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When ad valorem tax prevails in international tax competition

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ABSTRACT

The studies on capital tax competition have assumed that the governments compete for mobile capital in unit tax, and this assumption is partially justified by Lockwood (2004), which proves that unit tax competition is always welfare superior to ad valorem tax competition within a framework of symmetric tax competition. This paper presents the reexamination of governments' choice on tax method in the framework of asymmetric tax competition. The results show that asymmetric countries do not compete in the same tax instrument, as assumed in the literature. The capital importing countries compete in ad valorem tax, while the capital exporting countries compete in unit tax.

1. Introduction

In tax competition models, it has often been assumed that the governments compete for mobile capital in the unit tax. This assumption is justified by the equivalence property of the unit and ad valorem tax when the number of competing governments is sufficiently large. As the tax elasticity of capital is infinite when there are a sufficiently large number of countries, there exist no disparate impacts on equilibrium property between unit and ad valorem tax competition. Hence, assuming unit tax competition is a reasonable way for analytical tractability. Even in the case of tax competition among a small number of governments, a justification of unit-tax assumption has been offered by Lockwood (2004). He shows that unit tax competition is always welfare superior to ad valorem tax competition in symmetric tax competition, so that the assumption of unit tax competition is plausible. Extending this study, Akai, Ogawa, and Ogawa (2011) formally solves an endogenous choice problem of tax instruments and shows that selecting unit tax as a policy instrument is the dominant strategy of governments.

Departing from *symmetric* tax competition model, this paper reexamines the argument of Lockwood (2004) and Akai et al. (2011) and shows that ad valorem tax competition prevails in an *asymmetric* framework: if the number of countries competing for mobile capital is sufficiently small, and if the governments have some kind of regional asymmetries, such as in capital endowment and production technology, then one government competes in ad valorem tax and the other employs unit tax as a policy instrument. This result is quite a contrast to the preceding study of tax competition with regional asymmetries in which it has been assumed that different countries employ the same tax method, i.e., the unit tax (Bucovetsky, 1991, 2009 ; DePeter & Myers, 1994 ; Peralta & van Ypersele, 2005 ; Itaya et al., 2008; among others). Developing two-stage model of tax competition

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with regional asymmetries, this paper reexamines the tax method endogenously employed by asymmetric governments and clarifies the reason for the prevalence of tax competition with different tax methods. Although the results are derived under a specific functional form for analytical tractability, this result, at least, differs from known result and the conventional assumption in the literature.

The critical factor that leads us to obtain a different result from Lockwood (2004) and Akai et al. (2011) is that *asymmetric* countries have incentives to manipulate the terms of trade. They study the choice of tax instrument within a *symmetric* tax competition model, in which the terms of trade effect vanish altogether. In our model, the asymmetry among countries produces capital flow in the equilibrium. Hence, the capital importing country aims to lower the capital price so as to hold down the import cost, while the capital exporting country tries to raise the capital price to increase the income from capital exports. For an exactly contrary incentive to manipulate the capital price, the asymmetric countries employ a different tax instrument.

This paper is organized as follows. In the next section, we present a basic model. The equilibrium properties are presented to derive the main results in Section 3. Section 4 concludes the paper.

2. Model

There are two countries, and in each country i ($i = S, L$), there are homogeneous residents normalized by 1. The government in country i provides a public good that is solely financed by taxation on capital. Production of private goods in country i requires capital and labor. We assume that the production per capita in country i is conducted by the function $f(k_i) = (A - k_i)k_i$, where $A(> 0)$ is the parameter that represents the productive efficiency and k_i is the amount of capital per capita located in country i. A resident in country i has an initial endowment of capital, $\bar{k_i}$. We express the per capita capital endowments for countries S and L as $\bar{k_S} = \bar{k} - \epsilon$ and $\bar{k_L} = \bar{k} + \epsilon$, respectively, where $\epsilon \in (0, \bar{k})$ and $\bar{k} \equiv (\bar{k_S} + \bar{k_L})/2$. Capital is perfectly mobile among countries, while residents/workers are immobile.

The resident's preference in country i is given by

$$u(c_i, g_i) = c_i + g_i \tag{1}$$

defined over the consumption of a private numeraire good c_i and a public good g_i.[1] We employ the linear utility function because it can avoid the existence of the fiscal externality. This enables us to abstract the fundamental reason which makes governments compete in different tax methods. That is, by removing fiscal externality from the model, we can observe the role of the terms of trade effect in choosing the tax instrument as it happens.

The resident in country i receives labor income $f^i(k_i) - f^i_k(k_i)k_i (= k_i^2)$, and return from capital investment $r\bar{k_i}$. Hence, the budget constraint of the resident requires

$$c_i = k_i^2 + r\bar{k_i}. \tag{2}$$

To finance the public good, the government in each country can only use capital tax, but it can choose either the unit tax or ad valorem tax system. If government i selects the ad valorem tax method, the government budget constraint in country i is given by

$$g_i = \tau_i k_i f^i_k(k_i), \tag{3}$$

where $\tau_i (< 1)$ is the ad valorem tax rate. On the other hand, if the government imposes unit tax on mobile capital, the budget constraint of country i becomes

$$g_i = T_i k_i, \tag{4}$$

where T_i is the unit tax rate.

