International Review of Economics and Finance 49 (2017) 58–68

Contents lists available at ScienceDirect

International Review of Economics and Finance

journal homepage: www.elsevier.com/locate/iref

Mixed duopoly with foreign firm and subcontracting☆

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ARTICLE INFO

JEL classification:
L13
L32
L33

Keywords:
Mixed duopoly
Subcontracting
Foreign firm

ABSTRACT

We consider a Hotelling model, in which a public firm competes with a foreign firm, at the mean time cooperates with it through subcontracting. We find that when there exists subcontracting, the presence of a foreign firm raises social welfare. Comparing to competing with the domestic private firm, when the public firm competes with the foreign firm, social welfare is lower, but consumer welfare is higher. And a variation in firms' costs or tariffs has no effect on either firm's location. Tariff on inputs raises domestic social welfare and government will charge an input tariff to the extent of costs difference. Tariff on final good has no effect on welfare but will raise prices and thus hurt consumers. Compared to no tariff at all, tariff on inputs alone raises retail prices, but tariffs on both inputs and final goods may reduce prices and raise consumer surplus.

1. Introduction

Due to the development of economies, relationships among different firms within the same industry are not restricted to competition anymore. They may also cooperate with each other. One important way of cooperation is subcontracting.[1] Subcontracting not only exists among private firms within the same country, but also exists among firms from different countries with different ownership. For example, First Automobile Works (FAW), a leading public firm in automobile industry in China, uses Toyota's engine VVT-i for its Hongqi HQ3, which is the engine used for Toyota's Crown Majesta at the mean time[2]. Chery Automobile Co. Ltd, also a public automobile enterprise in China, uses Mini-Cooper's engine Tritec for its Chery Amulet.[3]

As of today, subcontracting between a public firm and a foreign firm is very popular in developing and transitional economies. But to our best knowledge, there has been no literature on this subject yet, leaving several interesting questions to be answered: what is the welfare implication in presence of a public firm cooperating with a foreign firm? What is the effect of the cooperation on firms' strategies. What is the optimal tariffs and their effects on welfare?

To address these questions, we discuss a mixed duopoly model à la Hotelling, in which one public firm competes against a foreign firm and cooperates with it through subcontracting at the same time. We find that first, in consideration of subcontracting, the presence of foreign firm raises social welfare. This is in contrast to the case when there is no subcontracting, under which social

☆ We wish to thank the editor Hamid Beladi, and two anonymous referees for helpful comments and suggestions. Financial support from the National Science Foundation of China (71603283) is gratefully acknowledged. The usual disclaimers apply.

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[1] For some reasons such as technology advantages, cost advantages etc., one firm may subcontract part of production to its competitors. See Spiegel (1993) for examples. There are also other ways of cooperation, see for example (Liu, Mukherjee, and Wang, 2016).

[2] Technical parameters of Hongqi HQ3 and Crown Mejesta. Retrieved February 18, 2013 from http://www.faw-hongqi.com.cn/http://www.faw-hongqi.com.cn/http://www.faw-hongqi.com.cn/ and http://toyota.jp/crownmajesta/dynamism/engine/index.html

[3] Jingyi Ma (August 19, 2012), Origin of Chery and Mini Sharing Tritec.

http://dx.doi.org/10.1016/j.iref.2017.01.006
Received 13 September 2016; Received in revised form 3 January 2017; Accepted 10 January 2017
Available online 21 January 2017

welfare is unaffected by the foreign firm. Comparing to competing with a domestic private firm, when a public firm competes with a foreign firm, social welfare is lower but consumer welfare is higher. Second, the public firm is less aggressive in location if there is subcontracting. And comparing to competing with a domestic private firm, when competing with a foreign firm, the public firm is more aggressive in its location and pricing choice. It locates close to the center of the market while the foreign firm locates close to the end. Both firms' locations and market shares are independent of variation in tariffs or production costs. Third, tariff on intermediate good increases social welfare. Tariff on final good has no effect on social welfare but lowers consumer welfare. Tariff on both intermediate and final goods may actually lower retail prices and raise consumer welfare comparing to no tariff at all. This is in sharp contrast to existing studies (e.g. Chao and Yu (2006) and Wang, Wang, and Lee (2010)) which usually find that tariffs hurt consumers.

Our paper is related to the literature on mixed oligopoly markets. This strand of literature can be traced back to at least (Merrill and Schneider, 1966). Since then, many economists have contributed to different aspects of this field. Some literature focuses on the regulating role of the public firm in mixed oligopoly markets (e.g. Harris and Wiens (1980) and Cremer, Marchand, and Thisse (1991), Matsushima and Matsumura (2003), Li (2006)), some focuses on the privatization (e.g. Anderson, de Palma, and Thisse (1997) and Matsumura and Matsushima (2004)) or partial privatization (e.g. George and La Manna (1996), Matsumura (1998) and Kumar and Saha (2008)) of a public firm. Particularly, in a Hotelling model, Cremer et al. (1991) examine one public firm competes with $n-1$ domestic private firm and find that when $n=2$, the existence of the public firm can guarantee the first best equilibrium. Thus privatization of the public firm will hurt social welfare in such setup. This result is reversed in Matsumura and Matsushima (2004). In a mixed duopoly Hotelling model with endogenous cost differential through firms' cost reducing R & D, they find that privatization improves social welfare by mitigating the loss arising from excessive cost reduction investments. The key difference between our paper and the aforementioned two papers are the nationality of the private firm. The private firm(s) is domestic in their papers but foreign in ours. In the literature on mixed oligopoly markets with foreign firm(s) (e.g. Matsumura, Matsumura, and Ishibashi (2009), Pal and White (1998), Chao and Yu (2006), Wang et al. (2010), Fjell and Pal (1996)); Heywood and Ye (2009) use Hotelling type model to examine spatial price discrimination in presence of a foreign firm. Our paper uses a Hotelling type model to examine the mixed market with foreign firm too, but is different from theirs in several aspects. First, firms use a delivered price to discriminate consumers according to their locations in their model, while we use mill price and there is no price discrimination. Second, we focus on the subcontracting relationship, and provide formal analysis for the subcontracting and tariff.

