Journal of Constructional Steel Research 92 (2014) 40–45

Contents lists available at ScienceDirect

Journal of Constructional Steel Research

Buckling of axially loaded castellated steel columns

Wei-bin Yuan [a], Boksun Kim [b,*], Long-yuan Li [b]

[a] College of Civil Engineering and Architecture, Zhejiang University of Technology, Hangzhou, China
[b] School of Marine Science and Engineering, Plymouth University, Plymouth PL4 8AA, UK

ARTICLE INFO

Article history:
Received 30 April 2013
Accepted 6 October 2013
Available online 7 November 2013

Keywords:
Castellated columns
Buckling stability
Critical buckling loads
Shear effects
Energy methods

ABSTRACT

The majority of the existing literature on castellated members is focused on beams. Very little work has been done on the stability of castellated columns although they have been increasingly used in buildings in recent years. This paper presents a new analytical solution for calculating the critical buckling load of simply supported castellated columns when they buckle about the major axis. This analytical solution takes into account the influence of web shear deformations on the buckling of castellated columns and is derived using the stationary principle of potential energy. The formula derived for calculating the critical buckling load is demonstrated for a wide range of section dimensions using the data obtained from finite element analyses published by others. It was found that the influence of web shear deformations on the critical buckling loads of castellated columns increased with the cross-sectional area of a tee section and the depth of web opening, but decreased with the length and the web thickness of the column. It is shown that the inclusion of web shear deformations significantly reduces the buckling resistance of castellated columns. Neglecting the web shear deformations could overestimate the critical buckling load by up to 25%, even if a reduced second moment of area is used.

1. Introduction

Castellated beams have been used as structural members in structural steel frames [1]. An example is shown in Fig. 1. A castellated beam or column is fabricated from a standard steel I-shape by cutting the web on a half hexagonal line down the centre of the beam. The two halves are moved across by one spacing and then rejoined by welding [1]. This process increases the depth of the beam and hence the major axis bending strength and stiffness without adding additional materials. This allows castellated beams to be used in long span applications with light or moderate loading conditions in floors and roofs. The fabrication process creates openings on the web, which can be used to accommodate services. Despite the increase in the beam depth the overall building height can hence be reduced, compared with a solid web solution, where services are provided beneath the beam. This leads to savings in the cladding costs. Despite the increase in the fabrication costs caused by cutting and welding, the advantages outweigh the disadvantages.

Some design guidance on the strength and stiffness of castellated beams is given [1–3]. Due to the opening in the web, castellated beams are more susceptible to lateral-torsional buckling. Intensive research on the lateral stability [4–13] of castellated beams started in the early 1980s. Experimental investigations [4–6,9,13] were carried out and finite element methods [6–8,10–12] were used to predict the buckling behaviour of such beams and to compare the predictions with the results from the experiments [6,12]. The effects of slenderness on the moment-gradient factor [7] and of elastic lateral bracing stiffness on the lateral-torsional buckling [8,13] of simply supported castellated beams were studied using 3-D finite element models. The failure modes [4–6,9–11] and the interaction of the buckling modes [10] of castellated beams were investigated. It was found that the web opening of castellated beams had little influence on the lateral-torsional buckling [4] and the failure mode by lateral-torsional buckling of castellated beams was shown to be similar to that for solid web beams [5], while web distortional buckling was prone when an effective lateral brace was provided at the mid-span of the compression flange [9,13] and this type of failure reduced significantly the failure load [10] of slender castellated beams.

In recent years castellated members have also been widely used as columns in buildings [14]. The main benefit of using a castellated column is to increase its buckling resistance about the major axis. However, because of the openings in the web, castellated columns have complicated sectional properties, which make it extremely difficult to predict their buckling resistance analytically. Compared to a solid web column, the castellated column has weak web shear stiffness and thus the shear deformations are more pronounced when the column has a flexural buckling, which can significantly reduce the buckling capacity of the columns [14]. The effect of shear on the buckling capacity of built-up columns was reported by Gjelsvik [15], who showed that the columns exhibit reduced shear stiffness and this reduces their buckling capacity due to the increase in the lateral deflection. This indicates that the buckling theory taking into account shear deformations developed by Timoshenko and Gere [16] for solid web columns may not be suitable for castellated columns.

* Corresponding author. Tel.: +44 1752 586135; fax: +44 1752 586101.
E-mail address: boksun.kim@plymouth.ac.uk (B. Kim).

