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A simplified approximation for seismic analysis of silo–bulk material system

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Abstract In order to estimate the distribution, as well as the magnitude, of dynamic material pressures on ground-supported silos a simplified seismic analysis procedure was utilized. The seismic analysis of silos can be complex, as the evaluation of several parameters must be taken into consideration, including the properties of bulk materials used and how the bulk materials and silo wall are joined together. It is therefore useful to develop an analytical approximation in order to better assess results. In addition to a simplified model for the seismic analysis of a silo–bulk material system being utilized, a three-dimensional finite element model was also incorporated. Using the finite element method, a more realistic representation of the structure is possible. Moreover, the finite element method also takes into consideration contact problems between the bulk material and the silo wall, which results in easier analyses. Both a squat and a slender silo were selected for this study. The results obtained in the study of selected examples were compared with those findings obtained via EN1998-4. Modified Veletsos and Younan approximations, which are commonly used for the analysis of grain silos, were also used. Results and analysis concluded that the proposed analytical model provided, overall, a good outcome, especially in regards to the analysis of dynamic material pressure. It should be noted that using the analytical method as proposed in Eurocode, the dynamic material pressure for squat silos can be underestimated, but the results for slender silos are stronger.

Keywords Cylindrical silos · Bulk material–silo wall interaction · Seismic response · EN1998-4

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1 Introduction

The failure rate of silos all over the world is much higher when compared to the failure rate of other industrial structures. Designing a silo requires a multidisciplinary cooperation of civil engineering, mechanical engineering, chemical engineering, as well as other areas of expertise such as experimental mechanics, dust explosion, simulation etc. A silo collapse can cause contamination of the contained material and, more worryingly, put human life and health at risk. Therefore, the reliability and stability of these special structures under seismic loads are of critical concern, and should be understood by those involved in design, implementation and management. Seismic events are of particular threat. The devastating effects of earthquakes make the problem more complicated suggesting that the static design procedures for silos need to be reassessed. It is important to consider that cylindrical shell structures require more sensitive boundary conditions due to their Eigen frequencies (Leissa 1973). Mode shapes are susceptible to boundary conditions imposed on the translational motion, whereas the influence of the boundary condition for the rotational degree of freedom is almost negligible, the exception being slender silos. For these reasons, a realistic three-dimensional finite element model should be used.

A few tests were performed in the late eighties, in which silo models were exposed to dynamic, typical earthquake loads (Shimamoto et al. 1982; Harris and von Nad 1985; Sasaki et al. 1986; Sasaki and Yoshimura 1992) and in all these investigations, researchers focused on the influence of effective bulk material mass on the dynamic response of the whole system. There are also a number of studies that look at the effects resulting from filling and discharging the flow of bulk materials. Rotter et al. (1998), using a wide literature review, investigated the efficiency of both finite element and discrete element approximation on these effects. However, little information can be found about the seismic behavior of cylindrical silos in the established literature. Indeed, the number of relevant studies is insufficient, which should encourage momentum for further studies to understand seismic effects on silos. Recently, Holler and Meskouris (2006) conducted a numerical and experimental study to describe the dynamic behavior of silos. Furthermore, during the last decade, there has been increased attention on the in situ modal analysis techniques in structural engineering to estimate the dynamic characteristics of this special structure. By using this technique, Dooms et al. (2006) carried out a study on empty silos in order to evaluate the reliability of the finite strip method, which allows the silo model to be reduced into two dimensions. They also used a finite element model, validated by means of the in situ modal analysis technique. Developing numerical analysis techniques and increasing technological capacity to provide scientists and researchers a wider perspective to represent and simulate the problems that arise from seismic behavior on the bulk material–silo system is crucial. Two general approaches for the analysis and design of silo walls exist (Abdel-Sayed et al. 1985). In the first approach, pressure induced by solids on the wall is established with no account for the interaction between the two materials. In the second approach, the system of the ensiled material and wall is regarded as a continuum and yet separated by a number of finite elements connected at the coinciding nodes. The first approach is still widely accepted amongst silo designers, whereas the second approach is largely restricted to research purposes (Abdel-Fattah et al. 2006). The researchers first modeled both the silo wall and bulk material using the elastic finite element, i.e. Rotter and Hull (1989). A cylindrical silo structure containing bulk solids was modeled using an elastic finite element analysis for solids with axisymmetric geometry. Seismic activity was induced by a quasi-static horizontal body force, which was utilized by Rotter and Hull (1989) and led to further recommendations on silo design by the researchers.

In relation to the importance of these structures and complexity of the problem it is remarkable at the lack of research that exists. The seismic behavior of silos is necessary in order to establish a stronger common understanding of the practical use of existing knowledge and of further research needs. The design of silo structures in reinforced concrete is now generally well understood. The problems identified are therefore usually related to either features associated with their high stiffness or to quality control of the construction process (Eibl 2009). In particular, in the case of inflexible structures, thermal actions and differential settlements arising from foundation problems will be more critical. Nevertheless, there is no established consensus when it comes to the calculation procedures available to take the effects of earthquakes into consideration. Very few national and international regulatory standards include explicit requirements in regards to the design of silos and fortifying them from earthquakes. Most standards do not cover the subject at all, or they refer to general building codes. The best example is the Uniform Building Code (1197), which states that flat bottom tanks or other tanks with supported bottoms, founded at or below grade, shall be designed using a procedure for rigid structures (those with period T less than 0.06 s) that take into consideration equivalent lateral forces (Briassoulis 2009). Eurocode, however, has recently introduced a simple seismic procedure for fortifying silos and tanks, although this is still far too general (EN1998-4 2006).

Evaluating silo seismic behavior falls into two main classes: they are either elevated above the ground, or are directly ground-supported, with both squat and slender possible designs. For ground-supported silos, seismic induced loading on the wall is significant and may control designs. Elevated silos respond in a similar manner to elevated tanks filled with fluid. When considering the interaction between a silo's wall and the bulk material, a silo's cross-sectional geometry, the slenderness ratio and many other similar parameters can be effective when it comes to the seismic performance of silos. These parameters should be further researched utilizing both bulk material–silo systems and those silos with soil/foundation systems. There are especially challenging issues concerning ground-supported silos, which need to be dealt with. Thus, this study looks at both a squat and a slender ground-supported cylindrical silo for two clearly defined purposes. Firstly, a simplified analytical model for the seismic design of the bulk material–silo system is undertaken in order to explore the current analytical models via a more realistic numerical model. Secondly, an evaluation of the simplified models for silos, as developed by researchers and recommended by current codes and guidelines, is assessed to discover their ability to represent adequate seismic responses in such silo systems.

2 Seismic analysis methods for silos

2.1 The code provisions and EN1998-4 approximation

A few simplified assumptions can be taken from international standards about loads on the walls, resulting from seismic interaction between the bulk material and the silo wall. As stated previously, there are two main approaches used to represent the pressure created by the bulk solids on the wall. The first approach provides no account for the interaction between the two materials. In the second approach, the composite system of the ensiled material and the wall is regarded as a continuum, modeled by a number of finite elements connected at their common nodes. The first approach is still widely accepted amongst silo designers, whereas the second one largely restricted to research purposes. This is perhaps due to the fact that the former is adopted by most silo design codes in which the lateral pressures on the wall are established using either of the two classic silo theories, namely Janssen's theory and Reimbert's theory. Janssen's theory is preferred in the United States of America, while Reimberts' is usually utilized in parts of Europe (Abdel-Fattah et al. 2006). In Eurocode (EN1998-4 2006), the silo is approximately considered as a cantilever beam with different point masses and these masses are then excited by an equivalent static load that can be calculated by spectral analysis, in which the soil condition, earthquake zone etc., are the input parameters. Holler and Meskouris (2006) showed that, by investigating silos numerically as described in the European standards and additionally by taking into account nonlinear effects, the provisions given in the Eurocode (EN1998-4 2003) yield good results for the slender silo, although results concerning squat silos are rather conservative. They also indicated that Eurocode provisions (EN1998-4 2003) for slender silos are quite good, whereas for squat silos a reduction of the assumed active mass would be necessary. On the other hand, American Code of Practice ACI 313-97 (1997), in computing lateral seismic force due to the mass of the stored granular material, assumes the silo to be full during seismic analysis. This approximation takes into consideration added mass, which may be useful when it comes to spectrum analysis. The first approximation is commonly used, whereby the bulk material is constituted as an inviscid fluid. However, the mechanical properties of these bulk materials cannot accurately be defined in this way. Therefore, their representation as ideal liquids may not generally be appropriate.

In light of the commonly used codes, the effects created by bulk material–silo interaction are used to determine the equivalent static force acting on the silo wall. Therefore, operational codes require that various parameters be taken into consideration, including geotechnical, structural and seismic characteristics. This point of view may be summarized as given in Eq. 1;

$$F_t = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_4 \cdot \alpha_5 \cdot \alpha_6 \cdot \alpha_7 \cdot a_0 \cdot M \tag{1}$$

where the factors (α) describe almost all the parameters that should be taken into account during design. The factors include: seismic zone (α_1), amplification due to supporting system-soil interaction (α_2), site specific response (α_3), damping effects (α_4), ductility effect of structural systems (α_5), risk (α_6), importance (α_7) and maximum ground accelerations (a_0). The use of the product of all these factors and mass of the system (M) is proposed by almost all well-known codes to estimate the equivalent lateral forces acting on the silo wall or even the frame of an ordinary structure. For the purpose of this study Eurocode provisions and related previous codes are summarized below.

