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Unemployment and optimal currency intervention in an open economy☆

Hailong Jin [a], Yoonho Choi [b], E. Kwan Choi [c,d,*]

[a] Black Hills State University, United States
[b] Iowa State University, United States
[c] Iowa State University, United States
[d] University of Macau, China

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ABSTRACT

This paper investigates whether China, with unemployed resources, can benefit from a trade surplus in one period and a deficit in the next by manipulating the yuan's peg. A country may be tempted to stimulate its economy temporarily by devaluation, but any surplus so generated subsequently must be expended with inescapable reverse output effect. It is shown that under reasonable conditions, nonintervention is the optimal policy and the optimal exchange rates are the equilibrium rates that yield a trade balance in each period. Numerical examples using the Cobb–Douglas utility function illustrate the main proposition.

1. Introduction

Due to mounting currency reserves since the 1990s, China's currency policy has been under intense scrutiny. According to the State Administration of Foreign Exchange of the People's Bank of China (PBC), China's foreign exchange reserve, which excludes gold, was $22 billion in 1993. China's foreign exchange reserve has increased steadily since, to $3.8 trillion as of March 2014. This dramatic rise in China's cumulative trade surplus has provoked much debate concerning China's currency valuation and misalignment. The common view is that China intentionally has depressed the value of its currency, the renminbi (RMB), to gain unfair advantages in the global market. (Cheung, 2012; Cheung, Chinn, & Fujii, 2009).

Most major currencies except the renminbi are floating freely vis-à-vis other currencies. It has been argued that China may be deliberately depressing the yuan in the hope of stimulating domestic production. In the celebrated Mundell (1963)–Fleming (1962) model, currency devaluation affects a country's balance of payments, thereby influencing production and unemployment. In a study of ten countries, Gylfason and Schmid (1983) showed that devaluation has positive output effects. However, any foreign currency reserve so accumulated must eventually be used up, which yields the reverse output effect.

In an open economy, the government may be more interested in the output effect of currency devaluation. For instance, Helpman (1976) considered a single-period framework with a nontraded good and showed that devaluation increases employment, while Cuddington (1981) investigated the contemporaneous effect of devaluation. More recently, Batra and Beladi (2013) suggested that

☆ The usual caveats apply.

* Corresponding author at: Department of Economics, Iowa State University, Ames, IA 50011, United States.
URL's: hailongjin@bhsu.edu (H. Jin), ecoyoon@iastate.edu (Y. Choi), kchoi@iastate.edu (E. Kwan Choi).

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both China and Japan kept their currency values low relative to those of other nations such as the United States and Europe in order to lower unemployment below a target rate.[1] Jin and Choi (2013, 2014) noted that while in the short run some profits might be generated by slightly deviating from the equilibrium exchange rates, in the long run excessive hoarding of reserve assets can only result in huge losses to the PBC's balance of payment account.

The purpose of this paper is to investigate whether a Keynesian open economy with unemployment can gain by devaluing its currency and temporarily maintaining a trade surplus, which reduces unemployment.[2] However, these macroeconomic models employ a single homogeneous good.[3] In order to consider currency devaluation and output effects, we consider an open economy that produces two tradable goods. In a stationary equilibrium, neither a permanent trade surplus nor a deficit is sustainable. Thus, any cumulative trade surplus eventually must be spent. If a trade surplus has a positive output effect in one period, the resulting trade deficit has a reverse output effect in the next period.

Section 2 considers the effect of yuan appreciation on a trade deficit. Section 3 investigates optimal currency pegging in a two-period framework. Section 4 compares stable exchange rates and yuan appreciation above the equilibrium rate for the case of Cobb–Douglas utility function. Section 5 offers the concluding remarks.

2. Production, consumption and trade deficit

In this section we first consider the effect of yuan appreciation on trade deficits and welfare in order to lay the basis for optimal currency pegging that maximizes utility over two periods. Let the exportable good C be the numéraire, i.e., its yuan price $b = 1$, and let ε denote the dollar price of yuan. The dollar price $b^* = \varepsilon$ of the exportable is equal to unity in the benchmark equilibrium. Let P be the yuan price of the importable good Z. The foreign price of the importable good $P^* = P\varepsilon$ is exogenous. Since there is no tariff, the relative foreign price of the importable is $P^*/\varepsilon = P$, equal to the domestic price of the importable.

Next, we consider the Keynesian model of a small open economy with unemployment. The country produces two goods over two periods. Let C and Z denote quantities of the exportable and importable that China produces. Money wage and interest rate are fixed. To lay the basis for analyzing the welfare effects of free trade for a small country with unemployment,[4] we adapt Batra and Beladi (1990) with the following assumptions:

(i) *China*'s production functions of the exportable, $C = F(K_C, L_C)$, and the importable, $Z = G(K_Z, L_Z)$, are monotone increasing and concave. The supply curves $C(b)$ and $Z(P)$ are positively sloped.

