European Journal of Political Economy 46 (2017) 15–25

Contents lists available at ScienceDirect

European Journal of Political Economy

journal homepage: www.elsevier.com/locate/ejpoleco

Quality contests

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ARTICLE INFO

JEL:
C72
D72

Keywords:
Optimal contest design
Expected effort of the winner
Contestants' exclusion
Leveling the playing-field

ABSTRACT

In noisy contests where only the winner's entry will eventually be implemented, the suitable objective is to maximize the expected quality of the entry of the winner. We compare the optimal set of rules in contests under such an objective to the one under maximization of the sum of contestants' efforts, which is commonly assumed in the literature, and find that it may be beneficial to exclude weak contestants, unlevel the playing field, and weaken the underdog.

1. Introduction

What should we focus on when analyzing contestants' behavior in contests? The answer depends on the applications we want to embrace. The first and most commonly adopted view in the literature is to focus on the sum of contestants' efforts as the key outcome of the contest. This focus is particularly suitable for applications where efforts are potentially wasteful, such as arms races among countries, political campaigns, lobbying over the outcome of a public policy and, in general, rent-seeking activities. The second approach is to focus on competitive balance, interpreted as the uncertainty of the contest outcome. The organizer of a sporting event benefits from outcome uncertainty in that it thrills the interest of the audience and thus it increases the organizer's sales. The third possible focus is on contestants' participation, which is to be maximized in online communities where users contribute to the content, and minimized in military conflicts.[1] The fourth focus, and the cornerstone of this paper, is on the quality of the winner's entry. This is of particular interest in contests where contestants submit entries but only the winner's entry is eventually implemented. Five examples fit this context: (i) the body of laws of a country concerning a specific matter is found to be flawed, obsolete or simply silent, and competing bills are proposed to fix the flaws, to reform the obsolete laws or simply to legislate that matter from scratch; (ii) lobbyists write persuasive policy proposals so as to convince a policymaker to implement their proposal over that of other lobbyists; (iii) parties, politicians, and mayors propose political programs so as to win the elections; (iv) lawyers collect evidence in favor of their client to persuade the court; (v) citizens submit urban development projects to their municipality. In these five examples, contestants—i.e., legislators, lobbyists, politicians, lawyers, citizens—exert costly effort in order to increase the quality of their entries—i.e., bills, policy proposals, political programs, evidence, urban development projects.[2] The common feature of these five examples is that, to some extent, the quality of the losing entries is of no interest, and in fact we claim that the suitable focus is on the quality of the winner's entry, namely (i) the quality of the only bill which will eventually become law, (ii) the quality of the policy

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[1] A comprehensive literature review of the proposed approaches is beyond the scope of this work. Papers focusing on the sum of efforts are, for instance, Baye et al. (1993), Moldovanu and Sela (2001), Moldovanu et al. (2007), and Brown (2011). As for competitive balance, see Falconieri et al. (2004), and Vrooman (2012). Contests targeting participation are analyzed, for instance, in Azmat and Möller (2009).

[2] We claim that examples (i)–(v) are contests in that entries are in direct competition and a greater entry quality increases one's probability of winning the contest and decreases that of the rival.

http://dx.doi.org/10.1016/j.ejpoleco.2016.10.008
Received 1 March 2016; Received in revised form 13 October 2016; Accepted 13 October 2016
Available online 15 October 2016
0176-2680/

proposal of the lobbyist who will manage to persuade the policymaker, (iii) the quality of the only political program which will win and thus presumably be implemented, (iv) the quality of the evidence brought by the lawyer who will eventually win the trial, seen as a proxy of the probability of misjudgment by the court, and (v) the quality of the only urban development project which will eventually be carried out.[3]

If the quality of the entries is perfectly observable by who selects the winner (i.e., perfectly discriminatory contest), the suitable objective is to maximize the quality of the highest quality entry, which coincides with the winner's entry. If instead the quality of the entries is observable with some noise (i.e., imperfectly discriminatory contest), then a low quality entry might win the contest and thus be implemented.[4] In this realm of noisy contests, rather than the quality of the *highest quality* entry, it seems reasonable to maximize the expected quality of the entry of the *winner*, which could well *not* be the highest quality entry because of the noise. Thus, we claim that, in noisy contests, the expected quality of the entry of the winner is the natural objective in settings where only the winner's entry will eventually be implemented.[5] Throughout the paper we conform to the literature and use the conventional word “effort” to define the costly and sunk investment of contestants—here, the words “quality”, “bid” and “effort” are interchangeable. We compare the optimal set of contest rules under the maximization of the expected effort of the winner and under the conventional maximization of the sum of efforts.[6]

In order to easily recall these two different objectives throughout the paper, we call an *aggregative contest* one where the objective is to maximize the sum of efforts, and we call a *quality contest* one where the objective is to maximize the expected effort of the winner. The reason why our benchmark is an aggregative contest, besides the fact that it is the most commonly adopted in the literature, is that there are real-life situations where both objectives are of interest; for instance, in a research contest, a benevolent government might want to stimulate the market of research and thus induce an aggregative contest, whereas the entity organizing the contest might probably only benefit from the quality of the winner's entry and thus induce a quality contest. From this point of view, this comparison sheds light on the government's need to impose certain policies of contest design on the entity organizing the contest.

Our model has two periods. In the first period, a set of rules which shapes the competitive environment that contestants face is chosen and publicly announced so as to maximize the objective of interest—i.e., the sum of efforts or the winner's effort. In the second period, the noisy contest is played. The way we add the noise to the winner selection process is by means of a standard lottery contest success function—see (1)—which is a tractable reduced-form corresponding to a contest where the winner is the contestant with the highest effort, multiplied by a noise following an inverse exponential distribution, see Jia (2008).[7] We cherry-pick three sets of contest rules for which the different results on how to optimally design an aggregative and a quality contest most greatly help us to grasp the intuitions behind the differences between these two types of contest. Delivering these intuitions is the main goal of the paper.

