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Hyung Sun Choi, Ohik Kwon, Manjong Lee

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Inflation, Credit, and Indexed Unit of Account

Hyung Sun Choi[1]

Department of Economics, Kyung Hee University, Seoul, South Korea

Ohik Kwon

Department of Economics, Korea University, Seoul, South Korea

Manjong Lee

Department of Economics, Korea University, Seoul, South Korea

Abstract

A simple monetary model is constructed to study the implications of an indexed unit of account (Indexed-UoA). In an economy with an Indexed-UoA, the credit-trade friction attributed to inflation can be resolved and unexpected inflation causes no redistribution effect between debtors and creditors. However, in an economy without an Indexed-UoA, credit trades occur only if inflation is not too high and unexpected inflation renders debtors better off, but creditors worse off. In a high-inflation economy, money is used as a unit of account for spot trades only and an Indexed-UoA emerges as a unit of account for deferred-payment trades.

Keywords: indexed unit of account, deferred payment, inflation, welfare
JEL Classifications: E31, E42, E50

1. Introduction

A unit of account anchored on real value (hereinafter Indexed-UoA) has a long history dating back to Mill (1848, p.349) who pointed out the

[1]Corresponding Author, Department of Economics, Kyung Hee University, 26 Kyungheedae-ro, Dongdaemun-gu, Seoul, 130-701, South Korea; hyungsunchoi@khu.ac.kr.

problem of an unstable monetary standard as follows:

> All variations in the value of the circulating medium are mischievous: they disturb existing contracts and expectations, and the liability to such changes renders every pecuniary engagement of long date entirely precarious.

In the same spirit, Jevons (1875, chapter XXV) emphasized the necessity of an Indexed-UoA, called the "Tabular Standard of Value," by referring to the proposals made by Lowe and Scorope:

> He [Joseph Lowe] proposes that persons should be appointed to collect authentic information concerning the prices at which the staple articles of household consumption were sold. ... . Having regard to the comparative quantities of commodities consumed in a household, he would then frame a table of reference, showing in what degree a money contract must be varied so as to make the purchasing power uniform. ... Mr. Scrope suggests ... that a standard might be formed by taking an average of the mass of commodities which, ... , might serve to determine and correct the variations of the legal standard. ... Such schemes for a tabular or average standard of value appear to be perfectly sound ... and the practical difficulties are not of a serious character.

In a similar vein, Friedman (1974), Fischer (1986), Tobin (1987), and Shiller (1999, 2002, 2003) advocate indexing payments to inflation. From a real-world perspective, indexed units of account have indeed been adopted in some Latin-American countries such as Brazil, Chile, Colombia, Ecuador, Mexico, and Uruguay.

This paper attempts to delve deeper into the nature of an Indexed-UoA using a widely used microfounded monetary model. More specifically, we try to elaborate on the mechanism by which an Indexed-UoA can affect real allocations and the circumstances under which an Indexed-UoA is essential. In order to do that, considering the concern raised by Mill (1848), we introduce deferred-payment trades into the model of Berentsen, Camera, and Waller (2005). In particular, we take notice of the key features of the deferred-payment trade, such as a credit-card payment, as follows: (i) the good-delivering point does not coincide with the trade-clearance point and (ii) the trade is typically made at the price of good-delivering point. To spotlight the key characteristics of an Indexed-UoA, we then compare two economies: one in which only money plays a role of a unit of account (No-Indexed-UoA economy) and another in which in addition to money, an Indexed-UoA can play a role of a unit of account (Indexed-UoA economy).

The No-Indexed-UoA economy is motivated by Shiller (1999, p.1): "The general public appears to have sufficient difficulty with indexation, ..., that they will do so only in rare or extreme situations. Even in times of moderate to high inflation, most people will not purchase inflation-indexed debt, will not borrow with an indexed mortgage, will not agree to indexed alimony or child support payments and will not push hard for indexed rent or wage contracts."[2]

[2]In a similar context, Freeman and Tabellini (1998) also mentioned that: "One of the greatest economic puzzles in an age of widely varying, random rates of inflation is

The INDEXED-UOA economy is motivated by the Chilean CPI-INDEXED-UOA called the *Unidad de Fomento* (UF). In 1967, the Chilean government introduced the UF which indicates the amount of Chilean Peso required to buy a representative basket of consumer goods. The UF has came into wide use as a unit of account since 1980s: real estate, rent payments, mortgages, car loans, long-term government securities, and pension payments are all priced using the UF, while wages, consumer goods prices, and stock prices are expressed in Peso terms.

Our comparison results suggest that while deferred-payment trades (hereinafter referred as "credit trades") occur only if inflation is not too high in a NO-INDEXED-UOA economy, they take place regardless of inflation in an INDEXED-UOA one. In an INDEXED-UOA economy, credit-trade balances are denominated in an INDEXED-UOA and settled according to realized inflation. That is, the presence of an INDEXED-UOA essentially facilitates inflation-contingent trades rendering credit trades independent of inflation. However, in a NO-INDEXED-UOA economy, credit-trade balances are denominated in money because it is the only available unit of account and are thus settled without adjustment. This implies that if inflation is too high, credit-trade balances repaid at the point of settlement may be insufficient to compensate for the cost borne by the seller at the good-delivering point and

the persistent use of nominal contracts, that is, of promises of a future payment of a prespecified, uncontingent sum of fiat money."

consequently sellers are reluctant to make credit trades.

In addition, in a No-Indexed-UoA economy, as suggested in most standard models, unexpected inflation renders debtors better off and creditors worse off. However, there is no such redistribution effect in an Indexed-UoA economy because credit-trade balances are denominated in an Indexed-UoA whose nominal value is contingent on an inflation rate.

The results above conform with the claim of Shiller (1999, 2002, 2003) that introducing an Indexed-UoA can resolve many problems caused by inflation. They are also somewhat in line with the view of Keynes (1923) that the role of money as a unit of account would deteriorate if its value were unstable, followed by the emergence of an alternative unit of account anchored to real value. As mentioned, this prediction has indeed come to pass in some Latin-American countries—that is, Brazil, Chile, Colombia, Ecuador, Mexico, and Uruguay introduced indexed units of account during episodes of high inflation (Shiller 2002).

We finally endogenize the choice of a unit of account for credit trades. In the baseline model, the terms of a credit trade are posted in money in a No-Indexed-UoA economy and in an Indexed-UoA in an Indexed-UoA economy. We relax this assumption to let a credit-market maker offer 2 submarkets: a money-posting one where the terms of a credit trade are quoted in money and an Indexed-UoA-posting one where the terms of a credit trade are quoted in an Indexed-UoA. An agent as a buyer or a seller then decides which submarket to enter, if any. A buyer participating in the INDEXED-UOA-posting submarket bears an operational cost of an INDEXED-UOA. This extension implies that if an inflation rate is not too high, money would be used as a unit of account for both spot and credit trades. If an inflation rate is too high, however, money would be used as a unit of account for spot trades only and an INDEXED-UOA would emerge as a unit of account for credit trades. Indeed, this prediction is somewhat consistent with real-world observation in Chile.

The paper proceeds as follows. Section 2 describes the model, followed by an equilibrium characterization in Section 3. Section 4 explores the credit friction associated with inflation and the implications of an INDEXED-UOA. Section 5 endogenizes a unit of account for deferred-payment trades. Section 6 summarizes the paper with a few concluding remarks, followed by the Appendix which contains the proofs of our main results.

2. Model

The background environment is that of Berentsen, Camera, and Waller (2005) with competitive markets.[3] Time is discrete and continues forever. There is a $[0, 1]$ continuum of infinitely lived agents with one perishable and

[3]Among the related models of competitive pricing in the framework of Lagos and Wright (2005) are Rocheteau and Wright (2005), Lagos and Rocheteau (2005), and Berentsen, Camera, and Waller (2007).

divisible good that can be produced and consumed by all agents. There is also an intrinsically useless, divisible, and durable object called money. Each agent is endowed with $M_0 > 0$ units of money at the beginning of the initial period. In each period, agents trade in three Walrasian markets, markets 1, 2, and 3, which open and close sequentially. Other than these, there is an auxiliary credit-trade submarket that is offered by the credit platform before market 1 closes. Agents discount across periods with factor $\beta \in (0, 1)$.