With two countries, there are four possible combinations of tax methods selected by the governments:

Case (i) both governments compete for mobile capital in the ad valorem tax;
Case (ii) both governments compete in the unit tax;
Case (iii) government S competes in the unit tax while government L competes in the ad valorem tax;
Case (iv) governments S and L compete in the ad valorem tax and unit tax, respectively.

[1] While the linear utility function is also used in Peralta and van Ypersele (2005), Itaya et al. (2008), Ogawa (2013), and Ogawa and Wang (2016), it might be plausible to denote g_i as the lump-sum public transfer to the resident since it may take a negative value. In this case, Eq. (1) represents the available income of the representative resident.

Table 1
Equilibrium values ($\Omega \equiv A - 2\bar{k}$).

	r	k_L	k_S
Case (i)	$\frac{2\Omega(1-\tau_L)(1-\tau_S)}{2-\tau_L-\tau_S}$	$\bar{k} - \frac{(\tau_L-\tau_S)\Omega}{2(2-\tau_L-\tau_S)}$	$\bar{k} - \frac{(\tau_S-\tau_L)\Omega}{2(2-\tau_L-\tau_S)}$
Case (ii)	$\Omega - \frac{T_L+T_S}{2}$	$\bar{k} - \frac{T_L-T_S}{4}$	$\bar{k} - \frac{T_S-T_L}{4}$
Case (iii)	$\frac{(1-\tau_L)(2\Omega-T_S)}{2-\tau_L}$	$\bar{k} - \frac{\Omega\tau_L-T_S}{2(2-\tau_L)}$	$\bar{k} - \frac{T_S-\Omega\tau_L}{2(2-\tau_L)}$
Case (iv)	$\frac{(1-\tau_S)(2\Omega-T_L)}{2-\tau_S}$	$\bar{k} - \frac{T_L-\Omega\tau_S}{2(2-\tau_S)}$	$\bar{k} - \frac{\Omega\tau_S-T_L}{2(2-\tau_S)}$

In Section 3, we analyze the four cases in turn and clarify the equilibrium properties.

Since capital is perfectly mobile among countries, in the equilibrium, the net (after tax) return to capital will be equalized across the countries, i.e., $r = f_k^i(k_i) - T_i$ if government i adopts a system of unit tax, and $r = (1-\tau_i)f_k^i(k_i)$ if it selects the ad valorem tax, where r is the price of capital. Under the quadratic technology, the marginal product of capital is given by $f_k^i(k_i) = A - 2k_i$, where $A - 2k_i > 0$ is assumed.

3. Equilibrium

Let us consider a simple two-stage tax competition model in which the issue of the tax method endogenously chosen by governments can be addressed. In our two-stage game model, the governments choose either the unit tax or ad valorem tax method in the first stage. In the second stage, they choose the tax rate of the tax instrument selected in the former stage.[2]

3.1. Capital market

Under the specification, the capital allocation in the equilibrium satisfies

$$2\bar{k} = k_L + k_S \tag{5}$$

and one of the following equations:

$$\text{Case (i)} \quad r = (A - 2k_L)(1 - \tau_L) = (A - 2k_S)(1 - \tau_S), \tag{6}$$

$$\text{Case (ii)} \quad r = A - 2k_L - T_L = A - 2k_S - T_S, \tag{7}$$

$$\text{Case (iii)} \quad r = (A - 2k_L)(1 - \tau_L) = A - 2k_S - T_S, \tag{8}$$

$$\text{Case (iv)} \quad r = A - 2k_L - T_L = (A - 2k_S)(1 - \tau_S). \tag{9}$$

Using Eqs. (5)–(9), we have the price and the amount of capital as a function of tax rates, which are summarized in Table 1.

3.2. Choice on tax rate

We start our analysis with the second stage and examine four possible equilibria in order. Let us begin with the case where two governments adopt a system of ad valorem tax.