0143-974X/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.jcsr.2013.10.013

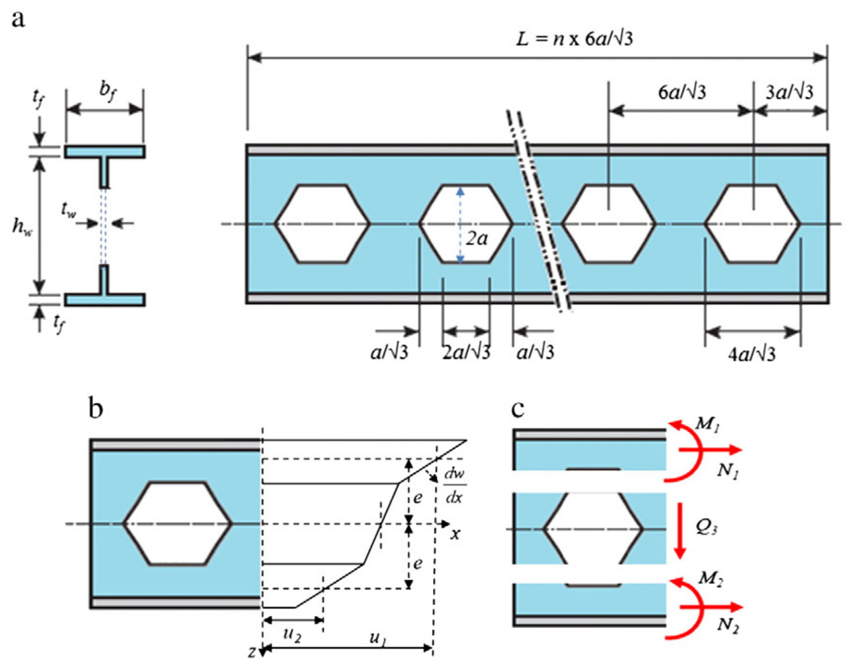

Fig. 1. Definitions of notations for (a) geometry where a is the half depth of a hexagon, (b) deformations, and (c) internal forces of a castellated member with hexagonal holes.

The above survey of the literature shows that the majority of the existing literature on castellated members dealt with the research into castellated beams using experiments and/or numerical studies. Very little work has been found on castellated columns. The stability of castellated columns was studied by El-Sawy et al. [14] using finite element methods. Their solution takes into account shear and flexural deformations for the calculation of the buckling capacity. However, their study used finite element methods and only numerical solutions were provided. To the best of the authors' knowledge, no analytical work is available to predict the buckling capacity of castellated columns.

In this paper an analytical solution using the energy method is presented to determine the buckling capacities of castellated columns. A simple close-form solution for determining the critical buckling load of simply supported castellated columns of doubly-symmetric sections, subject to axial compression load is developed. The critical buckling load derived is demonstrated using the results from the finite element analysis published in the literature.

2. Analytical study

The classical bending theory of beams, based on Bernoulli's hypothesis that the plane normal cross sections of a beam remain plane and normal to the deflected centroidal axis of the beam during deflection, ignores the deformation caused by shear forces. When a column buckles, however, the axial load causes not only bending moments in the cross sections, but also shear forces. This is particularly so in castellated columns because the web is flexible in shear. The deformations due to shear forces in castellated columns can be taken into account by using either the generalized form of the classical bending theory called Timoshenko beam theory [16] or the bending theory of sandwich beams [17]. In the former the assumption that the plane cross sections remain normal to the deflected beam axis is relaxed, that is, the slope of the deflected beam axis is no longer required to be equal to the rotation of the cross section. The difference of these two rotations is defined as the shear angle, which is produced by shear forces that are normal to the deflected beam axis. In the latter the outer layers of the sandwich beam are assumed to deform according to Bernoulli's hypothesis and the cross section of the middle layer behaves as a shear wall. However, the rotation of the middle layer due to shear forces does not need to be equal to the slope of the deflected beam axis.

In addition to the shear deformation, another difficult problem that arises in castellated members is the second moment of area that varies periodically from that of an "I-section" shaped beam (i.e. with no openings) to that of a "two-tee-section" shaped beam (i.e. with openings). This unique nature makes the castellated beam more like a sandwich beam, in which the two tee sections behave as the outer layers of the sandwich beam to take the bending moment, whereas the discontinuous parts of the web behave as the middle layer of the sandwich beam to take shear forces.

Consider a doubly-symmetric section castellated member shown in Fig. 1a, in which the flange width and thickness are b_f and t_f, the web depth and thickness are h_w and t_w, and the half depth of hexagons is a. The distance between the centroids of the top and bottom tee sections is $2e$ as shown in Fig. 1b. Let $u_1(x)$ and $u_2(x)$ be the axial displacements of the centroids of the top and bottom tee sections, and $w(x)$ be the transverse displacement of the section (i.e. all points on the section have the same transverse displacement). According to the displacement assumptions shown in Fig. 1b, the axial displacement at any point at the section with distance x from the origin can be expressed as follows:

For the top tee section, $-(h_w/2+t_f)\le z\le -a$

$$u(x,z)=u_1(x)-(z+e)\frac{dw}{dx}. \tag{1}$$

For the bottom tee section, $a\le z\le (h_w/2+t_f)$

$$u(x,z)=u_2(x)-(z-e)\frac{dw}{dx}. \tag{2}$$

For the middle part between the two tee sections, $-a\le z\le a$

$$u(x,z)=\frac{u_1(x)+u_2(x)}{2}-\frac{z}{a}\left(\frac{u_1(x)-u_2(x)}{2}-(e-a)\frac{dw}{dx}\right). \tag{3}$$