(ii) The yuan price b of good C is fixed by China's exporters, and the exportable is the numéraire, $b = 1$. The dollar price of good Z, P*, is set by U.S. exporters. Exchange rate pass-through is perfect, and the yuan price of good Z is $P = P^*/\varepsilon$.[5]

(iii) The domestic wage and rent are fixed and do not respond to random changes in domestic and foreign prices.

(iv) Perfect competition prevails in product markets, but resources are not fully employed in the factor markets.

2.1. Yuan profit maximization

Domestic outputs of the traded goods are given by $C = F(L_C, K_C)$ and $Z = G(L_Z, K_Z)$, where L_j and K_j denote labor and capital employed in sector j, $j = 1, 2$. For convenience, we suppress the time subscript i, and consider optimal production when the wage rate and rent are fixed and unemployment exists in both the capital and labor markets.

Let ε be the dollar price of the yuan. Yuan price of the numéraire is $b = 1$, and the dollar price of the exportable is $b^* = b\varepsilon$. Since the dollar price of the importable P^* is fixed, the yuan price of the importable is $P = P^*/\varepsilon$. Appreciation of the yuan lowers the domestic price of the importable P.

Chinese producers maximize yuan profits. Note the relationships between yuan and dollar prices: $b\varepsilon = b^*$ and $P\varepsilon = P^*$. If the exchange rate (or dollar price of yuan) rises, it has no effect on the yuan price of the exportable $b = 1$, but reduces the yuan price of the importable, $P = P^*/\varepsilon$.

Yuan profit of the export sector is:

$$\Pi_C = bF(K_C, L_C) - wL_C - rK_C, \tag{1}$$

where w and r are the yuan wage rate and rent, respectively. First order conditions are:

$$bF_{LC} - w = 0, \quad bF_{KC} - r = 0, \tag{2}$$

[1] Chinese currency devaluation may not be the only cause of U.S. trade deficits. For instance, Beladi and Oladi (2014) suggest that outsourcing may widen U.S. trade deficits. Also, Yue and Zhang (2013) emphasize that the U.S. trade deficit would be reduced very little by a change in the Chinese exchange rate.

[2] Of course, the first best policy is to remove wage and rent rigidity in the factor markets. Given this rigidity, Chinese government may be using yuan devaluation as a second best policy.

[3] In the same vein, Bruno (1976) considered a two-sector model, but defined the exchange rate as the ratio of the price of the tradable goods to that of the nontradable good.

[4] The real wage may be rigid or flexible, depending on the choice of the numéraire. Since the nominal wage w is fixed, the real wage, w/q, also is fixed, but w/p increases as p declines.

[5] Devereux (2000) analyzed the impact of devaluation on the trade balance when exchange rate pass-through is imperfect.

where $F_{LC} \equiv \partial F/\partial L_C$ is the marginal product of labor in the C sector, and other partial derivatives are similarly defined. From Eq. (2), input demand functions are written as $K_C = K_C(b,w,r)$ and $L_C = L_C(b,w,r)$. Indirect supply of the exportable is written as $C = C(b,w,r) = F(K_C(b,w,r),L_C(b,w,r))$. Since the export price $b = 1$ is fixed, and domestic factor prices are fixed in yuan, appreciation of the yuan has no effect on the supply of the exportable, $\partial C/\partial \varepsilon = 0$.

Profits can be calculated in terms of dollars. The export sector profit in dollars is:

$$\pi_C = b\varepsilon F(K_C, L_C) - w\varepsilon L_C - r\varepsilon K_C = \varepsilon \Pi_C. \tag{3}$$

Since there are no tariffs, for any given exchange rate ε yuan profit is maximized if, and only if, dollar profit is maximized. That is, an input combination (K_C, L_C) that maximizes yuan profit also maximizes dollar profit.

Yuan profit of the importable sector is:

$$\Pi_Z = (P*/\varepsilon)G(K_Z, L_Z) - wL_Z - rK_Z. \tag{4}$$

The first order conditions are:

$$(P*/\varepsilon)G_{LZ} - w = 0, \quad (P*/\varepsilon)G_{KZ} - r = 0. \tag{5}$$

Let the minimum production cost of Z be denoted by $\gamma(Z; w, r)$. Then profit in Eq. (4) can be written as:

$$\Pi_Z = (P*/\varepsilon)Z - \gamma(Z; w, r).$$

Profit maximization requires

$$P = \gamma'. \tag{6}$$

That is, the yuan price of the importable must be equal to its marginal cost. Differentiating Eq. (6) with respect to the exchange rate yields:

$$\frac{dZ}{dP} = \frac{1}{\gamma''} > 0.$$

It also follows that $\pi_Z = \varepsilon \Pi_Z$ and hence, maximizing the yuan profit is equivalent to maximizing the dollar profit in the import sector. Input demand functions can be written as $L_Z(P^*/\varepsilon, w, r)$ and $K_Z(P^*/\varepsilon, w, r)$. Indirect supply function of the importable good is $Z(P^*/\varepsilon, w, r) = G(K_Z(P^*/\varepsilon, w, r), L_Z(P^*/\varepsilon, w, r))$. Recall that the supply of the importable is positively sloped, i.e., $dZ/dP > 0$. Recall that the yuan price of the importable $P = P^*/\varepsilon$ falls as the yuan appreciates. Accordingly, yuan depreciation raises the supply of the importable Z, $\partial Z/\partial \varepsilon = (\partial Z/\partial P)(dP/d\varepsilon) < 0$, and thereby reduces unemployment, $\partial L_Z(P^*/\varepsilon, w, r)/\partial \varepsilon < 0$.