Previous design standards for silos like DIN 1055-6 (1987), which was the design standard for silos containing granular material and DIN 4149-1 (1981), which looked at the force created by seismic action on a silo, need to take into account both the mass of the structure and the bulk material. The system utilizes a cantilever beam in this approximation and its uniformly distributed mass is assumed as lumped masses at different levels along its height (Fig. 1). The seismic response of this equivalent system can be estimated by using a modal analysis technique.

Equation 2 can calculate the equivalent static forces stemming from this system.

$$H_{E,i,j} = \frac{G_j + P_j}{g} \cdot \beta(T_i) \cdot \Psi_{j,i} \cdot \frac{\sum_{i=1}^{n} m_j \cdot \Psi_{j,i}}{\sum_{i=1}^{n} m_j \cdot \Psi_{j,i}^2} \cdot a_0 \cdot \kappa \cdot \alpha \tag{2}$$

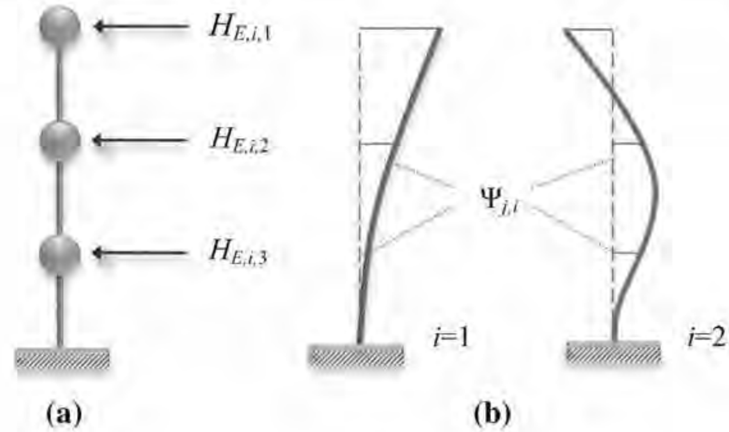

Fig. 1 **a** Lumped mass approximation for silos and **b** the first two mode shapes (DIN-4149 1981)

where i is the number of the modes and j is the mass number. $G_j + P_j$ and $\beta(T_i)$ represents total mass, including both live and dead loads, and the damping effects of ith mode shapes due to the 5 % damping ratio and period of the mode (T_i). The other well-known parameters such as g, a_0, κ and α are the gravity, the horizontal acceleration coefficient, the coefficient of the soil–structure system and the reduction factor of the supporting system of the silo and seismic zone, respectively. Consequently in this equation, $\psi_{j,i}$ is used for representing the modal displacement for ith mode at the level of jth mass.

EN1998-4 (2006) includes criteria and rules required for the seismic design of silos without restrictions on their size, structural type and other functional characteristics. For some types of tanks and silos, however, it also provides detailed methods.

This code does emphasize, however, that this standard may not be sufficient for facilities associated with large risks to the population or the environment, meaning that additional requirements should be established by the competent authorities. This standard evaluates silos in two main groups: silos directly supported by the ground and silos elevated with column and/or shell supports. For silos situated directly on the ground, the reaction of the bulk material pressure is concentrated on the seismic design due to the main effect of the seismic movement. These types of silos may be depicted as Fig. 2. Analysis of silos shall be in accordance with the following equations.

In circular silos (or silo compartments) the additional normal pressure (p_{hs}) on the wall may be evaluated using Eq. 3.

$$p_{hs} = p_{hso} \cos\theta \tag{3}$$

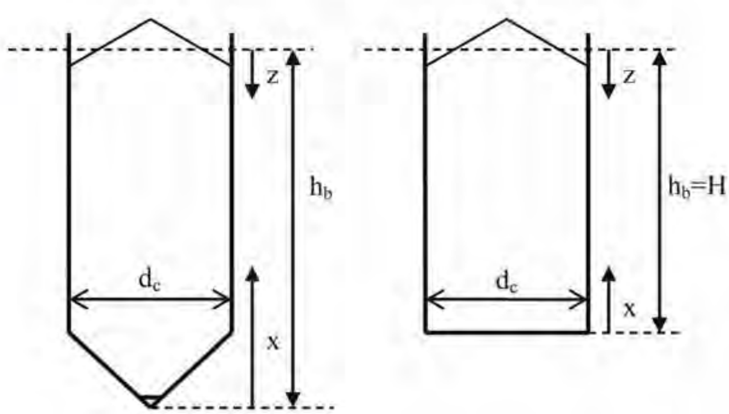

Fig. 2 Geometrical properties of silos due to EN1998-4 (2006)

where p_{hso} is the reference pressure, θ is the angle ($0° \leq \theta < 360°$) between the radial line and the point of interest on the wall. To estimate the pressure variation with the height of the wall, first the r_s^* dimension should be selected from Eq. 4.

$$r_s^* = \min(h_b, d_c/2) \tag{4}$$

where h_b is the overall height of the silo, from a flat bottom or the hopper outlet to the equivalent surface of the stored contents and d_c is the inside dimension of the silo parallel to the horizontal component of the seismic action (inside diameter, d_c, in circular silos or silo compartments, inside horizontal dimension b parallel to the horizontal component of the seismic action in rectangular ones). Using all these parameters, reference pressure acting on the silo wall throughout its height can be determined using Eq. 5.

$$p_{hso} = \alpha(z) \cdot \gamma \cdot \min(r_s^*; 3x) \tag{5}$$

where $\alpha(z)$ is defined as the ratio of the response acceleration of the silo at a vertical distance z from the equivalent surface of the stored contents, to the acceleration of gravity in the EN1998-4 (2006); γ is the bulk unit weight of the particulate material in the seismic design situation. In this study, $\alpha(z)$ was estimated using a single mass approximation of the silo–bulk material system analysis. Consequently, the defined response acceleration value at the level of mass was used to obtain the variation of these values along the height. The variation of acceleration throughout the height of a silo's wall was taken into consideration.

2.2 The Veletsos–Younan analytic approximations

By considering all outcomes in the literature and in practice, Veletsos and Younan (Younan and Veletsos 1998; Veletsos and Younan 1998) conducted studies taking bulk material effects into account in both static and dynamic cases. They modeled the storing material on a uniform viscoelastic solid that is free at its upper surface but is also bonded to a non-deformable base that experiences uniform horizontal motion. The tank was assessed as being vertical and of circular cross section. To present a simple, approximate, yet reliable method of analysis for this system, they included various parameters, such as tank-height to tank-radius ratio, the physical properties of the contained material, and a dimensionless measure of the flexibility of the wall relative to that of the contained material. Through the study of comprehensive numerical solutions to elucidate the underlying response mechanisms, they separated their studies into two parts. Part A of the study (Younan and Veletsos 1998) deals with rigid tanks, while Part B (Veletsos and Younan 1998) addresses the effects of wall flexibility with the assumption that the tank will respond as a cantilever shear beam with no change in its cross section. For rigid tanks, the interface of the tank wall and the contained material may be either smooth or rough. The combined version of these studies can be reviewed in the report by Veletsos et al. (1997).

2.3 Proposed analytical approximation

As mentioned previously, the need for simple determination of the seismic behavior of silos is crucial. When it comes to the determination of dynamic pressures, this and similar methods are used for not only practical purposes but also for scientific purposes. Although there are a considerable number of approximations on this subject, this study focused on the approach proposed by Veletsos and Younan (Veletsos et al. 1997; Younan and Veletsos 1998) and the method given by EN1998-4 (2006). However, the literature involving experimental and numerical approximations showed that the proposed method in EN1998-4 (2006) does not give appropriate results (Holler and Meskouris 2006; Tatko and Kobielak 2008). Therefore a more realistic and simplified approach is needed.

The analytical approach suggested by Veletsos and Younan (Younan and Veletsos 1998; Veletsos and Younan 1998) and based on harmonic motion, the realization of analyses according to earthquake motion becomes difficult. However, in this study, equations for both rigid and flexible silos, derived by Veletsos and Younan, and taking into consideration harmonic motion, were modified for use. Some corrections are needed on this approach, which will taken into consideration the effects of strong ground motion. These can be achieved by obtaining each response in the frequency domain. The ground motion is converted into the frequency domain from the time domain using the technique of fast Fourier transform. The dynamic material pressures, base shear force and overturning moment can be obtained, respectively, by using the obtained definition of the ground motion in the frequency domain. The responses obtained in the frequency domain are later converted to the time domain using the inverse Fourier transformation technique. Using this approach, which is derived for harmonic motion, the responses to the seismic actions of the silos for ground motion are obtained. A program code was written with respect to the procedure of this approximation within the scope of this study (EAS 2013).