Our paper is also related to the literature on subcontracting (e.g. Kamien, Li, and Samet (1989), Spiegel (1993), Arya, Mittendorf, and Sappington (2008) and Andaluz (2009)). Among these studies, the most related one is Liang and Mai (2006). They develop a variant of Hotelling's model with subcontracting to examine the principle of minimum differentiation. In their model, two spatially competing firms have different marginal costs, and the high cost firm can subcontract its input to the low cost firm. They find that if the ratio of transport rates between the subcontracted input and the final product is sufficiently large, the principle of minimum differentiation arises, otherwise the principle of maximum differentiation arises.

The rest of the paper proceeds as follows. Basic model is presented in Section 2 and analyzed in Section 3. Section 4 contains the concluding remarks. Proofs of Lemmas and Propositions are presented in Appendix A.

2. The model

Following the Hotelling model, we assume that there are two firms (1 and 2), each chooses a location $x_i (i = 1, 2)$ for its final product over an interval $[0, 1]$. Without loss of generality, we assume that $x_1 \le x_2$. There is a unit mass of consumers, whose preferences are uniformly distributed over the interval $[0, 1]$. Each consumer is assumed to buy exactly one unit of final product from either firm, and her utility loss of buying a not perfectly matched product (transport cost) takes a quadratic form. For example, consider a generic consumer located at x, her utility of buying from firm 1 and 2 are respectively:

$$u_1 = V - p_1 - t(x - x_1)^2, \quad u_2 = V - p_2 - t(x - x_2)^2,$$

where t is transport cost coefficient, $p_i (i = 1, 2)$ is firm i's price, and V is consumers' reservation price. It is readily shown the marginal consumer's location is:

$$\hat{x} = \frac{p_2 - p_1}{2t(x_2 - x_1)} + \frac{x_1 + x_2}{2}.$$

Firm 1 and 2's demands are then respectively:

$$q_1 = \frac{p_2 - p_1}{2t(x_2 - x_1)} + \frac{x_1 + x_2}{2}, \quad q_2 = 1 - \left(\frac{p_2 - p_1}{2t(x_2 - x_1)} + \frac{x_1 + x_2}{2}\right).$$

The two firms use an intermediate input to produce a final product. For simplicity we assume that firms employ one unit of input to produce one unit of final good. Firms' costs of producing final goods are the same, which we normalize to zero for simplicity. Their costs of producing input are different. Firm i produces input with a constant marginal cost c_i. Without loss of generality, we assume that $c_1 \ge c_2$. We assume that public firm is firm 1 and foreign firm is firm 2, so it is consistent with practice.

The foreign firm may produce its inputs and/or final goods either within the country or outside the country. In the latter case, the foreign firm needs to import the input and/or final good into the country, and the domestic government can charge a per unit tariff on the imported objects. The per unit tariff on input is denoted by t_m, and tariff on final goods is denoted by t_f.

Due to the cost difference, the public firm may subcontract out the production of input to the foreign firm. In this case, the public

firm is a consignor, and the foreign firm is a subcontractor in input market. The surplus from subcontracting, denoted by S, is the cost saved from subcontracting.

$$S = (c_1 - c_2 - t_m)q_s$$

where q_s is the quantity of input subcontracted, and t_m is the input tariff when applicable. The division of the surplus is assumed to be determined by a bargaining between the two firms. In a one vs. one bargaining game, the division can be determined via a Nash bargaining process, or through other mechanism. In fact, one may assume there is one stage before our following three stage game, in which the two firms sign a subcontracting contract and specify the division of the surplus. Clearly, the division of the surplus depends crucially on the bargaining power of the two firms in practice. As our focus is not this bargaining process, and our major results hold for various divisions, we assume the division is exogenous. We assume that firms 1 and 2 get proportions α and $1 - \alpha$ of the surplus respectively, with $\alpha \in [0, 1]$. Since we focus on the impact of subcontracting on market equilibrium in this paper, we assume that the cost difference is large enough so that the surplus is positive.[4]

Following (Liang and Mai, 2006), we consider the following three-stage game. In the first stage, each firm simultaneously chooses its location over the interval [0, 1]. In the second stage, firms choose their prices of final goods simultaneously and then each firm's demand is revealed. In the last stage, firms subcontract input production.[5] The equilibrium concept we adopt is sub-game perfect Nash equilibrium (hereafter SPNE). Since both firms can get a share of the surplus through subcontracting and the subcontracting decision will not affect its profit from the final good market. It can be easily shown that a SPNE must consist of both firms engaging in subcontracting and the subcontracted quantity is the public firm's demand from final goods market, i.e. $q_s = q_1$[6]. In some sense, it can be viewed that the whole market is served by the foreign firm, directly (the foreign firm sells to consumers directly) and indirectly (the foreign firm supplies inputs to the public firm, which then sells the products to the rest consumers).