2.2. Effect of yuan appreciation on dollar revenue

The yuan production cost of the importable is $wL_z + rK_z$and yuan profit is $\pi = (P^*/\varepsilon)G(L_Z(P,w,r)K_Z(P,w,r)) - wL_Z(P,w,r) - rK_Z(P,w,r)$. Thus, yuan revenue in the importable sector is: $(P^*/\varepsilon)Z(P,w,r)$. Likewise, yuan revenue in the exportable sector is $bC(b,w,r)$. The total yuan revenue is

$$R = bC(b, w, r) + PZ(P, w, r). \tag{7}$$

Since $b = 1$, producer revenue in dollars is.

$$R = \varepsilon C(b, w, r) + P * Z(P, w, r). \tag{8}$$

How does producer revenue in dollars respond to yuan appreciation? Recall that yuan appreciation has no effect on the supply of the exportable, i.e., $\partial C/\partial \varepsilon = 0$, but increases the dollar value of the exportable. Since the dollar price P^* is fixed, yuan appreciation effectively decreases the domestic price P, thereby reducing the domestic supply of the importable good. In general, the sign of $\partial R/\partial \varepsilon$ is indeterminate.[6]

[6] Differentiating Eq. (8) with respect to ε_i gives

$$\frac{dR}{d\varepsilon} = C + P*\frac{dZ}{d\varepsilon} = C + P*\frac{dZ}{dP}\frac{dP}{d\varepsilon} = C + P*\frac{Z}{P}\frac{dZ}{dP}\frac{P}{Z}\frac{dP}{d\varepsilon}$$
$$= C + P*\frac{\theta Z}{P}\frac{dP}{d\varepsilon} = C - (P*)^2\frac{\theta Z}{P}\frac{1}{\varepsilon^2} = C - \theta PZ,$$

where $\theta \equiv (P/Z)(dZ/dP)$ is the price elasticity of supply of the importable good.

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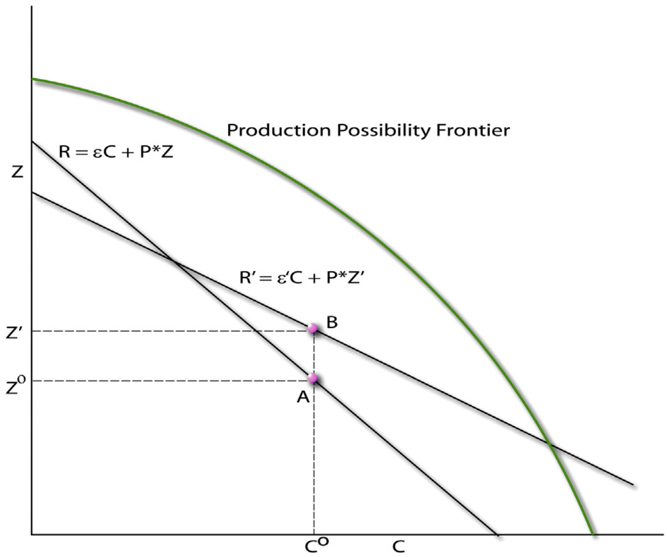

Fig. 1. Yuan devaluation and revenue in dollars.

It is important to note that an increase in the price of one product does not affect the supply of the other sector directly, i.e., $\frac{\partial C}{\partial P} = \frac{\partial Z}{\partial b} = 0$. That is, the output of the exportable good depends on its price b, but not on P. Likewise, domestic supply of the importable depends on the domestic price, $P = P^*/\varepsilon$, but not on b. Thus, yuan appreciation only affects the output of the importable sector. The two markets are mutually independent due to idle resources.

From the profit function of the export sector, we obtain input demand functions $L_C = L_C(b,w,r)$ and $K_C = K_C(b,w,r)$. Thus, indirect supply function can be written as: $C(b,w,r) = F(K_C(b,w,r), L_C(b,w,r))$. Likewise, from the profit function of the import sector, we obtain input demand functions, $L_Z = L_Z(P,w,r)$, and $K_Z = K_Z(P,w,r)$, and the indirect supply of the importable good is $Z(P,w,r) = G(K_Z(P,w,r), L_Z(P,w,r))$.

Fig. 1 illustrates the effect of yuan devaluation on production and dollar revenue. The slope of the dollar isorevenue line is ε/P^* and hence, yuan depreciation makes the line flatter, and only the output of the importable rises. Suppose that China has balanced trade at the equilibrium exchange rate $\varepsilon = 1$, and let the dollar revenue be $R = \varepsilon C^o + P^* Z^o$. The initial production occurs at point A well inside the production possibility frontier due to unemployed resources. When the yuan depreciates to ε', it does not affect the domestic price of the exportable or its output. However, it effectively raises the domestic price P of the importable, thereby increasing its supply to Z' and shifting production to point B. Both points A and B are inside the production possibility frontier.