The model developed in this study is able to take into account the behavior of the bulk material, the geometry and the rigidity of the load bearing system. In this proposed approach, it is assumed that a specific portion of the granular material is moved along with the silo wall. Moreover, the silo's wall contributes to the rigidity of the system when it comes to calculating the dynamic material pressure acting on the wall exposed to the horizontal component of the earthquake motion. It is assumed that the residual mass, which is not taken into consideration, has no effect on the wall pressure and on the stiffness of the system. In light of these assumptions, it is possible to think of the whole system as a single mass system by taking into consideration a specific part of the mass of bulk material with the mass of the silo structure. Supporting systems with a cylindrical geometry and bulk material shaped according to the geometry of the silo structure, are fixed to the base. The mass which is distributed along the silo height can be considered as a continuous mass. Thus, in terms of continuity of the structural system, although it is possible to model the system with distributed mass and stiffness, it can also be modeled as a system with equivalent mass. In this case, the considered model for a ground supported, flat bottomed bulk material–silo system is shown in Fig. 3.

According to this approach, the rigidity of the system can be obtained by considering the system as a cantilever beam. Horizontal stiffness of the system, k_s, taking into account the contribution of the wall and the bulk material stiffness, is determined with Eq. 6.

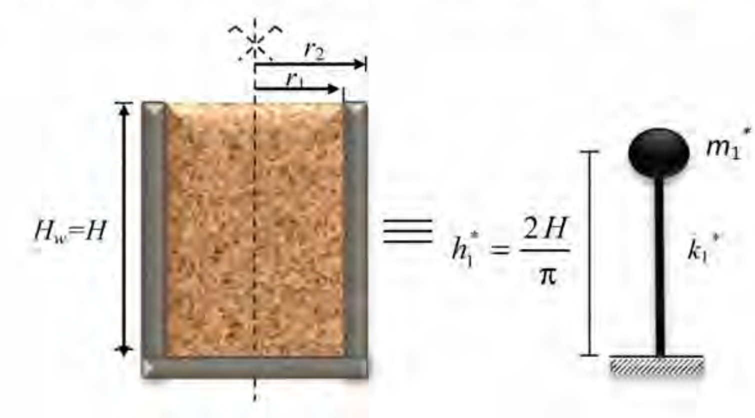

Fig. 3 Proposed analytical model for bulk material–silo system

$$k_s = \frac{3E_w\pi\left(r_2^4 - r_1^4\right)}{4H_w^3} + \frac{3E_m\pi r_1^4}{4H^3} \tag{6}$$

where E_w, E_m and H_w, H are the wall and the bulk material's elasticity modules and the heights, respectively. r_1 and r_2 are the internal and external radius of the silo, respectively.

Another important issue to be determined is the amount of the mass of bulk material. Both Eurocode and ACI, widely accepted codes, indicate that 80 % of the mass should be taken into consideration. In addition to these approaches, the Veletsos–Younan approach can be used and the amount obtained as illustrated by using Eq. 7.

$$\Omega = \frac{16}{\pi^3}\frac{H}{r_1}(\psi_o g_1 + h_1)\sqrt{\frac{1 + i\delta}{1 - \phi_1^2 + i\delta}} \tag{7}$$

where H and δ are the height and damping ratio of bulk material, respectively, and r_1 is the internal radius of the silo, and ϕ_1 is the ratio of angular frequency of the load to that of contained material. The required parameters in this equation can be determined as follows by using equations given in Veletsos–Younan approach (Eqs. 8–16).

$$\beta_1 = \frac{\pi}{2}\frac{r_1}{H}\sqrt{1 - \frac{\phi_1^2}{1 + i\delta}} \tag{8}$$

$$\psi_o = \sqrt{\frac{2}{1 - \upsilon}}, \alpha_1 = \frac{\beta_1}{\psi_o} \tag{9}$$

$$I_0(\alpha_1) = \sum_{m=0}^{\infty}\frac{(-1)^m \alpha_1^{2m}}{2^{2m}(m!)^2} \tag{10}$$

$$I_0(\beta_1) = \sum_{m=0}^{\infty}\frac{(-1)^m \beta_1^{2m}}{2^{2m}(m!)^2} \tag{11}$$

$$I_1(\alpha_1) = \sum_{m=0}^{\infty}\frac{(-1)^m \alpha_1^{2m+1}}{2^{2m+1}m!(m+1)!} \tag{12}$$

$$I_1(\beta_1) = \sum_{m=0}^{\infty}\frac{(-1)^m \beta_1^{2m+1}}{2^{2m+1}m!(m+1)!} \tag{13}$$

$$\Delta_1 = \alpha_1 I_0(\alpha_1)[\beta_1 I_0(\beta_1) - I_1(\beta_1)] - \beta_1 I_0(\beta_1) I_1(\alpha_1) \tag{14}$$

$$g_1 = \frac{1}{\Delta_1}\alpha_1 I_1(\alpha_1)[2I_1(\beta_1) - \beta_1 I_0(\beta_1)] \tag{15}$$

$$h_1 = \frac{1}{\Delta_1}\beta_1 I_1(\beta_1)[2I_1(\alpha_1) - \alpha_1 I_0(\alpha_1)] \tag{16}$$

where v is Poisson's ratio of bulk material. ψ_o, β_1, α_1, Δ_1, g_1 and h_1 are dimensionless factors. I_0 and I_1 are modified Bessel functions of the first kind and zero and first order, respectively.

In this proposed model, the performance of a parametric study is required for determination of the mass ratio. Here, a squat ($H/d_c = 1.5$) and a slender ($H/d_c = 2.5$) silo were selected and the following variations were obtained (Fig. 4). It is worth mentioning that these equations can also be used to calculate other ratios regarding slenderness of silos. As the figure illustrates, for possible dominant frequencies of ground motion, the mass ratio to be taken into consideration is around 80 %. This ratio can reach 100 % via a much lower frequency range for slender silos as compared to squat silos. However, when the actual dimensions of the silo and recorded frequencies of strong ground motions are considered, observations acknowledge that 80 % of this ratio is normal as stated in codes. Therefore, this study clearly shows that it is appropriate to say that this value can be considered as 80 % for the use of the proposed model in practice. In this study, the value of the aforementioned ratio was calculated via programming codes written specifically for the purposes of this study. The parameter values considered in the analysis for the determination of the mass ratio are given in Table 1.

The considered total mass can be obtained in terms of bulk material (m) and silo wall mass (m_w) by using the modal contribution factor, which can be selected from Fig. 4, with the following Eq. 17.

$$m_1^* = m_w + \Omega \cdot m \tag{17}$$

When appropriate simplifications are applied, an effective height can be obtained in terms of bulk material height, as in Eq. 18, by using the relations between moment-shear force derivations given by Veletsos–Younan (Younan and Veletsos 1998; Veletsos and Younan 1998).

$$h_1^* = \frac{2H}{\pi} \tag{18}$$

The dynamic characteristics of the proposed model can be determined using the system stiffness and the total mass obtained from Eqs. 6 and 17, respectively. Here, for example, the frequency of the system (f_1^*) is calculated from Eq. 19.

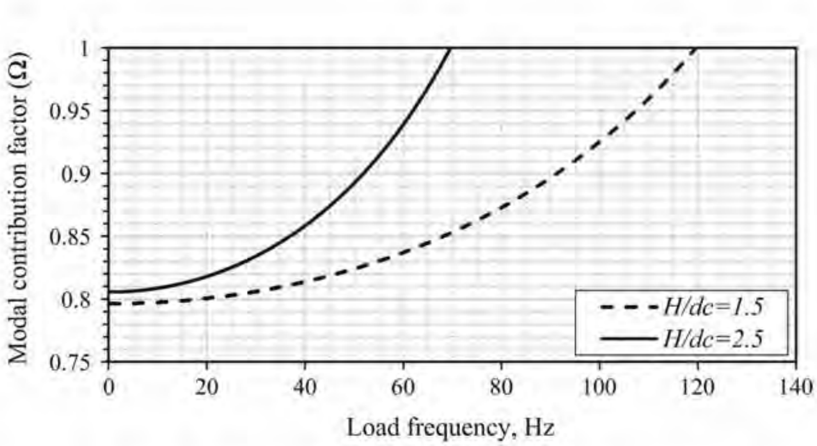

Fig. 4 The variation of modal contribution factors according to loading frequency for a squat ($H/d_c = 1.5$) and a slender ($H/d_c = 2.5$) silo

Table 1 The parameter values considered in the analysis for the determination of the mass ratio

Damping ratio of bulk material, δ	Poisson's ratio of bulk material, ν	Silo height, H (m)	Internal radius of silo, r_1 (m)
10 %	0.3	15 and 25	5

$$f_i^* = \frac{1}{2\pi}\sqrt{\frac{\frac{3E_w\pi\left(r_2^4-r_1^4\right)}{4H_w^3}+\frac{3E_m\pi r_1^4}{4H^3}}{(m+m_w)\cdot 2/3}} \tag{19}$$

In the case of lumped mass approaches for fixed based systems, a part of the total mass of height above the ground is assumed to vibrate with the ground as one part. Thus, this portion of the mass has no effect on the dynamic characteristics of the system. Accordingly, in this approach, the mass was reduced by a factor of 2/3, which is a common solution (ACI317R-98 1995; Housner 1963).