First, we study the *optimal selection of contestants*. From a given set of contestants—“applicants”—of known types (i.e., marginal cost of effort), the set of contestants—“finalists”—who are granted access to the contest is chosen. Once the choice of finalists is made and publicly announced, some of them might still drop out of the contest if better off doing so (in particular, if the other selected finalists are much stronger). In a quality contest, we find that it may be profitable to exclude some contestants, especially when types are sufficiently homogeneous. In fact, types homogeneity across contestants yields efforts homogeneity across contestants, and when efforts are homogeneous, the expected effort of the winner equals individual effort regardless of who wins. Individual effort decreases in the number of finalists, because each finalist has roughly $1/n$ probability of winning the prize, and thus as n increases, individual willingness to exert effort decreases. Therefore, if contestants are sufficiently homogeneous, excluding some applicants from the final is profitable in a quality contest.[8] However, the sum of efforts—contrary to the expected effort of the winner—decreases as contestants quit. This negative effect turns out to be stronger than any positive effect on individual efforts that exclusions may generate, and this is why Fang (2002) finds that no exclusion is profitable in aggregative contests, regardless of the heterogeneity of types. Thus, while in an aggregative contest we should not be concerned about excluding contestants, and rather be concerned about stimulating applications, we find that in a quality contest it might be optimal to exclude some applicants from the contest. In fact, for a significant set of contestants’ types—which numerical analysis suggests to be large—the optimal number of finalists of a quality contest is two, and for this reason we focus on a two-contestant contest in the subsequent analysis.[9]

[3] Note that focusing exclusively on the quality of the winner's entry implicitly assumes that the winner of the contest has no role in implementing her entry once it wins; thus, the reader can either think of a completely self-implementing entry, which is the case in (i), or see the present model as part of a bigger model where the winner will be involved in the implementation process whose success arguably depends on the winner's type, which is the case in (iii).

[4] The noise could arise, for instance, when the time to analyze the details of the entries in order to select the winner is limited or costly.

[5] This idea mirrors the highest bid maximization in perfectly discriminatory contests: see, for instance, Cohen et al. (2008), Segev and Sela (2014) or Jonsson and Schmutzler (2015). Maximization of the highest bid in noisy contest would correspond to maximize the quality of the best entry, but the best entry could lose because of the noise and, if this is the case, we claim its quality is of no value. The expected quality of the winner's entry takes this into account.

[6] To the best of our knowledge, there are no works performing such an analysis. Yet, we acknowledge that, at the end of p. 345 of Andreff and Szymanski (2006), the authors mention “[.] there are also situations in sports where the aim of the contest designer is not to maximise total expected effort but expected effort of the winner.” However, they do not run any analysis of it. Drugov and Ryvkin (2016) propose as the principal's objective function the expectation over types of a general function that may depend on the efforts, types, and bias of the playing field. The expected effort of the winner is thus a special case.

[7] A review of the contest success functions and their extensions is in Van Long (2013).

[8] If all contestants are *perfectly* homogeneous, the optimal number of contestants is two.

[9] Fang (2002) and the present paper adopt a lottery contest success function. In different models of contests, the no-exclusion result of Fang (2002) and our finding of profitability of exclusion “ from the bottom”—that is, starting from the weakest contestants—in quality contests need not hold. For instance, Baye et al. (1993) find that in an all-pay auction excluding top contestants might be beneficial in boosting competition among weaker contestants. However, their objective function is the sum of efforts.

Second, we analyze whether and how to optimally *level the playing field* by giving, for instance, a competitive advantage to the low type. We find that, unlike an aggregative contest, in a quality contest it is optimal to leave the playing field unleveled. The intuition is as follows. Start from a contest between two contestants—a high type and a low type—with a perfectly leveled playing field, such that both contestants have the same equilibrium probability of winning and competition for the prize is maximum. Let us move away from the perfectly leveled playing field, such that one of the two contestants is now more likely to win in equilibrium. Then, two effects simultaneously arise: (i) both contestants' efforts decrease because competition for the prize is no longer maximum,[10] and (ii) the probability of winning increases for one contestant and decreases for the other. An aggregative contest is harmed by (i) and not affected by (ii), and hence an unleveled playing field is always detrimental in aggregative contests, as found by the existing literature—see Nti (2004), Runkel (2006), Franke (2012), and Franke et al. (2013). On the contrary, a quality contest is harmed by (i), but benefits from (ii) if the high type is the one given the advantage, because this increases the probability that the high type's effort (which is greater than that of the low type) is the winning one. This trade-off is present only in quality contests and it explains the optimality of leaving some degree of advantage to the high type (i.e., a non-perfectly leveled playing field).

Third, we run *comparative statics on contestants' types*.[11] We find that while weakening the favorite—i.e., an increase in her marginal cost of effort—would be beneficial both in a quality contest and in an aggregative contest, weakening the underdog could be beneficial only in a quality contest, despite lowering both contestants' efforts. The intuition builds upon the above-mentioned playing field effect and adds the layer of individual effect; that is, changing a contestant's type affects the playing field—making it more or less leveled—as well as the affordability of effort of that contestant (individual effect). In particular, weakening the underdog is beneficial to a quality contest when types are sufficiently heterogeneous and the playing field is sufficiently leveled. The intuition of this result is given in Section 6.

The structure of the paper is as follows. In Section 2 we outline the model and formalize the difference between an aggregative and a quality contest. In Section 3 we analyze the optimal selection of contestants and find that for a significant set of contestants' types—which numerical analysis suggests to be large—the optimally selected set of contestants is two. Thus, in the remainder of the paper we focus on a two-contestant contest. In Section 4 we fully characterize its equilibrium and propose a way to disentangle, in the equilibrium efforts, the effect due to the contestant's individual type and the effect due to the contest playing field. In Sections 5 and 6 we build upon Section 4 to analyze the optimal contest design's differences between an aggregative and a quality contest, in particular: optimal leveling of playing field (Section 5) and comparative statics on contestants' types (Section 6). Section 7 concludes. The proofs are in the appendix.

2. Model

Consider a complete information contest with n risk-neutral contestants indexed by $i \in \{1, \ldots, n\}$. Contestants compete for a prize whose value we normalize wlog to 1. Each contestant i simultaneously chooses effort level e_i, and has a probability of winning the prize equal to

$$p_i(e_1, \ldots, e_n) = \begin{cases} 0 & \text{if } e_j = 0\ \forall\, j \in \{1, \ldots, n\} \\ \dfrac{\alpha_i e_i}{\sum_{j=1}^{n} \alpha_j e_j} & \text{otherwise} \end{cases} \tag{1}$$

with $p_i\colon \mathbb{R}_+^n \to [0, 1]$, and $\alpha_i \geq 0\ \forall\, i \in \{1, \ldots, n\}$.[12] The cost of effort is linear, and the possibly heterogeneous marginal cost is referred to as the contestant's type. Hence, contestant i chooses e_i to maximize

$$\frac{\alpha_i e_i}{\sum_{j=1}^{n} \alpha_j e_j} - \beta_i e_i \tag{2}$$

Prior to contestants' simultaneous choice of efforts, a set of contest rules is chosen and announced. The paper is about how the optimal set of rules changes from an aggregative to a quality contest. In Section 3, the rule to be chosen is $\alpha_i \in \{0, 1\}$ for $n \geq 2$; that is, it is possible to select the set of contestants who can participate in the contest, but it is not possible to level the playing field.[13] In Section 5, the rule to be chosen is (α_1, α_2), with $n = 2$; that is, a competitive advantage (or disadvantage) can be given to contestant i as opposed to contestant j (i.e., leveling the playing field). In Section 6, we run comparative statics on β_i's, with $n = 2$.