The slope of the dollar revenue line is now $\varepsilon'/P^* < \varepsilon/P^*$, and hence the new revenue line $R' = \varepsilon' C + P^* Z$ becomes flatter. Yuan devaluation caused a movement from point A to B inside the production possibility frontier. Thus, policy makers in China may use yuan devaluation as an instrument to stimulate the country's domestic economy. However, yuan devaluation below $\varepsilon = 1$ results in a trade surplus and expenditure falls below this revenue level.

We now consider the consumption effect of yuan devaluation. Consumer preferences are represented by a monotone-increasing utility function, $u = u(c,z)$. The dollar budget constraint of the consumer is $\varepsilon c + P^* z = I$, where I is dollar expenditure, which could exceed national (or producer) income R. In order to permit trade deficits and surpluses, it is more convenient to express the budget constraint in dollars. The yuan budget constraint is $c + (P^* z/\varepsilon) = y = I/\varepsilon$, where y is (yuan) income in terms of the numéraire good.

The first order condition is: $u_z/u_c = P = P^*/\varepsilon$. Let $c = c(\varepsilon, P^*, I)$ and $z = z(\varepsilon, P^*, I)$ denote the demand functions for c and z, respectively. Then the indirect utility function is written

$$V(\varepsilon, P*, I) \equiv u[c(\varepsilon, P*, I), z(\varepsilon, P*, I)].$$

3. Optimal exchange rates

We consider a two-period model of currency intervention. In a stationary equilibrium, a country cannot accumulate a trade surplus or deficit indefinitely. Recall that the dollar price of the importable, P^*, remains fixed when China changes its peg to the dollar.[7] This means that yuan appreciation only affects the dollar price of China's exports. Since the price of the importable is stable, $P_1^* = P_2^* = P^*$, and there is no tariff on imports, producer revenue in dollars depends on the exchange rates. Specifically, $R_1 = \varepsilon_1 C_1 + P^* Z_1$, and $R_2 = \varepsilon_2 C_2 + P^* Z_2$.

[7] China is not the only country that pegs its currency to the dollar. This theory applies to other countries such as India and Mexico.

In the benchmark scenario, the policy maker chooses stable exchange rates over the two periods, i.e., $\varepsilon_1=\varepsilon_2=\varepsilon^o=1$, and trade is balanced in both periods. Alternatively, the country may allow the yuan to appreciate above unity in the first period, which results in a trade deficit. However, any trade deficit D must be borrowed from the United States and the debt $D(1+r)$is paid off in the second period. That is, a trade deficit in one period must be offset by a trade surplus in the next period.

3.1. Two-period maximization problem

We are now ready to analyze a two-period utility maximization problem. Since the yuan price of the exportable C is unity, its dollar price is ε_i in period i. Since factor prices are not manipulated, we now suppress them. Thus, the supply functions are written as $C_i(b) = C_i(b,w,r)$ and $Z_i(P_i)=Z(P_i,w,r)$.

In each period, producers maximize yuan profits. If resources are fully employed, factor prices w and r yield zero profits, and the resulting output mix occurs on the production possibility frontier (PPF). However, since factor prices are fixed above the equilibrium rates, resources are not fully employed, and the output mix occurs somewhere inside the PPF. With no tariffs or quotas, dollar revenue is $R_i=\varepsilon_i C_i+P_i^* Z_i(P_i)$.

When trade is balanced, consumer expenditure exhausts revenue from production. When the yuan appreciates above the equilibrium exchange rate, a trade deficit occurs. Consumers incur trade deficits by spending more than the economy produces. On the other hand, when a trade surplus is generated, consumer spending is less in an amount equal to the trade surplus.[8]

Let x and q denote the physical volumes of exports and imports. The dollar value of imports is $qP^*=qP\varepsilon=Q\varepsilon$, where $Q(\varepsilon)=q(\varepsilon)P$ is the yuan value of imports. China's trade deficit in dollars is written as $D=g(\varepsilon)=qP^*-\varepsilon x=Q(\varepsilon)\varepsilon-X(\varepsilon)$, where $X=x\varepsilon$ is the dollar value of China's exports.

The relationship between the trade deficit and the exchange rate is given by $D=g(\varepsilon)$.[9] When trade is balanced, the budget constraint in dollars is given by: $\varepsilon_i c_i+P_i^* z_i=\varepsilon_i C_i+P_i^* Z_i$. Let $D_i=P_i^* q_i-X_i$ denote the trade deficit in dollars in period i, where X_i is the value of exports, and q_i is the physical volume of imports. Consumer expenditure in dollars is $\varepsilon_i c_i+P_i^* z_i$. When trade is balanced, $D_i=0$, and consumer expenditure is equal to producer revenue, R_i. Let $I_i=R_i+D_i$denote the consumer expenditure in dollars in period i. When the country has a trade deficit ($D_i>0$), the total expenditure reduces to

$$R_i+D_i=\varepsilon_i c_i+P_i^* z_i. \tag{9}$$

If China incurs a trade deficit ($D_i>0$), its expenditure will be greater than if trade is balanced.