The axial strains in the two tee sections can be obtained using the strain-displacement relation as follows:

For the top tee section, $-(h_w/2+t_f)\le z\le -a$

$$\varepsilon_{1x}(x,z)=\frac{\partial u}{\partial x}=\frac{du_1}{dx}-(z+e)\frac{d^2w}{dx^2}. \tag{4}$$

For the bottom tee section, $a\le z\le (h_w/2+t_f)$

$$\varepsilon_{2x}(x,z)=\frac{\partial u}{\partial x}=\frac{du_2}{dx}-(z-e)\frac{d^2w}{dx^2}. \tag{5}$$

The shear strain in the middle part between the two tee sections can also be obtained using the shear strain-displacement relation as follows:

$$\gamma_{xz}(x,z)=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}=-\frac{u_1-u_2}{2a}+\frac{e}{a}\frac{dw}{dx}. \tag{6}$$

The internal forces defined in Fig. 1c can be obtained as follows:

$$N_1(x)=Eb_f\int_{-(t_f+h_w/2)}^{-h_w/2}\varepsilon_{1x}dz+Et_w\int_{-h_w/2}^{-a}\varepsilon_{1x}dz=EA_{tee}\frac{du_1}{dx} \tag{7}$$

$$M_1(x)=Eb_f\int_{-(t_f+h_w/2)}^{-h_w/2}(z+e)\varepsilon_{1x}dz+Et_w\int_{-h_w/2}^{-a}(z+e)\varepsilon_{1x}dz=-EI_{tee}\frac{d^2w}{dx^2} \tag{8}$$

$$Q_3(x)=Gt_w\int_{-a}^{a}\gamma_{xz}dz=2Gt_w\left(e\frac{dw}{dx}-\frac{u_1-u_2}{2}\right) \tag{9}$$

$$N_2(x)=Et_w\int_{a}^{h_w/2}\varepsilon_{2x}dz+Eb_f\int_{h_w/2}^{t_f+h_w/2}\varepsilon_{2x}dz=EA_{tee}\frac{du_2}{dx} \tag{10}$$

$$M_2(x)=Et_w\int_{a}^{h_w/2}(z+e)\varepsilon_{2x}dz+Eb_f\int_{h_w/2}^{t_f+h_w/2}(z+e)\varepsilon_{2x}dz=-EI_{tee}\frac{d^2w}{dx^2} \tag{11}$$

where E is the Young's modulus, G is the shear modulus, A_{tee} and I_{tee} are the area and second moment of area of the tee-section defined in its own coordinate system as follows:

$$A_{tee}=b_ft_f+t_w\left(\frac{h_w}{2}-a\right) \tag{12}$$

$$I_{tee}=\frac{b_ft_f^3}{12}++b_ft_f\left(\frac{h_w+t_f}{2}-e\right)^2+\frac{t_w}{12}\left(\frac{h_w}{2}-a\right)^3+t_w\left(\frac{h_w}{2}-a\right)\left(\frac{h_w+2a}{4}-e\right)^2. \tag{13}$$

The strain energy of the member due to the axial and transverse displacements can be expressed as follows:

$$U=\frac{Eb_f}{2}\int_o^l\int_{-(t_f+h_w/2)}^{-h_w/2}\varepsilon_{1x}^2dzdx+\frac{Et_w}{2}\int_o^l\int_{-h_w/2}^{-a}\varepsilon_{1x}^2dzdx+\frac{k_{sh}Gt_w}{2}\int_o^l\int_{-a}^{a}\gamma_{xz}^2dzdx+\frac{Et_w}{2}\int_o^l\int_a^{h_w/2}\varepsilon_{2x}^2dzdx+\frac{Eb_f}{2}\int_o^l\int_{h_w/2}^{t_f+h_w/2}\varepsilon_{2x}^2dzdx \tag{14}$$

where l is the member length and $k_{sh}=1/4$ is the modified shear factor which is derived in the Appendix A. Substituting Eqs. (4) to (6) into Eq. (14) yields,

$$U=\frac{EA_{tee}}{2}\int_o^l\left[\left(\frac{du_1}{dx}\right)^2+\left(\frac{du_2}{dx}\right)^2\right]dx+EI_{tee}\int_o^l\left(\frac{d^2w}{dx^2}\right)^2dx+\frac{k_{sh}Gt_w}{a}\int_o^l\left(e\frac{dw}{dx}-\frac{u_1-u_2}{2}\right)^2dx. \tag{15}$$

For simplicity of presentation, the following two notations are introduced,

$$u_\alpha=\frac{u_1+u_2}{2} \tag{16}$$

$$u_\beta=\frac{u_1-u_2}{2}. \tag{17}$$

Hence, Eq. (15) can be rewritten in terms of $u_\alpha(x)$, $u_\beta(x)$, and $w(x)$ as follows:

$$U=EA_{tee}\int_o^l\left[\left(\frac{du_\alpha}{dx}\right)^2+\left(\frac{du_\beta}{dx}\right)^2\right]dx+EI_{tee}\int_o^l\left(\frac{d^2w}{dx^2}\right)^2dx+\frac{k_{sh}Gt_we^2}{a}\int_o^l\left(\frac{dw}{dx}-\frac{u_\beta}{e}\right)^2dx. \tag{18}$$

