A new type of photon-added squeezed coherent state and its statistical properties*

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We construct a new type of photon-added squeezed coherent state generated by repeatedly operating the bosonic creation operator on a new type of squeezed coherent state [Fan H Y and Xiao M 1996 *Phys. Lett.* A **220** 81]. We find that its normalization factor is related to single-variable Hermite polynomials. Furthermore, we investigate its statistical properties, such as Mandel's Q parameter, photon-number distribution, and Wigner function. The nonclassicality is displayed in terms of the intense oscillation of photon-number distribution and the negativity of the Wigner function.

Keywords: non-Gaussian state, Hermite polynomial, squeezed coherent state, Wigner function

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1. Introduction

As the most common Gaussian states of a radiation field, coherent states, and squeezed states are important topics in quantum optics and play significant roles in quantum information processing with continuous variables, such as quantum teleportation, dense coding, and quantum cloning.[1−3] Experimentally, Gaussian states, such as Fock states, coherent states, and squeezed states, have been generated, but there are some limitations in using them for quantum information processing.[4] Non-Gaussian coherent states and non-Gaussian squeezed states with exteme nonclassical properties may constitute powerful resources for the efficient implementation of quantum communication and computation.[5,6] It is well known that operating-photon addition or subtraction of a given Gaussian state is an effective method to generate non-Gaussian states.[7,8] Thus, subtracting or adding photon states, which have been successfully demonstrated experimentally by Parigi *et al.*,[9] has received increasing attention from both experimentalists and theoreticians recently[10−14] because it is possible to generate and manipulate various nonclassical optical fields by subtracting/adding photons from/to traditional quantum states.[13,15,16]

In Ref. [17], Fan and Xiao introduced a new type of squeezed coherent state (SCS), which differs from conventional squeezed states whose displacement parameter depends on its squeezing parameter and is more general. For instance, the SCS can be reduced to the simple coherent state, the coordinate eigenstate, or the momentum eigenstate. Then, a question of what properties the new type photon-added squeezed coherent state (PASCS) has is raised. In this paper, we will introduce the PASCS and investigate its statistical properties, such as Mandel's Q parameter, photon-number distribution (PND), as well as the Wigner function (WF). This work is arranged as follows. In Section 2, we give a brief review of the SCS. In Section 3, we introduce the PASCS and derive its normalized constant, which turns out to be related to a single-variable Hermite polynomial with a remarkable result. In Section 4, Mandel's Q parameter of the PASCS is calculated analytically. The PND and WF are discussed in Sections 5 and 6, respectively, and the results display the nonclassicality of the PASCS. Section 7 is devoted to projecting a scheme to product the PASCS. The main results are summarized in Section 8.

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2. Brief review of SCS

As is commonly known in quantum optics, single-mode coherent squeezed states are constructed either by[18]

$$\begin{aligned}|z,\lambda\rangle &= D(z)S(\lambda)|0\rangle \\ &= \operatorname{sech}^{1/2}\lambda\exp\Big[-\frac{1}{2}|z|^2+za^\dagger \\ &\quad+\frac{1}{2}\left(a^\dagger-z^*\right)\tanh\lambda\Big]|0\rangle,\end{aligned} \tag{1}$$

or by

$$\begin{aligned}|\lambda,z\rangle &= S(\lambda)D(z)|0\rangle \\ &= \operatorname{sech}^{1/2}\lambda\exp\Big[-\frac{1}{2}|z|^2+za^\dagger\operatorname{sech}\lambda \\ &\quad+\frac{1}{2}\tanh\lambda\left(a^{\dagger 2}-z^2\right)\Big]|0\rangle,\end{aligned} \tag{2}$$

where $a^\dagger$ and a are Bose creation and annihilation operators, respectively, $[a,a^\dagger]=1$, $|0\rangle$ is the vacuum state, and λ and z are the single-mode squeezing and displacement parameters, respectively. In these two kinds of states, λ and z are independent of each other. By constrast, in Ref. [17], Fan and Xiao introduced a special kind of unnormalized single-mode squeezed coherent state whose displacement is related to its squeezing, and the state also spans an overcompleteness space

$$||z\rangle_g=\exp\left[-\frac{1}{2}|z|^2+(fz+gz^*)a^\dagger-fga^{\dagger 2}\right]|0\rangle, \tag{3}$$

where z is the displacement parameter, and $f=|f|\exp(\mathrm{i}\phi_f)$ and $g=|g|\exp(\mathrm{i}\phi_g)$ are complex parameters satisfying $|f|^2+|g|^2=1$, where ϕ_f and ϕ_g are phase angles. Particularly, when $g=0$ or $f=0$, $|z\rangle_g$ will be reduced to a normalized coherent state. By virtue of the IWOP technique[19] and the vacuum projector $|0\rangle\langle 0|=:\mathrm{e}^{-a^\dagger a}:$, the overcompleteness can be easily proved as follows:

$$\begin{aligned}\int\frac{\mathrm{d}^2z}{\pi}||z\rangle_g\,{}_g\langle z|| &= \int\frac{\mathrm{d}^2z}{\pi}:\exp[-|z|^2 \\ &\quad+\left(fa^\dagger+ga\right)z+\left(f^*a+ga^\dagger\right)z^* \\ &\quad-fga^{\dagger 2}-f^*g^*a^2-a^\dagger a]: \\ &= 1.\end{aligned} \tag{4}$$