Recall that the exportable is the numéraire and its yuan price is unity, while its dollar price is ε. Since $b=1$, yuan revenue from production is $Y=C+PZ$. Production revenue in dollars is $R=\varepsilon C+P^*Z=\varepsilon Y$. In the absence of trade barriers, maximizing revenue in dollars is equivalent to maximizing revenue in yuan. From the budget constraint in Eq. (9), the total yuan expenditure in period i is given by

$$c_i+P_i z_i=(R_i+D_i)/\varepsilon_i. \tag{10}$$

The equilibrium condition for optimal consumption in each period is:

$$\frac{U_c^i}{U_z^i}=\frac{1}{P_i}=\frac{\varepsilon_i}{P_i^*}, i=1,2.$$

China's consumer demands in period i are written as: $c_i=c(\varepsilon_i,P_i^*,R_i+D_i)$, and $z_i=z(\varepsilon_i,P_i^*,R_i+D_i)$. Since yuan appreciation has no effect on the fixed dollar price of the importable good, we now suppress P_i^*. Consumer demands now are written as: $c_i=c(\varepsilon_i,R_i+D_i)$, and $z_i=z(\varepsilon_i,R_i+D_i)$. Likewise, the indirect utility is written as $V_i(\varepsilon_i,R_i+D_i)\equiv U(c(\varepsilon_i,R_i+D_i),z(\varepsilon_i,R_i+D_i))$.

3.2. Two-period budget constraint

Any exchange rate below (above) unity results in a trade surplus (deficit) in the first period. We investigate whether China can benefit from a trade deficit or surplus in the first period. This means China pegs the renminbi to the dollar above the equilibrium level ($\varepsilon_1>\varepsilon^o=1$) in the first period. A two-period budget constraint is given by

$$D_1(1+r)+D_2=0. \tag{11}$$

[8] In this model, China's trade surplus is exactly equal to its savings. That is, any trade surplus in dollars is converted into renminbi, but this extra amount is not spent on any domestic goods in China. Thus, an increase in the foreign exchange reserve is completely sterilized and causes no inflation.

[9] Differentiating the trade deficit with respect to ε gives $D_\varepsilon=Q+\varepsilon Q_\varepsilon-X_\varepsilon$,where subscripts denote partial derivatives. Let $\eta_{X\varepsilon}\equiv(\partial X/\partial\varepsilon)(\varepsilon/X)$and $\eta_{Q\varepsilon}\equiv-(\partial Q/\partial\varepsilon)(\varepsilon/Q)$ stand for the elasticity of exports and imports with respect to the exchange rate ε, respectively. As shown in Jin and Choi (2013), if the Marshall-Lerner condition is satisfied ($\eta_{X\varepsilon}+\eta_{Q\varepsilon}<1$), then yuan appreciation increases China's trade deficit in dollars ($D_\varepsilon>0$).

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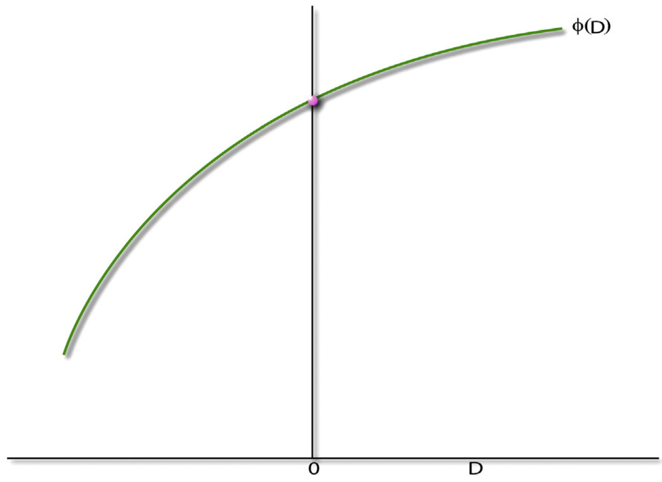

Fig. 2. Trade Deficit and Indirect Utility.

3.3. Devaluation and utility

Next, we consider the effect of yuan appreciation on consumer welfare. The consumer has an additive two period-utility function,

$$\Phi \equiv U(c_1, z_1) + \frac{1}{1+r} U(c_2, z_2). \tag{12}$$

Recall that the inverse trade deficit function $\varepsilon = f(D)$ is monotonically increasing and concave, i.e., $f'(D) > 0$ and $f''(D) > 0$. Let $\varepsilon_1 = f(D_1)$ and $\varepsilon_2 = f(D_2)$. Indirect utility in period i is

$$\phi(D_i) = V(\varepsilon_i(D_i), R_i + D_i). \tag{13}$$

Consider the effect of a trade deficit on consumer welfare in the current period, ignoring the subsequent effects of debt on future welfare. An increase in the trade deficit affects indirect utility in two ways. First, a trade deficit increases current expenditure, and the expenditure effect increases welfare in the current period. Next, the required yuan appreciation benefits consumers because China is an exporter of C. Since $\phi(D_i)$ is monotonically increasing in trade deficit, $\phi'(D_i) > 0$.