Case (i) (Both governments compete in the ad valorem tax). We refer to this case as the *AA* case. In this case, by substituting Eqs. (2) and (3) into Eq. (1) and using $f_k^i(k_i) = A - 2k_i$, government i's ($i = S, L$) problem at the beginning of the second stage is to maximize $u_i = (A - k_i)k_i + r(\bar{k}_i - k_i)$, with respect to τ_i. The objective function reveals that the capital importing country ($\bar{k}_i - k_i < 0$) has an incentive to reduce the capital price r, while the capital exporting country ($\bar{k}_i - k_i > 0$) benefits if r increases. By taking account the tax effects on capital allocation and price, which can be obtained by Eqs. (5) and (6), as $\partial k_i/\partial \tau_i = -\Omega(1-\tau_j)/(2-\tau_i-\tau_j)^2 < 0$ and $\partial r/\partial \tau_i = -2\Omega(1-\tau_j)^2/(2-\tau_i-\tau_j)^2 < 0$ where $\Omega \equiv A - 2\bar{k}$, the first-order conditions are obtain as follows:

$$\tau_L = \frac{\tau_S(\Omega + 2\epsilon) - 4\epsilon}{3\Omega - 2\epsilon}, \tag{10}$$

[2] The tax competition model with a two stage-game was first presented by Wildasin (1991) and extended by Persson and Tabellini (1992), Bayindir-Upmann (1998), Hindriks, Peralta, and Weber (2008), Akai et al. (2011), and Kawachi, Ogawa, and Susa (2015).

$$\tau_S = \frac{\tau_L(\Omega - 2\epsilon) + 4\epsilon}{3\Omega + 2\epsilon}. \tag{11}$$

To ensure strategic complementarity in tax competition, we make the following assumption.

Assumption 1. $\Omega > 2\epsilon$.

Eqs. (10) and (11) give us the stable equilibrium ad valorem tax rates as

$$\tau_L^{AA} = -\frac{\epsilon}{\Omega} < 0, \tau_S^{AA} = \frac{\epsilon}{\Omega} > 0. \tag{12}$$

Using Eqs. (5), (6) and (12), we have the equilibrium values as

$$k_L^{AA} = \bar{k} + \frac{\epsilon}{2}, k_S^{AA} = \bar{k} - \frac{\epsilon}{2}, \quad r^{AA} = \frac{(\Omega - \epsilon)(\Omega + \epsilon)}{\Omega} > 0. \tag{13}$$

From Eq. (13), we find that country L exports capital $\bar{k_L} - k_L^{AA} = \epsilon/2 > 0$, while country S imports it, $\bar{k_S} - k_S^{AA} = -\epsilon/2 < 0$.

Case (ii) (Both governments compete in the unit tax). We call this case the *UU* case. The government i's ($i = S, L$) problem at the beginning of the second stage is to maximize $u_i = (A - k_i)k_i + r(\bar{k_i} - k_i)$, with respect to T_i. Tax effects can be derived by Eqs. (5) and (7) as $\partial k_i/\partial T_i = -1/4 < 0$ and $\partial r/\partial T_i = -1/2 < 0$. Taking into account these effects, the first-order conditions are obtained as

$$T_L = \frac{T_S - 4\epsilon}{3}, T_S = \frac{T_L + 4\epsilon}{3}, \tag{14}$$

which give us the equilibrium unit tax rates as

$$T_L^{UU} = -\epsilon, T_S^{UU} = \epsilon. \tag{15}$$

Here we touch on the equivalent translation between the ad valorem tax and unit tax.[3] Assume that country i employs ad valorem tax τ_i. We can define the effective unit tax rate for country i as

$$T_i = \tau_i f_k^i(k_i). \tag{16}$$

by replacing the ad valorem tax with unit tax. This *equivalent unit tax* changes neither the capital price nor the government's tax revenue. Correspondingly, we can define the equivalent ad valorem tax rate for country i as $\tau_i = T_i/f_k^i(k_i)$ if country i employs the unit tax. Hence, a simple conversion of Eq. (14) gives the reaction function expressed by the converted (ad valorem) tax rate as follows.

$$\tau_L = \frac{(\Omega + 2\epsilon)\tau_S - 4\epsilon}{3\Omega - 2\epsilon - 2\Omega\tau_S}, \tag{17}$$

$$\tau_S = \frac{(\Omega - 2\epsilon)\tau_L + 4\epsilon}{3\Omega + 2\epsilon - 2\Omega\tau_L}. \tag{18}$$

From Eqs. (17) and (18), we confirm that we can depict the upward-sloping reaction curves; $\partial\tau_L/\partial\tau_S = (\Omega - 2\epsilon)(3\Omega + 2\epsilon)/(3\Omega - 2\epsilon - 2\Omega\tau_S)^2 > 0$, $\partial\tau_S/\partial\tau_L = (\Omega + 2\epsilon)(3\Omega - 2\epsilon)/(3\Omega + 2\epsilon - 2\Omega\tau_L)^2 > 0$. Solving Eqs. (17) and (18), when both governments compete in unit tax, the significant tax rate measured by the ad valorem tax is given by

$$\tau_L^{UU} = -\frac{\epsilon}{\Omega - \epsilon} < 0, \quad \tau_S^{UU} = \frac{\epsilon}{\Omega + \epsilon} > 0. \tag{19}$$

Using Eqs. (5), (7) and (15), or equivalently Eqs. (5), (6) and (19), we have the equilibrium values as

$$k_L^{UU} = \bar{k} + \frac{\epsilon}{2}, k_S^{UU} = \bar{k} - \frac{\epsilon}{2}, r^{UU} = \Omega. \tag{20}$$

At this stage, we obtain our first result, which can be contrasted with that of Lockwood (2004) and Akai et al. (2011).