Moreover, $||z\rangle_g$ can be normalized as follows:

$$\begin{aligned}|z\rangle_g &= (1-4|fg|^2)^{1/4}\exp\Big[-\frac{1}{2}|z|^2 \\ &\quad+(fz+gz^*)a^\dagger-fga^{\dagger 2}\Big]|0\rangle,\end{aligned} \tag{5}$$

whose overlap is

$$\begin{aligned}&{}_g\langle z''|z'\rangle_g \\ &= \int\frac{\mathrm{d}^2z}{\pi}\,{}_g\langle z''|z\rangle\langle z|z'\rangle_g \\ &= \exp\left[-\frac{1}{2}\left(|z'|^2+|z''|^2\right)\right]\exp\Big[\frac{(fz'+gz'^*)}{1-4|fg|^2} \\ &\quad\times\frac{(f^*z''+g^*z''^*)}{1-4|fg|^2}-\frac{f^*g^*(fz'+gz'^*)^2}{1-4|fg|^2} \\ &\quad-\frac{fg(f^*z''+g^*z''^*)^2}{1-4|fg|^2}\Big],\end{aligned} \tag{6}$$

where the overcompleteness relation of coherent state

$$\int\frac{\mathrm{d}^2z}{\pi}|z\rangle\langle z|=1 \tag{7}$$

has been used. Equation (6) indicates that the SCS is non-orthogonal. Especially, when $z''=z'$ and ${}_g\langle z'|z'\rangle_g=1$, Eq. (6) proves that the SCS is capable of making up a new quantum mechanical representation.

3. PASCS and its normalization

Enlightened by the above studies, we introduce a new type of PASCS. Theoretically, the PASCS can be obtained by repeatedly operating the photon creation operator on a SCS as follows:

$$|z\rangle_{g,m}=N_{g,m}a^{\dagger m}|z\rangle_g, \tag{8}$$

where $N_{g,m}$ is the normalization factor. Next, we shall determine the normalization constant $N_{g,m}$, which is the key to analyze the quantum statistical properties of the PASCS. For this purpose, using the overcompleteness relation of coherent state (Eq. (7)) and the normalization condition, we have

$$\begin{aligned}1 &= {}_{g,m}\langle z|z\rangle_{g,m} \\ &= N_{g,m}^2(1-4|fg|^2)^{1/2}\int\frac{\mathrm{d}^2\alpha}{\pi}\langle 0|\exp\Big[-\frac{1}{2}|z|^2 \\ &\quad+(f^*z^*+g^*z)a-f^*g^*a^2\Big]a^m|\alpha\rangle\langle\alpha|a^{\dagger m} \\ &\quad\times\exp\left[-\frac{1}{2}|z|^2+(fz+gz^*)a^\dagger-fga^{\dagger 2}\right]|0\rangle \\ &= N_{g,m}^2(1-4|fg|^2)^{1/2}\frac{\mathrm{d}^{2m}}{\mathrm{d}j^m\,\mathrm{d}k^m}\int\frac{\mathrm{d}^2\alpha}{\pi} \\ &\quad\times\exp[-|\alpha|^2+(f^*z^*+g^*z+j)\alpha-f^*g^*\alpha^2 \\ &\quad+(fz+gz^*+k)\alpha^*-fg\alpha^{*2}-|z|^2]|_{j=k=0}.\end{aligned} \tag{9}$$

Further using the following integral formula:[20]

$$\int \frac{\mathrm{d}^2\alpha}{\pi} \exp(\zeta |\alpha|^2 + \xi\alpha + \eta\alpha^* + f\alpha^2 + g\alpha^{*2}) = \frac{1}{\sqrt{\zeta^2 - 4fg}} \exp\left(\frac{-\zeta\xi\eta + \xi^2 g + \eta^2 f}{\zeta^2 - 4fg}\right), \tag{10}$$

whose convergent conditions are $\mathrm{Re}(\zeta \pm f \pm g) < 0$ and $\mathrm{Re}((\zeta^2 - 4fg)/(\zeta \pm f \pm g)) < 0$ (the conditions are physical and always satisfied), Eq. (9) can be rewritten as follows:

$$\begin{aligned} {}_{g,m}\langle z|\, z\rangle_{g,m} &= N_{g,m}^2 \frac{\mathrm{d}^{2m}}{\mathrm{d}j^m \mathrm{d}k^m} \exp\Bigg[A_1 k - A_2 k^2 \\ &\quad + A_1^* j - A_2^* j^2 + \frac{kj}{1 - 4|fg|^2}\Bigg]|_{j=k=0} \\ &= N_{g,m}^2 \frac{\mathrm{d}^{2m}}{\mathrm{d}j^m \mathrm{d}k^m} \sum_{l=0}^{\infty} \frac{k^l j^l}{(1 - 4|fg|^2)^l l!} \\ &\quad \times \exp(A_1 k - A_2 k^2 + A_1^* j - A_2^* j^2)|_{j=k=0} \\ &= N_{g,m}^2 \sum_{l=0}^{\infty} \frac{|A_2|^m}{(1 - 4|fg|^2)^l l!} \\ &\quad \times \left|\frac{\partial^l}{\partial A_1^l} H_m\left(\frac{A_1}{2\sqrt{A_2}}\right)\right|^2, \end{aligned} \tag{11}$$

where

$$\begin{aligned} A_1 &= \frac{g^* z + f^* z^* - 2g^* z |f|^2 - 2f^* z^* |g|^2}{1 - 4|fg|^2}, \\ A_2 &= \frac{f^* g^*}{1 - 4|fg|^2}. \end{aligned} \tag{12}$$

In the last step, we have used the generating function of single-variable Hermite polynomials

$$H_n(x) = \frac{\partial^n}{\partial t^n} \exp\left(2xt - t^2\right)|_{t=0}. \tag{13}$$

Noticing the relation

$$\frac{\partial^l}{\partial x^l} H_n(x) = \frac{2^l n!}{(n-l)!} H_{n-l}(x), \tag{14}$$

the compact form of $N_{g,m}$ is found to be

$$N_{g,m}^{-2} = \sum_{l=0}^{m} \frac{(m!)^2 |A_2|^m}{l![(m-l)!]^2 |fg|^l} \left|H_{m-l}\left(\frac{A_1}{2\sqrt{A_2}}\right)\right|^2, \tag{15}$$

which is related to a single-variable Hermite polynomial. Especially when $m = 0$, the normalization constant $N_{g,0} = 1$, as expected.