If $\phi(D_i)$ is convex in trade deficit, there exists no interior solution for any pair of trade deficit/surplus. In this case, a corner solution is optimal; i.e., zero consumption in one period and all income spent in the next period. To ensure the existence of optimal exchange rates and the associated pairing of the trade deficit and surplus, we assume that $\phi(D_i)$ is concave, i.e., $\phi''(D_i) < 0$, as shown in Fig. 2.

Using $\varepsilon = f(D)$, the indirect utility in period 1 is written[10] as:

$$\phi(D_1) \equiv V(\varepsilon_1, R_1 + D_1) = V(f(D_1), R_1 + D_1). \tag{14}$$

Note that a trade deficit changes the exchange rate, which in turn affects indirect utility through two channels. First, it affects the relative price of the importable, and this is the *price effect*. Second, it not only provides an additional amount of money the consumer can spend, but also affects nominal income or GDP through the change in the exchange rate. This can be called the *income effect*. Eq. (14) shows that similarly, the indirect utility in period 2 is:

$$\phi(D_2) \equiv V(\varepsilon_2, R_2 + D_2) = V(f(D_2), R_2 + D_2). \tag{15}$$

The total utility over two periods is given by:

$$\Phi \equiv \phi(D_1) + \frac{1}{1+r}\phi(D_2) = \phi(D_1) + \frac{1}{1+r}\phi(-(1+r)D_1). \tag{16}$$

[10] Let $P = P^*\delta\alpha$, where $\delta = 1/\varepsilon$. Then the reduced-form indirect utility is $\phi(D) \equiv V(P^*\delta\alpha, bC + (P^*\delta\alpha)Z + D)$, and suppressing subscripts for periods, we get $d\phi/dD = V_P\alpha\delta'(D) + V_I(dR/d\delta + 1) = V_I(-z\alpha\delta'(D) + \delta'\alpha Z + Z_D + 1) = V_I(-q\alpha\delta'(D) + Z_D + 1)$, which reduces to $V_I(-q\delta'(D) + Z_D + 1)$ when exchange rate pass-through rate for import price is 100%. Thus, the first order condition reduces to: $\phi'(D_1) - \phi'(D_2) = V_{I_1}(Z_{D_1} + 1 - q_1\alpha\delta'(D_1)) - V_{I_2}(Z_{D_2} + 1 - q_2\alpha\delta'(D_2))$. When evaluated at $D_1 = D_2 = 0$, we have $V_{I_1} = V_{I_2}$, $Z_{D_1} = Z_{D_2}$, and $\delta'(D_1) = \delta'(D_2)$. Thus, $D_1 = D_2 = 0$ is an optimal solution when exchange rate pass-through is imperfect.

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The first order condition for an optimal trade deficit is.

$$\phi'(D_1)-\phi'(D_2)=0. \tag{17}$$

First, note that when evaluated at $D_1=D_2=0$, the left hand side of Eq. (17) reduces to zero for all $r>0$. We now show that this solution is unique. Concavity of $\phi(D)$ means $\phi''<0$ everywhere. Suppose there exists another solution $D_1>0$. Then $D_2<0$ and $\phi''<0$imply $\phi'(D_1)<\phi'(0)=0$, and $-\phi'(D_2)<0$, and hence $\phi'(D_1)-\phi'(D_2)<0$. Thus, utility must be decreasing at $D_1>0$, meaning it is not an optimal trade deficit. This implies that the optimal trade deficit is zero in each period, i.e., $D_1=D_2=0$.[11]

In the absence of exchange rate effects, any extra expenditure D raises indirect utility, $V(\varepsilon,R+D)$. However, if China is an open economy with unemployed resources, a trade deficit in the first period not only raises consumer expenditure, but also the dollar price of the exportable. Thus, this price effect raises utility further. The resulting trade surplus in the second period, however, not only reduces consumer expenditure, but also depresses the price of China's exportable, and hence utility declines. The total effect of unbalanced trade is shown to be less than that of balanced trade.

Proposition 1. : *Assume that a Keynesian open economy is subject to a two-period budget constraint, $D_1(1+r)+D_2=0$ and that indirect utility $\phi(D)\equiv V(\varepsilon(D),R+D)$ and the inverse trade deficit function, $\varepsilon=f(D)$, are monotonically increasing and concave in each period. Then the optimal trade deficit is zero in each period and the associated exchange rates maximize the total utility over two periods, regardless of the interest rate.*

Since the indirect utility is assumed to be concave, a stable consumption stream is preferred to variable or fluctuating consumption bundles over the two periods. This proposition yields an important policy implication for countries such as China, Japan and South Korea, which tend to accumulate foreign exchange reserves. It is not optimal, for instance, for China to maintain a large trade surplus in any period, even if it is to be offset by a trade deficit in the following period.