Physically, the first term in Eq. (18) represents the membrane strain energy, the second term is the bending strain energy, whereas the third term stands for the shear strain energy. For the case where the castellated member is subjected to an axial compression load, P, the potential change of the applied load due to the axial and transverse displacements can be expressed as follows:

$$W=-P\int_o^l\left[\frac{du_\alpha}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2\right]dx. \tag{19}$$

The buckling load can be determined by finding the value of the applied load at which bifurcation buckling occurs, that is, the load at which the castellated member can be in equilibrium both in a straight configuration ($u_\beta(x)=w(x)=0$) and in a slightly bent configuration about its major axis (i.e. bending occurs in the plane of the web). This is accomplished through setting the variational of the total potential energy $\Pi=U+W$ equal to zero. This operation results in an eigenvalue problem that can be solved for nontrivial solutions that are discrete values of the applied load. The lowest eigenvalue is the actual physical buckling load, and is defined to be the critical buckling load.

The examination of Eqs. (18) and (19) reveals that u_α is independent of u_β and w, and thus will vanish after the variational. In this case, the total potential energy for finding the critical buckling load can be simplified as follows:

$$\Pi*=EA_{tee}\int_o^l\left(\frac{du_\beta}{dx}\right)^2dx+EI_{tee}\int_o^l\left(\frac{d^2w}{dx^2}\right)^2dx+\frac{k_{sh}Gt_we^2}{a}\int_o^l\left(\frac{dw}{dx}-\frac{u_\beta}{e}\right)^2dx-\frac{P}{2}\int_o^l\left(\frac{dw}{dx}\right)^2dx. \tag{20}$$

For a simply supported castellated column, $u_\beta(x)$ and $w(x)$ can be assumed to be as follows:

$$u_\beta(x) = C_1 \cos\frac{\pi x}{l} \tag{21}$$

$$w(x) = C_2 \sin\frac{\pi x}{l} \tag{22}$$

where C_1 and C_2 are constants. Obviously, $u_\beta(x)$ and $w(x)$ satisfy the simply supported displacement boundary conditions, $w = \frac{d^2w}{dx^2} = \frac{du_\beta}{dx} = 0$, at both ends. Substituting Eqs. (21) and (22) into Eq. (20) yields,

$$\Pi * = \frac{EA_{tee}l}{2}\left(\frac{\pi}{l}\right)^2 C_1^2 + \frac{EI_{tee}l}{2}\left(\frac{\pi}{l}\right)^4 C_2^2 + \frac{k_{sh}Gt_we^2l}{2a}\left(\frac{\pi C_2}{l} - \frac{C_1}{e}\right)^2 - \frac{Pl}{4}\left(\frac{\pi}{l}\right)^2 C_2^2. \tag{23}$$

The variational of Eq. (23) with respect to C_1 and C_2 results in the following two algebraic equations,

$$EA_{tee}C_1 - \frac{k_{sh}Gt_wel}{\pi a}\left(C_2 - \frac{lC_1}{\pi e}\right) = 0 \tag{24}$$

$$EI_{tee}\left(\frac{\pi}{l}\right)^2 C_2 + \frac{k_{sh}Gt_we^2}{a}\left(C_2 - \frac{lC_1}{\pi e}\right) - \frac{PC_2}{2} = 0. \tag{25}$$

Eqs. (24) and (25) are the eigen-equations where C_1 and C_2 are the eigenvector and P is the corresponding eigenvalue. The smallest eigenvalue, P_{cr} of Eqs. (24) and (25) can be obtained as follows:

$$P_{cr} = \frac{2\pi^2 EI_{tee}}{l^2} + \frac{2\pi^2 EA_{tee}e^2}{l^2}\frac{1}{1 + \frac{\pi^2 aEA_{tee}}{k_{sh}t_wGl^2}}. \tag{26}$$

It can be seen from Eq. (26) that, if $G \approx 0$, then the whole second term vanishes, and thus the critical buckling load can be calculated by considering that the two tee-section beams are independent of each other, while if $G \approx \infty$, the second part of the second term vanishes, and thus the critical buckling load can be calculated by considering the two tee beams as a whole (i.e. they are rigidly assembled together). The actual critical buckling load of a castellated column thus is in between these two extreme cases. It also can be seen from Eq. (26) that the influence of the shear deformation on the critical buckling load increases with the area of the tee section and the depth of web opening but decreases with the column length and the web thickness.