4. Mandel's Q parameter of the PASCS

By virtue of the analytical expression of $N_{g,m}$, it is easy for us to examine the sub-Poissonian statistics of the PASCS in terms of Mandel's Q parameter

$$Q = \frac{\langle a^{\dagger 2} a^2\rangle}{\langle a^\dagger a\rangle} - \langle a^\dagger a\rangle, \tag{16}$$

which measures the deviation of the variance of the photon number distribution of the field state under consideration from the Poissonian distribution of the coherent state. It is well known that if $Q = 0$, the field has Poissonian photon statistics, if $Q > 0$ ($Q < 0$), the field has super- (sub-)Poissonian photon statistics. In addition, the case of $Q < 0$ means that a state is nonclassical. However, the positive value of Mandel's Q parameter cannot be used to conclude that a state is classical because a state may be nonclassical even though Q is positive, as pointed out in Ref. [21].

To calculate Mandel's Q parameter, we begin with converting operators $a^\dagger a$ and $a^{\dagger 2} a^2$ into its anti-normal ordering form

$$\begin{aligned} a^\dagger a &= a a^\dagger - 1, \\ a^{\dagger 2} a^2 &= a^2 a^{\dagger 2} - 4 a a^\dagger + 2. \end{aligned} \tag{17}$$

One can easily obtain

$$\begin{aligned} \langle a^\dagger a\rangle &= N_{g,m}\ {}_g^2\langle z| a^{m+1} a^{\dagger m+1} |z\rangle_g - 1 \\ &= \frac{N_{g,m}^2}{N_{g,m+1}^2} - 1, \end{aligned} \tag{18}$$

and

$$\begin{aligned} \langle a^{\dagger 2} a^2\rangle &= N_{g,m}^2 {}_g\langle z|\, a^m \left(a^2 a^{\dagger 2} - 4 a a^\dagger + 2\right) a^{\dagger m} |z\rangle_g \\ &= \frac{N_{g,m}^2}{N_{g,m+2}^2} - 4\frac{N_{g,m}^2}{N_{g,m+1}^2} + 2. \end{aligned} \tag{19}$$

Substituting Eqs. (18) and (19) into Eq. (16), we have

$$\begin{aligned} Q &= \frac{N_{g,m}^2 N_{g,m+2}^{-2} - 4N_{g,m}^2 N_{g,m+1}^{-2} + 2}{N_{g,m}^2 N_{g,m+1}^{-2} - 1} - \frac{N_{g,m}^2}{N_{g,m+1}^2} + 1 \\ &= \frac{N_{g,m}^2 N_{g,m+1}^2 - 4N_{g,m}^2 N_{g,m+2}^2 + 2N_{g,m+1}^2 N_{g,m+2}^2}{N_{g,m}^2 N_{g,m+2}^2 - N_{g,m+1}^2 N_{g,m+2}^2} \\ &\quad - \frac{N_{g,m}^2}{N_{g,m+1}^2} + 1, \end{aligned} \tag{20}$$

which implies that Mandel's Q parameter of the PASCS is related to the difference values of the phase angles ϕ_f and ϕ_g. From Fig. 1, one can clearly see that all Mandel's Q parameters become positive when the modulus of f is larger than a certain threshold value, and the threshold value achieves the maximum when the values of the phase angles ϕ_f and ϕ_g are $\pi/2$.

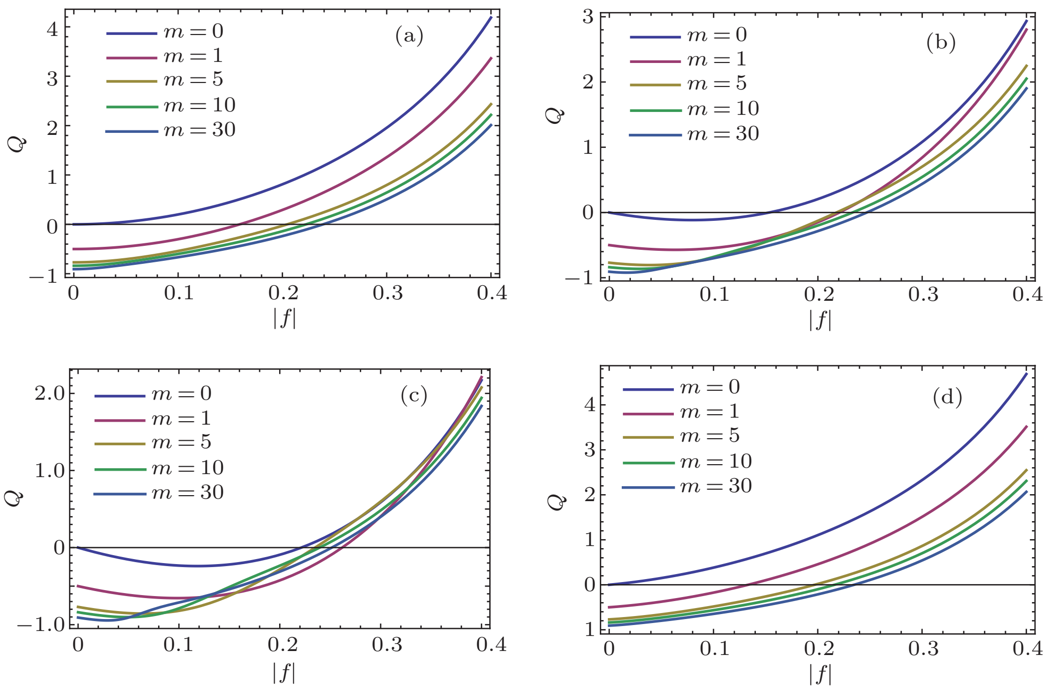

Fig. 1. (colour online) Mandel's Q parameter of the PASCS as a function of r with $z=(1+\mathrm{i})/\sqrt{2}$ and $m=0$, 1, 5, 10, and 30 for (a) $\phi_g-\phi_f=0,\pi$; (b) $\phi_g-\phi_f=\pi/4, 3\pi/4$; (c) $\phi_g-\phi_f=\pi/2$; (d) $\phi_g-\phi_f=3\pi/2$.