[3] See, for instance, Salanié (2002, p.21) and Lockwood (2004).

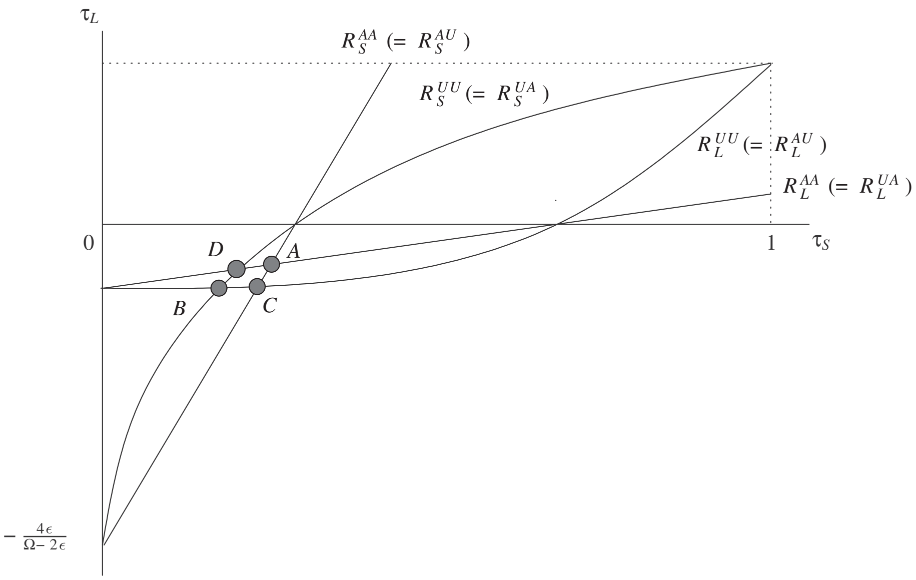

Fig. 1. Reaction curves. Note. *A* and *B* represent the equilibrium when both governments compete in ad valorem tax and unit tax, respectively. *C* represents the equilibrium when country *L* competes in ad valorem tax while country *S* competes in unit tax, and *D* represents the equilibrium when country *L* competes in unit tax and country *S* competes in ad valorem tax.

Proposition 1. *If both countries use the same tax method, capital-exporting country L has a preference for unit tax competition, while capital importing country S prefers ad valorem tax competition.*

Proof. Eqs. (13) and (20) reveal that $k_i^{AA} = k_i^{UU}$ and $r^{AA} < r^{UU}$. The equilibrium utility in tax competition is given by $u_i^{jj} = (A - k_i^{jj})k_i^{jj} + r^{jj}(\bar{k}_i - k_i^{jj})$, where $j = A$ denotes the case (i) and $j = U$ denotes the case (ii). Then, we have $u_L^{UU} > u_L^{AA}$ and $u_S^{AA} > u_S^{UU}$ since $r^{AA} < r^{UU}$, $\bar{k}_L > k_L^{jj}$, and $\bar{k}_S < k_S^{jj}$ for $j = A, U$.

While Lockwood (2004) argues that unit tax competition yields higher utility than ad valorem tax competition for two governments in the symmetric equilibrium, Proposition 1 shows that one of the two countries prefers to compete in unit tax, but the other prefers to compete in ad valorem tax.[4]

In Fig. 1, the reaction curves are depicted. A straight line labeled R_i^{AA} is the reaction curve of country *i* when it competes in ad valorem tax. An intersecting point *A* represents a Nash equilibrium, where $(\tau_L^{AA}, \tau_S^{AA}) = (-\epsilon/\Omega, \epsilon/\Omega)$. A curved line labeled R_i^{UU} represents the country *i*'s reaction curve expressed by the ad valorem tax rate when it competes in unit tax. An intersecting point *B* denotes the equilibrium, where $(\tau_L^{UU}, \tau_S^{UU}) = (-\epsilon/(\Omega - \epsilon), \epsilon/(\Omega + \epsilon))$. Comparing the point *A* with *B*, Proposition 1 indicates that country *L* prefers *B*, since it is capital exporter so that it prefers lower tax rate which results in higher price of capital in the equilibrium. By contrast, country *S* prefers the point *A* since it is capital importer so that it prefers lower capital price which will be achieved when countries choose higher tax rate.

Next, we derive the equilibrium outcome when the governments select different tax methods.