The reaction of the proposed single mass system exposed to ground motion can be obtained by solving the equation of motion in general terms. Although the three components of ground motion can be included in this equation, the reactions obtained according to one of the horizontal components of movement will generally be sufficient to calculate pressure. Therefore, the other components regarding ground motion may be omitted. The equation of motion can be written using Eq. 20.

$$\ddot{u}_1(t) + 2\zeta\omega\dot{u}_1(t) + \omega^2 u_1(t) = -\ddot{u}_g(t) \tag{20}$$

The relationships found between ground motion and variation of displacement, velocity and acceleration in time, occurred at mass level, which were determined by the ground motions and dynamic characteristics of the established system. This equation can be solved easily using numerical integration techniques or direct solution methods. The variation of the acceleration in time, occurring at the mass level and according to the ground motion, will change according to the height of the silo. This calculation can be obtained by using a console beam with a uniform and distributed mass. To this end, acceleration can be obtained from Eq. 21, according to the assumption that the acceleration will reach the maximum value at the lumped mass level.

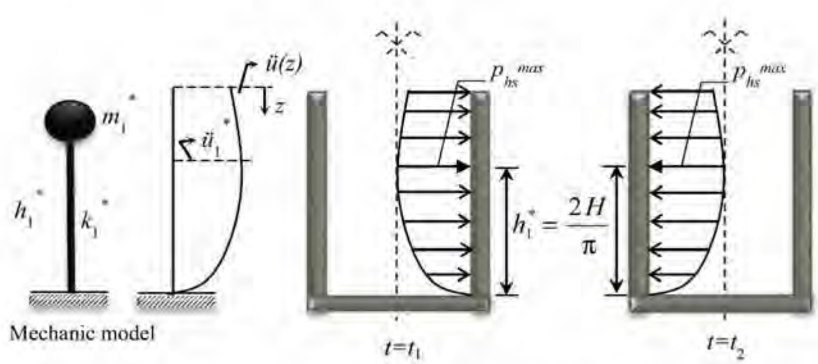

Fig. 5 The distribution of dynamic material pressures along the height of the silo

$$\ddot{u}(z,t_i) = \frac{-3{,}363 \cdot z^4 + 0{,}481 \cdot H^3 \cdot z + 2{,}882 \cdot H^4}{3 \cdot H^4} \tag{21}$$

Here, H shows the height from the upper surface of the silo base to the upper surface of the bulk material. Thus, the distribution of acceleration along the height of the silo along with time (t) are obtained. The dynamic material pressure (p_{hs}) on the per unit area of the wall can be obtained from Eq. 22 (see Fig. 5).

$$p_{hs}(z,t) = R \cdot \rho \cdot \left[\ddot{u}_1(z,t) + \ddot{u}_g(t)\right] \tag{22}$$

In addition to displacement, velocity and acceleration responses, equivalent shear force and equivalent bending moments can be obtained for the dynamic response of a single degree of freedom system by means of the proposed model. It is worth mentioning that the values, occurrence instants and heights of responses are equal for opposite sides of the silo wall. When it comes to earthquake direction, according to the proposed analytical model and due to the simplifications in the model, such as neglecting the contact mechanism, the opposite walls of the silo will experience the same factors.

2.4 Finite element model for numerical approximation

In addition to the above-mentioned simplified approximations, the finite element model was also used (Fig. 6). The seismic action effects were calculated on the basis of an elastic approximation, since the considered simplified approximations are based on elastic behavior. Bulk solids exhibit complex mechanical behavior such as anisotropy, stress or strain dependency, plasticity, dilatancy and so on. Most of these characteristics are present during earthquake loadings. However, any of these features needs sophisticated mathematical modelling and in parallel, a rigorous measurement and determination of the mechanical properties of the bulk material (Rombach and Martinez 2009). The silo walls are exposed to additional stress resulting from asymmetrical pressure distributions that take place in the silo during seismic loadings. These pressure arrangements can lead to ovalization of the silo wall, especially for silos with a diameter to height ratio equal to or less than one. Later, dynamic loads lead to compactions of the bulk material as well as changes

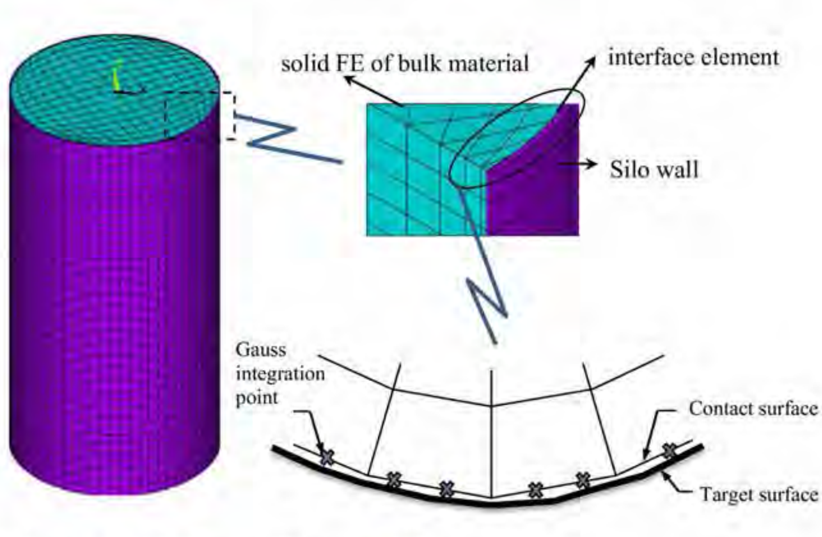

Fig. 6 Finite element models for silos (Durmuş 2013)

of material parameters such as the angle of internal friction (Braun and Eibl 2009). Elastic material models can not simulate such situations. When comparing the various approaches found in the technical literature in regards to varying versions of the hypo-plasticity theory the most useful approach is undoubtedly the intergranular strain approach as developed by Niemunis and Herle (1997). It is the most suitable material law available when it comes to describing the time dependent cyclic behavior of granular material (Holler and Meskouris, 2006). Moreover, although visco-elastic, elasto-plastic and hypoplastic material models have been used to represent bulk material, material tests and existing databases show that granular materials stored within a silo have scattered mechanical properties, which vary due to their conditions. Even bulk mass density may change as much as 40 % because of the effects of moisture on the contents. Present studies on the design of silos as outlined in the literature are not satisfactory, however, meaning that new studies with different approaches to the problem are essential to understanding the seismic behavior of the system. For this purpose, granular material was used as a model for solid elements and it was assumed that the behavior of the material would remain within elastic range. An isoparametric eight-node-brick element was used both for the bulk material and the silo wall. Finally, to determine the seismic behavior and response of the bulk material–silo system, a full transient dynamic analysis was carried out using the ANSYS (2012). In this analysis, Rayleigh damping approximation was chosen in order to take damping into account.

Another critical issue to be taken into consideration, when it comes to system interaction, is the contact mechanism. The literature outlines several procedures that can be used for simulating contact between two separate surfaces, however, not all of them will be appropriate for use as it depends on the given situation. Couto (2000) showed that the surface-to-surface contact algorithm is the most suitable method for three-dimensional analyses of silos. For this study, this contact algorithm was selected. Both the target surface and the contact surface have to be specified in this contact algorithm. Due to greater rigidity, the target surface was selected as the silo wall and the surface of the bulk material was designated as contact surface (Fig. 6). The contact status between these two surfaces was regularly determined at Gauss integration points. The Coulomb friction model was used to model the interaction between the bulk material and the silo wall. According to the chosen contact behavior, the interface elements enable load transfer by friction while allowing local loss of contact and separation. If contact is present, the appropriate shear forces according to the Coulomb friction law are transferred within the element pair.

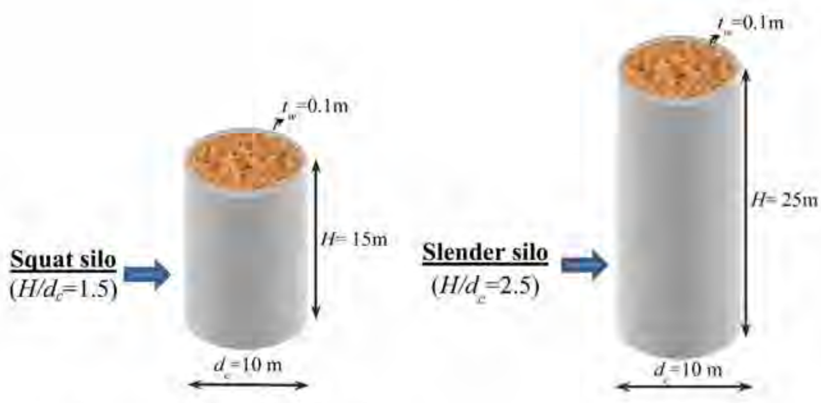

Fig. 7 Geometrical properties of considered silos

Conversely, normal pressure is equal to zero if separation occurs. Thus, tension stresses cannot occur at the interface as in reality.

3 Description of considered bulk material–silo system

The silo investigated in this study constituted a flat bottom cylindrical type silo, situated on the ground (Fig. 7). Two different silo heights were selected to simulate squat and slender silo behavior. Both silos have 10 m diameter and their heights are 15, and 25 m, respectively (Fig. 7). The material of the silo is reinforced concrete (RC) and different storage capacities were looked at, in relation to their geometrical properties as given in Fig. 7. The Young's modulus, unit mass, Poisson ratio and the material damping ratio of RC, were interpreted as 28,000 MPa, 2500 kg/m^3, 0.2 and 5 %, respectively. All the systems were assessed as having been filled with granular materials such as wheat. Additionally, the Young's modulus, unit mass, Poisson ratio and the material damping ratio of considered stored material, were interpreted as 5 MPa, 900 kg/m^3, 0.3 and 10 %, respectively (Ayuga et al. 2001; EN-1991-4 2006).