5. Photon-number distribution of PASCS

Next, we discuss the photon-number distribution of the PASCS. Using the un-normalized coherent state $|\alpha\rangle=\exp\left(\alpha a^{\dagger}\right)|0\rangle$, we have $|n\rangle=\frac{\mathrm{d}^n}{\mathrm{d}\alpha^n}|\alpha\rangle|_{\alpha=0}/\sqrt{n!}$ and $\langle\beta|\alpha\rangle=\exp(\alpha\beta^*)$. Based on Eq. (8), the probability of finding m photons in the field is given by

$$\begin{aligned}\mathcal{P}(n) &= N_{g,m}^2\,\langle n|\,a^{\dagger m}|z\rangle_{g\ g}\langle z|a^m\,|n\rangle \\ &= \frac{N_{g,m}^2(1-4|fg|^2)^{1/2}\exp(-|z|^2)}{n!}\frac{\mathrm{d}^{2n}}{\mathrm{d}\beta^{*n}\mathrm{d}\alpha^n} \\ &\quad\times[\beta^{*m}\alpha^m\exp\left(fz+gz^*\right)\beta^*-fg\beta^{*2} \\ &\quad+(f^*z^*+g^*z)\,\alpha-f^*g^*\alpha^2]|_{\alpha=\beta^*=0} \\ &= \frac{N_{g,m}^2 n!\exp(-|z|^2)(1-4|fg|^2)^{1/2}|fg|^{n-m}}{[(n-m)!^2]} \\ &\quad\times\left|H_{n-m}\left(\frac{fz+gz^*}{2\sqrt{fg}}\right)\right|^2, \qquad (21)\end{aligned}$$

which is a compact result of a single-variable Hermite polynomial. In particular, when $m=0$ ($N_{g,m}=1$), Eq. (21) is reduced to

$$\begin{aligned}\mathcal{P}(n)_{m=0} &= \frac{\exp(-|z|^2)(1-4|fg|^2)^{1/2}|fg|^n}{n!} \\ &\quad\times\left|H_n\left(\frac{fz+gz^*}{2\sqrt{fg}}\right)\right|^2, \qquad (22)\end{aligned}$$

which is exactly the PND of SCS. From Eqs. (21) and (22), it is easy to see that, when z, $|f|$, and $|g|$ are fixed, the value of $\mathcal{P}(n)$ or $\mathcal{P}(n)_{m=0}$ is only related to the difference values of the phase angles ϕ_f and ϕ_g.

Now we firstly analyze the changes of the PND of SCS with the difference values of the phase angles ϕ_f and ϕ_g for given z, $|f|$, and $|g|$. Using Eq. (22), the change of the PND of SCS with $(\phi_g-\phi_f)$ in the range of $[0,2\pi]$ is shown in Fig. 2, from which we can see that the PND is extremely sensitive to the difference values of phase angles ϕ_f and ϕ_g. When $(\phi_g-\phi_f)\in[0,\pi]$, the value of $\mathcal{P}(n)_{m=0}$ is symmetrical with respect to $(\phi_g-\phi_f)=\pi/2$, as shown in Figs. 2(a) and 2(b). However, if $(\phi_g-\phi_f)\in[\pi,2\pi]$, the value of $\mathcal{P}(n)_{m=0}$ is symmetrical with respect to $(\phi_g-\phi_f)=3\pi/2$, as shown in Fig. 2(d). Especially, if $(\phi_g-\phi_f)=\pi/2$, the PND of SCS obviously shows squeezed vacuum oscillations for strong squeezing in Fig. 2(c), or else the PND of SCS shows Schleich–Wheeler oscillations[22,23] (nonclassical intense oscillations) which reflect two different oscillating behaviors for even and odd photon numbers of the PND of squeezed states.