As discussed, our focus is on the objective to maximize when choosing the contest rules, which is either the sum of efforts in an aggregative contest, denoted by A, as prevalent in the literature

$$[\textbf{Aggregative contest}] \quad A = \sum_{i=1}^{n} e_i \tag{3}$$

or the expected effort of the winner in a quality contest, denoted by Q:

[10] The fact that an unleveled playing field reduces contestants' efforts holds in our class of lottery contests, but needs not hold in different contest models (see, Drugov and Ryvkin, 2016).

[11] All exercises are considered separately. Thus, for instance, when we run comparative statics on types, the playing field cannot be leveled.

[12] Note that one of the α_i's can be normalized to 1. This will be particularly useful in the two-player model, where we will define the ratio of α_i's as α.

[13] Note that if $\alpha_i = 0$, then $e_i^* = 0$, and thus choosing $\alpha_i = 0$ is equivalent to excluding contestant i from the contest.

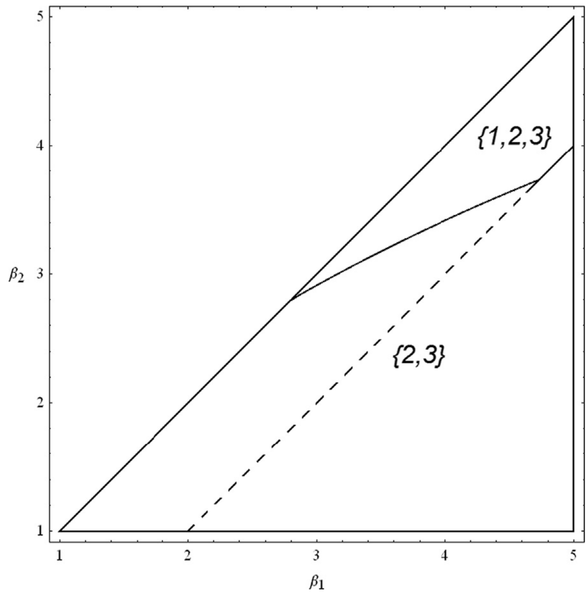

Fig. 1. Optimal set of finalists in a three-contestant quality contest.

$$[\textbf{Quality contest}] \quad Q = \sum_{i=1}^{n} p_i(e_1, \ldots, e_n) e_i \tag{4}$$

3. Contestants' selection

In this section we analyze the optimal selection of contestants for a given set of contestants of known types, fixing $\alpha_i = 1$ $\forall\, i \in \{1, \ldots, n\}$. We analyze whether it is optimal to exclude some contestants from the contest. It should be noted that once the choice of contestants has been made and announced, contestants might drop out of the contest by exerting zero effort if they find it profitable—namely, if too weak as opposed to the other selected contestants. When selecting contestants, we should take into consideration this endogenous participation and compare payoffs—either A or Q—under different subsets of contestants.

We first deliver the main intuition of this section with a three-contestant example, which is the simplest non-trivial case. Consider a contest between three contestants with marginal costs equal to $\beta_1 \geq \beta_2 \geq \beta_3 > 0$. Set the marginal cost of the high type, $\beta_3 = 1$. The results of the three-contestant optimal contestant selection in a quality contest are shown in Fig. 1, where we denote the regions by the optimal set of finalists' identities.

First, consider contestants' participation choice. If two contestants are selected, no contestant exerts 0 effort in equilibrium. If all three contestants are selected, then the low type—contestant 1—is better off quitting when she is too weak compared to other two contestants, the medium and the high type. This occurs when β_1 is sufficiently large; that is, in the {2,3}-region below the dashed line. Thus, only medium and high types {2,3} exert positive effort, and the contest is a two-contestant one.[14] In the remaining region—i.e., between the two parallel lines—the low type is sufficiently strong to be willing to participate, if selected. This region is in turn separated into two regions, one belonging to the {2,3}-region and one to the {1,2,3}-region.

In the {1,2,3}-region, β_1 and β_2 are between 3 and 5, and sufficiently close to each other, whereas $\beta_3 = 1$; that is, the low and medium types are sufficiently similar in type, but significantly weaker than the high type (the " superstar"). Here, it turns out to be optimal to admit all the contestants to the contest in order to create competition between the two (similar) weak contestants, which has positive spillovers on the effort of the superstar. Thus, no exclusion is optimal. This is why this region is called {1,2,3}.

In the {2,3}-region (above the dashed line), the three contestants are sufficiently homogeneous in types — that is, β_1 and β_2 are sufficiently close to $\beta_3 = 1$. This means that exerted efforts are also sufficiently similar. Hence, $Q \approx e_i$ regardless of the number of active contestants — i.e., the identity of the winner negligibly affects Q because individual efforts are similar. In this situation, excluding a contestant yields higher competition between the remaining two because their individual equilibrium probabilities of winning become approximately 1/2 instead of 1/3, and thus each of them exerts more effort: that is, exclusion increases e_i. Since $Q \approx e_i$, a quality contest benefits from excluding a contestant. For this reason we find that in this region it is optimal to exclude the low type, even though she would be willing to exert positive effort if selected, and thus the optimal set of finalists is {2,3}. Note that the lack of effort by the excluded contestant in this region is not detrimental to Q. On the contrary, in an aggregative contest, the lack

[14] In this region one could think of excluding the high or the medium type contestants in order to make the low type willing to participate. However, this is never optimal in either contest.