Each firm's objective can be characterized as the following. The foreign firm's objective is to maximize its profit.

$$\pi_2 = (p_2 - c_2 - t_j)q_2 + (1 - \alpha)S,$$

where $t_j (j = m, f)$ is the tariff on foreign firm. When foreign firm only produces input outside the country, government charges tariff on input, i.e. $t_j = t_m$. When foreign firm only produces final good outside the country, government charges tariff on final goods, i.e. $t_j = t_f$, and in subcontracting surplus S, $t_m = 0$. When foreign firm produces both inputs and final goods outside the country, it will only import a quantity of inputs required by public firm q_1 and quantity of final good equals to its demand q_2. In this case, $t_j = t_f$, and t_m in S is not zero.

The public firm wants to maximize social surplus that consists of its profit, consumer surplus and tariff income[7]. Its profit can be written as follows:

$$\pi_1 = (p_1 - c_1)q_1 + \alpha S.$$

Consumer surplus is

$$CS = V - TC - p_1 q_1 - p_2 q_2, \tag{1}$$

where TC is consumers' total transportation cost,

$$TC = \int_0^{\hat{x}} t(x - x_1)^2 dx + \int_{\hat{x}}^1 t(x_2 - x)^2 dx.$$

And social welfare is:

$$SS = \pi_1 + CS + t_f q_2 + t_m q_I, \tag{2}$$

where q_I is the input imported. It equals to 1 when the foreign firm only has input produced outside the country, equals to 0 when inputs are produced within the country, and equals to q_1 when both inputs and final goods are produced outside the country.

3. Analysis

According to the places where the foreign firm produces its inputs and final goods, we have following four cases:

- Case 1: Both inputs and final goods are produced within the country.
- Case 2: Inputs are produced outside the country but final goods are produced within the country.

[4] Liang and Mai (2006) use a similar setup.

[5] Alternatively, the three stage game can be understood as follows. The two firms sign a subcontracting contract in the first stage, which specifies the division of the share of subcontracting surplus. And then the two firms compete in locations and prices. In the last stage, the foreign firm fulfils the contract by supplying the quantity of inputs demanded by the public firm. This is in accordance with examples in practice. Firms usually sign a subcontracting contract to specify the input price or other instruments to determine the division of the subcontracting surplus, but not the exact amount of quantity, which they do not know before the market demand is revealed. When they fulfil the contract, the subcontracting firm then decides how many inputs to acquire from the subcontracted firm.

[6] Choosing vertical foreclosure is not a reliable threaten to make the public firm compromising in location and/or price, and thus is not an optimal strategy for the foreign firm in the last stage. An alternative game structure, in which foreign firm can reliably commit to vertical foreclosure is discussed in a separated Appendix.

[7] The public firm does not collect tariff, but as we assumed in the model, it is owned by the government, so it has the same objective as the government, i.e. maximizing social welfare.

- Case 3: Inputs are produced within the country but the final goods are produced outside the country.
- Case 4: Both inputs and final goods are produced outside the country.

We start with case 1, in which there is no tariff on foreign firm at all.

3.1. No tariff on foreign firm

When the foreign firm produces both inputs and final goods within the country, the government can not charge tariff on either. The unique pure strategy SPNE is characterized in the following lemma.

Lemma 1. *When both inputs and final goods are produced within the country, in equilibrium*:

- *The public firm and the foreign firm's locations are respectively*:

$$x_1 = \frac{3\sqrt{7} - 7}{2} \approx 0.49, \quad x_2 = \frac{\sqrt{7} - 1}{2} \approx 0.82;$$

- *The public firm and foreign firm's prices are respectively*:

$$p_1 = (1 - \alpha)c_1 + \alpha c_2, \quad p_2 = (1 - \alpha)c_1 + \alpha c_2 + (16 - 6\sqrt{7})t \approx (1 - \alpha)c_1 + \alpha c_2 + 0.13t.$$

- *The public firm and the foreign firm's demands are respectively*:

$$q_1 = \frac{\sqrt{7} - 1}{2} \approx 0.82, \quad q_2 = \frac{3 - \sqrt{7}}{2} \approx 0.18.$$

- *The social surplus, consumer surplus and foreign firm's profit are*:

$$SS = CS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_1 + \alpha(c_1 - c_2), \quad \pi_2 = (45 - 17\sqrt{7})t + (c_1 - c_2)(1 - \alpha).$$

Proof. See Appendix. □

Comparing our equilibrium locations with (Heywood and Ye, 2009), we share a common result that equilibrium locations are independent of transport cost coefficient[8]. But the equilibrium locations of public firm and foreign firms are respectively (0.5,0.83) in their model[9]. And they conclude that the presence of foreign firm has no effect on the location of public firm nor the domestic welfare. It can be easily verified that when there is only public firm in our model, the public firm locates at the center of the market ($x_1 = 0.5$) and social welfare is $V - \frac{t}{12} - c_1$. We compare social welfare with and without foreign firm:

$$\left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_1 + \alpha(c_1 - c_2) - (V - \frac{t}{12} - c_1) \approx 0.01t + \alpha(c_1 - c_2) > 0.$$

We conclude this as following lemma.

Lemma 2. *In consideration of subcontracting, public firm is less aggressive in location choice, and social welfare is higher.*

We next present an alternative case–the public firm competes against domestic private firm–to show that how the public firm will behave differently when its competitor is domestic firm. We assume that the public firm is still firm 1 and the private is firm 2[10]. When the market consists of a public firm and a domestic private firm, the private firm's profit is accounted into domestic welfare. The public firm's objective therefore is to minimize consumers' transport cost TC[11]. The corresponding SPNE is characterized in following Lemma.

Lemma 3. *When public firm competes against a domestic private firm, and subcontracts its input to the private firm, in equilibrium*:

- *The public firm and private firm's locations are respectively*:

[8] Since they normalize both firms' production costs to zero, whether equilibrium locations depend on costs in their model is unknown.

