value of each attribute. The only information it requires from the DM is a weak ordering over a set of alternatives. The results from the illustrative example and the experiment shed light on the usefulness and effectiveness of the proposed method.

The proposed method is applied to real brand image data of to delineate the role of brand colour in brand perception. The results support a significant contribution of all three colour components in almost all brand image attributes. We also find that colour value and saturation dominate the colour hue role in brand perception by customers.

As future work, we are interested in analysing several product categories to study how the association between brand colour and brand image changes across industries. It would also be interesting to compare the colour pattern for each brand image attribute across product categories. It would be particularly interesting to look for a universal rule of brand perception by brand colour regardless of the product category.

References

[1] B.J. Babin, D.M. Hardesty, T.A. Suter, Color and shopping intentions: the intervening effect of price fairness and perceived affect, J. Bus. Res. 56 (2003) 541–551.

[2] P.A. Bottomley, J.R. Doyle, The interactive effects of colors and products on perceptions of brand logo appropriateness, Mark. Theory 6 (2006) 63–83.

[3] J.C. Chebat, M. Morrin, Colors and cultures: exploring the effects of mall dcor on consumer perceptions, J. Bus. Res. 60 (2007) 189–196.

[4] D.K. Despotis, C. Zopounidis, Building additive utilities in the presence of non-monotonic preferences, Adv. Multicri. Anal. 5 (1995) 101–114.

[5] J. Devaud, G. Groussaud, E. Jacquet-Lagrèze, UTADIS: Une méthode de construction de fonctions d'utilité additives rendant compte de jugements globaux, in: European Working Group on Multicriteria Decision Aid, Bochum, 1980.

[6] M. Doumpos, Learning non-monotonic additive value functions for multicriteria decision making, OR Spectr. 34 (2012) 89–106.

[7] A. Eckhardt, T. Klieger, Preprocessing algorithm for handling non-monotone attributes in the UTA method, in: Preference Learning: Problems and Applications in AI (PL-12) Workshop, 2012.

[8] L.M. Falomir Zoe, L. Gonzalez-Abril, A model for colour naming and comparing based on conceptual neighbourhood. an application for comparing art compositions, Knowl.-Based Syst. 81 (2015) 1–21.

[9] L.L. Garber, E.M. Hyatt, R.G. Starr, The effects of food color on perceived flavor, J. Mark. Theory Pract. (2000) 59–72.

[10] J. Hoegg, J.W. Alba, Taste perception: more than meets the tongue, J. Consum. Res. 33 (2007) 490–498.

[11] E. Jacquet-Lagrèze, Y. Siskos, Assessing a set of additive utility functions for multicriteria decision making: the UTA method, Europ. J. Operat. Res. 10 (1982) 151–164.

[12] T. Kliegr, Uta-nm: explaining stated preferences with additive non-monotonic utility functions, in: Preference Learning (PL-09), ECML/PKDD-09 Workshop, 2009.

[13] L.I. Labrecque, G.R. Milne, Exciting red and competent blue: the importance of color in marketing, J. Acad. Mark. Sci. 40 (2012) 711–727.

[14] L.I. Labrecque, V.M. Patrick, G.R. Milne, The marketers prismatic palette: a review of color research and future directions, Psychol. Mark. 30 (2013) 187–202.

[15] C. Lightfoot, R. Gerstman, Brands: the new wealth creators, in: S. Hart, J. Murphy (Eds.), Brand Packaging, Basingstoke: Macmillan, 1998, pp. 46–55.

[16] G.L. Lohse, D.L. Rosen, Signaling quality and credibility in yellow pages advertising: the influence of color and graphics on choice, J. Advert. 30 (2001) 73–83.

[17] G.D. Long, N. Cliff, Confidence intervals for Kendall's tau, Br. J. Math. Stat. Psychol. 50 (1997) 31–41.

[18] M. Lovett, R. Peres, R. Shachar, A data set of brands and their characteristics, Mark. Sci. 33 (2014) 609–617.

[19] I. Patiniotakis, D. Apostolou, G. Mentzas, Fuzzy UTASTAR: a method for discovering utility functions from fuzzy data, Expert Syst. Appl. 38 (2011) 15463–15474.

[20] P.S. Schindler, Color and contrast in magazine advertising, Psychol. Mark. 3 (1986) 69–78.

[21] Wu, Chia-Chien, F.A. Wick, M. Pomplun, Guidance of visual attention by semantic information in real-world scenes, Front. Psychol. 5 (2014).

[22] P.M. Zaltner, John Dewey's Aesthetic Philosophy, John Benjamin Company, Amsterdam, The Netherlands, 1975.