Note that $k_{sh} = 1/4$ and if Poisson's ratio, $v = 1/3$ then $E = 8G/3$. For $t_wl^2 >> aA_{tee}$ Eq. (26) can be simplified into,

$$P_{cr} = \frac{2\pi^2 EI_{tee}}{l^2} + \frac{2\pi^2 EA_{tee}e^2}{l^2}\left(1 - \frac{\pi^2 aEA_{tee}}{k_{sh}t_wGl^2}\right) = P_o\left(1 - \frac{64\pi^2 A_{tee}^2 ae^2}{3I_ot_wl^2}\right) \tag{27}$$

where P_o is the critical buckling load of the column calculated based on a reduced constant second moment of area, I_o, due to the existence of web openings but ignoring the shear deformation as follows:

$$P_o = \frac{\pi^2 EI_o}{l^2} \tag{28}$$

$$I_o = 2\left(I_{tee} + A_{tee}e^2\right) = 2\left[\frac{b_ft_f^3}{12} + b_ft_f\left(\frac{h_w + t_f}{2}\right)^2\right] + \frac{t_w}{12}\left[h_w^3 - (2a)^3\right]. \tag{29}$$

Eqs. (27) and (28) indicate that the critical load of a castellated column can be calculated using the simple Euler formula if a proper reduction factor due to the shear is applied. The reduction factor, however, is dependent upon several dimensions, including column length, the cross sectional area of a tee section, open depth, web thickness, the second moment of area, and the distance between the centroids of the two tee sections.

For battened lattice columns, $I_{tee} \approx 0$, $I_o \approx 2e^2A_{tee}$, and $e \approx a$. In this case, Eq. (26) can be simplified,

$$P_{cr} = \frac{P_o}{1 + \frac{2P_o}{at_wG}}. \tag{30}$$

This is similar to Engesser's buckling formula for battened built-up columns but with a slight different pre-factor [15,18]. The reason for this is because Eq. (30) involves both shear and bending of web posts.

3. Comparison of Eq. (27) with finite element analysis results

The critical buckling load formula, Eq. (27), is demonstrated using the results from the finite element analysis obtained by El-Sawy et al. [14]. The cross sectional dimensions and lengths of the columns used for the demonstration are given in Table 1, which are based on those used commonly in practice, with relative flange-to-web flexural stiffness ratios varying from 0.62 to 24, flange width-to-thickness ratios varying from 4.0 to 25, and web open depth-to-web height ratios ($2a/h_w$) varying from 0.43 to 0.79.

The finite element analysis [14] was performed using the ANSYS finite element software package to determine the critical elastic bifurcation buckling loads and the associated buckling modes of castellated columns. Three-dimensional 6-noded and 8-noded structural solid elements (SOLID45) with three translational degrees of freedom at each node were used to model the geometrical details of the analysed columns. Across the thickness direction there are two elements used in the flanges and three elements used in the web. The number and sizes of the elements used were obtained based on the numerical convergence test of buckling loads. Fig. 2 shows a typical finite element mesh of a modelled castellated column.

Due to the symmetry in the column geometry, loading and response, only a quarter of the column (a half length and a half cross-section of the column) is modelled (see Fig. 2). Zero lateral displacement was applied to every node on the plane of symmetry. Zero axial displacement was applied to every node on the section of symmetry. Nodes on the section of simple supports were assumed to have zero lateral and transverse displacements and subjected to a uniformly distributed axial compressive stress.

The comparison between the present analytical solutions and the results from the finite element analysis [14] is plotted in Fig. 3. The critical buckling loads calculated using Eq. (28) are also superimposed in the figure to demonstrate the effect of the shear deformation of the web on the critical buckling load of a castellated column. It is evident from Fig. 3 that, ignoring web shear deformations could overestimate the critical buckling load by up to 25%, even if a reduced second moment of area (I_o) is used. However, when web shear deformations are taken into account, the critical buckling loads calculated are in good agreement with those obtained from the finite element analysis. This demonstrates that the analytical model proposed in the present study is appropriate to predict the buckling capacity of castellated columns.

It should be noticed from the results shown in Table 1 (data with a star) that, for several sections the critical stresses obtained from the finite element analyses are slightly greater than those calculated from Eqs. (27) and (28). This is interesting since this indicates that the effect of web shear deformations is overtaken by the reduction of the second moment of area by ignoring the solid part in the middle layer of the web. Nevertheless, the differences between the three critical stresses calculated using different models/methods for these sections are not remarkable as demonstrated in Fig. 3.

The elastic buckling solution developed in the preceding section, i.e. the critical buckling load given in Eqs. (26) or (27) together with the

Table 1
Dimensions of the castellated columns analysed and the corresponding critical buckling stress results ($E = 200$ GPa, $\sigma_y = 275$ MPa).