Case (iii) (Country *L* competes in the ad valorem tax, while country *S* competes in the unit tax). We denote this case as *AU* case. In this case, government *i*'s ($i = S, L$) problem at the beginning of the second stage is to maximize $u_i = (A - k_i)k_i + r(\bar{k}_i - k_i)$, subject to Eqs. (5) and (8). The instrument variable of country *L* is the ad valorem tax rate τ_L, and the variable of country *S* is the unit tax rate T_S. The first-order conditions are $\tau_L = (T_S - 4\epsilon)/(3\Omega - 2\epsilon - T_S)$ and $T_S = (1 - \tau_L)(4\epsilon + \Omega\tau_L - 2\epsilon\tau_L)/(3 - 2\tau_L)$. With the same procedure in case (ii), using Eqs. (16), the reaction function expressed by the converted (ad valorem) tax rate are obtained as

$$\tau_L = \frac{(\Omega + 2\epsilon)\tau_S - 4\epsilon}{3\Omega - 2\epsilon - 2\Omega\tau_S}, \tag{21}$$

$$\tau_S = \frac{\tau_L(\Omega - 2\epsilon) + 4\epsilon}{3\Omega + 2\epsilon}. \tag{22}$$

[4] As we assume the linear utility function, no fiscal externality arises in our model. Thus, governments are indifferent between unit tax and ad valorem tax when $\epsilon = 0$.

Notice that Eq. (21) coincides with Eq. (17), indicating that while country L tries to control capital price r by way of the tax rate, the effective tax rate chosen by country L does not differ between the ad valorem and unit tax as long as country S uses the unit tax. Since Eq. (22) is identical to Eq. (11), the same argument applies to country S's choice on tax rate when country L employs the ad valorem tax. Hence, in case (iii), the reaction curve of country L, R_L^{AU}, corresponds to R_L^{UU} and the reaction curve of country S, R_S^{UA}, coincides with R_S^{AA} in Fig. 1, where a point C represents the equilibrium in this case.

Solving Eqs. (21) and (22), the (ad valorem) tax rates when country L competes in ad valorem tax and country S competes in unit tax are given by

$$\tau_L^{AU} = \frac{2(\Omega - F - \epsilon)}{\Omega - 2\epsilon} < 0, \tau_S^{AU} = \frac{2(\Omega - F + \epsilon)}{3\Omega + 2\epsilon} > 0, \tag{23}$$

where $\Omega > F \equiv \sqrt{\Omega^2 - \epsilon(\Omega + \epsilon)}$.[5] The superscript AU denotes that country L competes in ad valorem tax while country S competes in unit tax. Using Eqs. (5), (6) and (23), we have the equilibrium values as $k_L^{AU} = k_L(\Omega, \epsilon)$, $k_S^{AU} = k_S(\Omega, \epsilon)$, and $r^{AU} = r(\Omega, \epsilon)$ (see Appendix A).

Case (iv) (Country L competes in the unit tax while country S competes in the ad valorem tax). This case is denoted by UA case, in which government i's ($i = S, L$) problem at the beginning of the second stage is to maximize $u_i = (A - k_i)k_i + r(\bar{k}_i - k_i)$, subject to Eqs. (5) and (9). The policy variable of country L is the unit tax T_L, and the variable of country S is the ad valorem tax τ_S. The first-order conditions are $T_L = (1 - \tau_S)(\Omega\tau_S + 2\epsilon\tau_S - 4\epsilon)/(3 - 2\tau_S)$ and $\tau_S = (4\epsilon + T_L)/(3\Omega + 2\epsilon - T_L)$. Using Eq. (16), the reaction function is expressed by the converted (ad valorem) tax rate as

$$\tau_L = \frac{(\Omega + 2\epsilon)\tau_S - 4\epsilon}{3\Omega - 2\epsilon}, \tag{24}$$

$$\tau_S = \frac{(\Omega - 2\epsilon)\tau_L + 4\epsilon}{3\Omega + 2\epsilon - 2\Omega\tau_L}. \tag{25}$$

We find that Eqs. (24) and (25) are identical to Eqs. (10) and (18), respectively. This indicates that while country L tries to control capital price r by way of the tax rate, the effective tax rate makes no difference whether country L employs ad valorem or unit tax as long as country S uses ad valorem tax. The same argument applies to country S's choice on tax rate when country L employs the unit tax. Hence, in Fig. 1, the reaction curve of country L, R_L^{UA}, and that of country S, R_S^{UA}, accords with R_L^{AA} and R_S^{UU}, respectively.

Solving Eqs. (24) and (25), the (ad valorem) tax rates when country L uses the unit tax and country S uses the ad valorem tax are given by

$$\tau_L^{UA} = -\frac{2(\epsilon + E - \Omega)}{3\Omega - 2\epsilon} < 0, \tau_S^{UA} = \frac{2(\epsilon + \Omega - E)}{\Omega + 2\epsilon} > 0, \tag{26}$$

where $E \equiv \sqrt{\Omega^2 + \epsilon(\Omega - \epsilon)} > \Omega$.[6] The equilibrium in case (iv) is depicted as a point D in Fig. 1. Furthermore, using Eqs. (5), (6) and (26), we have the equilibrium values as $k_L^{UA} = k_L(\Omega, \epsilon)$, $k_S^{UA} = k_L(\Omega, \epsilon)$, and $r^{UA} = r(\Omega, \epsilon)$ (see Appendix B).