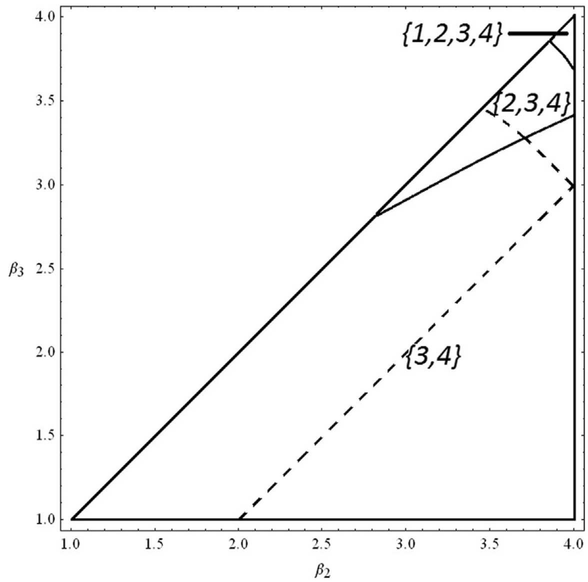

Fig. 2. Optimal set of finalists in a four-contestant quality contest.

of effort by the excluded contestant would be detrimental to A, and in fact this negative effect of exclusion turns out to prevail on any other positive effect of exclusion on individual efforts in aggregative contests. This is why (Fang, 2002) finds that no exclusion is beneficial to aggregative contests; that is, the whole region between the two parallel lines would be a {1,2,3}-region in an aggregative contest.

Proposition 1. [(Fang, 2002), *Aggregative contest*] *In an aggregative contest with* $\alpha_i = 1\ \forall\ i$, *and* $\beta_1 \geq \beta_2 \geq \ldots \geq \beta_n > 0$ *exclusion of contestants is not beneficial.*

From the three-contestant example we see that for a wide variety of parameters, only medium and high types participate in the contest (the {2,3}-region). This happens either because the low type is too weak to be willing to participate (the {2,3}-region below the dashed line) or because the exclusion of the low type from the contest is beneficial in a quality contest (the {2,3}-region above the dashed line).

If we do the same exercise with four contestants, the conclusion is similar. Set $4 = \beta_1 \geq \beta_2 \geq \beta_3 \geq \beta_4 = 1$. The result is in Fig. 2. Again, along the dashed lines a different degree of exclusion is needed to reach the final set of contestants. In particular, within each region, the more we move upwards, the more exclusion is required to reach the final set of contestants. For instance, the finalists set {3,4} in the {3,4}-region is achieved with different exclusion policies which divide this region into three subregions: in the big triangle, neither contestant 1 nor contestant 2 are willing to exert positive effort; in the irregular pentagon, contestant 1 is excluded and contestant 2 is not willing to exert effort; and in the small triangle, contestant 1 and contestant 2 are both excluded.

Extending the graphical analysis to more contestants would either require the addition of dimensions to the graph or giving arbitrary values to all but two β_i's. In the former case we lose visualizability, in the latter we lose generality of the analysis. However, profitability of exclusion in a quality contest carries over. The intuition learnt in the three-contestant and four-contestant examples above is that exclusion is particularly profitable when contestants are of sufficiently homogeneous types. This is the idea behind the proof of Proposition 2.

Proposition 2. [*Quality contest*] *In a quality contest with* $\alpha_i = 1\ \forall\ i$, *for any* $n \geq 3$ *there exists an open set containing* $\{\beta_1, \ldots, \beta_n\}$ *with* $\beta_1 \geq \beta_2 \geq \ldots \geq \beta_n > 0$ *such that the exclusion of contestant (s)* $\{1, \ldots, j\}$ *with* $1 \leq j \leq n-2$ *is strictly beneficial (i.e., expected effort of the winner strictly increases).*

The main conclusion of this section is that for a non-negligible set of contestants' types the optimal number of finalists is two. Furthermore, the numerical exercises we made with three and four contestants seem to suggest that this set of types is large. Then, it seems natural to analyze in depth a two-contestant quality contest, which we do in the remainder of the paper. Prior to discussing the optimal choice of contest rules in aggregative and quality contests, in the next section we analyze equilibrium efforts and propose a notation which is convenient for delivering intuitions later on.

4. Equilibrium with n=2

The unique NE of the contest described above with two contestants is well-known in the literature[15]

[15] See, for example, Nti (2004).

$$e_i^* = \frac{\alpha_1\alpha_2\beta_1\beta_2}{(\alpha_1\beta_2+\alpha_2\beta_1)^2}*\frac{1}{\beta_i}$$

We define the following

$$\alpha = \frac{\alpha_1}{\alpha_2}\beta = \frac{\beta_1}{\beta_2}S = \frac{\alpha}{\beta}$$

and in the special case of n=2—which is the focus of Sections 4, 5, and 6—the choice of (α_1, α_2) coincides with the choice of α. Using the above notation, we can write equilibrium efforts as

$$e_i^* = \underbrace{\frac{S}{(1+S)^2}}_{\text{playing field effect}} * \underbrace{\frac{1}{\beta_i}}_{\text{individual effect}} \tag{5}$$

and the equilibrium probabilities of winning as

$$p_1^* = \frac{S}{S+1} \quad \text{and } p_2^* = \frac{1}{S+1} \tag{6}$$

Thus, we name contestants according to (6): we name *the favorite*the contestant more likely to win in equilibrium, and we name *the underdog* the contestant less likely to win. The value of S determines who is the favorite between the contestants (i.e., $S \gtreqless 1 \Leftrightarrow \alpha \gtreqless \beta \Leftrightarrow p_1^* \gtreqless p_2^*$). We refer to the contestant with the lowest (greatest) β_i as the high (low) type. Thus, being a high or low type is different from being the favorite or the underdog and, for instance, contestant 1 might be the high type (when $\beta \le 1$), and yet less likely to win in equilibrium than her rival if the effect of α overcompensates the asymmetry of types (when $\alpha \le \beta$).

This notation allows us to distinguish in the equilibrium efforts (5) between:

- the playing field effect: $\frac{S}{(1+S)^2}$. This term is maximized at S=1, that is when $p_1^* = p_2^* = \frac{1}{2}$. This situation is often referred to as a " perfectly leveled playing field" .
- the individual effect: $\frac{1}{\beta_i}$. This term accounts for the contestant's idiosyncratic type: the greater the contestant's marginal cost of effort, the more severe the negative individual effect.

Changes in α_i's (Section 5) have only playing field effects—through S. Changes in β_i's (Section 6) have both playing field effects—through S—and individual effects.

Substitute (5) into (3) in order to write the sum of efforts as

$$A = \frac{S}{(1+S)^2}\left(\frac{1}{\beta_1}+\frac{1}{\beta_2}\right) \tag{7}$$

and substitute (5) and (6) into (4) in order to write the expected effort of the winner as

$$Q = \frac{S}{(1+S)^3}\left(\frac{S}{\beta_1}+\frac{1}{\beta_2}\right) \tag{8}$$

Note that in (8) the playing field effect and individual effect are not (multiplicatively) separable; that is, in quality contests the playing field and the individual effects are intertwined, as will be made clear in the analysis below.

With the preliminaries developed in this sections, we analyze the optimal leveling of playing field and we run comparative statics on type in aggregative and quality contests in the next sections.