[9] In Heywood and Ye (2009), public firm locates to the right of foreign firm, and the equilibrium locations are $\left(\frac{1}{2}, \frac{1}{6}\right)$ for public firm and foreign firm respectively. Due to symmetry, when assuming public firm locates to the left of foreign firm, this equilibrium location is equivalent to $\left(\frac{1}{2}, \frac{5}{6}\right)$ for public firm and foreign firm respectively.

[10] It is usually assumed that the private firm is more effective than the public firm. We thus assume the private firm is the low cost firm which produces input for the public firm.

[11] With subcontracting, total production cost is fixed.

$$x_1 = \frac{1}{4}, \quad x_2 = \frac{3}{4}.$$

- *The public firm and private firms' prices are respectively*:

$$p_1 = \frac{1}{2}t + c_1 - \alpha(c_1 - c_2) \quad p_2 = \frac{1}{2}t + c_1 - \alpha(c_1 - c_2).$$

- *The two firms share the market equally, i.e.* $q_1 = q_2 = \frac{1}{2}$.
- *The social surplus, consumer surplus and private firm's profit are*:

$$SS = V - \frac{1}{48}t - c_2, \quad CS = V - \frac{25}{48}t - (1-\alpha)c_1 - \alpha c_2, \quad \pi_v = \frac{1}{4}t + (1-\alpha)(c_1 - c_2).$$

Our Lemma 3 is a special case of Cremer et al. (1991) and Shuai (2016), we thus skip the proof and refer interested readers to his paper for details of proof and discussion.

Comparing these two cases, we have following lemma.

Lemma 4. *Comparing to competing with a domestic firm, when a public firm competes with a foreign firm, it is more aggressive in both location and pricing choices. Social welfare is lower but consumer surplus is higher.*

Since the foreign firm's profit is not accounted in domestic welfare, the public firm is more aggressive in order to lower foreign firm's profit. Comparing to domestic private firm, the presence of foreign firm affects consumer welfare in two aspects: first it causes two firms deviating from the locations that minimize transport costs, which increases consumers' transport cost and lowers consumer welfare, *travel cost effect*; second, it drives down prices, which raises consumer welfare, *price effect*. Overall, our results suggest the price effect dominates the travel cost effect, and consumer welfare always increases with foreign firm. Social surplus is undoubtedly lower in presence of foreign firm as whose profit is not accounted in domestic welfare.

3.2. Tariff on foreign firm

In the remaining 3 cases, the government will charge tariff on either the inputs or the final goods or both. Under this situation, the government chooses optimal tariff rates to maximize social surplus. We therefore add a pre-stage (stage zero) before our three-stage game, in which the government chooses its tariff with the objective of maximizing social welfare.

Tariff on input

When the inputs are produced outside the country but the final goods are produced within the country, the government can only charge tariff on the inputs. The unique pure strategy equilibrium is summarized in following lemma.

Lemma 5. *When the government can only charge tariff on the inputs, in the unique SPNE*:

- *The public firm and foreign firm's locations are respectively*:

$$x_1 = \frac{3\sqrt{7} - 7}{2} \approx 0.49, \quad x_2 = \frac{\sqrt{7} - 1}{2} \approx 0.82.$$

- *The public firm and foreign firm's prices are respectively*:

$$p_1 = c_1, \quad p_2 = c_1 + (16 - 6\sqrt{7})t \approx c_1 + 0.13t.$$

- *The public and the foreign firm's demands are respectively*:

$$q_1 = \frac{\sqrt{7} - 1}{2} \approx 0.82, \quad q_2 = \frac{3 - \sqrt{7}}{2} \approx 0.18.$$

- *The tariff rate on input is* $t_m = c_1 - c_2$, *and total tariff income equals to* $c_1 - c_2$
- *The social surplus, consumer surplus and foreign firm's profit are*:

$$SS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_2, \quad CS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_1, \quad \pi_2 = (45 - 17\sqrt{7})t \approx 0.02t.$$

Proof. See Appendix.□

Tariff on final goods

When the foreign firm chooses to produce the inputs within the country but produces the final goods outside the country, the

government can only charge tariff on the final goods. The unique pure strategy SPNE is presented in the next lemma.

Lemma 6. *When the government can only charge tariff on the final goods, in the unique SPNE*:

- *The public firm and foreign firm's locations are respectively*:

$$x_1 = \frac{3\sqrt{7} - 7}{2} \approx 0.49, \quad x_2 = \frac{\sqrt{7} - 1}{2} \approx 0.82.$$

- *The public firm and foreign firm's prices are respectively*:

$$p_1 = (1 - \alpha)c_1 + \alpha c_2 + t_f, \quad p_2 = (1 - \alpha)c_1 + \alpha c_2 + t_f + (16 - 6\sqrt{7})t.$$

- *The public firm and the foreign firm's demands are respectively*:

$$q_1 = \frac{\sqrt{7} - 1}{2} \approx 0.82, \quad q_2 = \frac{3 - \sqrt{7}}{2} \approx 0.18.$$

- *The tariff rate on final goods has no effect on social surplus, the total tariff income equals to* $t_f q_2$.
- *The social surplus, consumer surplus and foreign firm's profit are*:

$$SS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_1 + \alpha(c_1 - c_2),$$

$$CS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_1 + \alpha(c_1 - c_2) - t_f\pi_2 = (45 - 17\sqrt{7})t + (c_1 - c_2)(1 - \alpha)$$

Proof. See Appendix.□

Tariff on both inputs and final goods

When both inputs and final goods are produced outside the country, the government can charge tariffs on both. The unique pure strategy SPNE then is summarized in following lemma.