Dimensions of castellated columns (mm)						σ_{cr}/σ_y		
b_f	t_f	h_w	t_w	$2a$	l	FEA [1]	Eq. (27)	Eq. (28)
20	5	100	5	43.30	3000	1.5263	1.5237	1.5415
80	20	400	20	173.20	12000	1.5263	1.5237	1.5415
100	4	200	10	86.60	6000	1.4739	1.4724	1.4897
200	8	400	20	173.20	12000	1.4739	1.4724	1.4897
200	50	200	10	86.60	6000	2.6104	2.6474	3.0453
800	200	800	40	346.40	24000	2.6104	2.6474	3.0453
250	10	100	5	43.30	3000	1.9690	2.0364	2.3472
500	20	200	10	86.60	6000	1.9690	2.0364	2.3472
20	5	100	5	51.96	3000	1.6409	1.6273	1.6485
80	20	400	20	207.84	12000	1.6409	1.6273	1.6485
100	4	200	10	103.92	6000	1.5846	1.5710	1.5916
200	8	400	20	207.84	12000	1.5846	1.5710	1.5916
200	50	200	10	103.92	6000	2.5982	2.5883	3.0668
800	200	800	40	415.68	24000	2.5982	2.5883	3.0668
250	10	100	5	51.96	3000	1.9530	1.9895	2.3628
500	20	200	10	103.92	6000	1.9530	1.9895	2.3628
20	5	100	5	60.62	3000	1.7649	1.7351	1.7593*
80	20	400	20	242.48	12000	1.7649	1.7351	1.7593*
100	4	200	10	121.24	6000	1.6980	1.6728	1.6962*
200	8	400	20	242.48	12000	1.6980	1.6728	1.6962*
200	50	200	10	121.24	6000	2.5912	2.5285	3.0870
800	200	800	40	484.96	24000	2.5912	2.5285	3.0870
250	10	100	5	60.62	3000	1.9236	1.9418	2.3773
500	20	200	10	121.24	6000	1.9236	1.9418	2.3773
20	5	100	5	69.28	3000	1.8899	1.8452	1.8716*
80	20	400	20	277.12	12000	1.8899	1.8452	1.8716*
100	4	200	10	138.56	6000	1.8151	1.7753	1.8007*
200	8	400	20	277.12	12000	1.8151	1.7753	1.8007*
200	50	200	10	138.56	6000	2.5893	2.4678	3.1059
800	200	800	40	554.24	24000	2.5893	2.4678	3.1059
250	10	100	5	69.28	3000	1.8753	1.8933	2.3902
500	20	200	10	138.56	6000	1.8753	1.8933	2.3902
20	5	100	5	77.94	3000	2.0273	1.9540	1.9818*
80	20	400	20	311.76	12000	2.0273	1.9540	1.9818*
100	4	200	10	155.88	6000	1.9266	1.8744	1.9011*
200	8	400	20	311.76	12000	1.9266	1.8744	1.9011*
200	50	200	10	155.88	6000	2.5736	2.4063	3.1230
800	200	800	40	623.52	24000	2.5736	2.4063	3.1230
250	10	100	5	77.94	3000	1.8074	1.8439	2.4012
500	20	200	10	155.88	6000	1.8074	1.8439	2.4012
20	5	50	10	21.65	1500	1.6130	1.6135	1.6322
60	15	450	10	194.85	13500	1.4985	1.4949	1.5125
50	2	50	10	21.65	1500	1.5096	1.5064	1.5240
150	6	450	10	194.85	13500	1.4632	1.4613	1.4785
100	25	50	10	21.65	1500	3.8003	3.8934	4.4605
300	75	450	10	194.85	13500	2.2624	2.2953	2.6435
250	10	50	10	21.65	1500	2.2483	2.4323	2.8000
750	30	450	10	194.85	13500	1.8691	1.9145	2.2074
20	5	50	10	38.97	1500	2.1817	2.0935	2.1232
60	15	450	10	350.73	13500	1.9797	1.9094	1.9366*
50	2	50	10	38.97	1500	1.9871	1.9271	1.9546*
150	6	450	10	350.73	13500	1.9092	1.8571	1.8836*
100	25	50	10	38.97	1500	3.7540	3.5594	4.5862
300	75	450	10	350.73	13500	2.2145	2.0819	2.7076
250	10	50	10	38.97	1500	2.1479	2.2080	2.8694
750	30	450	10	350.73	13500	1.6826	1.7320	2.2567

yield load can be used to determine the slenderness of a castellated column, from which one can calculate the resistance of a compressed castellated column based on the "design buckling curves", which account for imperfections, specified in various design standards, for example, AISC 360, Eurocode 3, AS 4100 etc.

4. Conclusions

This paper has presented an analytical solution for determining the critical buckling load of simply supported castellated columns subject to axial compression. The present analysis has highlighted the importance of taking into account the effect of web shear deformations on the critical buckling load of castellated columns when they buckle about the major axis. The present analytical solution has been demonstrated for a wide range of section dimensions using the published data obtained from finite element analysis. From the present study the following conclusions can be drawn:

- The inclusion of web shear deformations significantly reduces the buckling resistance of castellated columns. Neglecting the web shear deformations could overestimate the critical buckling load by up to 25%, even if a reduced second moment of area is used.
- The influence of web shear deformations on the critical buckling loads of castellated columns increases with the cross-sectional area of a tee section and the depth of web opening, but decreases with the length and the web thickness of the column.
- The analytical solution agrees well with the finite element solutions.
- Finally, although the present study discusses only simply supported castellated columns, the method and principle presented in this