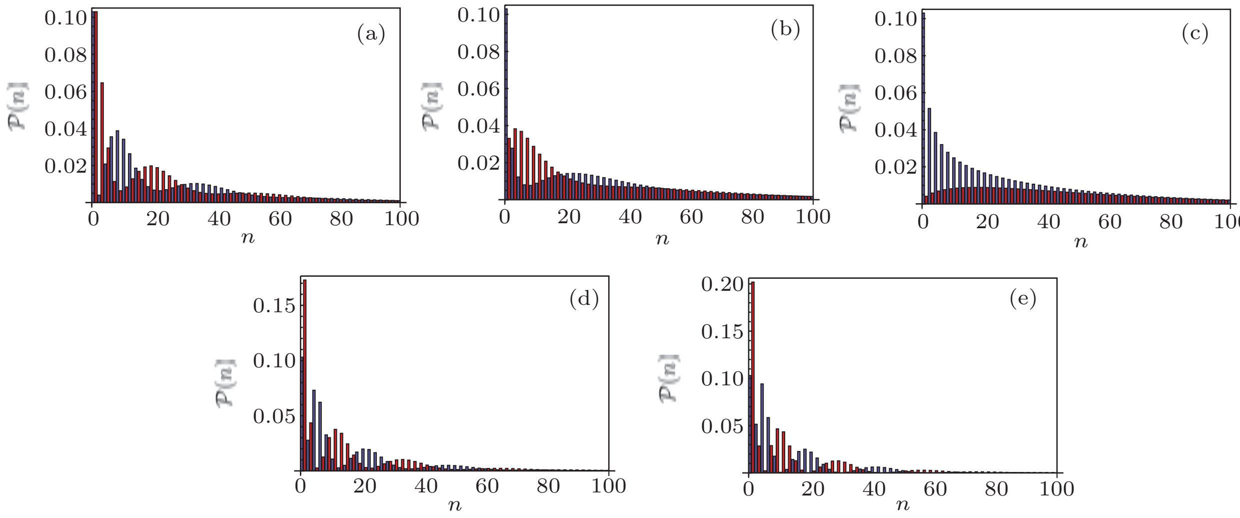

Fig. 2. (colour online) Photon-number distributions of the SCS with $|f| = 0.6$, $|g| = 0.8$, $z = (1 + \mathrm{i})/\sqrt{2}$, and $m = 0$ for (a) $\phi_f - \phi_g = 0, \pi$; (b) $\phi_f - \phi_g = \pi/4, 3\pi/4$; (c) $\phi_f - \phi_g = \pi/2$; (d) $\phi_g - \phi_f = 5\pi/4, 7\pi/4$; (e) $\phi_g - \phi_f = 3\pi/2$.

Next we plot the PND of PASCS as a function of m for given z, $|f|$, $|g|$, and $(\phi_g - \phi_f)$ in Fig. 3. From Figs. 2(a) and 3(a)–3(d), we can see that as m increases, the nonclassical intense oscillations of the PND are more and more unconspicuous, and almost disappear in Fig. 3(d). In addition, all peaks of the PND move toward a larger photon-number region, and the curves of the PND become more and more flat and wide.

Finally, we plot the nonclassical oscillations of the PND of PASCS at various values of $\phi_g - \phi_f$ for given z, $|f|$, $|g|$, and m in Fig. 4. It is easy to see that the symmetry of the PND with respect to $\phi_g - \phi_f$ is similar to that in Fig. 2, but the PND exhibits squeezed vacuum oscillations when $\phi_g - \phi_f = 3\pi/2$. These results indicate distinctly the nonclassicality of PASCS.

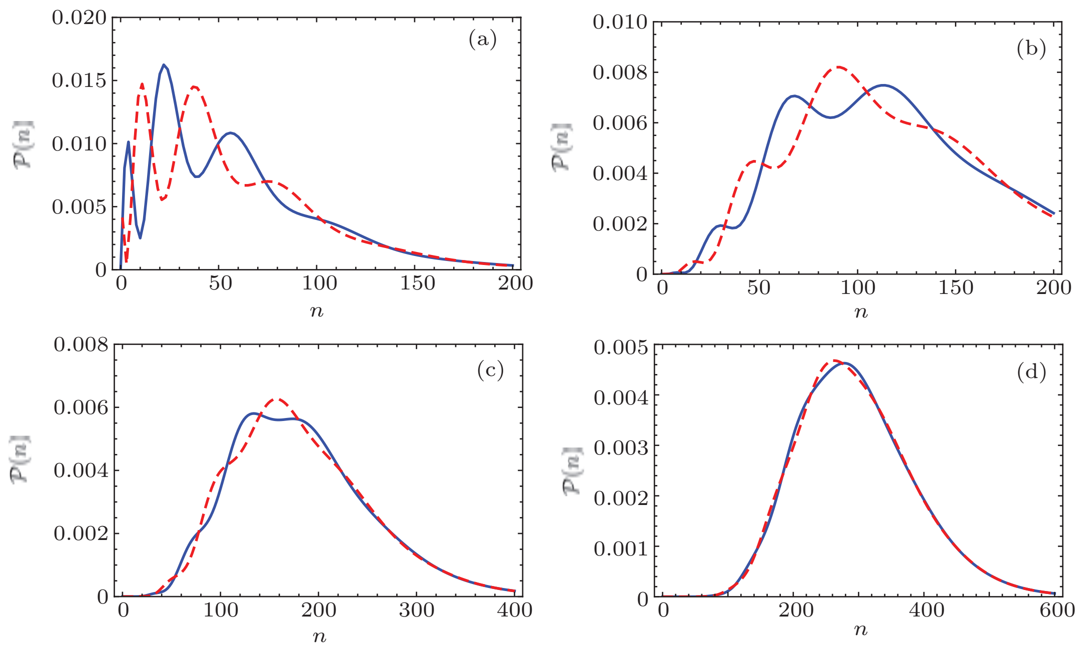

Fig. 3. (colour online) Photon-number distributions of the SCS with $|f| = 0.6$, $|g| = 0.8$, $z = (1 + \mathrm{i})/\sqrt{2}$, and $\phi_g - \phi_f = 0$ for (a) $m = 1$; (b) $m = 3$; (c) $m = 5$; (d) $m = 9$. Blue solid lines represent the even-photon-number distribution, while red dotted lines denote odd-photon-number distribution.