5. Leveling the playing field

In this section a competitive advantage (or disadvantage) can be given to one of the two contestants by choosing the parameter $\alpha \ge 0$. There is an abundance of examples of such policies: court-appointed attorney in trials where a party cannot afford a private attorney, governmental subsidies to start-ups, public procurement auctions with bidding advantage depending on the firm size, sport competitions where the weak—or losing—player is given a competitive advantage,[16] hiring procedures of international organizations that guarantee a minimal share of employees from developing countries, and so on.

In an aggregative contest, the problem $\max_\alpha A$ is equivalent to $\max_S \frac{S}{(1+S)^2}$ because α enters A only in S. The maximum of $\frac{S}{(1+S)^2}$ is at S=1, or equivalently when $\alpha = \beta$. This result is already present, for instance, in Nti (2004).

Proposition 3. [(Nti, 2004), *Aggregative contest*] *The optimal* α *in an aggregative contest is*

$$\alpha^A = \beta$$

[16] Examples of these are golf tournaments where the performance of low-performing players is normalized, or horse handicap race where horses carry different weights and a faster horse is given a heavier weight, or football where the ball is given to the team that has just conceded a goal.

Intuitively, the optimal design of an aggregative contest is when competition between contestants is maximum, and this is achieved by setting an α such that contestants are equally likely to win in equilibrium (S=1) regardless of their marginal costs. We denote by α^A the optimal α in an aggregative contest, which is the one that perfectly levels the playing field.

In a quality contest, we retrieve the optimal α building upon the analysis of Section 4.

Proposition 4. [*Quality contest*] *The optimal* α *in a quality contest is*

$$\alpha^Q = \beta(1 - \beta + \sqrt{(1 - \beta)^2 + \beta}) \tag{9}$$

and thus, $\beta \lesseqqgtr 1 \iff \alpha^Q \gtreqqless \beta \iff p_1^* \gtreqqless p_2^*$.

The intuition is as follows. Each contestant's effort is maximum when the playing field is perfectly leveled, that is, $\alpha^A = \beta$. Setting α lower or greater than β would decrease both e_1 and e_2 (i.e., negative playing field effect). This would decrease the sum of efforts $e_1 + e_2$, and thus moving away from $\alpha^A = \beta$ in an aggregative contest is harmful (Proposition 3). On the other hand, the objective function of a two-contestant quality contest is $p_1e_1 + p_2e_2$, [17] thus it is not trivial that setting α lower or greater than β—which decreases both e_1 and e_2—is detrimental, because p_1 and p_2 also change. Say wlog that $\beta \le 1$, such that contestant 1 is the high type, contestant 2 is the low type, and $e_1 \ge e_2$ (see (5)). Setting α lower than β is not optimal because, besides the negative playing field effect of decreasing e_1 and e_2, it also decreases p_1 and increases p_2 (in fact, $\alpha < \beta \Rightarrow S < 1 \Rightarrow p_1^* < p_2^*$, see (6)) which has a negative effect on $p_1e_1 + p_2e_2$ because $e_1 \ge e_2$. Setting α greater than β would instead increase p_1 and decrease p_2, which increases $p_1e_1 + p_2e_2$ because $e_1 \ge e_2$. Thus, while the detrimental decrease of e_1 and e_2 occurs both as α becomes lower or greater than β, the change of p_1 and p_2 are detrimental if α becomes lower than β and beneficial if α becomes greater than β. Both the detrimental effect on e_1 and e_2 and the beneficial effect on p_1 and p_2 when α increases are small at $\alpha = \beta$, but the former is only of second order because $\alpha = \beta$ reaches the maximum of $e_1 + e_2$. Therefore, $\alpha^Q \ge \beta$.[18] The same argument obviously holds if we swap identities: that is, if $\beta \ge 1$, $\alpha^Q \le \beta$. Hence, in a quality contest it is optimal to level the playing field, but not perfectly, and to leave the high type more likely to win in equilibrium (Proposition 4).[19]

6. Comparative statics on contestants' types

In this section we use the tools developed in Section 4 to pin down the different channels through which changes of contestants' types affect first individual efforts, and second A and Q. In order to gradually build intuition, it is crucial to distinguish between the playing field effect and the individual effect of changes of β_i.[20] Below, we weaken contestant i, who is the underdog or the favorite.[21]

Weakening the underdog increases the asymmetry between contestants, and hence the playing field effect is negative for both contestants (the favorite is more favorite, and the underdog is less likely to win). Moreover, the effort of the underdog drops further because her marginal cost of effort increases—i.e., negative individual effect. Thus, the overall effect of weakening the underdog is negative on both the underdog's and the favorite's efforts.

Weakening the favorite decreases the asymmetry between contestants, and hence the playing field effect is positive for both contestants (a more leveled playing field). However, the effort of the favorite undergoes a negative individual effect, because her marginal cost of effort increases. All in all, the negative individual effect overcomes the positive playing field effect, and thus the effort of the favorite decreases as she is weakened. Thus, the overall effect of weakening the favorite is negative on the favorite's effort and positive on the underdog's effort.

What are the final effects on A and Q of increasing β_i? The only straightforward conclusion from the above analysis is that A decreases when the underdog is weakened because both contestants' efforts decrease. The other comparative statics depend on values of (α, β) and are contained in Propositions 5 and 6. We first report the results and then discuss the intuition behind them.[22]

Proposition 5. [*Aggregative contest*] *Weakening the underdog is not beneficial to A.*

Weakening the favorite is beneficial to A iff

$$\beta \le \alpha - 2 \tag{10}$$

where contestant 1 is the favorite.

[17] We often omit dependency of p_i's on e_i's in the paper.

[18] One might wonder why, if in general not leveling perfectly the playing field is optimal, distorting an already leveled field is not optimal (i.e., why if $\beta = 1$, $\alpha^Q = 1$). When $\beta = 1$, setting $\alpha = 1$ makes contestants exert the *same* effort, say $\bar{e}$, and hence the expected effort of the winner is $\bar{e}$. Setting an $\alpha \ne 1$ rather than $\alpha = 1$ would make both contestants exert less than $\bar{e}$, and hence the winner's effort will certainly be lower than $\bar{e}$ regardless of how p_1 and p_2 change. We thank Mikhail Drugov for raising this point.

[19] For instance, if $\beta = 2$, $\alpha^Q = 2(\sqrt{3} - 1) \approx 1.46$, thus $p_1^* \approx 0.42$, $p_2^* \approx 0.58$ and $\frac{p_2^*}{p_1^*} = \frac{1}{\sqrt{3} - 1} \approx 1.36$. That is,if the high type is twice as strong as the low type ($\beta = 2$) then the optimally leveled quality contest leaves the high type 36% more likely to win than the low type in equilibrium.