At the beginning of market 1, each agent receives one of two equally probable preference shocks such that an agent becomes either a buyer who can consume but cannot produce or a seller who can produce but cannot consume. A seller suffers disutility q from producing $q \in \mathbb{R}_+$ units of a good and can trade them with anonymous buyers. Trades with anonymous buyers cannot be recorded and hence they should be *quid pro quo*.[4] The buyer obtains utility $u(q)$ from consuming $q \in \mathbb{R}_+$ units of the good where $u'' < 0 < u'$, $u'(\infty) = 0$, $u(0) = 0$, and $u'(0) = \infty$.

The credit platform can record trades associated with it only in market 1 and opens a submarket where it posts the deferred-payment trades such that goods are delivered on the spot at the current market price whereas the relevant balances are cleared at the end of the period (i.e., market 3). Specifically, the credit platform purchases goods from sellers who are willing

[4]See, for example, Kocherlakota (1998), Wallace (2001), Corbae, Temzelides, and Wright (2003), and Aliprantis, Camera, and Puzzello (2007).

to participate in this submarket. The platform then transforms one unit of good purchased into one unit of so-called deferred-payment good at no cost and resells it at the purchasing price to buyers who wish to consume it with a delayed payment in market 3.[5] Credit-trade balances are denominated in money in an economy where only money plays a role of a unit of account, whereas they are denominated in an INDEXED-UOA in an economy with an INDEXED-UOA whose value is indexed to a realized price level. The buyer obtains utility $\upsilon(q)$ from consuming $q \in \mathbb{R}_+$ units of a deferred-payment good where $\upsilon'' < 0 < \upsilon'$, $\upsilon(0) = 0$, and there exists $\bar{q} > 0$ such that $\upsilon(\bar{q}) = \bar{q}$.

After closing market 1 but before opening market 2, new money is injected in a lump-sum manner. That is, the money stock evolves according to $M_t = \mu_t M_{t-1}$ over the period where M_t denotes the money supply at the end of period t and μ_t is a random variable such that $\mu_t^h = \bar{\mu}(1+\varepsilon)$ with probability ρ and $\mu_t^l = \bar{\mu}(1-\varepsilon)$ with probability $1-\rho$. We here assume $\varepsilon \in (0,1)$ and $\rho = 1/2$ so that $\mathbb{E}(\mu_t) = \bar{\mu}$.

With money balances after the trades in market 1 and the lump-sum transfer, each agent moves on to market 2 where she again receives an idiosyncratic preference shock such that she becomes either a buyer or a seller with equal probability. As in market 1, a buyer obtains utility $u(q)$ from consuming $q \in \mathbb{R}_+$ units of a good and a seller suffers disutility q from pro-

[5] Although, for simplicity, a single submarket maker is introduced in our model, the assumed terms of trade are indeed reminiscent of those with competitive price posting.

ducing $q \in \mathbb{R}_+$ units of the good. Since in this market the credit platform cannot access record-keeping technology and agents cannot commit to future actions, all trades should be on the spot. It is also worthwhile noting that the unavailability of record-keeping technology implies that the credit-trade balances made in market 1 cannot be used to purchase the market-2 good. This feature distinguishes money acquired in market 1 from the balance of credit—that is, the former is liquid in market 2, whereas the latter is illiquid.

In market 3, all agents can consume, produce, and obtain utility $U(q)$ from consuming $q \in \mathbb{R}_+$ units of a good and suffer disutility q from producing $q \in \mathbb{R}_+$ units of the good where $U'' < 0 < U'$, $U'(\infty) = 0$, $U(0) = 0$, and $U'(0) = \infty$.[6] In addition, all the credit trades made in market 1 are settled in this market and we assume that the credit platform can force repayment at no cost. Hence according to the record, the platform collects the credit balances from debtors who consume via credit trades in market 1 and transfers them to creditors who produce for credit trades.

[6]As discussed in Berentsen, Camera, and Waller (2005), the different preference in market 3 is simply a technical device to ensure a degenerate distribution at the beginning of each period. In particular, the scaling of $U(q)$ so that $q_3^* \geq 2q^* + q_d^*$ is required to guarantee the result where $q_3^* = \arg\max[U(q_3) - q_3]$, $q^* = \arg\max[u(q) - q]$, and $q_d^* = \arg\max[\upsilon(q_d) - q_d]$.

3. Stationary Equilibrium

We will consider a stationary monetary equilibrium in which the end-of-period real money balance is constant over time: i.e., $\phi_{t-1}M_{t-1} = \phi_t^h \mu_t^h M_{t-1} = \phi_t^l \mu_t^l M_{t-1}$ where ϕ^i for $i \in \{h, l\}$ is the real price of money in market 3 when the realized money growth shock is μ^i. Hereinafter we drop the time subscript t and index the next-period (previous period) variable by $+1$ (-1) if there is no risk of confusion.

3.1. No-Indexed-UoA *Economy*

We first study an economy in which only money plays a role of a unit of account. Let $V_j(m_j, d)$ denote the expected value for an agent entering market $j \in \{2, 3\}$ with m_j and d amount of money and credit balances, respectively, and let $V_1(m_1)$ denote the expected value for an agent entering market 1 with m_1 amount of money. Then the lifetime utility of an agent entering market 3 with $m_3 \in \mathbb{R}_+$ and $d \in \mathbb{R}$ is given by

$$V_3(m_3, d) = \max_{(q_3^b, q_3^s, m_{1,+1})} \left[U(q_3^b) - q_3^s + \beta V_{1,+1}(m_{1,+1})\right] \tag{1}$$

$$\text{s.t. } q_3^b + \phi m_{1,+1} = q_3^s + \phi(m_3 + d)$$

where q_3^b (q_3^s) is consumption (production) in market 3, $\phi = 1/p_3$ with p_3 denoting the nominal price of the market-3 good, and d is positive (negative) for a creditor (debtor). Substituting q_3^s from the constraint, we have

$$V_3(m_3, d) = \phi(m_3 + d) + \max_{(q_3^b, m_{1,+1})} \left[U(q_3^b) - q_3^b - \phi m_{1,+1} + \beta V_{1,+1}(m_{1,+1})\right].$$

The first order conditions for $(q_3^b, m_{1,+1}) \in \mathbb{R}^2_{++}$ are

$$U'(q_3^b) = 1 \tag{2}$$

$$\beta V'_{1,+1}(m_{1,+1}) = \phi \tag{3}$$

where $V'_{1,+1}$ is the marginal value of an additional unit of money taken into market 1. The envelope condition is

$$V'_{3,i}(m_3, d) = \phi \tag{4}$$

where $V'_{3,i}$ for $i \in \{m, d\}$ is the marginal value of an additional unit of i taken into market 3. As in Lagos and Wright (2005), regardless of (m_3, d), all agents consume $q_3^b = q_3^* = \arg\max[U(q_3^b) - q_3^b]$ and exit market 3 with an identical balance of money. This conveniently allows us to restrict our attention to the case where the distribution of money holdings is degenerate at the beginning of each period.