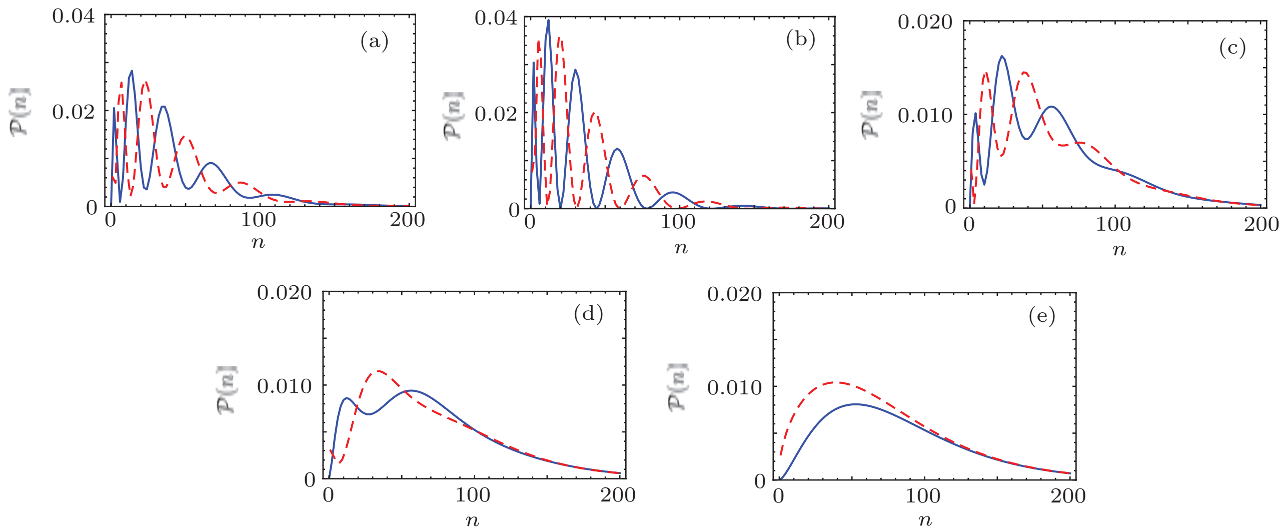

Fig. 4. (colour online) Photon-number distributions of the SCS with $|f| = 0.6$, $|g| = 0.8$, $z = (1 + \mathrm{i})/\sqrt{2}$, and $m = 1$ for (a) $\phi_g - \phi_f = \pi/4, 3\pi/4$; (b) $\phi_g - \phi_f = \pi/2$; (c) $\phi_g - \phi_f = 0, \pi$; (d) $\phi_g - \phi_f = 5\pi/4, 7\pi/4$; (e) $\phi_g - \phi_f = 3\pi/2$. Blue solid lines represents the even-photon-number distribution, while red dotted lines denote odd-photon-number distribution.

6. Wigner function of PASCS

The WF[24] is a powerful tool to investigate the nonclassicality of optical fields. Its partial negativity implies the highly nonclassical properties of quantum states and is often used to describe the decoherence of quantum states. In this section, we derive the analytical expression of the WF for the PASCS. For this purpose, we first recall the Wigner operator in the coherent state $|z\rangle$ representation[25,26]

$$\Delta(\beta) = \mathrm{e}^{2|\beta|^2} \int \frac{\mathrm{d}^2\alpha}{\pi^2} |\alpha\rangle \langle -\alpha| \, \mathrm{e}^{-2(\alpha\beta^* - \alpha^*\beta)}, \quad (23)$$

where $\beta = (q + \mathrm{i}p)/\sqrt{2}$, the expression of the WF

$$W(\beta) = \mathrm{tr}[\rho\Delta(\beta)], \quad (24)$$

as well as the density operator of the PASCS

$$\rho = N_{g,m}^2 a^{\dagger m} |z\rangle_{gg} \langle z| a^m. \quad (25)$$

Substituting Eqs. (23) and (25) into Eq. (24), we have

$$\begin{aligned} W(\beta) &= N_{g,m}^2 \mathrm{e}^{2|\beta|^2} \int \frac{\mathrm{d}^2\alpha}{\pi^2} \langle -\alpha| a^{\dagger m} |z\rangle_{gg} \langle z| \\ &\quad \times a^m |\alpha\rangle \, \mathrm{e}^{-2(\alpha\beta^* - \alpha^*\beta)} \\ &= (-1)^m N_{g,m}^2 (1 - 4|fg|^2)^{1/2} \mathrm{e}^{2|\beta|^2} \int \frac{\mathrm{d}^2\alpha}{\pi^2} \\ &\quad \times \alpha^{*m}\alpha^m \exp[-|\alpha|^2 + (f^*z^* + g^*z \\ &\quad - 2\beta^*)\alpha - (fz + gz^* - 2\beta)\alpha^* - f^*g^*\alpha^2 \\ &\quad - fg\alpha^{*2} - |z|^2] \\ &= \frac{N_{g,m}^2 \mathrm{e}^{B_1}}{\pi} \frac{\mathrm{d}^{2m}}{dr^m ds^m} \sum_{p=0}^{\infty} \frac{r^p s^p}{(4|fg|^2 - 1)^p p!} \\ &\quad \times \exp\left(B_2 r + B_2^* s - B_3 r^2 - B_3^* s^2\right)|_{r=s=0} \\ &= \frac{N_{g,m}^2 \mathrm{e}^{B_1}}{\pi} \sum_{p=0}^{m} \frac{(-1)^p (m!)^2 |B_3|^m}{[(m-p)!]^2 p! |fg|^p} \\ &\quad \times \left| H_{m-p}\left(\frac{B_2}{2\sqrt{B_3}}\right)\right|^2, \end{aligned} \quad (26)$$

where

$$\begin{aligned} B_1 &= [-2|z|^2 - 2|\beta|^2 - 4\mathrm{Re}(fg^*z^2 - \beta g^*z \\ &\quad - f\beta^*z - 2f\beta^*|g|^2z - 2\beta g^*|f|^2z \\ &\quad + 2fg\beta^{*2})]/(1 - 4|fg|^2), \\ B_2 &= (2\beta - gz^* - fz + 4fg\beta^* - 2gz^*|f|^2 \\ &\quad - 2fz|g|^2)/(1 - 4|fg|^2), \\ B_3 &= \frac{fg}{1 - 4|fg|^2}. \end{aligned} \quad (27)$$