[20] The meaning of playing field and individual effects can be seen in (5). Note that changes in α_i's as in Section 5 only have playing field effects, whereas changes in β_i's also have individual effects. This is why we first analyzed changes in α_i's, and now in β_i's, and we keep these two analyses separate.

[21] Comparative statics on individual efforts are straightforward from (5), and thus proofs are omitted; we instead focus on results and intuitions. Comparative statics on A and Q are proved in Propositions 5 and 6.

[22] While the comparative statics on individual efforts are well-known (see Malueg and Yates, 2005), to the best of our knowledge we are the first to provide comparative statics on A, and thus in case of weakening the underdog the trade-off between a positive playing field effect and a negative individual effect is highlighted.

Proposition 6. [*Quality contest*] *Weakening contestant 1, who is either the favorite or the underdog, is beneficial to Q iff*

$$3\alpha + \beta^2 - 2\alpha\beta \leq 0 \tag{11}$$

Propositions 5 and 6 convey the main message of this section: while weakening the favorite could be beneficial both in a quality contest and in an aggregative contest, weakening the underdog could only be beneficial in a quality contest. We plot conditions (10) and (11) in the (α, β)-space in order to have a graphical support when building intuition. Above the 45° line contestant 1—who is weakened in the propositions above—is the favorite, and below the 45° line contestant 1 is the underdog. We rewrite in two boxes in the figures the implications on the efforts discussed above. Furthermore, assume $\beta \geq 1$, so that 1 (2) is the high (low) type.[23] This has a clear implication to keep in mind: $e_1^* \leq e_2^*$ (see (5)).

Fig. 3. Weakening the underdog (below the 45 ° line) implies $e_1\downarrow$ and $e_2\downarrow$, and thus A decreases. On the other hand, weakening the favorite (above the 45 ° line) implies $e_1\downarrow$ and $e_2\uparrow$, thus A might increase or decrease. As explained above, this is due to the positive playing field effect and the negative individual effect. The former should prevail over the latter in order to have that $\frac{\partial A}{\partial \beta_1} \geq 0$. The playing field effect is stronger the more unleveled the playing field is: that is, leveling the playing field becomes the "priority" of an aggregative contest in that it is the main driver of efforts.[24] Very unleveled playing fields occur when sufficiently far from S=1 (i.e., the 45 ° line), and this is why $\frac{\partial A}{\partial \beta_1} \geq 0$ in the top-left area of Fig. 3, corresponding to (10).

Fig. 4. The region (11) is partially above and partially below the 45° line—that is, both weakening the underdog or the favorite could increase Q. Weakening the favorite yields $\overset{\downarrow}{p}_1\overset{\downarrow}{e}_1 + \overset{\uparrow}{p}_2\overset{\uparrow}{e}_2$, and weakening the underdog yields $\overset{\downarrow}{p}_1\overset{\downarrow}{e}_1 + \overset{\uparrow}{p}_2\overset{\downarrow}{e}_2$, thus it is intuitive that the region where the former is beneficial (above the 45° line) is bigger than the region where the latter is beneficial (below the 45° line). Consider these two cases separately. First, weakening the favorite ($\overset{\downarrow}{p}_1\overset{\downarrow}{e}_1 + \overset{\uparrow}{p}_2\overset{\uparrow}{e}_2$) is beneficial unless p_1, which is the weight of the low and decreasing effort e_1, is high enough to overcome the positive effect of $\overset{\uparrow}{e}_2$. Now, p_1 is high enough in points sufficiently close to the vertical axis.[25] This is why the region where $\frac{\partial Q}{\partial \beta_1} \geq 0$ is qualitatively sufficiently far from the vertical axis. Second, weakening the underdog ($\overset{\downarrow}{p}_1\overset{\downarrow}{e}_1 + \overset{\uparrow}{p}_2\overset{\downarrow}{e}_2$) is rather thorny. Remember that $e_1 \leq e_2$, so if there is sufficient difference between e_2 and e_1, the positive effect of $\overset{\uparrow}{p}_2$ prevails on the negative $\overset{\downarrow}{e}_1$ and $\overset{\downarrow}{e}_2$.[26] Sufficient difference between e_2 and e_1 is achieved when β is sufficiently far from 1. In fact, a necessary condition for (11) is that $\beta \geq 3$, [27] which implies $e_2 \geq 3e_1$—see (5). At the same time, now the playing field effect is negative (the underdog is even more less likely to win) and thus the argument behind Fig. 3, namely to avoid very unleveled playing field, carries over. This is why this region is also sufficiently far from the horizontal axis. In fact, a necessary condition for (11) is that $S \geq \frac{1}{2}$.[28] We provide a numerical example consistent with the above reasoning; if types are sufficiently heterogeneous and the playing field sufficiently leveled, weakening the underdog is beneficial in a quality contest.

Example 1. Consider $(\beta_1, \beta_2) = (5, 1)$ and $S = 4/5$, so that: i) types are sufficiently heterogeneous, ii) the playing field sufficiently leveled, and iii) contestant 1 is the underdog.[29] Then, $Q = \frac{116}{729}$, see (8).

Now, consider $\beta_1 = 6$ (weakening the underdog) and keep the rest unchanged. Then, $Q = \frac{4}{25} > \frac{116}{729}$. Thus, weakening the underdog yielded an increase in Q. In fact, this numerical example lies in the region of Fig. 4 below the 45° line where $\frac{\partial Q}{\partial \beta_1} \geq 0$.

7. Conclusions

In a standard model of lottery contest we study how the optimal set of rules changes when the objective is to maximize the expected effort of the winner (quality contest) as opposed to the conventional case where the objective is the sum of every contestant's effort (aggregative contest). We find that, contrary to an aggregative contest, in a quality contest it may be beneficial to exclude weak contestants, unlevel the playing field, and weaken the underdog. The results of this paper are a first step meant to stress the importance of specifying the focus of the contest according to the applications one wants to embrace, as well as to pave the way for future works in this direction.

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[23] Assuming $\beta \geq 1$ is wlog since we can swap i and j's identities and retrieve the same qualitative graph in the $(\alpha, \beta) \in [0, \infty) \times [0, 1]$ space, rather than in the $(\alpha, \beta) \in [0, \infty) \times [1, \infty)$ space. For instance, condition (10), $\beta \leq \alpha - 2$, if we swap identities, reads $\frac{1}{\beta} \leq \frac{1}{\alpha} - 2$, which could be depicted in the $(\alpha, \beta) \in [0, \infty) \times [0, 1]$ space but would correspond to the one we depict in the $(\alpha, \beta) \in [0, \infty) \times [1, \infty)$ space.