We next turn to market 2. The lifetime utility of an agent entering market 2 with $m_2 \in \mathbb{R}_+$ amount of money and $d \in \mathbb{R}$ amount of credit balance is given by

$$\begin{aligned} V_2(m_2, d) = \; & \frac{1}{2}\left\{\max_{q_2^b}\left[u(q_2^b) + V_3(m_2 - p_2 q_2^b, d)\right]\right\} + \\ & \frac{1}{2}\left\{\max_{q_2^s}\left[V_3(m_2 + p_2 q_2^s, d) - q_2^s\right]\right\} \end{aligned} \tag{5}$$

where q_2^b (q_2^s) is consumption (production) in market 2 and p_2 is the nominal price of the market-2 good. Taking $p_2 \in \mathbb{R}_{++}$ as given, a seller chooses

$q_2^s \in \mathbb{R}_{++}$ that solves the second term of the right-hand side in (6), which yields the optimality condition

$$V'_{3,m}(m_2 + p_2 q_2^s, d) = (p_2)^{-1}. \tag{6}$$

Then (4) immediately gives

$$p_2 = p_3 = \phi^{-1}. \tag{7}$$

Similarly, a buyer chooses $q_2^b \in \mathbb{R}_{++}$ that solves the first term of the right-hand side in (6), which yields the optimality condition

$$u'(q_2^b) = p_2 \left[V'_{3,m}(m_2 - p_2 q_2^s, d) + \lambda_2 \right] \tag{8}$$

where $\lambda_2 \in \mathbb{R}_+$ is the Lagrangian multiplier on the buyer's budget constraint $(p_2 q_2^b \leq m_2)$. Using (4) and (6)-(7), (8) reduces to

$$u'(q_2^b) = 1 + \lambda_2 \phi^{-1}. \tag{9}$$

Notice that $q_2^b = q^* = \arg\max[u(q) - q]$ if $\lambda_2 = 0$. In addition, (6) and (7), together with $(\partial q_2^b / \partial m_2) = (1/p_2)$ for $\lambda_2 \neq 0$ and $u'(q_2^b) = 1$ for $\lambda_2 = 0$, imply that the marginal value of an additional unit of money at the beginning of market 2 is given by

$$V'_{2,m}(m_2, d) = \begin{cases} \phi & \text{for } \lambda_2 = 0 \\ \frac{\phi}{2} \left[u'(q_2^b) + 1 \right] & \text{otherwise.} \end{cases} \tag{10}$$

The marginal value of an additional unit of credit balance at the beginning of market 2 is given by

$$V'_{2,d}(m_2, d) = V'_{3,d} = \phi. \tag{11}$$

We now move on to market 1. The lifetime utility of an agent entering market 1 with $m_1 \in \mathbb{R}_+$ amount of money is given by

$$
\begin{aligned}
V_1(m_1) \;=\; & \frac{1}{2}\left\{\max_{(q^b_{1,m},q^b_{1,d})}\left[u(q^b_{1,m}) + \upsilon(q^b_{1,d}) + \mathbb{E}V_2\left(m_1 - p_1 q^b_{1,m} + T, -p_1 q^b_{1,d}\right)\right]\right\} + \\
& \frac{1}{2}\left\{\max_{(q^s_{1,m},q^s_{1,d})}\left[\mathbb{E}V_2(m_1 + p_1 q^s_{1,m} + T, p_1 q^s_{1,d}) - (q^s_{1,m} + q^s_{1,d})\right]\right\} \qquad (12)
\end{aligned}
$$

where p_1 is the nominal price of the market-1 good, $q_{1,m}$ ($q_{1,d}$) is the quantity of a good traded for money (deferred-payment), and T denotes the lump-sum transfer after the market-1 trade such that either $T = (\mu^h - 1)M_{-1}$ or $T = (\mu^l - 1)M_{-1}$ with equal probability. Taking $p_1 \in \mathbb{R}_{++}$ as given, a seller chooses $(q^s_{1,m}, q^s_{1,d})$ and a buyer chooses $(q^b_{1,m}, q^b_{1,d})$. The choice problems of $(q^s_{1,m}, q^b_{1,m}) \in \mathbb{R}^2_{++}$ yield the optimality conditions

$$p_1 \mathbb{E}V'_{2,m}(m_1 + p_1 q^s_{1,m} + T, p_1 q^s_{1,d}) = 1 \qquad (13)$$

$$p_1 \mathbb{E}V'_{2,m}(m_1 - p_1 q^b_{1,m} + T, -p_1 q^b_{1,d}) = u'(q^b_{1,m}) \qquad (14)$$

in which we use the result in Lemma 2 of Berentsen, Camera, and Waller (2005) that $p_1 q^b_{1,m} = m_1$ cannot happen because $u'(0) = \infty$. Using (13)-(14) and the market clearing condition ($q^b_{1,m} = q^s_{1,m} = q_{1,m}$), we then have

$$u'(q_{1,m}) = \frac{\mathbb{E}V'_{2,m}(m_1 - p_1 q_{1,m} + T, -p_1 q^b_{1,d})}{\mathbb{E}V'_{2,m}(m_1 + p_1 q_{1,m} + T, p_1 q^s_{1,d})}. \qquad (15)$$

Furthermore, (13)-(14) and $p_1 q^b_{1,m} < m_1$ imply the marginal valuation of an additional unit of money at the beginning of market 1 as follows:

$$V'_1(m_1) = \frac{1}{2}\left[\frac{u'(q^b_{1,m}) + 1}{p_1}\right]. \qquad (16)$$

Finally, noting that credit-trade balances are not available in market 2, sellers will participate in a credit-trade submarket if $p_1 \mathbb{E} V'_{3,d} \geq 1$ where $p_1 \mathbb{E} V'_{3,d}$ is the expected marginal revenue from a credit trade and 1 is the marginal disutility from producing a unit of deferred-payment good. This inequality holds if a seller in market 1 has enough money at the beginning of market 2 so that p_1 relies on an expected inflation only (i.e., $\lambda_2 = 0$). That is, $V'_{3,d} = \phi$ from (4) and $p_1 = 1/\mathbb{E} V'_{2,m}$ from (13) imply that $p_1 \mathbb{E} V'_{3,d} = 1$ if $\lambda_2 = 0$ because $p_1 = 1/\mathbb{E}\phi$ from (10). However, if $\lambda_2 \neq 0$, $p_1 \mathbb{E} V'_{3,d} = (1/\mathbb{E} V'_{2,m})\mathbb{E}(\phi) < 1$ because, as we can see from (10), p_1 now depends on the expected inflation as well as a kind of liquidity premium. Hence, in this case, sellers in market 1 are unwilling to participate in a credit-trade submarket because it is cheaper for them to acquire money in market 3 than in market 1. All in all, in the credit-trade submarket, $(q^s_{1,d}, q^b_{1,d}) \in \mathbb{R}^2_+$ should satisfy

$$\left.\begin{aligned} & p_1 \mathbb{E} V'_{2,d}(m_1 + p_1 q_{1,m} + T, p_1 q^s_{1,d}) = 1 \\ & p_1 \mathbb{E} V'_{2,d}(m_1 - p_1 q_{1,m} + T, -p_1 q^b_{1,d}) = \upsilon'(q^b_{1,d}) \\ & q^b_{1,d} = q^s_{1,d} = q_{1,d} \end{aligned}\right\} \quad \text{if } p_1 \mathbb{E} V'_{3,d} \geq 1 \tag{17}$$

and

$$q^b_{1,d} = q^s_{1,d} = q_{1,d} = 0 \qquad \text{if } p_1 \mathbb{E} V'_{3,d} < 1. \tag{18}$$

Now a stationary monetary equilibrium of an economy without an Indexed-UoA can be defined as follows.

Definition 1. *A stationary monetary equilibrium for a* No-Indexed-UoA *economy is a list of* $[(p_j)_{j=1}^3, (q_{1,m}, q_{1,d}, q_2, q_3), \lambda_2, m_{1,+1}]$ *that satisfies (2)-(3), (7)-(9), (13)-(14), (17), and (18).*

3.2. Indexed-UoA *Economy*

We now consider an Indexed-UoA economy where, as in Chile, money (e.g., the Chilean Peso) as well as an Indexed-UoA (e.g., the *Unidad de Fomento*) play roles as units of account. That is, like the *Unidad de Fomento* (CPI indexed unit of account in Chile), an Indexed-UoA of which exchange rate with money is indexed to a realized price level is introduced. Specifically, one unit of an Indexed-UoA in the market $j \in \{1, 2, 3\}$ is converted into p_j units of money. For on-the-spot trades, there is no reason for traders to use an alternative unit of account in place of money (medium of exchange). However, for credit trades, since money-supply shock is realized between the point of delivering the good and the point of settlement, there is exposure to an inflation risk. This suggests that if using an Indexed-UoA incurs no extra cost, there is no reason for credit traders not to use an Indexed-UoA as a unit of account. Hence, without loss of generality, we can assume that credit-trade balances are denominated in the Indexed-UoA.