3.3. Choice on tax method

In the first stage, there are four candidates for equilibrium. Let us list the payoff of both countries in each case as a matrix table (Table 2). By comparing the utility levels, we first obtain the following result when countries differ in the initial capital endowment, $\epsilon > 0$.

Lemma 1. *Selecting the ad valorem tax method is a dominant strategy for country S.*

Proof. We have $u_S^{AU} < u_S^{AA}$ and $u_S^{UU} < u_S^{UA}$. See Appendix C.

The intuitive explanation on Lemma 1 can be obtained by considering the incentive to increase or decrease the capital price in two countries: Since capital-rich country L is a capital exporter, it tries to raise capital price r, which brings country L a higher capital income, while capital-poor country S is a net capital importer, and therefore it prefers low capital price to save the cost of capital borrowing. To manipulate the capital price, country S (L) tries to induce country L (S) to set a high (low) tax rate. Lemma 1

[5] Another solution that satisfies Eq. (23) is inconsistent with the assumption that $\tau_i < 1$. Under Assumption 1, $\Omega^2 - \epsilon(\Omega + \epsilon) > 0$ is ensured, and $\Omega - F - \epsilon < 0$ holds since $(\Omega - \epsilon + F)(\Omega - \epsilon - F) = 2\epsilon - \Omega < 0$.

[6] $\epsilon + \Omega - E > 0$ holds since $(\epsilon + \Omega - E)(\epsilon + \Omega + E) = \epsilon(\Omega + 2\epsilon) > 0$.

Table 2
Payoff matrix.

	Country S chooses U	Country S chooses A
Country L chooses U	u_L^{UU}, u_S^{UU}	u_L^{UA}, u_S^{UA}
Country L chooses A	u_L^{AU}, u_S^{AU}	u_L^{AA}, u_S^{AA}

shows that, to induce country L to set a high tax rate, an effective strategy for country S would be to adopt a system of ad valorem tax. This is because any outflow of capital from country L induced by the increased tax in country L will lower the effective unit tax in country S, which is favorable for country L. This suggests that country L easily raises its tax rate, which produces favorable situation for country S.[7] The reduction of effective unit tax rate in country S is arrived at as follows (Lockwood, 2004; Akai et al., 2011). Since the effective (unit) tax rate of country S is given by Eq. (16), $T_S = \tau_S f_k^S(k_S)$, the tax increase in country L increases k_S and decreases $f_k^S(k_S)$, which lowers the effective tax rate of country S, given τ_S is fixed. The reduction of tax rate in country S is favorable for country L, resulting in country L choosing a higher tax rate. As country S tries to increase the tax rate of country L, and owing to the presence of this effect that gives country L an incentive to increase its tax rate, country S commits itself to employing ad valorem tax.

Now, we find the Sub-game perfect Nash equilibrium. To find it, we study the choice of tax method for country L, given country S selects the ad valorem tax. The comparison of utilities gives the main result.

Proposition 2. *The unique sub-game perfect equilibrium consists of country L, which competes for mobile capital by way of the unit tax and country S, using the ad valorem tax.*

Proof. We have $u_L^{UA} > u_L^{AA}$. See Appendix C.

The reason for country L choosing unit tax, given country S chooses the ad valorem tax, can be deduced similarly. Assume that country L employs the ad valorem tax. In this case, country S understands that, as it increases its tax rate, the capital in country L increases, which reduces the effective (unit) tax rate in country L, following $T_L = \tau_L f_k^L(k_L)$. The decrease in the effective tax rate in country L increases the capital price, which is favorable for country S, so that country S tends to choose a higher tax rate. Since country L wants country S to choose a lower tax rate, it is not a good strategy for country L to employ the ad valorem tax, and hence, it selects the unit tax.

To give a more detailed explanation, let us provide an alternative expression that coincides with the above intuitive explanation, using Fig. 1, where the ad valorem tax rate of country i, $\tau_i \in (0, 1)$, is taken as the axis. In Fig. 1, R_i^{AA} represents the reaction curve when both governments compete in the ad valorem tax. The stable equilibrium is given by point A. Point B shows the (converted) ad valorem tax rates in the equilibrium when both governments employ the unit tax. Furthermore, government i's reaction function when country L adopts a system of ad valorem tax and country S selects the unit tax is plotted by the curve R_i^{AU}. Symmetrically, curve R_i^{UA} represents government i's reaction curve when country L adopts a system of unit tax, while country S selects the ad valorem tax. Points C and D are the equilibria obtained in the tax competition in which one government employs the unit tax while the other government employs the ad valorem tax.