Lemma 7. *When the government can charge tariff on both, in the unique SPNE*:

- *The public firm and foreign firm's locations are respectively*:

$$x_1 = \frac{3\sqrt{7} - 7}{2} \approx 0.49, \quad x_2 = \frac{\sqrt{7} - 1}{2} \approx 0.82.$$

- *The public firm and foreign firm's prices are*:

$$p_1 = c_2 + t_f, \quad p_2 = c_2 + t_f + (16 - 6\sqrt{7})t.$$

- *The public firm and the foreign firm's demands are respectively*:

$$q_1 = \frac{\sqrt{7} - 1}{2} \approx 0.82, \quad q_2 = \frac{3 - \sqrt{7}}{2} \approx 0.18.$$

- *The government will charge a tariff of* $t_m = c_1 - c_2$ *on the inputs and the tariff on final goods has no effect on social surplus. The total tariff income is* $(c_1 - c_2)q_1 + t_f q_2$.
- *The social surplus, consumer surplus and foreign firm's profit are*:

$$SS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_2, \quad CS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_2 - t_f, \quad \pi_2 = (45 - 17\sqrt{7})t$$

Proof. See Appendix.□

When government can charge tariff on both inputs and final goods, in equilibrium, firms' locations and market shares are independent of tariffs. The government will charge an input tariff to the extent of extracting all surplus from subcontracting. And the tariff on final goods has no effect on social welfare or foreign firm's profit. What is interesting here is that when government charges tariff on both inputs and final goods, the retail prices are lower than those in the cases where the government charges tariff only on inputs or final goods. Moreover, since the tariff on final goods has no effect on social welfare but lowers consumer surplus,

government can charge zero tariff on final goods, i.e. $t_f = 0$. In this case, the prices are even lower than the case with no tariff at all. Tariffs lower retail prices and help consumers! This is in sharp contrast to existing studies. For example, Chao and Yu (2006) and Wang et al. (2010) find that tariffs lower consumer surplus[12]. Why is that? Note that when government charges tariff on inputs, it leaves no surplus for the foreign firm from subcontracting[13]. In the government's perspective, the actual domestic production cost is c_2 (public firm's production cost after subcontracting plus per unit tariff on inputs). From the government's point of view, after tariff, the lowest price it can take is lowered to c_2. And consumers benefit from lower prices.[14].

From all four cases, we can see that firms' locations and their market shares are constant, so is the prices difference between foreign firm and public firm. We summarize consumer surplus, social surplus, and foreign firm's profit in Table 1. To ease exhibition, we convert fractions into approximate decimal points.

Based on Table 1 we have following proposition.

Proposition 1. *Comparing the cases that government can charge tariff on inputs or final goods or both,*

- *Firms' locations and market share are independent of production costs or tariff choice.*
- *Government tariff on inputs can increase social surplus and decrease foreign firm's profit. An optimal input tariff is to the extent of cost difference.*
- *Government tariff on final goods has no effect on social surplus or foreign firm's profit, but will raise prices by its value and thus lower consumers surplus.*
- *Compared to no tariff at all, if government only charges tariff on inputs, retail prices are raised, and if government charges tariff on inputs as well as final goods, retail prices may be lower. Consumer surplus is higher when prices are lower.*

In the past three decades, developed countries have moved their production sector to developing countries to enjoy a lower production cost or avoid tariffs. At the mean time, many countries have joined World Trade Organization which lowers tariffs for its member countries. Our results indicate that in either case, a variation in tariff will not change firms' positioning. If there exists subcontracting between domestic firm and foreign firm, the government can use input tariff to extract some benefit from subcontracting, which will increase domestic social surplus but lower consumer welfare. On the other hand, in the perspective of the foreign firm, the tariff on final good is equivalent to an increase in its cost. With perfect inelastic demand, an increment in cost is completely shifted to consumers by increasing the price by the same amount. Thus the tariff on final good increases government revenue by $t_f q_2$, but lowers consumer welfare by the same amount, leaving social welfare unchanged.[15] Based on this, in some instances, it may be better to exempt final good tariff[16]. When government can charge tariff on both inputs and final goods, consumer surplus can be higher with tariff comparing to the case without tariff.

Next, we examine how the division of the surplus (α) affects firms' strategies, consumer surplus and welfare. We summarize this as following proposition.

Proposition 2. *In terms of the division of the subcontracting surplus*

- *it has no effect on either firm's equilibrium location;.*
- *when there is no tariff on inputs, prices decrease, but consumer surplus and social welfare increase with α;.*
- *when there is tariff on inputs, it has no effect on either* firm's *price, nor consumer surplus or social welfare.*

If the division of the subcontracting surplus is determined by a bargaining between the two firms, it is reasonable to believe the firm with relative larger bargaining power will get a larger share. Our results indicate that firms' locations are independent of the bargaining power, and moreover, a larger bargaining power by the public firm will benefit consumer and social welfare only when there is no tariff on input.

Endogenous decision of the production location by foreign firm

It is interesting to know how the foreign firm responds to government tariff by choosing its production location, domestic or foreign country. We endogenize the foreign firm's production location decision by adding one stage before the four-stage game. From Table 1, we can see the foreign firm's profit are higher when there is no tariff or there is only tariff on final good. That is, if the foreign firm chooses to produce inputs domestically, its profit will be higher. And where to produce its final good is irrelevant. We summarize this as following proposition.

Proposition 3. *When the production location of the foreign firm is endogenously determined, the foreign firm chooses to produce the input domestically, and the final good domestically or in a foreign country.*

[12] (Chao and Yu, 2006) do not explicitly analyze tariff's effect on consumer surplus, but from Eq. (7) and (8) of their paper, it is easily to verity that tariff lowers quantity supplied which in turn raises price and hurts consumers.