[24] In fact, $\lim_{S\to 0} e_i^* = \lim_{S\to\infty} e_i^* = 0$ for i=1,2 regardless of β_1 and β_2. See (5).

[25] Recall that $S = \alpha/\beta$, and thus S=0 along the horizontal axis, $S = 1$ along the 45° line and $S \to \infty$ along the vertical axis. Also, from (6) we know $p_1^* = \frac{S}{S+1}$.

[26] Start from $Q = p_1e_1 + p_2e_2$, with $e_1 \leq e_2$. Increase p_2 by δ and decrease both efforts by a (this decrease needs not be symmetric). Then for Q to increase, we need

$$\begin{aligned} & p_1e_1 + p_2e_2 \leq (p_1 - \delta)(e_1 - a) + (p_2 + \delta)(e_2 - a) \\ & \iff 0 \leq -p_1a - \delta(e_1 - a) - p_2a + \delta(e_2 - a) \\ & \iff 0 \leq -a - \delta(e_1 - e_2) \\ & \iff \delta \geq \frac{-a}{e_1 - e_2} \end{aligned}$$

which holds if δ is sufficiently large as opposed to the efforts' drop relative to the efforts' difference. δ being sufficiently large means that p_2 increases sufficiently.

[27] $3\alpha + \beta^2 - 2\alpha\beta \leq 0 \iff 3\alpha - \alpha\beta + \beta^2 - \alpha\beta \leq 0 \iff \alpha(3 - \beta) + \beta(\beta - \alpha) \leq 0$. Since $\beta \geq \alpha$, it has to be that $\beta \geq 3$.

[28] (11) is equivalent to $3\alpha + \beta(\beta - 2\alpha) \leq 0$. Since $\alpha \geq 0$, it has to be that $\beta \leq 2\alpha$, or equivalently that $S \geq 1/2$.

[29] In fact, when $(\beta_1, \beta_2) = (5, 1)$, any $\alpha \leq 5$ guarantees that contestant 1 is the underdog. We set $\alpha = 4$, so that $S = 4/5$, consistent with the need for a sufficiently leveled playing field in order for the result to hold.

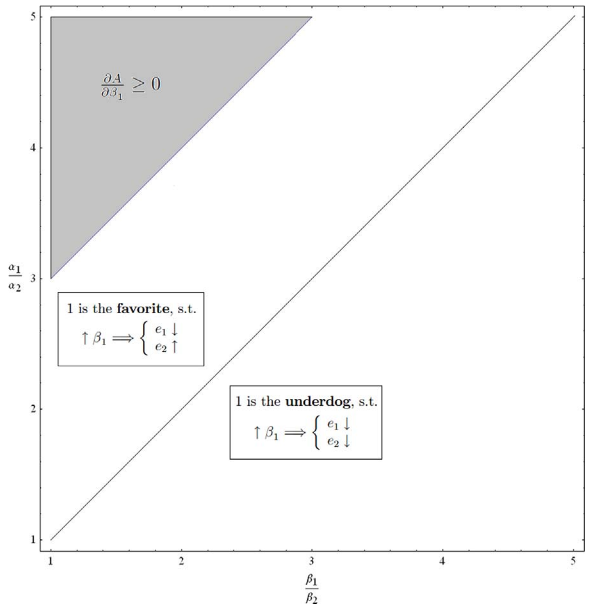

Fig. 3. Region where weakening contestant 1 is beneficial to A.

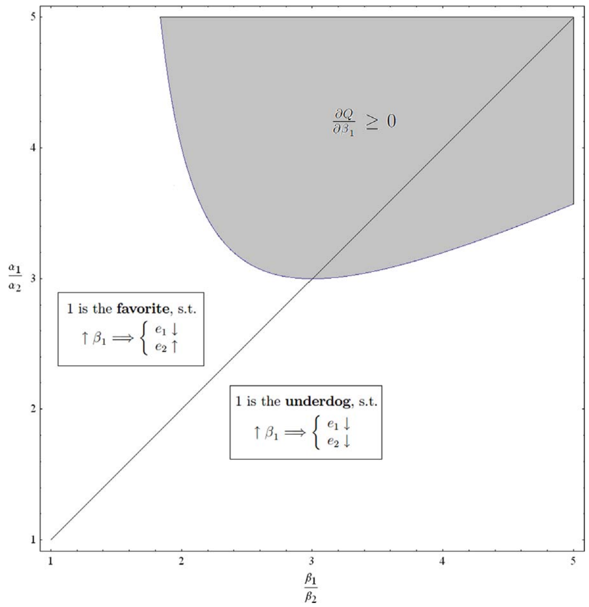

Fig. 4. Region where weakening contestant 1 is beneficial to Q.

Acknowledgments

I am grateful to Luis Corchón, René Kirkegaard, Diego Moreno and John Morgan for useful discussions. I am also grateful to Vincenzo Galasso (the editor) and two anonymous reviewers for comments which significantly improved the quality of the paper. Additionally, I benefited from comments by participants at the UC3M-CEMFI 3rd Microeconomics Workshop, Luis Corchón's internal seminar, and MPI informal research meetings. I acknowledge financial support from ECO2011-25330/ECON. Errors are mine.

Appendix. Proofs

Proof of Proposition 2. First, it is well-known (see, for instance, Fullerton and McAfee, 1999) that the equilibrium effort is

$$e_i^* = \frac{m-1}{\sum_{j\in M}\beta_j}\left[1-\frac{\beta_i(m-1)}{\sum_{j\in M}\beta_j}\right] \tag{12}$$

when m contestants exert positive effort. We denote the set of m contestants by $M=\{n-m,\ldots,n\}$.