Now the lifetime utility of an agent entering market 3 with $m_3 \in \mathbb{R}_+$ amount of money and $d^u \in \mathbb{R}$ amount of credit balance denominated in the Indexed-UoA is given by

$$V_3(m_3, d^u) = \phi(m_3 + p_3 d^u) + \max_{(q_3^b, m_{1,+1})} \left[U(q_3^b) - q_3^b - \phi m_{1,+1} + \beta V_{1,+1}(m_{1,+1}) \right]. \tag{1'}$$

The first order conditions for $(q_3^b, m_{1,+1}) \in \mathbb{R}^2_{++}$ are identical to those for a No-Indexed-UoA economy, (2) and (3). Noting that $\phi p_3 = 1$, the envelope conditions are given by

$$V'_{3,m}(m_3, d^u) = \phi, \quad V'_{3,d^u}(m_3, d^u) = 1. \tag{4'}$$

The lifetime utility of an agent entering market 2 with $m_2 \in \mathbb{R}_+$ amount of money and $d^u \in \mathbb{R}$ amount of credit balance denominated in the Indexed-UoA is given by

$$\begin{aligned} V_2(m_2, d^u) =& \frac{1}{2}\left\{\max_{q_2^b}\left[u(q_2^b) + V_3(m_2 - p_2 q_2^b, d^u)\right]\right\} + \\ & \frac{1}{2}\left\{\max_{q_2^s}\left[V_3(m_2 + p_2 q_2^s, d^u) - q_2^s\right]\right\} \end{aligned} \tag{5'}$$

and the relevant optimality conditions are identical to (6)-(8).

The lifetime utility of an agent entering market 1 with $m_1 \in \mathbb{R}_+$ amount of money is given by

$$\begin{aligned} V_1(m_1) = & \frac{1}{2}\left\{\max_{(q_{1,m}^b, q_{1,d}^b)}\left[u(q_{1,m}^b) + \upsilon(q_{1,d}^b) + \mathbb{E}V_2\left(m_1 - p_1 q_{1,m}^b + T, -q_{1,d}^b\right)\right]\right\} + \\ & \frac{1}{2}\left\{\max_{(q_{1,m}^s, q_{1,d}^s)}\left[\mathbb{E}V_2\left(m_1 + p_1 q_{1,m}^s + T, q_{1,d}^s\right) - (q_{1,m}^s + q_{1,d}^s)\right]\right\} \end{aligned} \tag{12'}$$

where the credit balances, the second argument of V_2 on the right-hand side in (12′), is expressed in terms of the Indexed-UoA, $q_{1,d} = (p_1 q_{1,d})/p_1$. The first order conditions for $(q_{1,m}^s, q_{1,m}^b)$ are the same as (13)-(14) which together with the market clearing condition $(q_{1,m}^b = q_{1,m}^s = q_{1,m})$ imply that

$$u'(q_{1,m}) = \frac{\mathbb{E}V'_{2,m}(m_1 - p_1 q_{1,m} + T, -q_{1,d}^b)}{\mathbb{E}V'_{2,m}(m_1 + p_1 q_{1,m} + T, q_{1,d}^s)}. \tag{15'}$$

Finally, sellers will participate in a credit-trade submarket if $\mathbb{E}V'_{3,d^u} \geq 1$. If $\mathbb{E}V'_{3,d^u} < 1$, however, sellers would not be willing to participate in the submarket because the expected revenue from a credit trade would fall short of its disutility cost. The participation constraints eventually imply that in the submarket, $(q^s_{1,d}, q^b_{1,d}) \in \mathbb{R}^2_+$ should satisfy

$$\left.\begin{aligned} &\mathbb{E}V'_{2,d^u}(m_1 + p_1 q_{1,m} + T, q^s_{1,d}) = 1 \\ &\mathbb{E}V'_{2,d^u}(m_1 - p_1 q_{1,m} + T, -q^b_{1,d}) = \upsilon'(q^b_{1,d}) \\ &q^b_{1,d} = q^s_{1,d} = q_{1,d} \end{aligned}\right\} \quad \text{if } \mathbb{E}V'_{3,d^u} \geq 1 \tag{17$'$}$$

and

$$q^b_{1,d} = q^s_{1,d} = q_{1,d} = 0 \qquad \text{if } \mathbb{E}V'_{3,d^u} < 1. \tag{18$'$}$$

Now a stationary monetary equilibrium of an Indexed-UoA economy can be defined as follows.

Definition 2. *A stationary monetary equilibrium for an* Indexed-UoA *economy is a list of* $[(p_j)^3_{j=1}, (q_{1,m}, q_{1,d}, q_2, q_3), \lambda_2, m_{1,+1}]$ *that satisfies (2)-(3), (7)-(9), (13)-(14),* (17′)*, and* (18′)*.*

4. Inflation, Credit Trade, and Welfare

We are now ready to explore the implications of an Indexed-UoA. The following proposition suggests that an essential role of an Indexed-UoA lies in the facilitation of inflation-contingent credit trades.

Proposition 1. *Suppose* $\bar{\mu} \geq \beta^* = [\beta/(1-\varepsilon^2)]$.

1. *In a* NO-INDEXED-UOA *economy, there is a stationary monetary equilibrium with* $q_{1,d} = q_d^* = \arg\max[\upsilon(q_{1,d}) - q_{1,d}]$ *if* $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$*, but* $q_{1,d} = 0$ *if* $\bar{\mu} > \bar{\mu}^*$ *where* $\bar{\mu}^*$ *is defined in the Appendix.*
2. *In an* INDEXED-UOA *economy, there is a stationary monetary equilibrium with* $q_{1,d} = q_d^* = \arg\max[\upsilon(q_{1,d}) - q_{1,d}]$ *regardless of* $\bar{\mu}$*.*

Proof. See Appendix. □

In an INDEXED-UOA economy, a credit trade is indeed independent of inflation because it is denominated in an INDEXED-UOA of which the nominal value is indexed to realized inflation. However, in a NO-INDEXED-UOA economy, money is the only unit of account. Hence, besides spot trades, credit trades are also denominated in money and repaid in market 3 without any adjustment. This implies that if inflation is too high, the credit balances repaid in market 3 will fall short of the cost borne by a seller in market 1. Hence, in a high-inflation economy, sellers are not willing to participate in the credit-trade submarket.

Proposition 1 supports the claim of Shiller (1999, 2002, 2003) that introducing an INDEXED-UOA could resolve the problem caused by inflation. Furthermore, it somewhat conforms to the view of Keynes (1923) that the role of money as a unit of account would deteriorate if its value were unstable and then an alternative unit of account anchored to real value would emerge. Indeed, this prediction came to pass in the real world: for instance, during the episode of German hyperinflation in the early 1920s, prices were typically posted in terms of goldmarks (1 goldmark=358 mg of pure gold), rather than circulated currency (see Wolf 2002); Brazil introduced a kind of indexed unit of account following hyperinflation in the 1980s and other Latin American countries such as Chile, Colombia, Ecuador, Mexico, and Uruguay also introduced one during episodes of high inflation (Shiller 2002).

We now compare the welfare of the two economies where the welfare is defined as the expected lifetime utility of a representative agent: i.e., welfare $\mathbb{W}$ can be expressed as

$$\mathbb{W} = \frac{1}{1-\beta}\left\{ \begin{array}{l} \frac{1}{2}\left[u(q_{1,m}) + \upsilon(q_{1,d}) - (q_{1,m} + q_{1,d})\right] + \\ \frac{1}{4}\sum_{i=\{p,r\}}\left[u(q_2^i) - \bar{q}_2\right] + \left[U(q_3) - q_3\right] \end{array} \right\}$$

where q_2^p (q_2^r) is the consumption of a poor (rich) buyer in market 2 and $\bar{q}_2 = (1/2)(q_2^p + q_2^r)$. The following result shows that regarding welfare, a No-Indexed-UoA economy is the same as an Indexed-UoA economy when inflation is sufficiently low, but dominated by an Indexed-UoA economy when inflation is sufficiently high.