[13] This makes foreign firm indifferent between with and without subcontracting, and here we assume that foreign firm will engage in subcontracting. Alternatively, one may think the input tariff can be a little bit lower and leaves foreign firm an infinite small gain from subcontracting. Under our current model setup, the subcontracting decision is made at the last stage of the game, thus strategic foreclosure is not a SPNE for foreign firm. An alternative structure of the game which allows strategic foreclosure is discussed in a separate Appendix. And we find that our major results continue to hold qualitatively.

[14] Note that at the price of c_2, the public firm operates at a loss. In practice, many public firms operate at a loss, such as United States Postal Service in the U.S., China Ocean Shipping (Group) Company (COSCO), etc.

[15] In market with inelastic but not perfect inelastic demand, one should expect most tariffs on final goods are shift to consumers, but not all. In such case, the effect of final good tariff on social welfare will be small, but not none.

[16] In some instances, tariff is used to protect domestic industry from the competition of foreign firms. Our model does not take this fact into account.

Table 1
Tariff impact.

Tariff	CS	SS	π_2
No tariff	$V - c_1 + \alpha(c_1 - c_2) - 0.07t$	$V - c_1 + \alpha(c_1 - c_2) - 0.07t$	$(c_1 - c_2)(1 - \alpha) + 0.02t$
On input	$V - c_1 - 0.07t$	$V - c_2 - 0.07t$	$0.02t$
On good	$V - c_1 + \alpha(c_1 - c_2) - 0.07t - t_f$	$V - c_1 + \alpha(c_1 - c_2) - 0.07t$	$(c_1 - c_2)(1 - \alpha) + 0.02t$
On both	$V - c_2 - 0.07t - t_f$	$V - c_2 - 0.07t$	$0.02t$

4. Conclusion

Existing literatures on mixed oligopoly market usually focus on the competition relationship among firms, while in practice (partly) public firms may cooperate with domestic/foreign private firms. One important way of cooperation is through subcontracting. To fill the gap between theory and reality, we examine both competition and cooperation between a public firm and a foreign firm. We find that comparing to no subcontracting, the public firm is less aggressive in location choice, and social welfare is higher, when it subcontracts input production to the foreign firm. And the public firm will be more aggressive in its location and pricing choice when competing with foreign firm comparing to competing with domestic private firm. Social welfare is lower in presence of foreign firm, consumer welfare is higher.

We also consider the case that foreign firm produces outside the country and thus the government can charge tariffs on either inputs or final goods or both. We find that firms' locations and market shares are constant, i.e. independent of production costs or tariffs. We also find that the tariff on final goods has no effect on social welfare but hurts consumers. And the tariff on input can raise social welfare, but its effect on consumer welfare depends. If government only tariffs inputs, consumer surplus is lower, but if government tariffs both inputs and final goods, consumer surplus can be higher.

In the paper, we assume perfect inelastic industry demand so that our model is comparable to existing literature which typically uses the same assumption[17]. Relaxing of this assumption may alter our results and requires further investigation. Since our focus is on the firms interaction at the final products market, as in Liang and Mai (2006), we treat the subcontracting in a relative simple way[18]. Future researches can introduce a more complex subcontracting contract and examine its effect on firms' decision about subcontracting.

Appendix A: Proof of propositions

Proof of Lemma 1. When there is no tariff on foreign firm, the public firm chooses its location and price to maximize social surplus characterized as following:

$$SS = \pi_p + CS = (p_1 - c_1)q_1 + \alpha s + V - UL - p_1q_1 - p_2q_2,$$

while foreign firm maximize its profit. We use backward induction to solve for SPNE. Starting from the second stage, we calculate two first order conditions and set them equal to zero:

$$\frac{\partial SS}{\partial p_1} = \frac{p_1 - c_1 + \alpha c_1 - \alpha c_2}{2t(x_1 - x_2)} = 0, \frac{\partial \pi_2}{\partial p_2} = \frac{2 - x_1 - x_2}{2} + \frac{2p_2 - p_1 - c_1 + (c_1 - c_2)\alpha}{2t(x_1 - x_2)} = 0.$$

we get[19]

$$p_1 = c_1(1 - \alpha) + \alpha c_2, \quad p_2 = t(-x_1 + x_2) + c_1(1 - \alpha) + \alpha c_2 + \frac{t(x_1^2 - x_2^2)}{2}.$$

we plug in the optimal prices and calculate first order conditions of the first stage:

$$\frac{\partial SS}{\partial x_1} = -\frac{3x_1^2 + 24x_1 + 2x_1x_2 - x_2^2 - 12)t}{16} = 0, \quad \frac{\partial \pi_2}{\partial x_2} = -\frac{(x_1^2 - 2x_1x_2 - 3x_2^2 - 4 + 8x_2)t}{8} = 0.$$

Solving first order conditions we get[20]:

$$x_1 = \frac{3\sqrt{7} - 7}{2}, \quad x_2 = \frac{\sqrt{7} - 1}{2}.$$

we then plug x_1 and x_2 back into p_1 and p_2, we get

[17] Our results apply to some industries with perfect inelastic demand or nearly perfect inelastic demand, such as agriculture market, crude oil market, etc.
[18] The share of surplus from subcontracting is divided by Nash bargaining of the two firms.
[19] Second order conditions are satisfied as $\frac{\partial^2 SS}{\partial p_1^2} = \frac{1}{2t(x_1 - x_2)} < 0$, and $\frac{\partial^2 \pi_2}{\partial p_1^2} = \frac{1}{t(x_1 - x_2)} < 0$.
[20] Second order conditions are satisfied as $\frac{\partial^2 SS}{\partial x_1^2} = -\frac{(3x_1 + 12 + x_2)t}{8} < 0$ and $\frac{\partial^2 \pi_2}{\partial x_2^2} = \frac{(x_1 + 3x_2 - 4)t}{4} < 0$.