From (12),

$$p_i^* = \frac{\left[1-\frac{\beta_i(m-1)}{\sum_{j\in M}\beta_j}\right]}{\sum_{k\in M}\left[1-\frac{\beta_k(m-1)}{\sum_{j\in M}\beta_j}\right]} = \frac{\sum_{j\in M}\beta_j-\beta_i(m-1)}{m\sum_{j\in M}\beta_j-(m-1)\sum_{j\in M}\beta_j} = \frac{\sum_{j\in M}\beta_j-\beta_i(m-1)}{\sum_{j\in M}\beta_j} = 1-\frac{\beta_i(m-1)}{\sum_{j\in M}\beta_j}$$

We can now provide a general formula for the expected effort of the winner given that m contestants exert positive effort, which we denote Q_m

$$Q_m = \sum_{i\in M}p_i^*e_i^* = \frac{m-1}{\sum_{j\in M}\beta_j}\sum_{i\in M}\left[1-\frac{\beta_i(m-1)}{\sum_{j\in M}\sum\beta_j}\right]^2 = \frac{m-1}{\sum_{j\in M}\beta_j}\left[m+(m-1)^2\frac{\sum_{i\in M}\beta_i^2}{\left(\sum_{j\in M}\beta_j\right)^2}-2(m-1)\frac{\sum_{i\in M}\beta_i}{\sum_{j\in M}\beta_j}\right]$$

$$= \frac{m-1}{\sum_{j\in M}\beta_j}\left[m+(m-1)^2\frac{\sum_{i\in M}\beta_i^2}{\left(\sum_{j\in M}\beta_j\right)^2}-2(m-1)\right] = \frac{m-1}{\sum_{j\in M}\beta_j}\left[2-m+(m-1)^2\frac{\sum_{i\in M}\beta_i^2}{\left(\sum_{j\in M}\beta_j\right)^2}\right] \tag{13}$$

Second, the intuition behind the three-contestant and four-contestant examples suggests looking for a profitable exclusion under homogeneity of types. Thus, start from $\beta_1=\beta_2=...=\beta_n=\overline{\beta}$. It is trivial to see that $e_i^*\geq 0$ for all contestants—see (12). We now prove that $Q_{m-1}>Q_m$, applying formula (13),

$$Q_{m-1}>Q_m \iff \frac{m-2}{(m-1)\overline{\beta}}\left[3-m+(m-2)^2\frac{(m-1)\overline{\beta}^2}{(m-1)^2\overline{\beta}^2}\right] > \frac{m-1}{m\overline{\beta}}\left[2-m+(m-1)^2\frac{m\overline{\beta}^2}{m^2\overline{\beta}^2}\right] \iff \frac{m-2}{(m-1)^2}$$
$$[(3-m)(m-1)+(m-2)^2] > \frac{m-1}{m^2}[(2-m)m+(m-1)^2] \iff \frac{m-2}{(m-1)^2} > \frac{m-1}{m^2} \iff m^2(m-2)>(m-1)^3 \iff$$
$$(m-1)^2>m$$

which holds true for all $m\geq 3$. Since Q is continuous in β_i's, for sufficiently small changes in the vector of β's the inequality $Q_{m-1}>Q_m$ continues to hold. This completes the proof.□

Proof of Proposition 4. The first-order condition of $\max_\alpha Q$ is

$$\frac{\partial Q}{\partial\alpha}=0 \iff \left[\frac{1-2S}{(1+S)^4}*\left(\frac{S}{\beta_1}+\frac{1}{\beta_2}\right)+\frac{S}{(1+S)^3\beta_1}\right]\frac{1}{\beta}=0 \tag{14}$$

$$\iff \frac{1-2S}{1+S}\left(\frac{S}{\beta_1}+\frac{1}{\beta_2}\right)+\frac{S}{\beta_1}=0 \iff (1-2S)(S+\beta)+S(S+1)=0 \iff S^2+2(\beta-1)S-\beta=0 \tag{15}$$

One root of (15) is negative, and the positive one is

$$S^q = 1-\beta+\sqrt{(1-\beta)^2+\beta}$$

which corresponds to (9). S^q is the unique maximum of (8) because $\lim_{S\to 0}Q=\lim_{S\to\infty}Q=0$, Q is continuously differentiable in S, (14) is strictly positive when S=0, and S^q is the unique positive root.□

Proof of Proposition 5. Consider $\frac{\partial A}{\partial\beta_1}$, where contestant 1 is either the favorite or the underdog. In the following steps we use $\frac{\partial S}{\partial\beta_1}=-\frac{S}{\beta_1}$ and $S=\frac{\alpha}{\beta}$.

$$\frac{\partial A}{\partial\beta_1}\geq 0 \iff \frac{\partial}{\partial\beta_1}\left[\frac{S}{(1+S)^2}\left(\frac{1}{\beta_1}+\frac{1}{\beta_2}\right)\right]\geq 0 \iff \frac{1-S}{(1+S)^3}\left(\frac{1}{\beta_1}+\frac{1}{\beta_2}\right)\left(-\frac{S}{\beta_1}\right)-\frac{S}{(1+S)^2}\frac{1}{\beta_1^2}\geq 0 \iff \frac{1-S}{1+S}(1+\beta)+1\leq 0 \tag{16}$$

$$\iff \frac{\beta-\alpha}{\beta+\alpha}(1+\beta)+1\leq 0 \iff (\beta-\alpha)(1+\beta)\leq -\alpha-\beta \iff 2\beta+\beta^2-\alpha\beta\leq 0 \iff \beta\leq\alpha-2 \tag{17}$$

Suppose $S \leq 1$, such that contestant 1 is the underdog. The left-hand side of (16) is positive, and thus $\frac{\partial A}{\partial \beta_1} \leq 0$. That is, weakening the underdog is detrimental to A. Suppose $S \geq 1$, such that contestant 1 is the favorite. Then $\alpha \geq \beta$, and condition (17) might or might not hold.

Proof of Proposition 6. In the following we use $\frac{\partial S}{\partial \beta_1} = -\frac{S}{\beta_1}$ and $S = \frac{\alpha}{\beta}$.

$$\frac{\partial Q}{\partial \beta_1} \geq 0 \Longleftrightarrow \frac{\partial}{\partial \beta_1}\left[\frac{S}{(1+S)^3}\left(\frac{S}{\beta_1}+\frac{1}{\beta_2}\right)\right] \geq 0 \Longleftrightarrow \frac{1-2S}{(1+S)^4}\left(\frac{S}{\beta_1}+\frac{1}{\beta_2}\right)\left(-\frac{S}{\beta_1}\right)+\frac{S}{(1+S)^3}\left(\frac{(-S/\beta_1)\beta_1 - S}{\beta_1^2}\right) \geq 0 \Longleftrightarrow$$

$$\frac{1-2S}{1+S}(S+\beta)+2S \leq 0 \Longleftrightarrow \frac{\beta-2\alpha}{\beta+\alpha}(\alpha+\beta^2)+2\alpha \leq 0 \Longleftrightarrow \alpha\beta+\beta^3-2\alpha\beta^2 \leq -2\alpha\beta \Longleftrightarrow 3\alpha+\beta^2-2\alpha\beta \leq 0 \tag{18}$$

Note that condition (18) could hold or not regardless of whether $S \gtreqless 1$. That is, weakening the underdog or the favorite could be beneficial to Q.□

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