Choosing the tax method in the first stage is equivalent to choosing the reaction curve faced by the other country in the second stage. As country L wants to make country S lower the tax rate, focusing on the third quadrant, it is more favorable for country L if country S faces the reaction curve $R_S^{UU} (= R_S^{UA})$. Country L ensures this environment if it commits to employ the unit tax in the first-stage. By contrast, as country S wants to make country L choose a higher tax rate, it wants country L to face the reaction curve represented by $R_L^{AA} (= R_L^{UA})$. To realize this environment, in the first stage, country S commits to the ad valorem tax.

4. Concluding remarks

We attempt to reexamine the conventional assumption on the tax method employed in the tax competition model. It has been assumed that all governments compete in the unit tax even though they have different characteristics in technology and capital endowments. In contrast to the traditional argument, the main result of our study perhaps sheds a new light on the choice on tax instrument in tax competition analysis. In an asymmetric tax competition model, the best choice for capital importing countries could be to use the unit tax, but the best tax instrument for capital importing countries is the ad valorem tax. Our result can provide an alternative explanation as to why countries, in fact, employs the ad valorem tax method in a tax competition environment. If countries are net capital importers, it is the best strategy for them to employ ad valorem tax, so that ad valorem tax competition prevails.

[7] In this case, country L faces with the larger elasticity of capital stock with respect to its tax rate since additional capital outflow occurs in association with lowered effective tax rate in country S.

Finally, we touch on the robustness of our result. First, the choice of tax method is effective only when the number of countries competing for mobile capital is small. If the number of countries is sufficiently large, each country's market power to manipulate the capital price becomes negligible, indicating that our results are limited to the tax competition with market power. Second, to figure out our main argument, we develop the model without fiscal externality. The existence of fiscal externality, which can be studied in our model by considering the cost of public funds, leads governments to choose lower tax rate, and thereby the unit tax method, as shown in Lockwood (2004) and Akai et al. (2011).[8] Thus, from our result, it is easily presumed that if the effect of fiscal externality outweighs the effect of terms of trade, our result is violated; the capital importer uses the unit tax as with the capital exporter does. By contrast, if the effect of fiscal externality is sufficiently small and is outweighed by the effect of terms of trade, our result still preserves.

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Appendix A

Using Eqs. (5), (6) and (23), we have

$$k_L^{AU} = \bar{k} + \frac{\Omega(\Omega - F) - 2F\epsilon}{\Omega - 4F + 2\epsilon}, k_S^{AU} = \bar{k} - \frac{\Omega(\Omega - F) - 2F\epsilon}{\Omega - 4F + 2\epsilon}, \quad \text{and} \quad r^{AU} = \frac{(2F + \Omega)(2F - \Omega)}{(2F - \Omega) + 2(F - \epsilon)} > 0.$$

We can easily confirm that country L exports and country S imports capital. The net capital flow in country L is $k_L^{AU} - \bar{k}_L = (\Omega - 2\epsilon)(\Omega - F + \epsilon)/(\Omega + 2\epsilon - 4F)$. Under Assumption 1, the numerator is positive. Furthermore, since $(\Omega + 2\epsilon - 4F)(\Omega + 2\epsilon + 4F) = -5(\Omega - 2\epsilon)(3\Omega + 2\epsilon) < 0$, the denominator is negative, indicating $k_L^{AU} < \bar{k}_L$ and $k_S^{AU} > \bar{k}_S$. Furthermore, $r^{AU} > 0$ holds since $2F > \Omega$ and $F - \epsilon > 0$ under Assumption 1;$F > \epsilon$ holds because, by raising to the second power and subtracting ϵ^2 from both sides of $F \equiv (\Omega^2 - \epsilon(\Omega + \epsilon))^{1/2}$, we have $(F - \epsilon)(F + \epsilon) = (\Omega - 2\epsilon)(\Omega + \epsilon) > 0$. Furthermore, $2F > \Omega$ holds under Assumption 1 since $(2F + \Omega)(2F - \Omega) = (3\Omega + 2\epsilon)(\Omega - 2\epsilon) > 0$.

Appendix B

Using Eqs. (5), (6) and (26), we have

$$k_L^{UA} = \bar{k} + \frac{2\epsilon E - \Omega\,(E - \Omega)}{(E - \Omega) + 2\epsilon + 3E}, k_S^{UA} = \bar{k} - \frac{2\epsilon E - \Omega\,(E - \Omega)}{(E - \Omega) + 2\epsilon + 3E}, \quad \text{and} \quad r^{UA} = \frac{(2E + \Omega)\,(2E - \Omega)}{(E - \Omega) + 2\epsilon + 3E} > 0.$$

We can confirm that country L exports and country S imports capital since the net capital flow in country L is $k_L^{UA} - \bar{k}_L = (\Omega + 2\epsilon)(E - \Omega + \epsilon)/((\Omega - E) - 2\epsilon - 3E) < 0$, indicating $k_L^{UA} < \bar{k}_L$ and $k_S^{UA} > \bar{k}_S$.