$$p_1 = (1-\alpha)c_1 + \alpha c_2, \quad p_2 = (1-\alpha)c_1 + \alpha c_2 + (16-6\sqrt{7})t,\ q_1 = \frac{\sqrt{7}-1}{2},$$
$$q_2 = \frac{3-\sqrt{7}}{2}.SS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_1 + \alpha(c_1 - c_2), \quad \pi_2 = (45-17\sqrt{7})t + (c_1 - c_2)(1-\alpha).$$

It can be easily verified that the public firm's price equals to its cost after subcontracting, thus its profit is zero and consumer welfare equals social welfare.□

Proof of Lemma 5. When the government can only charge tariff on the inputs, it can choose tariff on inputs t_m and the public firm's location and price to maximize social surplus characterized as following:

$$SS = \pi_p + CS + t_m(q_s + q_2) = (p_1 - c_1)q_1 + \alpha s + V - UL - p_1q_1 - p_2q_2 + t_m(q_s + q_2),$$

while foreign firm maximizes its profit. We use backward induction to solve for SPNE. Different from the section above, we have three stages here. Starting from the third stage, we calculate two first order conditions and set them equal to zero:

$$\frac{\partial SS}{\partial p_1} = \frac{p_1 - c_1 + \alpha c_1 - \alpha c_2 - \alpha t_m}{2t(x_1 - x_2)} = 0,\ \frac{\partial \pi_2}{\partial p_2} = \frac{-2tx_1 + 2tx_2 + p_1 + tx_1^2 - 2p_2 - tx_2^2 + c_1 - \alpha c_1 + \alpha c_2 + \alpha t_m}{2t(-x_1 + x_2)} = 0.$$

we get[21]

$$p_1 = c_1(1-\alpha) + \alpha c_2 + \alpha t_m, \quad p_2 = t(-x_1 + x_2) + \alpha t_m + c_1(1-\alpha) + \alpha c_2 + \frac{t(x_1^2 - x_2^2)}{2}.$$

we plug in the optimal prices and calculate first order conditions of the second stage:

$$\frac{\partial SS}{\partial x_1} = -\frac{3x_1^2 + 24x_1 + 2x_1x_2 - x_2^2 - 12)t}{16} = 0, \quad \frac{\partial \pi_2}{\partial x_2} = -\frac{(x_1^2 - 2x_1x_2 - 3x_2^2 - 4 + 8x_2)t}{8} = 0.$$

Solving first order conditions we get[22]:

$$x_1 = \frac{3\sqrt{7}-7}{2}, \quad x_2 = \frac{\sqrt{7}-1}{2}.$$

we then plug in the optimal locations and calculate first order condition of the first stage:

$$\frac{\partial SS}{\partial t_m} = 1 - \alpha > 0$$

So in order to maximize social surplus and also make subcontracting possible, government needs to put tariff at:

$$t_m = c_1 - c_2$$

we then plug t_m, x_1 and x_2 back into p_1 and p_2, we get

$$p_1 = c_1, \quad p_2 = c_1 + (16-6\sqrt{7})t,\ q_1 = \frac{\sqrt{7}-1}{2}, \quad q_2 = \frac{3-\sqrt{7}}{2}.SS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_2,$$
$$\pi_2 = (45-17\sqrt{7})t.CS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_1.\ \square$$

Proof of Lemma 6. When the government can only charge tariff on the final goods, it can choose tariff on final goods t_f and the public firm's location and price to maximize social surplus characterized as following:

$$SS = \pi_p + CS + t_fq_2 = (p_1 - c_1)q_1 + \alpha s + V - UL - p_1q_1 - p_2q_2 + t_fq_2,$$

while foreign firm maximizes its profit. We use backward induction to solve for SPNE. Exactly as the section above, we have three stages here. Starting from the third stage, we calculate two first order conditions and set them equal to zero:

$$\frac{\partial SS}{\partial p_1} = \frac{p_1 - c_1 + \alpha c_1 - \alpha c_2 - t_f}{2t(x_1 - x_2)} = 0,\ \frac{\partial \pi_2}{\partial p_2} = \frac{-2tx_1 + 2tx_2 + p_1 + tx_1^2 - 2p_2 - tx_2^2 + c_1 - \alpha c_1 + \alpha c_2 + t_f}{2t(-x_1 + x_2)} = 0.$$

we get[23]

[21] Second order conditions are satisfied as $\frac{\partial^2 SS}{\partial p_1^2} = \frac{1}{2t(x_1 - x_2)} < 0$, and $\frac{\partial^2 \pi_2}{\partial p_1^2} = \frac{1}{t(x_1 - x_2)} < 0$.

[22] Second order conditions are satisfied as $\frac{\partial^2 SS}{\partial x_1^2} = -\frac{(3x_1 + 12 + x_2)t}{8} < 0$ and $\frac{\partial^2 \pi_2}{\partial x_2^2} = \frac{(x_1 + 3x_2 - 4)t}{4} < 0$.