In analysis of the numerical model utilized, between the walls of the silo and the stored material, the contact mechanism was taken into consideration. The wall friction coefficient is the only parameter needed for this model, and it can be defined via two different ways. It can be obtained by a simple shear test or it can be obtained by using the standard values given in well-known guidelines. Eurocode's friction coefficient between concrete and wheat (EN1998-4 2006) identified as 0.57, was used as the coefficient in the study.

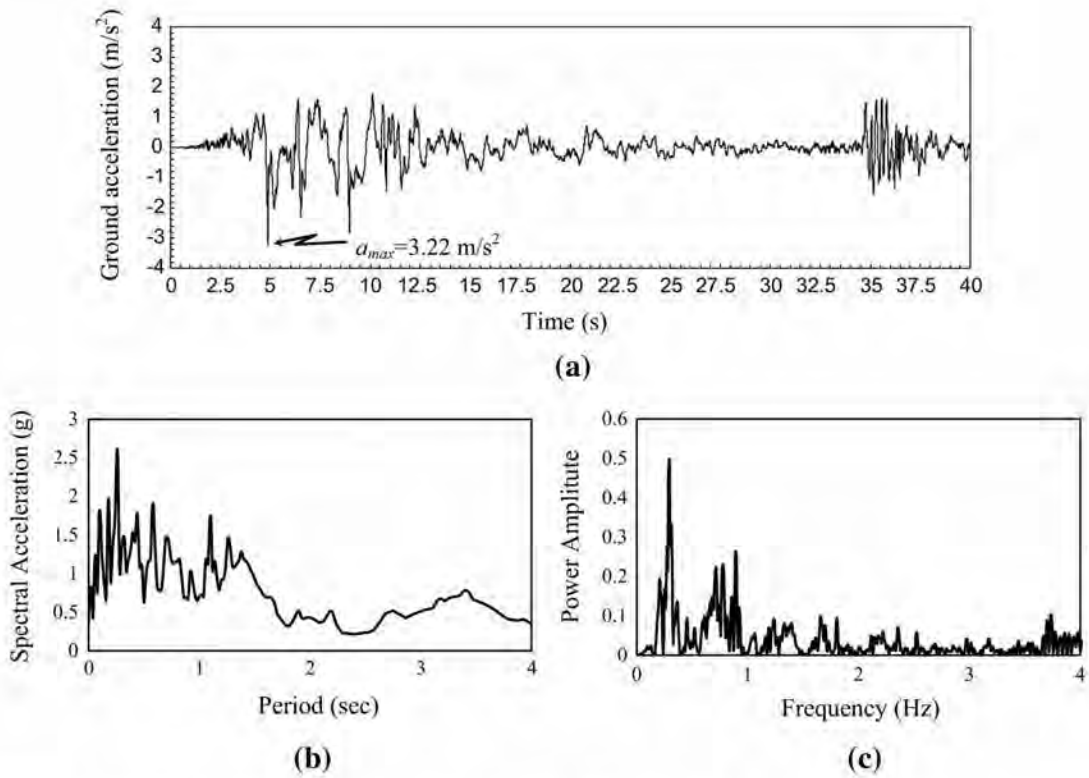

Fig. 8 Acceleration time history (**a**), response spectrum (**b**) and power spectrum (**c**) of N–S component of Yarimca Station–Izmit Earthquake

A destructive seismic event, the 1999 Marmara Earthquake, was considered in the time domain transient analyses. The İzmit-Yarimca station N–S component was selected for use in the analyses. Figure 8 depicts this acceleration time history and frequency spectrum of the ground motion. The horizontal earthquake time histories were applied to the base of the model shown in Fig. 6. The vertical components were neglected in the analyses.

4 Discussion of the results from proposed analytical and numerical approximation

Determination of seismic behavior of silos requires the consideration of many factors such as, bulk material characteristics and the variability of contact status between the bulk material and the silo wall. For these reasons, the seismic analysis of silos becomes complicated. Such analyses can produce realistic results by using appropriate material models, realistic methods and properly defined boundary conditions. Despite the important developments in computer technology and numerical methods, designers generally do not prefer this approach due to the difficulty and extra time needed for modeling. In addition, any small errors made in modeling can lead to serious inaccuracies in determining the effects and the responses of the system. Therefore, producing analytical models that give approximate results by simplifying this complex problem with the help of a number of assumptions becomes inevitable for both designers and researchers. Several countries propose easily applicable methods for engineers by taking advantage of these analytical methods in their codes. Therefore, an analytical model for a bulk material–silo system has been proposed in this study. This proposed analytical approximation (PAM) was evaluated with the proposed numerical approximation (NM) that can take into account the bulk material–silo wall contact problem. Thus to achieve a more realistic representation of the complex system which have bulk material–silo interaction by the finite element method is possible in comparison with the analytical methods.

The use of analytical approaches, by researchers and designers alike, are important for referral. The assumptions and the limits of the model must be made aware to users. To address this, the results are discussed parametrically for silo systems investigated in this study. For bulk material–silo system the proposed analytical model (PAM; see Fig. 3) and the proposed finite element model (NM; see Fig. 6) will be compared in terms of various parameters. Under the following subtitles, the variations of dynamic material pressure,

Table 2 Maximum dynamic pressures, their occurrence instants and heights according to NM and PAM

Slenderness ratio (H/d_c)	Maximum dynamic material pressure, p_{hs}^{max} (kN/m^2)								
	NM						PAM		
	A side			B side					
	t (s)	H_o (m)	p_{hs}^{max}	t (s)	H_o (m)	p_{hs}^{max}	t (s)	H_o(m), $(2/\pi)H$	p_{hs}^{max}
1.5	7.0	9.5 (0.63H)	24.97	9.0	8.5 (0.57H)	38.60	8.96	10.0 (0.637H)	30.58
2.5	7.0	9.5 (0.38H)	25.22	9.0	7.5 (0.30H)	38.02	9.09	17.0 (0.637H)	23.42

horizontal displacement and the equivalent base shear force obtained from these models are given comparatively.

4.1 Dynamic material pressure

According to PAM and NM, the obtained peak values of dynamic material pressure at the opposite sides of the wall in the earthquake direction ($p_{hs}^{\max}$), and their occurrence instants (t) and heights from the bottom of the silo (H_o) are presented in Table 2. These values obtained from the heightwise variation of the dynamic material pressures at the time step which gives the maximum resultant force.

It is seen from Table 1 that NM gives different values for maximum dynamic material pressures and their occurrence instants at both sides in the loading direction, due to the contact problem. While this behavior can be clearly noticed for the NM, the PAM, however, is not capable of such behavior. In addition, the values of the occurrence heights of these pressures are different at both sides and for different slenderness ratios. The maximum dynamic material pressure was obtained at a constant height, 0.637*H*, for all slenderness ratios according to PAM. This is an assumption of this model. It should also be noted that the earthquake load acts in proportion to the mass. There is a suggestion that the pressure differences can be the result of the asymmetric earthquake load, in addition to the contact mechanism. Thus, the appropriate contact algorithm must be selected. The opposite sides of the silo walls are identified according to the earthquake direction and a cross-sectional area of the center of the silo. The left side is identified as the A side and the right side is identified as the B side.

Maximum dynamic pressures along the height of the silos for PAM and NM, are illustrated in Fig. 9. Here, x shows the vertical distance from the bottom of the stored content. These comparisons were realized by considering the dynamic pressure variations along the silo height that gives the maximum base shear force and then determining their occurrence instants for each silo. These occurence instants were the same as those of equivalent base shear forces. It can be observed from Fig. 9 that the occurrence height of the maximum pressure decreases by the increase of slenderness ratio according to NM. In

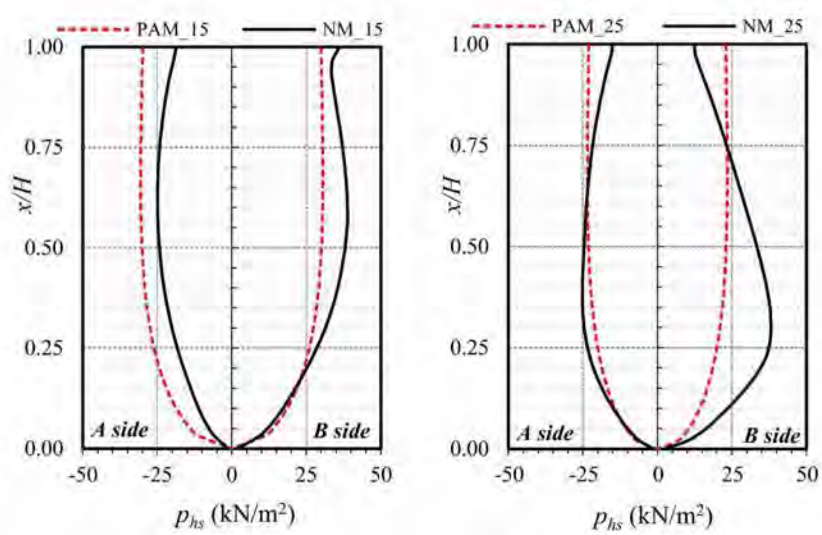

Fig. 9 Comparisons of the maximum dynamic pressures throughout the height of the silo for two different slenderness ratios via PAM and NM

addition, assuming the occurrence height of maximum dynamic pressure as constant, $H_o = (2/\pi)H$, for PAM means that it is able to represent the behavior sufficiently for a squat silo while for a slender silo this relation is not valid. This case does not cause any significant difference for equivalent shear force but it does calculate a larger equivalent bending moment, according to PAM, due to determining the resultant force position from the base at a higher level.