4. The case of the Cobb–Douglas utility function

Given that the interest rate is $r=0.05$, a two-period Cobb–Douglas utility function is[12]

$$U(c_1,z_1)+\frac{U(c_2,z_2)}{1+r}=c_1^{.25}z_1^{.25}+\frac{c_2^{.25}z_2^{.25}}{1+r}. \tag{18}$$

Supply functions of the exportable and the importable are given by

$$C=1,\quad Z=P^{\theta}. \tag{19}$$

4.1. Balanced trade

First, consider the case where trade is balanced, i.e., $\varepsilon^o=1$. Then $Z^o=\left(\frac{P*}{\varepsilon^o}\right)^{\theta}=P*^{\theta}$, and $C^o=1$. Equilibrium condition requires

$$\frac{U_1}{U_2}=\frac{z}{c}=\frac{1}{P*},$$

and $c=P^*z$. Autarky price of the importable is $P^*=1$. For Z to be imported, $P^*<1$. Let $P^*=.5$. Then $C^o=1$ and $Z^o=.5^{1/4}\simeq0.84$. GDP or consumer income under balanced trade is $I^o=R^o=C^o+0.5\times Z^o=1.42$. Consumption under balanced trade is $c^o=.5z^o$, and consumer expenditure is $c^o+.5z^o=1.42$. Thus $c^o=0.71=0.5\times1.41$, and $z^o=1.42=I^o$. Trade deficit is $D=P^*(z^o-Z^o)-(C^o-c^o)=(1/2)(1.42-0.84)-(1-0.71)=0$, as expected.[13] The utility under balanced trade is $U^o=(c^oz^o)^{.25}=1.002$, and the total utility over two periods is $\phi=1.002\frac{2+r}{1+r}$, as shown in Table 1.

4.2. Trade imbalance

The equilibrium condition for optimal consumption is:$\frac{z}{c}=\frac{\varepsilon}{P*}$. Thus, $\varepsilon c=P^*z$.

Budget constraint is: $\varepsilon c+P^*z=\varepsilon c+z/2=2\varepsilon c=I$. Equilibrium demands for goods are:$c=\frac{I}{2\varepsilon}$, $z=I$. While the output of the exportable is fixed, $C=1$, that of the importable depends on the exchange rate, $Z_1=\left(\frac{P*}{\varepsilon_1}\right)^{\theta}=\left(\frac{1}{2\varepsilon_1}\right)^{\theta}$. Thus, the dollar revenues from

[11] This model assumes invariant indirect utility$\phi_1(D)=\phi_2(D)=\phi(D)$ and no economic growth. If economic growth occurs, the consumer will have an incentive to borrow and to incur a trade deficit in the first period. Also, if the consumer is biased toward future over present consumption, i.e., $\phi_2(D)=(1+s)\phi_1(D)$, $s>0$,then devaluation in the first period is optimal, because it reduces current consumption.

[12] Indirect utility is $V=(cz)^{\alpha}=I^{2\alpha}/(4\varepsilon)^{\alpha}$. If $\alpha>.5$, then indirect utility could become convex in expenditure, and an optimal exchange rate may not exist.

[13] The deviation in the amount of 0.01 is due to rounding errors.

Table 1
Simulation ($r = 0$).

R	ε_1	ε_2	V_1	V_2	$\Phi = V_1 + V_2$
0	0.8	1.2	0.908738	1.078425	1.987164
0	0.82	1.18	0.919114	1.071391	1.990505
0	0.84	1.16	0.929229	1.064239	1.993468
0	0.86	1.14	0.939097	1.056965	1.996062
0	0.88	1.12	0.94873	1.049565	1.998295
0	0.9	1.1	0.95814	1.042032	2.000173
0	0.92	1.08	0.967338	1.034363	2.001702
0	0.94	1.06	0.976335	1.026552	2.002886
0	0.96	1.04	0.985138	1.018592	2.00373
0	0.98	1.02	0.993758	1.010477	2.004235
0	1	1	1.002202	1.002202	2.004404
0	1.02	0.98	1.010477	0.993758	2.004235
0	1.04	0.96	1.018592	0.985138	2.00373
0	1.06	0.94	1.026552	0.976335	2.002886
0	1.08	0.92	1.034363	0.967338	2.001702
0	1.1	0.9	1.042032	0.95814	2.000173
0	1.12	0.88	1.049565	0.94873	1.998295
0	1.14	0.86	1.056965	0.939097	1.996062
0	1.16	0.84	1.064239	0.929229	1.993468
0	1.18	0.82	1.071391	0.919114	1.990505
0	1.2	0.8	1.078425	0.908738	1.987164

production in the two periods are:

$$R_1 = \varepsilon_1 C_1 + P * Z_1 = \varepsilon_1 + \frac{1}{2}\left(\frac{1}{2\varepsilon_1}\right)^{\theta}, \quad R_2 = \varepsilon_2 C_2 + P * Z_2 = \varepsilon_2 + \frac{1}{2}\left(\frac{1}{2\varepsilon_2}\right)^{\theta}.$$

In the first period, the export volume is $C_1 - c_1 = 1 - \frac{I_1}{2\varepsilon_1}$, and the dollar value of exports is $\varepsilon_1 - \frac{I_1}{2}$. The dollar value of import is $P * (z - Z) = \frac{I_1}{2} - \frac{1}{2}\left(\frac{1}{2\varepsilon_1}\right)^{\theta}$.