Corollary 1. *Let* $\mathbb{W}$ *and* $\tilde{\mathbb{W}}$ *denote the welfare for a* No-Indexed-UoA *economy and an* Indexed-UoA *economy, respectively.* $\mathbb{W} = \tilde{\mathbb{W}}$ *for* $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$*, whereas* $\mathbb{W} < \tilde{\mathbb{W}}$ *for* $\bar{\mu} > \bar{\mu}^*$*.*

Proof. See Appendix. □

Intuitively, this result is straightforward. Other than $q_{1,d}$, allocations are the same between economies with and without an INDEXED-UOA. Now from Proposition 1, $q_{1,d} = q_d^*$ in both a NO-INDEXED-UOA and an INDEXED-UOA economy if $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$. If $\bar{\mu} > \bar{\mu}^*$, however, the credit-trade submarket is inactive in a NO-INDEXED-UOA economy, whereas $q_{1,d}$ still remains at q_d^* in an INDEXED-UOA economy.

Finally, the following proposition shows that in an economy where inflation ($\bar{\mu}$) is low enough, an unexpected injection of money can boost aggregate market-2 consumption temporarily. In addition, in an INDEXED-UOA economy, price is adjusted relatively flexibly in response to monetary shock.

Proposition 2. *Suppose $\mu = \bar{\mu}(1-\varepsilon) = \mu^l$ is realized but there is an unexpected injection of money $\Delta = 2\varepsilon\bar{\mu}M_{-1}$ so that $M = \bar{\mu}(1+\varepsilon)M_{-1} = \mu^h M_{-1}$.*

1. *If $\bar{\mu} \in (\beta^*, \bar{\mu}^*]$, market-2 aggregate consumption strictly increases both in an* INDEXED-UOA *and a* NO-INDEXED-UOA *economy.*
2. *An unexpected money injection renders debtors better off but creditors worse off in a* NO-INDEXED-UOA *economy, whereas there is no such a redistribution effect in an* INDEXED-UOA *economy.*
3. *The aggregate price level in an* INDEXED-UOA *economy is closer to μ^h than that in a* NO-INDEXED-UOA *economy.*

Proof. See Appendix □

As discussed in Berentsen, Camera, and Waller (2005), Craig and Rocheteau (2006) and Molico (2006), the real effect of unexpected inflation is due to its asymmetric effect on the real balances of the poor and the rich in market 2. Notice that inflation is basically a proportional tax on money holdings. Then for the rich, since the extra money received because of an unexpected lump-sum injection is not sufficient to offset the inflation-tax burden triggered by the unexpected money injection, their real balances decline. However, for the poor, it is more than enough to offset the inflation-tax burden and hence their real balances increase. Now under the Friedman rule ($\bar{\mu} = \beta^*$), both the rich and the poor are not binding in market 2 and hence unexpected inflation has no effect on consumption in market 2. If inflation is sufficiently low that the poor are always binding, unexpected inflation increases the consumption of the poor. If inflation is so high that even the rich are always binding, the negative effect of unexpected inflation on the rich offsets its positive effect on the poor.

The second result in Proposition 2 suggests that in a No-Indexed-UoA economy, unexpected inflation causes wealth redistribution at the point of clearing credit trades (i.e., market 3). This conforms with the claim of Shiller (1999) that if prices stay fixed in money terms across periods, wealth redistribution occurs in times of economic change.

The last result in Proposition 2 implies that in a No-Indexed-UoA economy, the price is adjusted relatively slowly in response to monetary shock. This alludes to the possibility that the nominal stickiness observed in the real world can be partly attributed to the widespread use of nominal contracts for deferred payments.

5. Endogenous Unit of Account for Credit Trades

In the baseline model, a credit trade is posted in money in a No-Indexed-UoA economy and in an Indexed-UoA in an Indexed-UoA economy. We here relax this assumption and endogenize the choice of a unit of account for credit trades as follows.

A credit platform opens 2 submarkets: one is a money-posting submarket in which the terms of a credit trade are quoted in money and the other is an Indexed-UoA-posting submarket in which the terms of a credit trade are quoted in an Indexed-UoA. An extra cost is borne by the credit platform to operate an Indexed-UoA. Indeed, in Chile, the Bank of Chile calculates the Peso-to-UF exchange rate and makes it available to the general public on a daily basis, which is costly. This operational cost is imposed to buyers in the form of an entrance fee: i.e., in order to enter the Indexed-UoA-posting submarket, a buyer should pay an entrance fee θ. An agent as a buyer or a seller then decides which submarket to enter, if any. If an agent is indifferent towards the choice of which submarket to enter, she enters the socially costless money-posting submarket.

Now the following proposition suggests that in an economy where an inflation rate is not too high, the money-posting submarket is active while the Indexed-UoA-posting submarket is inactive. However, in an economy where an inflation rate is sufficiently high, the INDEXED-UOA-posting submarket is active while the money-posting submarket is inactive.

Proposition 3. *(i) If $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$, the money-posting submarket is active and money is in use as a unit of account for credit trades. (ii) If $\bar{\mu} > \bar{\mu}^*$ and $\theta < \bar{\theta} = v(q_d^*) - q_d^*$, the* INDEXED-UOA*-posting submarket is active and an* INDEXED-UOA *is in use as a unit of account for credit trades.*

Proof. See Appendix. □

Notice that, as shown in Proposition 1, sellers are willing to participate in the credit-trade submarket in a low-inflation economy even if there is no INDEXED-UOA and buyers then do not need to enter the costly INDEXED-UOA-posting submarket. Hence, money is used as a unit of account for spot trades as well as credit trades. This result is complementary to Freeman and Tabellini (1998) in which nominal contracts are optimal under certain conditions. However, if inflation is too high, sellers are willing to participate in the INDEXED-UOA-posting submarket in which inflation-contingent trades are available. Since credit trades render buyers better off if the relevant cost is not too high, buyers then willingly enter the INDEXED-UOA-posting submarket by paying an entrance fee. As a consequence, an INDEXED-UOA emerges endogenously as a unit of account for credit trades. This prediction is somewhat consistent with real-world observation in Chile. In addition, it provides the rationale for the recent discontinuation assertion of *Unidad de*

Fomento based on stabilized prices. (See, for instance, Shiller 2002, pp.7-8.) That is, a continuation of an Indexed-UoA might lead to only a deadweight loss unless inflation once again becomes so high that people are reluctant to make deferred-payment trades.

6. Concluding Remarks

We have set out a simple monetary model suitable for studying the nature of an Indexed-UoA. Our results suggest that the presence of an Indexed-UoA facilitates credit trades against inflation and hence eventually improves welfare in a high-inflation economy.

Different complications relevant to credit trades could be added to our model. For example, as in Berentsen, Camera, and Waller (2007), credit trades could be introduced not in the form of a deferred-payment contract but in the form of lending and borrowing.[7] In such a variant, due to interest payments being determined endogenously in a financial market, the transaction cost of credit would be proportional to the transaction amount. But it is not believed that such a change would alter our main results qualitatively. The framework could also be extended to study the relationship between

[7]Berentsen, Camera, and Waller (2007) point out that default is a critical issue for models dealing with credit. Our baseline model simplifies such an issue by assuming full enforcement. However, it can be shown that if agents are sufficiently patient, our main results are still valid even if the credit platform cannot force repayment. The proof for this claim is available on request.

default risk and inflation by incorporating financial market frictions such as limited commitment or private information.