Figure 10 shows the deviations of maximum dynamic material pressure responses obtained via two different approximations, PAM and NM. As can be seen from Fig. 10, at A side PAM and NM findings are quite similar. As for B side, the maximum difference reached 22 % for a squat silo and 38 % for a slender silo. Nevertheless, when deviation throughout height is considered, PAM gives similar results to NM for squat silos as well as giving smaller response values compared to NM for slender silos at B side.

As it can be seen from comparisons, PAM gives rather close results to NM at the A side when it comes to determining dynamic pressures. Also, it can be determined from this figure that PAM gives quite good results at both sides for a squat silo and it can represent similar distribution for a slender silo.

4.2 Horizontal displacement

According to PAM and NM, the obtained peak values of lateral displacements at the opposite sides of the wall in the earthquake direction ($u^{\max}$) and their occurrence instants (t) are presented in Table 3. In fact, these horizontal displacements at opposite sides are expected to happen in the opposite direction to each other, at different instants and values due to the interaction of the bulk material and the silo wall. However, PAM is not capable of considering such behavior. Especially for 2.5 of slenderness ratio, obtaining the same occurrence instants of these responses at B side via PAM and NM may be interpreted that PAM is able to represent the behavior for this slenderness ratio more sufficiently for the considered ground motion and material properties. Observations revealed that the displacement response of squat silos is rather small and this response values can be ignored. However, the displacement response didn't reached to significant levels with increasing slenderness ratio, as would normally be expected.

The differences in behavior can be seen much more clearly from the results of the numerical model by examining the variation of horizontal displacements with time. As

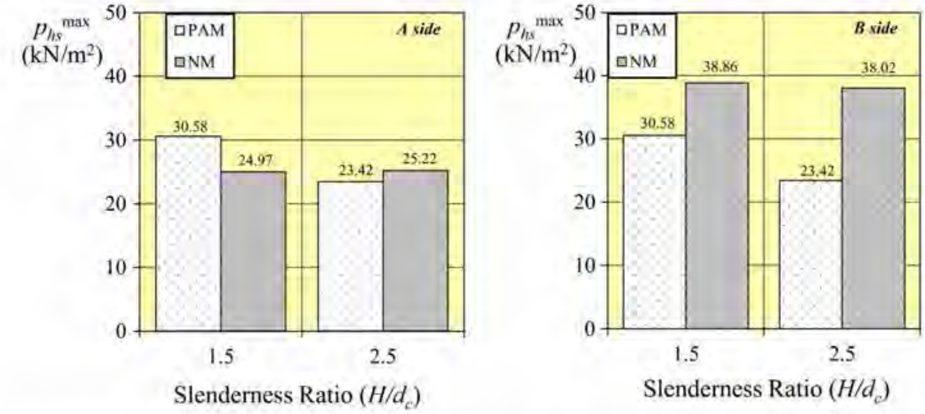

Fig. 10 Maximum dynamic material pressure variations for two different slenderness ratios according to PAM and NM

Table 3 Maximum horizontal displacements and their occurrence instants for two different slenderness ratios via NM and PAM

Slenderness ratio (H/d_c)	Maximum horizontal displacement, u^{max} (m)					
	NM				PAM	
	A side		B side			
	t (s)	u^{max}	t (s)	u^{max}	t (s)	u^{max}
1.5	7.05	−0.0032	4.95	0.0051	8.97	0.0031
2.5	7.10	−0.0101	9.10	0.0148	9.08	0.0244

Fig. 11 shows, the horizontal displacements of silo wall were obtained in the normal directions for both sides towards the outside of the silo and the horizontal displacement results were not obtained in the other direction during the ground movements experienced by squat silo via NM. Movement is concentrated on the outside as opposed to the inside of the silo. This is a fact that arises due to the slenderness ratio of the silo. In this case, the silo's dominant modes are related to their cylindrical geometry. Because of the mode shapes of the squat silos, the silo does not behave as a cantilever beam and the mode shapes is so different in comparison with those from the slender silos. These characteristic behaviour triggered by the contact behaviour considered in this study cause one-sign displacement. It should also be noted that cantilever behavior is dominant when it comes to a slender silo. Thus, slender silos behave like a cantilever beam for especially initial modes and this characteristic makes two-sign vibration possible.

The results obtained from PAM exhibited differences in character from those obtained from NM for $H/d_c = 1.5$ (Fig. 11), but behaviors according to NM and PAM almost overlapped for $H/d_c = 2.5$ (Fig. 12). From the two methods utilized, it is clear that response magnitudes are different along the same instants, while the occurrence instants of the silo response to the earthquake, are similar. Therefore, differences in character mean that both occurrence instants of the responses and response magnitudes are different according to these two methods. This occurs because of the mode shapes of the squat silos where the silo does not behave as a cantilever beam. Indeed, the mode shapes of squat silos

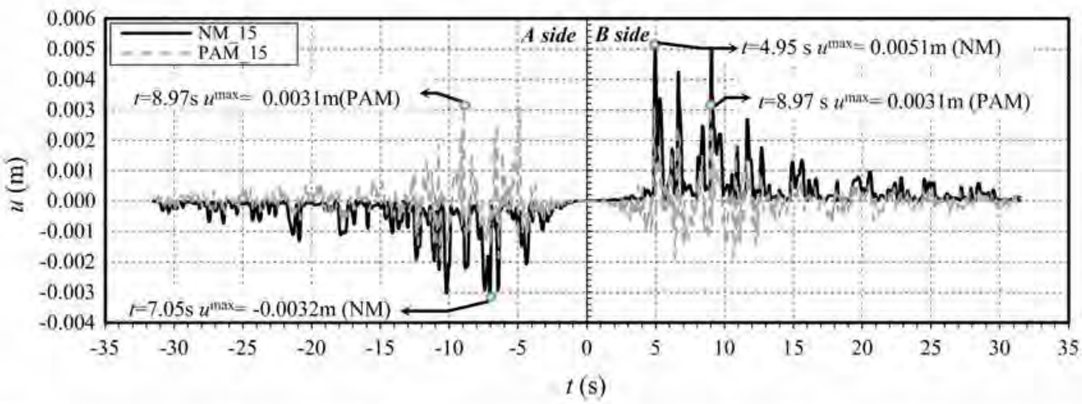

Fig. 11 Variations of horizontal displacements in time at opposite sides of the wall in the earthquake direction for PAM_15 and NM_15

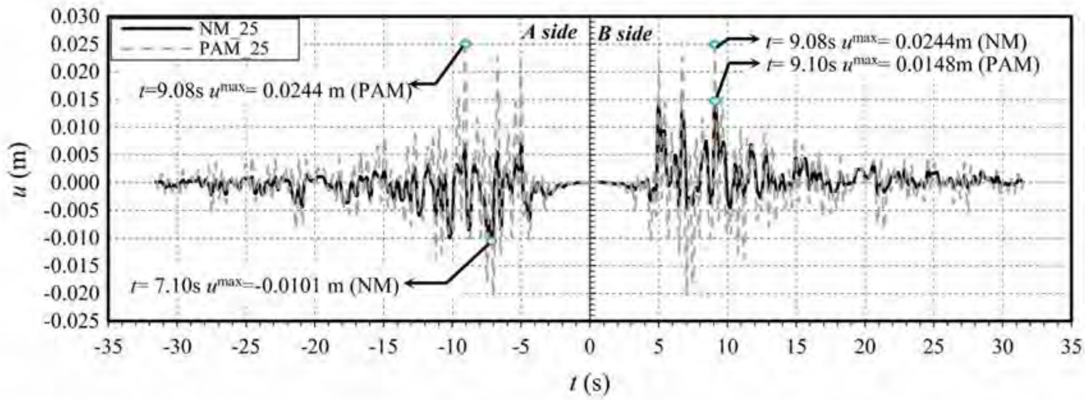

Fig. 12 Variations of horizontal displacements in time at opposite sides of the wall in the earthquake direction for PAM_25 and NM_25

are very different to those of slender silos and, in fact, squat silos do not have complete flexural modes. This characteristic behavior, triggered by the contact behavior considered in this study, causes one-sign displacement. Conversely, slender silos behave like a cantilever beam, especially for initial modes meaning this characteristic makes bi-directional vibration possible.