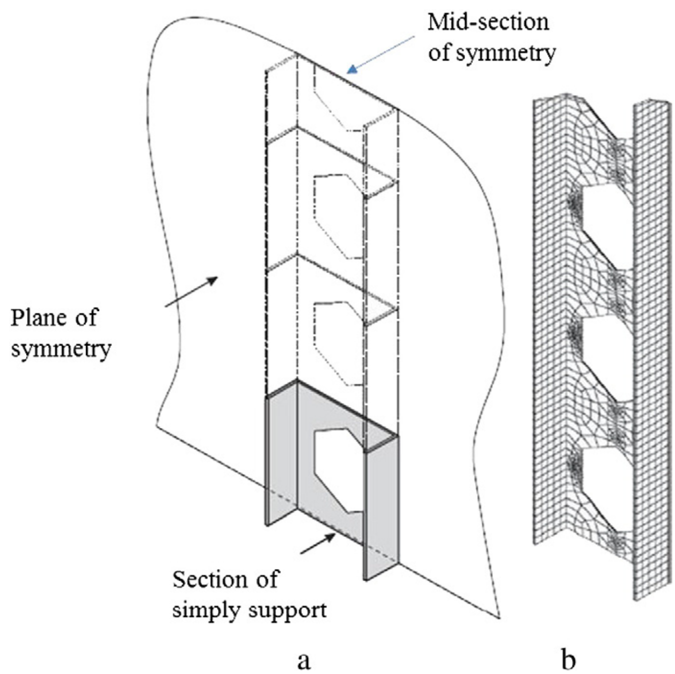

Fig. 2. (a) Finite element analysis model and (b) finite element mesh ([14]).

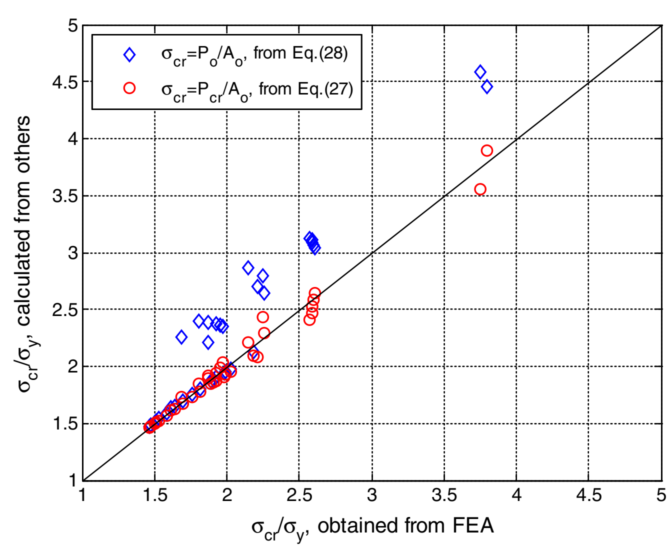

Fig. 3. Comparison of the critical buckling stresses of castellated columns obtained from different methods ($A_o = 2A_{tee}$, $E = 200$ GPa, $\sigma_y = 275$ MPa, the FEA data from [14]).

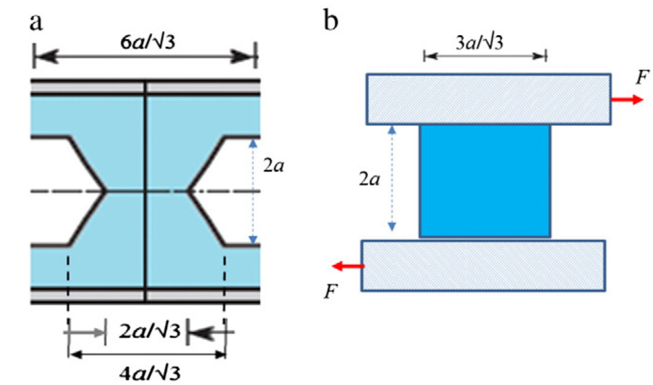

Fig. A. Shear strain energy calculation model: (a) unit considered and (b) shear deformation calculation model.

paper could be applied for castellated columns with other boundary conditions.

Acknowledgement

The authors would like to acknowledge Professor Amr M. I. Sweedan in the Department of Civil and Environmental Engineering at the UAE University, United Arab Emirates for kindly providing his finite element analysis data to be used to validate the present analytical solution.