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Appendix

Appendix A. Proof of Proposition 1

In a No-Indexed-UoA economy, we have $p_1 = [1/\mathbb{E}V'_{2,m}(m_1 + p_1 q^s_{1,m} + T, \cdot)] \leq (1/\mathbb{E}\phi)$ from (10) and (13). Then, if $p_1 \mathbb{E}\phi = 1$, a seller's expected return from making a credit trade is enough to compensate for the cost incurred from producing a unit of good. However, if $p_1 \mathbb{E}\phi < 1$, it is cheaper for a seller to acquire money in market 3 and then she is not willing to participate in the credit-trade submarket. Hence, only in the equilibrium with $\mathbb{E}V'_{2,m}(m_1 + p_1 q^s_{1,m} + T, \cdot) = \mathbb{E}\phi$, the credit-trade submarket is active. Notice that from (10), $\mathbb{E}V'_{2,m}(m_1 + p_1 q^s_{1,m} + T, \cdot) = \mathbb{E}\phi$ if $(\lambda^r_2)_{\mu^h} = (\lambda^r_2)_{\mu^l} = 0$ where $(\lambda^r_2)_{\mu^i}$ for $i \in \{h, l\}$ is the Lagrangian multiplier for a rich buyer in market 2. Since $\phi^h \mu^h = \phi^l \mu^l$ and $\phi^l > \phi^h = \phi^l(\mu^l/\mu^h)$ in a steady state,

$\phi^l(\mu^l M_{-1} - p_1 q_{1,m}) < \phi^h(\mu^h M_{-1} - p_1 q_{1,m}) < \phi^h(\mu^h M_{-1} + p_1 q_{1,m}) < \phi^l(\mu^l M_{-1} + p_1 q_{1,m})$. This implies that the candidate equilibrium with credit trades would then be the following cases: (1) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [0, 0, 0, 0]$, (2) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [0, 0, 0, +]$, and (3) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [0, 0, +, +]$. Now we will check each case in sequence in compliance with Berentsen, Camera, and Waller (2005).

We first consider case (1). From (13), $p_1 \mathbb{E}\phi = 1$ and from (16), $V_1'(m_1) = \mathbb{E}\phi$. Then from (3), we have $\phi_{-1}[(\beta/\bar{\pi}) - 1] \leq 0$ where $1/\bar{\pi} = (\mathbb{E}\phi/\phi_{-1}) = [1/\bar{\mu}(1 - \varepsilon^2)]$. Now if $(\beta/\bar{\pi}) > 1$ or if $(\beta/\bar{\pi}) < 1$, there is no monetary equilibrium because $m_{1,+1} = \infty$ for the former and $m_{1,+1} = 0$ for the latter. If $(\beta/\bar{\pi}) = 1$ (i.e., $\bar{\mu} = [\beta/(1 - \varepsilon^2)] = \beta^*$), there are an infinite number of monetary equilibria with $q_{1,d} = q_d^*$ and $q_{1,m} = q^*$.

We next consider case (2). Since $(\lambda_2^r)_{\mu^h} = (\lambda_2^r)_{\mu^l} = (\lambda_2^p)_{\mu^h} = 0$ in this equilibrium, $(q_{2,r}^b)_{\mu^h} = (q_{2,r}^b)_{\mu^l} = (q_{2,p}^b)_{\mu^h} = q^*$ and $V_{2,m}'(m_1 + p_1 q_{1,m}^s + T, \cdot) = \phi$ from (10). We then have $(1/p_1) = \mathbb{E}\phi$ from (13). As regards $q_{1,m}$, since $q_{1,m} = q_{1,m}^s = q_{1,m}^b$ in equilibrium, $2V_1'(m_1) = \mathbb{E}\phi[u'(q_{1,m}) + 1]$ from (16) and $V_1'(m_1) = (\phi_{-1}/\beta)$ from (3). These together with $(\phi_{-1}/\mathbb{E}\phi) = \bar{\pi} = \bar{\mu}(1 - \varepsilon^2)$ imply that

$$u'(q_{1,m}) = \frac{2}{\beta}(\bar{\pi} - \beta) + 1 \tag{A.1}$$

which gives a unique value $q_{1,m}$. In addition, since $1 = p_1 \mathbb{E}\phi$, (18) implies that $q_{1,d} = q_d^* = \arg\max[\upsilon(q_{1,d}) - q_{1,d}]$. Now, a stationary condition, $\phi^l \mu^l =$

$\phi^h \mu^h = \phi_{-1}$, gives

$$(q^b_{2,p})_{\mu^l} = \Phi - (\phi^l/\mathbb{E}\phi)q_{1,m} = \Phi - (1+\varepsilon)q_{1,m} \tag{A.2}$$

where Φ is the real balance of money ($\Phi = \phi M = \phi_{+1}M_{+1}$), $(q^b_{2,p})_{\mu^l}$ is consumption of a poor buyer in market 2 with the realized money-supply shock μ^l. Then, from (15), we have

$$4u'(q_{1,m}) = 2(1-\varepsilon) + (1+\varepsilon)\left[u'\left(\Phi - (1+\varepsilon)q_{1,m}\right) + 1\right] \tag{A.3}$$

which determines a unique value of Φ for a given $q_{1,m}$ from (A.1). Using the solution for $(q_{1,m}, \Phi)$, we can obtain $(q^b_{2,p})_{\mu^l}$ from (A.2), $\phi^l\mu^l = \Phi/M_{-1}$, $\phi^h\mu^h = \Phi/M_{-1}$, and $(1/p_1) = (1/2)(\phi^h + \phi^l)$. Finally, for this equilibrium to exist, it must be the case that $(q^b_{2,p})_{\mu^l} = \Phi - (1+\varepsilon)q_{1,m} < q^*$, $(q^b_{2,p})_{\mu^h} = q^* \leq \Phi - (1-\varepsilon)q_{1,m}$, $(q^b_{2,r})_{\mu^l} = q^* \leq \Phi + (1+\varepsilon)q_{1,m}$, and $(q^b_{2,r})_{\mu^h} = q^* \leq \Phi + (1-\varepsilon)q_{1,m}$. Combining all inequalities, the sufficient condition for this equilibrium is

$$q^* + (1-\varepsilon)q_{1,m} \leq \Phi < q^* + (1+\varepsilon)q_{1,m}. \tag{A.4}$$

Notice that as $\bar{\pi} \to \beta$, $q_{1,m} \to q^*$ from (A.1) and $\Phi \to q^* + (1+\varepsilon)q^*$ from (A.3). In addition, $(\partial\Phi/\partial q_{1,m}) > (1+\varepsilon)$ from (A.3). Therefore for $\bar{\pi}$ that is sufficiently close to β, the inequalities in (A.4) hold. However, as $\bar{\pi}$ is far away from β, the left-hand inequality in (A.4) will be violated, although the right-hand inequality is preserved. That is, there exists $\bar{\pi}_1 > \beta$ such that $q^* + (1-\varepsilon)q_{1,m} = \Phi$ at $\bar{\pi} = \bar{\pi}_1$ and for $\bar{\pi} > \bar{\pi}_1$, $q^* + (1-\varepsilon)q_{1,m} > \Phi$. Now let

$\bar{\mu}_1$ be average inflation corresponding to $\bar{\pi}_1$, $\bar{\mu}_1 = \bar{\pi}_1/(1-\varepsilon^2)$. Then, type-(2) equilibrium with $q_{1,d} = q_d^*$ exists if $\bar{\mu} \in (\beta^*, \bar{\mu}_1]$.