Consider a trade deficit function

$$D(\varepsilon) = \varepsilon - 1. \tag{20}$$

Then we have.

$$I = R + D = \varepsilon + \frac{1}{2}\left(\frac{1}{2\varepsilon}\right)^{\theta} + D = 2\varepsilon + \frac{1}{2}\left(\frac{1}{2\varepsilon}\right)^{\theta} - 1. \tag{21}$$

Table 2
Simulation ($r = 0.05$).

r	ε_1	ε_2	V_1	V_2	$\Phi = V_1 + V_2/(1 + r)$
0.05	0.8	1.21	0.908738	1.0819	1.939119
0.05	0.82	1.189	0.919114	1.074571	1.942515
0.05	0.84	1.168	0.929229	1.067115	1.945528
0.05	0.86	1.147	0.939097	1.059526	1.948169
0.05	0.88	1.126	0.94873	1.051799	1.950443
0.05	0.9	1.105	0.95814	1.043928	1.952357
0.05	0.92	1.084	0.967338	1.035908	1.953918
0.05	0.94	1.063	0.976335	1.027733	1.955128
0.05	0.96	1.042	0.985138	1.019395	1.95599
0.05	0.98	1.021	0.993758	1.010887	1.956507
0.05	1	1	1.002202	1.002202	1.95668
0.05	1.02	0.979	1.010477	0.993331	1.956507
0.05	1.04	0.958	1.018592	0.984266	1.955988
0.05	1.06	0.937	1.026552	0.974998	1.955121
0.05	1.08	0.916	1.034363	0.965515	1.953902
0.05	1.1	0.895	1.042032	0.955808	1.952326
0.05	1.12	0.874	1.049565	0.945864	1.950387
0.05	1.14	0.853	1.056965	0.93567	1.94808
0.05	1.16	0.832	1.064239	0.925213	1.945395
0.05	1.18	0.811	1.071391	0.914478	1.942323
0.05	1.2	0.79	1.078425	0.903448	1.938852

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If $\theta = 0.25$, then

$$I = 2\varepsilon + \left(\frac{1}{2^5\varepsilon}\right)^{0.25} - 1.$$

In the special case where $r = 0$, and the budget constraint $(1+r)D_1 + D_2 = 0$, we have $\varepsilon_1 + \varepsilon_2 = 2$. Also, indirect utility is:

$$V = (cz)^{0.25} = \left(\frac{I^2}{2\varepsilon}\right)^{0.25} = \left(\frac{\left(2\varepsilon + (1/32\varepsilon)^{0.25} - 1\right)^2}{2\varepsilon}\right)^{0.25}. \tag{22}$$

The two-period indirect utility function can be written as

$$\Phi = V_1 + \frac{1}{1+r}V_2 = \left(\frac{\left(2\varepsilon_1 + (1/32\varepsilon_1)^{0.25} - 1\right)^2}{2\varepsilon_1}\right)^{0.25} + \frac{1}{1+r}\left(\frac{\left(2\varepsilon_2 + (1/32\varepsilon_2)^{0.25} - 1\right)^2}{2\varepsilon_2}\right)^{0.25}$$
$$= \left(\frac{\left(2\varepsilon_1 + (1/32\varepsilon_1)^{0.25} - 1\right)^2}{2\varepsilon_1}\right)^{0.25} + \frac{1}{1+r}\left(\frac{\left(3 - 2\varepsilon_1 + (1/(64 - 32\varepsilon_1))^{0.25}\right)^2}{4 - 2\varepsilon_1}\right)^{0.25}.$$

Table 1 shows that in the special case where $r = 0$, balanced exchange rates ($\varepsilon_1 = \varepsilon_2 = 1$) achieve the highest utility over the two periods. Table 2 also confirms that balanced trade yields the highest utility over two periods when $r = 0.05$.

5. Concluding remarks

China has been criticized for maintaining a large amount of foreign exchange reserve to take unfair advantage of its trading partners. This paper investigated whether a country can benefit from generating a trade deficit or surplus by arbitrarily changing the yuan-dollar peg. In a two-period framework, a Keynesian open economy with high unemployment may devalue its currency below the equilibrium rate in order to stimulate output, but the accumulated trade surplus must be spent later, which reverses the output effect. Alternatively, the country may incur a trade deficit in one period, and the principal plus interest subsequently must be paid off. It is shown that under reasonable conditions, the optimal exchange rate yields a trade balance over both periods. Therefore, there are no gains from temporarily stimulating output as the output effect is reversed when the trade surplus subsequently is spent.

China's acquisition of a large foreign exchange reserve suggests that the current exchange rate policy may be harmful to consumer welfare, contrary to the prevailing view that China pursues its own self-interest by taking unfair advantage in the global market.

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