Maximum horizontal displacements along the height of the silo for two different slenderness ratios via PAM and NM are illustrated in Fig. 13. The evaluations of these comparisons were similar to the evaluations of Figs. 11 and 12. It can be seen from Fig. 13, that the distributions along the height of the silo via PAM and NM have almost similar shapes for $H/d_c = 2.5$, in behavior, but are different in behavior where $H/d_c = 1.5$. As mentioned before this characteristic behavior, triggered by the contact behavior and slender silos behave like a cantilever beam. However for squat silos the behaviour is so different because of the initial modes meaning this characteristic makes bi-directional vibration possible. For this purpose, a modification factor should be used to determine the displacement responses for especially squat silos. Results obtained from this study and the

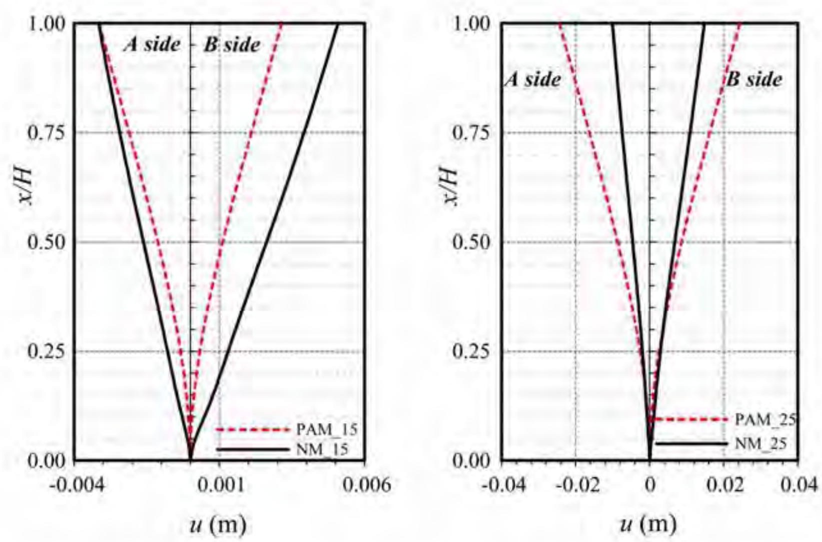

Fig. 13 Comparisons of the maximum horizontal displacements throughout the height of the silo for two different slenderness ratios via PAM and NM

other studies carried out by authors, show that displacement changes considerably in conjunction with a decreasing slenderness ratio. In this study, a modification factor of 3 is suggested; however, this only takes into consideration a strong ground motion result in the design process. The factor can be calculated at 2 when looking at the mean result from more than three strong ground motion records.

4.3 Equivalent base shear force

The obtained peak values and their occurrence instants for the maximum equivalent base shear via PAM and NM are given in Table 4. The presented equivalent base shear is not the same with the total base shear force. The equivalent base shear represents the force which occurs throughout the unit width of the silos' wall, however, it is the same as the behavior and character of the total base shear force. It is worth mentioning that these responses were obtained from the maximum dynamic pressure, as observed throughout the height of the silo, for each time step.

The deviations of maximum equivalent base shear responses obtained for two different slenderness ratios via PAM and NM are given in Fig. 14, and their variations in time are given in Figs. 15 and 16, comparatively. The equivalent base shear variations for both sides of the silo wall reached their peak values at different instants, similar to the dynamic material pressure results.

As it can be seen from comparing peak values, PAM gives quite similar results to those obtained via NM at the B side, while NM gives quite small results for the squat silo compared to PAM at the A side. Equivalent base shear value of the PAM, in this study, is determined as being 34 % larger than NM for the squat silo, while the value is quite similar to NM for the slender silo at the A side. As for the B side, this response value, respectively, is 14 and 23 % smaller than NM for the squat silo and the slender silo.

It can be seen from Figs. 15 and 16 that both models estimated the behavior quite similarly when examining the variation of equivalent base shears in time. As mentioned earlier, during seismic activity the pressure on the silo wall undergoes change due to separation and renewed contact of these materials. These effects can be modeled via NM; the equivalent base shears resulting in the maximum value in one direction and resulting in zero in the other direction.

As the comparisons show, the equivalent base shear response variations in time overlapped each other. It is worth mentioning that PAM will give negative values for dynamic pressures due to the lack of contact modeling. This is not a realistic approach for silo bulk

Table 4 Maximum equivalent base shear and their occurrence instants for two different slenderness ratios obtained from PAM and NM

Slenderness ratio (H/d_c)	Maximum equivalent base shear force, V_e^{max} (kN/m)					
	NM				PAM	
	A side		B side			
	t (s)	V_e^{max}	t (s)	V_e^{max}	t (s)	V_e^{max}
1.5	7.00	301.69	9.00	471.53	8.96	−404.44
2.5	7.00	516.08	9.00	671.18	9.09	−517.19

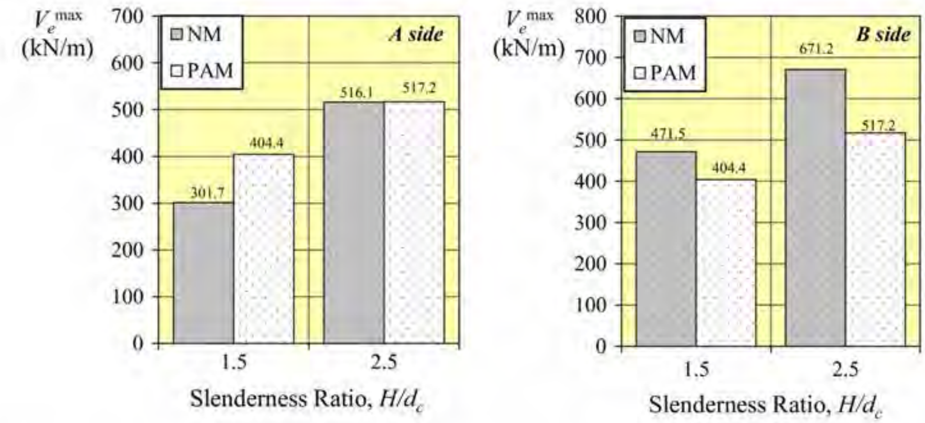

Fig. 14 Maximum equivalent base shears for two different slenderness ratios at the opposite sides of the silo wall in earthquake direction via PAM and NM

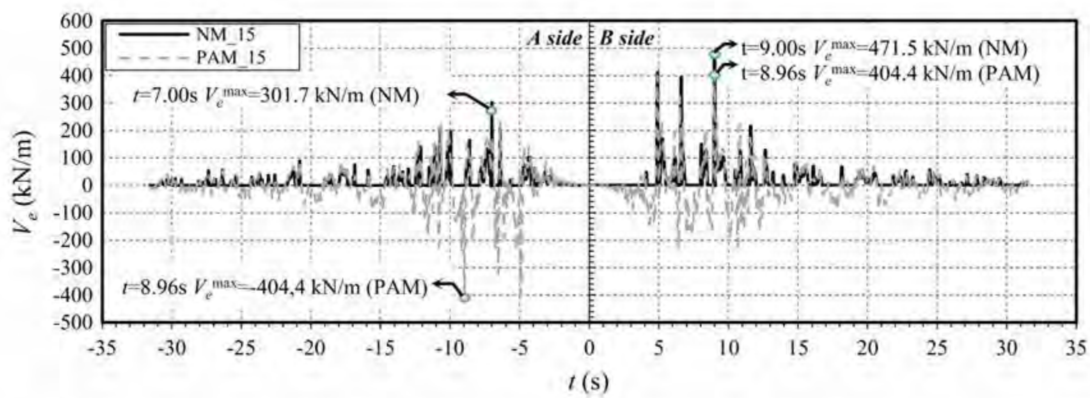

Fig. 15 Variations of equivalent base shears in time at opposite sides of the silo wall in the earthquake direction for slenderness ratio 1.5 via PAM and NM

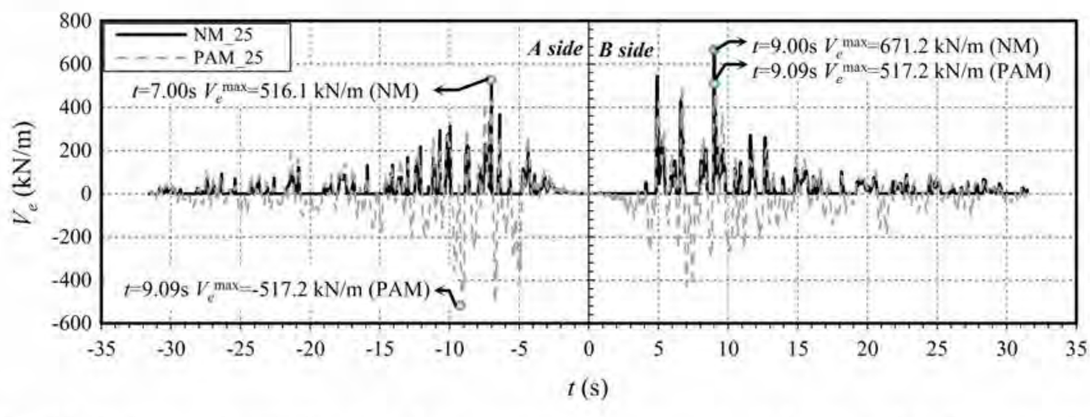

Fig. 16 Variations of equivalent base shears in time at opposite sides of the silo wall in the earthquake direction for slenderness ratio 2.5 via PAM and NM

material interaction. Besides, it can be also seen that the obtained positive peak values of equivalent base shears resulted in a similar direction.The peak value of this response was obtained at 7 s via NM at A side of the silo wall for both slenderness ratios. However, when this response was evaluated for H/dc = 2.5 via PAM at A side around 9 s, this response was obtained as 432 kN/m and PAM reached the maximum positive value at 7 s, similar to NM. The same conclusion can be obtained for a squat silo system.