We next consider case (3). Since $(\lambda_2^r)_{\mu^h} = (\lambda_2^r)_{\mu^l} = 0$ in this equilibrium, $(q_{2,r}^b)_{\mu^h} = (q_{2,r}^b)_{\mu^l} = q^*$ and $V'_{2,m}(m_1 + p_1 q_{1,m}^s + T, \cdot) = \phi$ from (10). We then have $(1/p_1) = \mathbb{E}\phi$ from (13). Since $2V_1'(m_1) = \mathbb{E}\phi[u'(q_{1,m}) + 1]$ from (16) and $V_1'(m_1) = (\phi_{-1}/\beta)$ from (3), the solution for $q_{1,m}$ is again given by (A.1). In addition, since $1 = p_1\mathbb{E}\phi$, (18) implies that $q_{1,d} = q_d^* = \arg\max[\upsilon(q_{1,d}) - q_{1,d}]$. Next, from (15), we have

$$4u'(q_{1,m}) = (1-\varepsilon)\left\{u'\left[\Phi - (1-\varepsilon)q_{1,m}\right] + 1\right\} + (1+\varepsilon)\left\{u'\left[\Phi - (1+\varepsilon)q_{1,m}\right] + 1\right\} \tag{A.5}$$

where we use $(q_{2,p}^b)_{\mu^h} = \Phi - (\phi^h/\mathbb{E}\phi)q_{1,m} = \Phi - (1-\varepsilon)q_{1,m}$ and $(q_{2,p}^b)_{\mu^l} = \Phi - (\phi^l/\mathbb{E}\phi)q_{1,m} = \Phi - (1+\varepsilon)q_{1,m}$. Then (A.5) determines a unique value of Φ for a given $q_{1,m}$ from (A.1). Using the solutions for $(q_{1,m}, \Phi)$, we can obtain $[(q_{2,p}^b)_{\mu^h}, (q_{2,p}^b)_{\mu^l}]$, $\phi^l\mu^l = \Phi/M_{-1}$, $\phi^h\mu^h = \Phi/M_{-1}$, and $(1/p_1) = (1/2)(\phi^h + \phi^l)$. Finally, for this equilibrium to exist, it must be the case that $(q_{2,p}^b)_{\mu^l} = \Phi - (1+\varepsilon)q_{1,m} < q^*$, $(q_{2,p}^b)_{\mu^h} = \Phi - (1-\varepsilon)q_{1,m} < q^*$, $(q_{2,r}^b)_{\mu^l} = q^* \leq \Phi + (1+\varepsilon)q_{1,m}$, and $(q_{2,r}^b)_{\mu^h} = q^* \leq \Phi + (1-\varepsilon)q_{1,m}$. Combining all inequalities, the sufficient condition for this equilibrium is

$$q^* - (1-\varepsilon)q_{1,m} \leq \Phi < q^* + (1-\varepsilon)q_{1,m}. \tag{A.6}$$

Notice that at $\bar{\pi} = \bar{\pi}_1$, $q^* - (1-\varepsilon)q_{1,m} < \Phi = q^* + (1-\varepsilon)q_{1,m}$. As $\bar{\pi}$ increases above $\bar{\pi}_1$, $q_{1,m}$ decreases from (A.1) and $(\partial\Phi/\partial q_{1,m}) > (1-\varepsilon)$ from (A.5).

Hence, the right-hand inequality is preserved for $\bar{\pi} > \bar{\pi}_1$ but the left-hand inequality binds. That is, there exists $\bar{\pi}^* > \bar{\pi}_1$ such that $q^* - (1-\varepsilon)q_{1,m} = \Phi$ at $\bar{\pi} = \bar{\pi}^*$ and for $\bar{\pi} > \bar{\pi}^*$, $q^* - (1-\varepsilon)q_{1,m} > \Phi$. Let $\bar{\mu}^*$ be average inflation corresponding to $\bar{\pi}^*$, $\bar{\mu}^* = \bar{\pi}^*/(1-\varepsilon^2)$. Then, type-(3) equilibrium with $q_{1,d} = q_d^*$ exists if $\bar{\mu} \in (\bar{\mu}_1, \bar{\mu}^*]$.

Now, if there exists a monetary equilibrium for $\bar{\mu} > \bar{\mu}^*$, it will be either (4) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [0,+,+,+]$ or (5) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [+,+,+,+]$. In any case, $\mathbb{E}\phi p_1 < 1$ because $\mathbb{E}V'_{2,m}(m_1 + p_1 q^s_{1,m} + T, \cdot) > \mathbb{E}\phi$. Then it is cheaper for a seller to acquire money in market 3 and hence she is not willing to participate in the credit-trade submarket.

Finally, in order to characterize an equilibrium for an Indexed-UoA economy, it suffices to identify $q_{1,d}$ with an Indexed-UoA because except for it, an Indexed-UoA and a No-Indexed-UoA economy are identical. Noting that $\mathbb{E}V'_{2,d^u} = 1$ from (11) and (4′), (18′) implies that $q_{1,d} = q_d^* = \arg\max[\upsilon(q_{1,d}) - q_{1,d}]$ regardless of $\bar{\mu}$ which completes the proof.

Appendix B. Proof of Corollary 1

As mentioned above, other than $q_{1,d}$, consumption in markets 2 and 3 is the same between economies with and without an Indexed-UoA. Then $q_{1,d} = q_d^*$ both in a No-Indexed-UoA and an Indexed-UoA economy for $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$ implies the first claim, $\mathbb{W} = \tilde{\mathbb{W}}$ if $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$. In addition, for $\bar{\mu} > \bar{\mu}^*$, $q_{1,d} = 0$ in a No-Indexed-UoA economy but $q_{1,d} = q_d^*$ in an

Indexed-UoA economy implies the second claim, $\mathbb{W} < \tilde{\mathbb{W}}$, if $\bar{\mu} > \bar{\mu}^*$.

Appendix C. Proof of Proposition 2.1

Conditional on the realization of μ^l, suppose there is a surprise injection of money Δ so that M increases to $\bar{\mu}(1+\varepsilon)M_{-1}$ from $\bar{\mu}(1-\varepsilon)M_{-1}$: i.e., $\Delta = 2\varepsilon\bar{\mu}M_{-1}$. Since it is an unanticipated change at the beginning of market 2, it does not have any effect on $(p_1, q_{1,m})$. Notice also that there is no difference in market-2 consumption between economies with and without an Indexed-UoA. Then from Proposition 1, we can consider the following five types of monetary equilibria: (1) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [0,0,0,0]$, (2) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [0,0,0,+]$, (3) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [0,0,+,+]$, (4) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [0,+,+,+]$, and (5) $[(\lambda_2^r)_{\mu^l}, (\lambda_2^r)_{\mu^h}, (\lambda_2^p)_{\mu^h}, (\lambda_2^p)_{\mu^l}] = [+,+,+,+]$.

In case (1), $(q^b_{2,r})_{\mu^h} = (q^b_{2,r})_{\mu^l} = (q^b_{2,p})_{\mu^h} = (q^b_{2,p})_{\mu^l} = q^*$. Then $[(q^b_{2,r})_{\mu^h} + (q^b_{2,p})_{\mu^h}] = 2q^* = [(q^b_{2,r})_{\mu^l} + (q^b_{2,p})_{\mu^l}]$ implies that an aggregate consumption in market 2 is unaffected by a surprise injection Δ. In case (2), $(q^b_{2,r})_{\mu^h} = (q^b_{2,r})_{\mu^l} = (q^b_{2,p})_{\mu^h} = q^*$. In addition, since $\mathbb{E}V'_{2,m}(m_1 + p_1 q^s_{1,m} + T, \cdot) = \mathbb{E}\phi$, $(q^b_{2,p})_{\mu^l} = \Phi - (1+\varepsilon)q_{1,m} < q^*$. Then $[(q^b_{2,r})_{\mu^h} + (q^b_{2,p})_{\mu^h}] = 2q^* > [(q^b_{2,r})_{\mu^l} + (q^b_{2,p})_{\mu^l}]$ implies that a surprise injection Δ increases aggregate consumption in market 2. In case (3), $(q^b_{2,r})_{\mu^h} = (q^b_{2,r})_{\mu^l} = q^*$. In addition, since $\mathbb{E}V'_{2,m}(m_1 + p_1 q^s_{1,m} + T, \cdot) = \mathbb{E}\phi$, $(q^b_{2,p})_{\mu^h} = \Phi - (1-\varepsilon)q_{1,m}$ and $(q^b_{2,p})_{\mu^l} = \Phi - (1+\varepsilon)q_{1,m}$. Then $[(q^b_{2,r})_{\mu^h} + (q^b_{2,p})_{\mu^h}] = q^* + \Phi - (1-\varepsilon)q_{1,m} > q^* +$