Appendix C

Using the equilibrium values, the utility level in each case can be obtained as follows.

$$u_L^{AA} = \bar{k}^2 + \Omega\left(\bar{k} + \epsilon\right) - \frac{\epsilon^2}{4} - \frac{\epsilon^3}{2\Omega}, \tag{27}$$

$$u_S^{AA} = \bar{k}^2 + \Omega\left(\bar{k} - \epsilon\right) - \frac{\epsilon^2}{4} + \frac{\epsilon^3}{2\Omega}, \tag{28}$$

$$u_L^{UU} = \bar{k}^2 + \Omega(\bar{k} + \epsilon) - \frac{\epsilon^2}{4}, \tag{29}$$

$$u_S^{UU} = \bar{k}^2 + \Omega(\bar{k} - \epsilon) - \frac{\epsilon^2}{4}, \tag{30}$$

$$u_L^{AU} = \bar{k}^2 + \Omega(\bar{k} + \epsilon) - \frac{2F[3(\Omega + \epsilon)(\Omega - 2\epsilon)^2 - 4\epsilon^2(\Omega + 2\epsilon)] + 6\Omega^4 - 21\Omega^3\epsilon - 5\Omega^2\epsilon^2 + 24\Omega\epsilon^3 + 12\epsilon^4}{(4F - \Omega - 2\epsilon)^2}, \tag{31}$$

[8] See Bucovetsky (2009) and Keen and Konrad (2013) for other variants of the linear models which account for the cost of public funds.

$$u_S^{AU} = \bar{k}^2 + \Omega(\bar{k} - \epsilon) + \frac{(\Omega - \epsilon)\left(2\Omega^2 - \Omega\epsilon - 2F\Omega - 2\epsilon^2\right)}{4F - \Omega - 2\epsilon}, \tag{32}$$

$$u_L^{UA} = \bar{k}^2 + \Omega(\bar{k} + \epsilon) + \frac{(\Omega + \epsilon)\left(2\Omega^2 + \Omega\epsilon - 2E\Omega - 2\epsilon^2\right)}{4E - \Omega + 2\epsilon}, \tag{33}$$

$$u_S^{UA} = \bar{k}^2 + \Omega(\bar{k} - \epsilon) - \frac{2E[3(\Omega - \epsilon)(\Omega + 2\epsilon)^2 - 4\epsilon^2(\Omega - 2\epsilon)] + 6\Omega^4 + 21\Omega^3\epsilon - 5\Omega^2\epsilon^2 - 24\Omega\epsilon^3 + 12\epsilon^4}{(4E - \Omega + 2\epsilon)^2}. \tag{34}$$

Proof of $u_S^{AA} > u_S^{AU}$. The comparison of Eqs. (28) and (32) gives

$$u_S^{AA} \gtrless u_S^{AU} \Longleftrightarrow 4F\left((\Omega - 2\epsilon)\left(2\Omega^2 + 3\epsilon^2 + 2\Omega\epsilon\right) + 8\epsilon^3\right) \gtrless 8\Omega^4 - 12\Omega^3\epsilon - 5\Omega^2\epsilon^2 + 8\Omega\epsilon^3 + 4\epsilon^4$$

The both sides are multiplied by itself, and using $F \equiv \sqrt{\Omega^2 - \epsilon(\Omega + \epsilon)}$, to rewrite

$$u_S^{AA} \gtrless u_S^{AU} \Longleftrightarrow \epsilon^3(3\Omega + 2\epsilon)^2\left(8\Omega^3 - 33\Omega^2\epsilon + 44\Omega\epsilon^2 - 20\epsilon^3\right) \gtrless 0$$

Since $8\Omega^3 - 33\Omega^2\epsilon + 44\Omega\epsilon^2 - 20\epsilon^3 = (\Omega - 2\epsilon)[8(\Omega - 2\epsilon)^2 + 15\epsilon(\Omega - 2\epsilon) + 8\epsilon^2] > 0$ under Assumption 1, we have $u_S^{AA} > u_S^{AU}$.

Proof of $u_S^{UA} > u_S^{UU}$. The proof can be made in a parallel manner. The comparison of Eqs. (30) and (34) gives

$$u_S^{UA} \gtrless u_S^{UU} \Longleftrightarrow 9\epsilon^3\,(8\Omega + 25\epsilon)\,(3\Omega - 2\epsilon)^2(\Omega + 2\epsilon)^2 \gtrless 0.$$

Hence, $u_S^{UA} > u_S^{UU}$.

Proof of $u_L^{UA} > u_L^{AA}$. The comparison of Eqs. (27) and (33) gives

$$u_L^{UA} \gtrless u_L^{AA} \Longleftrightarrow \epsilon^3(3\Omega - 2\epsilon)^2\left(8\Omega^3 + 33\Omega^2\epsilon + 44\Omega\epsilon^2 + 20\epsilon^3\right) \gtrless 0.$$

Hence, $u_L^{UA} > u_L^{AA}$.

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