5 Discussion of the results from proposed analytical approximation, EN-1998-4 and Veletsos–Younan approximation

Obtained dynamic material pressure responses from proposed analytical model (PAM), the approach proposed by EN1998-4 (2006, EC), and the model proposed by Veletsos–Younan modified for ground motion (VY) are examined here, comparatively. While the considered ground motion record was used for a time history analysis of the PAM and VY models, the acceleration response spectrum of the record was used to obtain the results along its height for the approach proposed by EN1998-4 (2006) code. It is worth noting that the response acceleration values, corresponding to angular frequency values obtained from PAM, were considered for these comparisons. The dynamic material pressure acted as the reference point in these evaluations and the accuracy of the obtained responses according to these models was discussed. The obtained maximum dynamic material pressure responses at the opposite sides of the silo wall in earthquake direction and their occurrence instants and heights from the base according to these models for two different slenderness ratios are presented in Table 5. These maximum dynamic pressure variations and comparisons of them along the silo heights are given in Figs. 17 and 18, respectively.

The dynamic material pressure distributions, their variations in time and the occurrence instants of the peak values of these responses can be estimated sufficiently via PAM by comparing these response values with NM results while considering the contact problem. Therefore, PAM will be regarded as the reference model in evaluating the comparisons with NM here. Thus, it can be said that for all the silo systems modeled, the dynamic pressure responses obtained by the VY model were rather small. Observations also showed that dynamic pressure values obtained from EC were smaller than PAM, and accordingly smaller for NM squat silos as well. Therefore, the EC method becomes less safe than PAM and NM for squat silos in this regard.

It can be suggested that the VY method would always give rather small pressure values for such silo systems. On the other hand, it can also be said that, using EC produced unsafe results compared to the results of using PAM. Moreover, due to the separation found between the bulk material and the silo wall, there will be different dynamic pressures at opposite sides of the silo's walls in the earthquake direction and that the estimated pressures sometimes would be larger than those identified by PAM. In this case, the smaller

Table 5 Maximum dynamic material pressures, their occurrence instants and heights, from base, for two different slenderness ratios obtained from EC, VY and PAM

Slenderness ratio (H/d_c)	Maximum dynamic material pressure, p_{hs}^{max} (kN/m^2)							
	EC		VY			PAM		
	H_o (m)	p_{hs}^{max}	t (s)	H_o (m)	p_{hs}^{max}	t (s)	H_o (m), $(2/\pi)H$	p_{hs}^{max}
1.5	10.0 (0.637H)	19.75	4.90	15 (H)	7.62	8.96	10.0 (0.637H)	30.58
2.5	17.0 (0.637H)	20.12	11.56	25 (H)	11.70	9.09	17.0 (0.637H)	23.42

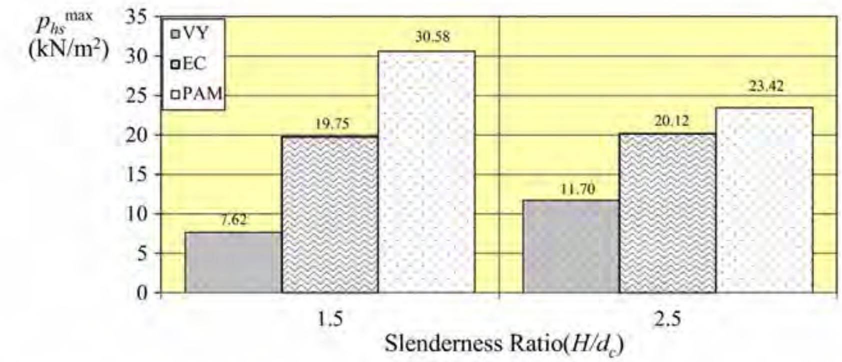

Fig. 17 Maximum dynamic material pressures for two different slenderness ratios via PAM, EC and VY

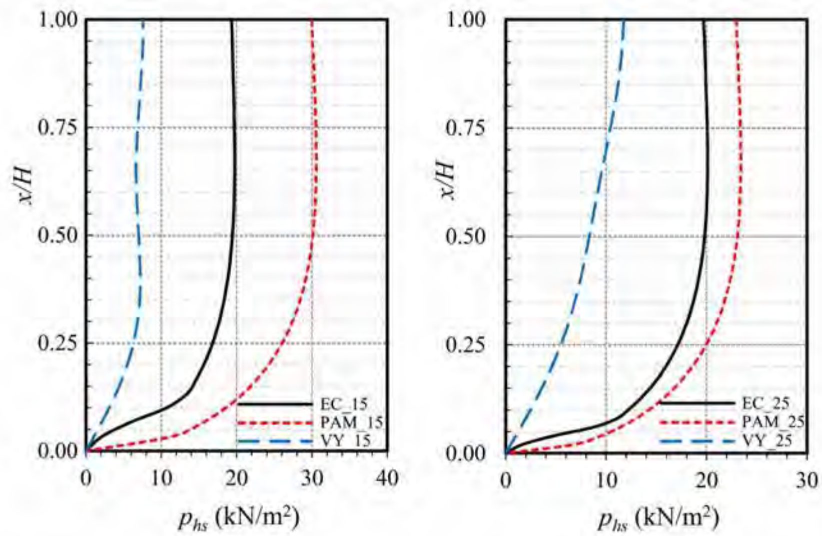

Fig. 18 Comparison of the maximum dynamic material pressure along the height of the silo wall for two different slenderness ratios via VY, EC and PAM

dynamic pressure response, as estimated by EC, is even more critical and smaller than PAM. Proportionally, the dynamic material pressure results obtained from EC are 35 % smaller than those obtained from PAM for squat silos. As for slender silos, this ratio is 14 %. Thus, it would be appropriate to say that EC gives safer results for slender silos when compared to squat silos.

These analyses can be clearly understood from Fig. 18. When looking at these comparisons, the VY and the EC methods produce quite small response values for squat silos and also have difficulty in representing the behaviour of such systems. The responses via EC and PAM are quite similar for slender silos.

6 Conclusions

In this paper, a simplified analytical model and a three dimensional finite element model considering bulk material–silo wall interaction effects, were proposed to evaluate the seismic behavior of the bulk material–silo system. The procedures can be used to determine not only the structural response of the silo system but dynamic material pressure distribution along the height of the wall. The results obtained from the proposed analytical model were compared with those obtained from numerical models to prove the reliability of the proposed analytical model. Later findings, obtained from the proposed analytical model, were compared with those obtained via EN1998-4 and the modified Veletsos and Younan approximations in order to check the procedures which are commonly used and/or based on the other current approximations in the literature. Analyses and discussions deduced the following conclusions:

Bulk material–silo wall interaction considerably affects the seismic behavior of silos. It is clearly seen from the findings of the analytical and numerical analyses, that consideration of the contact mechanism is quite significant both for determining the behavior of the system and the magnitudes of the responses. Therefore, due to the contact mechanism taken into account in the numerical model (NM), different responses occured at the opposite sides of the silo wall in the earthquake direction in terms of magnitude, occurrence instant and height. It is obvious that silo seismic behavior varies depending on the characteristics and interaction of two different physical medium. Therefore, in case of a reversible but antisymmetric loading activity, such as an earthquake, it is clear that different pressure values are obtained at both sides of the wall in the loading direction and accordingly system behavior will cause different responses at the sections symmetrical to the center of the silo. When it comes to dynamic loading, the dominant modes that control the response of the system will be different according to the medium used meaning that the resulting effects will vary at opposite sides.

Proposed analytical model (PAM) gives considerably good results in terms of dynamic material pressure and equivalent base shear, both for the maximum values of the responses and the behavior of the system. However, due to the assumption that the occurrence height of the maximum dynamic pressure is constant, the method can result in large bending moments.

Dynamic pressure results obtained from PAM are quite close to those from NM. This means that PAM estimates the behavior of the stock material and accordingly, the variation of the pressures are considerably safer than those obtained from the other methods. However, displacements of the silo structure can be represented, via significant approximation, only for slender silos due to the significant differences in dominant modes depending on the geometry of the silo wall.

The dynamic material pressure, equivalent base shear and bending moment values obtained from the Veletsos–Younan approach, modified in this study for earthquake loading, resulted in small response values when compared to the other methods. Therefore, it can be said that this method is less safe than the other methods in predicting responses for earthquake behavior on silos.

The proposed approach by EN1998-4 (2006, EC) produced unsafe results for squat silos according to PAM. As for slender silos, EC can represent the behavior with a certain approximation. PAM results determined that the dynamic material pressure along the height of a silo does not cause fatal errors for a squat silo.

These conclusions are valid only for the considered models under the ground motion used in this study. In this regard, to generalize these conclusions, investigation on this subject should be carried out with different ground motions.