Equation (25) seems to be a new result related to single-variable Hermite polynomials. In particular, when the photon-added number $m = 0$, Eq. (25) is reduced to

$$\begin{aligned} W_{m=0}(\beta) &= \frac{1}{\pi} \exp\left[\frac{-2|z|^2 - 2|\beta|^2}{1 - 4|fg|^2}\right. \\ &\quad - 4\mathrm{Re}(fg^*z^2 - \beta g^*z - f\beta^*z - f\beta^*z \\ &\quad - 2f\beta^*|g|^2z - 2\beta g^*|f|^2z \\ &\quad \left. + 2fg\beta^{*2})/(1 - 4|fg|^2)\right]. \end{aligned} \quad (28)$$

Now we shall analyze the changes of the Wigner function of PASCS as we change the complex squeezing parameters f and g and the photon-added number m. The complex squeezing parameters f and g are not independent of each other and can be adjusted to achieve different degrees of squeezing. Using Eq. (26), we depict the WFs of PASCS in Fig. 5 for several different values of complex squeezing parameters f and g with $m = 0$ and $z = (1+\mathrm{i})/\sqrt{2}$. From Figs. 5(a) and 1(b), we can see that the WF with $m = 0$ is Gaussian in phase space and has one positive peak which is squeezed along the q direction and expanded along the p direction as $|g|$ increases. Figures 5(c)–5(f) display that the peak will rotate clockwise when either ϕ_g or ϕ_f decreases.

In addition, the Wigner functions are depicted in Fig. 6 for different values of m with $|g| = 0.1$, $\phi_g = \phi_f = 0$, and $z = (1+\mathrm{i})/\sqrt{2}$. It is easy to see that the WF is non-Gaussian in phase space when $m \neq 0$. As evidence of the nonclassicality of the state, some negative regions of the WF are clearly seen in Figs. 6(a)–6(c), and they are gradually enlarged but their numerical fluctuations become inconspicuous gradually with the increasing number of photon addition.

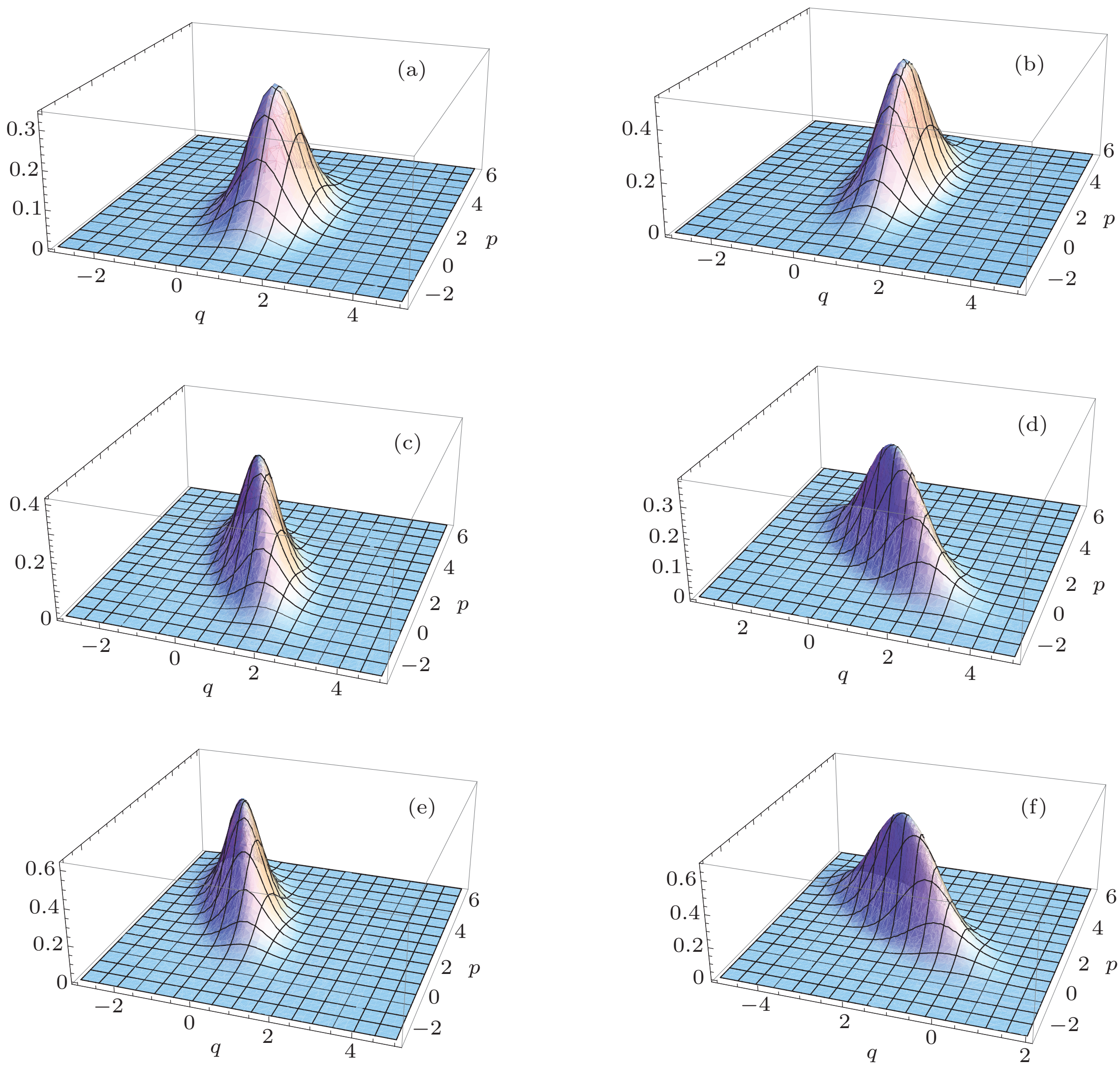

Fig. 5. (colour online) Wigner-function distributions of the PASCS for different f and g. (a) $|g| = 0.1$, $\phi_f = \phi_g = 0$; (b) $|g| = 0.2$, $\phi_f = \phi_g = 0$; (c) $|g| = 0.2$, $\phi_g = \pi/4$, $\phi_f = 0$; (d) $|g| = 0.2$, $\phi_g = \pi/2$, $\phi_f = 0$; (e) $|g| = 0.2$, $\phi_g = 0$, $\phi_f = \pi/4$; (f) $|g| = 0.2$, $\phi_g = 0$, $\phi_f = \pi/2$.