[23] Second order conditions are satisfied as $\frac{\partial^2 SS}{\partial p_1^2} = \frac{1}{2t(x_1 - x_2)} < 0$, and $\frac{\partial^2 \pi_2}{\partial p_1^2} = \frac{1}{t(x_1 - x_2)} < 0$.

$$p_1 = c_1(1-\alpha) + \alpha c_2 + t_f, \quad p_2 = t(-x_1 + x_2) + t_f + c_1(1-\alpha) + \alpha c_2 + \frac{t(x_1^2 - x_2^2)}{2}.$$

We plug in the optimal prices and calculate first order conditions of the second stage:

$$\frac{\partial SS}{\partial x_1} = -\frac{3x_1^2 + 24x_1 + 2x_1x_2 - x_2^2 - 12)t}{16} = 0, \quad \frac{\partial \pi_2}{\partial x_2} = -\frac{(x_1^2 - 2x_1x_2 - 3x_2^2 - 4 + 8x_2)t}{8} = 0.$$

Solving first order conditions we get[24]:

$$x_1 = \frac{3\sqrt{7} - 7}{2}, \quad x_2 = \frac{\sqrt{7} - 1}{2}.$$

we then plug in the optimal locations and calculate first order condition of the first stage:

$$\frac{\partial SS}{\partial t_m} = 0$$

So imposing a tariff on final goods has no effect on social surplus, government doesn't have motivation to put tariff on final good.We then plug x_1 and x_2 back into p_1 and p_2, we get

$$p_1 = c_1 + t_f - \alpha c_1 + \alpha c_2, \quad p_2 = c_1 + (16 - 6\sqrt{7})t + t_f - \alpha c_1 + \alpha c_2, \; q_1 = \frac{\sqrt{7} - 1}{2},$$

$$q_2 = \frac{3 - \sqrt{7}}{2}.SS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_1(1-\alpha) - \alpha c_2,$$

$$\pi_2 = (45 - 17\sqrt{7})t + (c_1 - c_2)(1-\alpha).CS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_1(1-\alpha) - \alpha c_2 - t_f. \; \square$$

Proof of Lemma 7. When the government can charge tariff on the inputs and final goods, it can choose tariff t_m,t_f and the public firm's location and price to maximize social surplus characterized as following:

$$SS = \pi_p + CS + t_m q_s + t_f q_2 = (p_1 - c_1)q_1 + \alpha s + V - UL - p_1q_1 - p_2q_2 + t_m q_s + t_f q_2,$$

while foreign firm maximizes its profit. We use backward induction to solve for SPNE. As with the section above, we have three stages here. Starting from the third stage, we calculate two first order conditions and set them equal to zero:

$$\frac{\partial SS}{\partial p_1} = \frac{p_1 - c_1 + \alpha c_1 - \alpha c_2 - \alpha t_m + t_m - t_f}{2t(x_1 - x_2)} = 0, \; \frac{\partial \pi_2}{\partial p_2} = \frac{-2tx_1 + 2tx_2 + p_1 + tx_1^2 - 2p_2 - tx_2^2 + t_f + c_1 - t_m - \alpha c_1 + \alpha c_2 + \alpha t_m}{2t(-x_1 + x_2)} = 0.$$

we get[25]

$$p_1 = c_1(1-\alpha) + \alpha c_2 - t_m(1-\alpha) + t_f, \quad p_2 = t(-x_1 + x_2) + \alpha t_m + c_1(1-\alpha) + \alpha c_2 - t_m + t_f + \frac{t(x_1^2 - x_2^2)}{2}.$$

we plug in the optimal prices and calculate first order conditions of the second stage:

$$\frac{\partial SS}{\partial x_1} = -\frac{3x_1^2 + 24x_1 + 2x_1x_2 - x_2^2 - 12)t}{16} = 0, \quad \frac{\partial \pi_2}{\partial x_2} = -\frac{(x_1^2 - 2x_1x_2 - 3x_2^2 - 4 + 8x_2)t}{8} = 0.$$

Solving first order conditions we get[26]:

$$x_1 = \frac{3\sqrt{7} - 7}{2}, \quad x_2 = \frac{\sqrt{7} - 1}{2}.$$

we then plug in the optimal locations and calculate first order condition of the first stage:

$$\frac{\partial SS}{\partial t_m} = 1 - \alpha > 0, \; \frac{\partial SS}{\partial t_f} = 0.$$

So tariff on final goods again does not have effect on social surplus and in order to maximize social surplus and also make subcontracting possible, government needs to put tariff on inputs at:

$$t_m = c_1 - c_2$$

we then plug t_m, x_1 and x_2 back into p_1 and p_2, we get

[24] Second order conditions are satisfied as $\frac{\partial^2 SS}{\partial x_1^2} = -\frac{(3x_1 + 12 + x_2)t}{8} < 0$ and $\frac{\partial^2 \pi_2}{\partial x_2^2} = \frac{(x_1 + 3x_2 - 4)t}{4} < 0$.

[25] Second order conditions are satisfied as $\frac{\partial^2 SS}{\partial p_1^2} = \frac{1}{2t(x_1 - x_2)} < 0$, and $\frac{\partial^2 \pi_2}{\partial p_1^2} = \frac{1}{t(x_1 - x_2)} < 0$.

[26] Second order conditions are satisfied as $\frac{\partial^2 SS}{\partial x_1^2} = -\frac{(3x_1 + 12 + x_2)t}{8} < 0$ and $\frac{\partial^2 \pi_2}{\partial x_2^2} = \frac{(x_1 + 3x_2 - 4)t}{4} < 0$.

$$p_1 = c_2 + t_f, \quad p_2 = c_2 + t_f + (16 - 6\sqrt{7})t, \; q_1 = \frac{\sqrt{7} - 1}{2}, \quad q_2 = \frac{3 - \sqrt{7}}{2}.SS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_2,$$

$$\pi_2 = (45 - 17\sqrt{7})t.CS = \left(\frac{7\sqrt{7}}{2} - \frac{28}{3}\right)t + V - c_2 - t_f. \; \square$$

Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.iref.2017.01.006.

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