$\Phi - (1+\varepsilon)q_{1,m} = [(q^b_{2,r})_{\mu^l} + (q^b_{2,p})_{\mu^l}]$ implies again that a surprise injection Δ boosts aggregate consumption in market 2. In case (4), $(q^b_{2,r})_{\mu^l} = q^*$, $(q^b_{2,r})_{\mu^h} = \Phi + \phi^h p_1 q_{1,m}$, $(q^b_{2,p})_{\mu^l} = \Phi - \phi^l p_1 q_{1,m}$, and $(q^b_{2,p})_{\mu^h} = \Phi - \phi^h p_1 q_{1,m}$. Notice that $(q^b_{2,r})_{\mu^l} = q^* \leq \Phi + \phi^l p_1 q_{1,m}$ and hence $[(q^b_{2,r})_{\mu^h} + (q^b_{2,p})_{\mu^h}] = 2\Phi = [\Phi + \phi^l p_1 q_{1,m} + \Phi - \phi^l p_1 q_{1,m}] \geq q^* + \Phi - \phi^l p_1 q_{1,m} = [(q^b_{2,r})_{\mu^l} + (q^b_{2,p})_{\mu^l}]$. Therefore, in this type of an equilibrium, aggregate consumption in market 2 is weakly increasing in response to a surprise injection Δ. Finally, in case (5), $(q^b_{2,r})_{\mu^l} = \Phi + \phi^l p_1 q_{1,m}$, $(q^b_{2,r})_{\mu^h} = \Phi + \phi^h p_1 q_{1,m}$, $(q^b_{2,p})_{\mu^l} = \Phi - \phi^l p_1 q_{1,m}$, and $(q^b_{2,p})_{\mu^h} = \Phi - \phi^h p_1 q_{1,m}$. Then $[(q^b_{2,r})_{\mu^h} + (q^b_{2,p})_{\mu^h}] = 2\Phi = [(q^b_{2,r})_{\mu^l} + (q^b_{2,p})_{\mu^l}]$ implies that an aggregate consumption in market 2 is unaffected by a surprise injection Δ. In summary, in cases of (2)-(3) with the realized money growth shock is μ^l, market-2 aggregate consumption is strictly increasing in response to a surprise injection of money and such equilibria exist if $(\beta^*, \bar{\mu}^*]$.

Appendix D. Proof of Proposition 2.2

In a No-Indexed-UoA economy, the nominal balance of credit trade is fixed at $p_1 q_{1,d} = q_{1,d}/[\mathbb{E}V'_{2,m}(m_1 + p_1 q_{1,m} + T, \cdot)]$ and is not adjusted in response to a change in the money stock. Hence, the real value of the credit-trade balance redeemed to a creditor when $\mu^i = \mu^l$ is $p_1 q_{1,d}\phi^l$, whereas that for $\mu^i = \mu^h$ is $p_1 q_{1,d}\phi^h$. Then $\phi^l > \phi^h$ immediately implies that a surprise injection Δ in a No-Indexed-UoA economy renders creditors worse off but debtors better off. However, in an Indexed-UoA economy, the credit-trade balances in market 1 is recorded as $q_{1,d}$ units of INDEXED-UOA which is redeemed in market 3 by $q_{1,d}(1/\phi^i)\phi^i = q_{1,d}$ units of a market-3 good where $p_3 = 1/\phi^i$ for $i \in \{h, l\}$. Therefore, the real value of a credit-trade balance is independent of a surprise injection Δ.

Appendix E. Proof of Proposition 2.3

First consider the case in which $q_{1,d} = 0$ in a NO-INDEXED-UOA economy. Noting that other than $q_{1,d}$, $(q_{1,m}, q_2, q_3^*, p_1, p_3)$ are the same between the two economies and $p_2 = p_3$ from (7), the aggregate price level for a NO-INDEXED-UOA economy ($\mathbf{P}_t$) and that for an INDEXED-UOA economy ($\tilde{\mathbf{P}}_t$) can be expressed as

$$\begin{aligned} \mathbf{P}_t &= \left(\frac{q_2 + q_3^*}{q_{1,m} + q_2 + q_3^*}\right) p_{3,t} + \left(\frac{q_{1,m}}{q_{1,m} + q_2 + q_3^*}\right) p_{1,t} \\ \tilde{\mathbf{P}}_t &= \left(\frac{q_d^* + q_2 + q_3^*}{q_d^* + q_{1,m} + q_2 + q_3^*}\right) p_{3,t} + \left(\frac{q_{1,m}}{q_d^* + q_{1,m} + q_2 + q_3^*}\right) p_{1,t}. \end{aligned} \tag{E.1}$$

Notice also that $p_{1,t}$ is not affected at all by an unexpected change in money supply Δ but $(p_{3,t}/p_{3,t-1}) = (\phi_{t-1}/\phi^h) = \mu^h$ in a stationary equilibrium. Hence the first part of the right-hand side in (E.1) captures the fraction that the price is adjusted proportionally in response to Δ, while the second part of it captures the fraction that is independent of Δ. Since the first part of $\tilde{\mathbf{P}}_t$ is greater than that of $\mathbf{P}_t$ and the second part of $\tilde{\mathbf{P}}_t$ is less than that of $\mathbf{P}_t$, the variation of $\tilde{\mathbf{P}}_t$ is closer to μ^h than that of $\mathbf{P}_t$. Suppose now $q_{1,d} > 0$

in a No-Indexed-UoA economy. Then $\mathbf{P}_t$ and $\tilde{\mathbf{P}}_t$ can be expressed as

$$\mathbf{P}_t = \left(\frac{q_2 + q_3^*}{q_d^* + q_{1,m} + q_2 + q_3^*}\right) p_{3,t} + \left(\frac{q_{1,m} + q_d^*}{q_d^* + q_{1,m} + q_2 + q_3^*}\right) p_{1,t}$$
$$\tilde{\mathbf{P}}_t = \left(\frac{q_d^* + q_2 + q_3^*}{q_d^* + q_{1,m} + q_2 + q_3^*}\right) p_{3,t} + \left(\frac{q_{1,m}}{q_d^* + q_{1,m} + q_2 + q_3^*}\right) p_{1,t}$$

Exactly the same argument as the above implies the claim.

Appendix F. Proof of Proposition 3

The first claim is an obvious consequence of Proposition 1 and the tie-breaking rule assumed. That is, in an economy with $\bar{\mu} \in [\beta^*, \bar{\mu}^*]$, a seller will enter the socially costless money-posting submarket because it is indifferent for her to enter either the money-posting submarket or the Indexed-UoA-posting submarket. A buyer is then also willing to participate in the money-posting submarket because there is no entrance fee. Hence, the money-posting submarket is active and money is used as a unit of account for spot trades as well as credit trades. As regards the second claim, as shown in Proposition 1, a seller in an economy with $\bar{\mu} > \bar{\mu}^*$ is not willing to participate in the money-posting submarket. But if there is an Indexed-UoA, an expected revenue from a credit trade is enough to compensate for the relevant cost and hence a seller is willing to participate in the Indexed-UoA-posting submarket. For a buyer, the gain from a credit trade is $\upsilon(q_d^*)- q_d^*$ which is strictly positive and hence, if θ is less than $\bar{\theta} \equiv \upsilon(q_d^*)- q_d^*$, she is also willing to enter the Indexed-UoA-posting submarket by paying an entrance fee θ.

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Research Highlights

Inflation, Credit, and Indexed Unit of Account

- The nature of an indexed unit of account is investigated using a standard monetary model.
- An indexed unit of account turns out to resolve credit-trade friction attributed to inflation.
- Also, it removes the redistribution effect between debtors and creditors caused by unexpected inflation.
- If inflation is low enough, money is used as a unit of account for even deferred-payment trades.
- If inflation is high, an indexed unit of account emerges as a unit of account for deferred-payment trades.