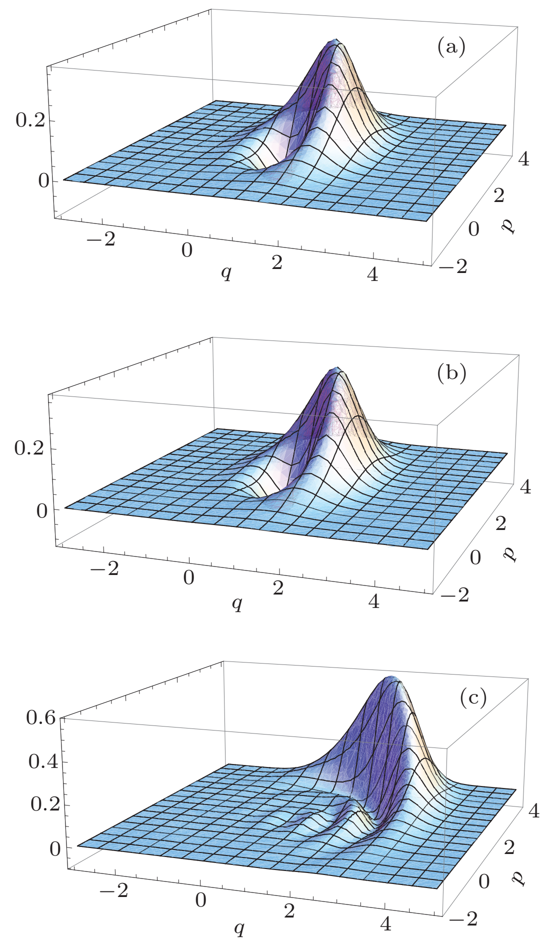

Fig. 6. (colour online) Wigner-function distributions of the PASCS for different values of m: (a) 1; (b) 3; (c) 5.

7. Production of the PASCS

We finally consider how the PASCS can be produced in optical processes. The whole process may be divided into two steps, and the first one is to produce the $|z\rangle_g$ state. In order to achieve this aim, we propose a scheme that an optical parametric down-conversion process coexists with a process of generating a coherent state. For instance, one may consider a composite system consisting of a gain medium and a parametric optical parametric oscillator in the same optical cavity. Then, the laser mode (through a nonlinear crystal) will generate a second harmonic field ($\omega \to 2\omega$), which will, in turn, generate (through another nonlinear crystal) a sub-harmonic mode ($\omega \to 2\omega$). If the variables for the second harmonic mode can be adiabatically eliminated, the system Harmiltonian will have the form that may produce the $|z\rangle_g$ state given by Eq. (5). The parameter f depends on the lasing gain and the parameter g is determined by the pumping-field strength and the second-order susceptibility of the down-conversion process. By carefully adjusting all the experimental parameters, the condition $|f|^2 + |g|^2 = 1$ will be satisfied and the light representing $|z\rangle_g$ is produced. The second step is the process of m-photon excitation on this light, letting this beam inject into a cavity including an atom with two levels, the interaction between them is described by the effective Harmiltonian

$$H_{\text{eff}} = \hbar\left(ga^{\dagger m}s^{-} + g^{*}s^{\dagger}a^{m}\right), \tag{29}$$

where g is the coupling factor between m photons and the atom, s^{-} is the operator which represents the absorption of the atom. To lower the order of the coupling constant, the wave function of the system at time t is

$$\begin{aligned}|\psi(t)\rangle = {} & |\psi_{\rm R}\rangle|G\rangle - \mathrm{i}t(ga^{\dagger m}s^{-} \\ & + s^{\dagger}g^{*}a^{m})|\psi_{\rm R}\rangle|G\rangle,\end{aligned} \tag{30}$$

where $|\psi_{\rm R}\rangle$ ($|G\rangle$) is the initial state of the field (atom). Suppose that at time t, the atom is measured to be in the absorption state, then the state of the field is reduced to

$$\rho_{\text{field}} \propto a^{\dagger m}|\psi(t)\rangle\langle\psi(t)|a^{m}, \tag{31}$$

which is apart from a normalization constant. Thus, if the initial state of the radiation field is $|z\rangle_g$, Eq. (31) becomes

$$\rho_{\text{field}} \propto a^{\dagger m}|z\rangle_{gg}\langle z|a^{m}. \tag{32}$$

Thus, the photon-added squeezed coherent state of the field we need is obtained.

8. Conclusion

In summary, we introduce a new type of photon-added squeezed coherent state, which is obtained by repeatedly exerting the photon creation operator on a new type of squeezed coherent state. We first derive the normalization factor of PASCS, which is related to single-variable Hermite polynomials. Based on the technique of integration within an ordered product of operators, we investigate its statistical properties such as Mandel's Q parameter, photon-number distribution, and the Wigner function, whose changes with the phase difference are analyzed graphically. Furthermore, the nonclassicality is discussed in terms of intense oscillations of the PND and the negativity of the Wigner function after deriving the explicit expressions of the PND and WF, and the results demonstrate explicitly the nonclassical properties of the PASCS.

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