Appendix A. Derivation of the modified shear factor k_{sh}

Consider the unit of length, $6a/\sqrt{3}$ of a castellated column, shown in Fig. A(a). The relative displacement of the top and bottom layers of the unit due to a pair of shear forces F, shown in Fig. A(b) can be calculated in terms of Timoshenko beam theory as follows:

$$\Delta = \frac{\alpha F l_b}{G A_b} + \frac{F l_b^3}{12 E I_b} \tag{A-1}$$

where $\alpha = 1.5$ is the shear coefficient for beams with a rectangular cross section, $l_b = 2a$ is the length of the beam, $A_b = t_w\sqrt{3}a$ is the cross-sectional area of the beam, $I_b = t_w(\sqrt{3}a)^3/12$ is the second moment of the cross-sectional area of the beam, which, for simplicity, is based on the average value of depths as shown in Fig. A(b). From Eq. (A-1) and using $E = 8G/3$ for $\nu = 1/3$, the combined stiffness of the beam due to the bending and shear thus can be expressed as follows,

$$k_b = \frac{F}{\Delta} = \frac{\sqrt{3}}{4} G t_w. \tag{A-2}$$

The strain energy of a beam due to bending and shear can be expressed in terms of the relative displacement Δ as follows:

$$U_b = \frac{1}{2} k_b \Delta^2 = \frac{\sqrt{3}}{8} G t_w \Delta^2. \tag{A-3}$$

Note that the relative displacement Δ can be expressed in terms of the shear strain as follows:

$$\Delta = 2a\gamma_{xz}. \tag{A-4}$$

Hence, Eq. (A-3) can be expressed as follows:

$$U_b = \frac{\sqrt{3}}{2} G t_w a^2 \gamma_{xz}^2. \tag{A-5}$$

For a castellated column of n units, of length $6na/\sqrt{3}$, the total strain energy of the middle layer of the column due to the shear strain γ_{xy} can be calculated as follows:

$$U_{sh} = \frac{\sqrt{3}}{2} G t_w a^2 \sum_{k=1}^{n} \gamma_{xz}^2 = \frac{\sqrt{3}}{2} \frac{G t_w a^2}{6a/\sqrt{3}} \int_o^l \gamma_{xz}^2 dx = \frac{G t_w a}{4} \int_o^l \gamma_{xz}^2 dx. \tag{A-6}$$

Let $k_{sh} = 1/4$, Eq. (A-6) can be expressed as follows:

$$U_{sh} = \frac{G t_w a}{4} \int_o^l \gamma_{xz}^2 dx = \frac{G t_w}{8} \int_o^l \int_{-a}^{a} \gamma_{xz}^2 dz dx = \frac{k_{sh} G t_w}{2} \int_o^l \int_{-a}^{a} \gamma_{xz}^2 dz dx. \tag{A-7}$$

References

[1] Harper C. Design in steel 4: castellated & cellular beams. British Steel; 1991.
[2] Knowles PR. Design of castellated beams. Constrado; 1985.
[3] Knowles PR. Castellated beams. Proceedings of Institution of Civil Engineers: Structural and Building Board, part 1, 90; 1991. p. 521–36.
[4] Nethercot D, Kerdal D. Lateral-torsional buckling of castellated beams. J Struct Eng 1982;60:53–61.
[5] Kerdal D, Nethercot D. Failure modes for castellated beams. J Constr Steel Res 1984;4(4):295–315.
[6] Redwood R, Zaarour W, Megharief J. Web post buckling in castellated beams. In: Chan SL, Teng JG, editors. Advances in steel structures (ICASS '96). Oxford: Pergamon; 1996. p. 67–71.
[7] Mohebkhah A. The moment-gradient factor in lateral-torsional buckling on inelastic castellated beams. J Constr Steel Res 2004;60(10):1481–94.
[8] Mohebkhah A, Showkati H. Bracing requirements for inelastic castellated beams. J Constr Steel Res 2005;61(10):1373–86.
[9] Zirakian T, Showkati H. Distortional buckling of castellated beams. J Constr Steel Res 2006;62(9):863–71.
[10] Ellobody E. Interaction of buckling modes in castellated steel beams. J Constr Steel Res 2011;67(5):814–25.
[11] Gholizadeh S, Pirmoz A, Attarnejad R. Assessment of load carrying capacity of castellated steel beams by neural networks. J Constr Steel Res 2011;67(5):770–9.
[12] Soltani MR, Bouchaïr A, Mimoune M. Nonlinear FE analysis of the ultimate behaviour of steel castellated beams. J Constr Steel Res 2012;70:101–14.
[13] Showkati H, Ghazijahani TG, Noori A, Zirakian T. Experiments on elastically braced castellated beams. J Constr Steel Res 2012;77:163–72.
[14] El-Sawy K, Sweedan A, Martini M. Major-axis elastic buckling of axially loaded castellated steel columns. Thin-Walled Struct 2009;47:1295–304.
[15] Gjelsvik A. Stability of built-up columns. ASCE J Eng Mech 1991;117(6):1331–45.
[16] Timoshenko SP, Gere JM. Theory of elastic stability. 2nd ed. New York: McGraw-Hill Book Company; 1961.
[17] Vasic Z, Raicevic V, Jovic S. Sandwich beam stability analysis applying bending theory of the sandwich constructions. Mech Autom Control Robot 2005;4(17):333–43.
[18] Bazant ZP. Shear buckling of sandwich, fiber composite and lattice columns, bearings, and helical springs: paradox resolved. J Appl Mech 2003;70:75–83.