Slender steel columns: How they are affected by imperfections and bracing stiffness

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Anders Klasson *, Roberto Crocetti, Eva Frühwald Hansson

Division of Structural Engineering, Lund University, Box 118, 22100 Lund, Sweden

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ABSTRACT

Finite-element programs can be used for designing columns and their bracing systems. It is well known, however, that the output obtained from such programs is highly dependent upon the input (such as imperfections and stiffness properties). In the present study, the effects of imperfections on the predicted strength and stiffness requirements of steel columns and of their bracing systems are investigated. Two different systems are analyzed: 1) a braced non-sway column and 2) a braced sway column. It was found that a poor choice of the shape of the initial imperfections can provide unrealistic results in terms of both the buckling load on the columns and the predicted reactions of the bracings. It was also found that superimposing different imperfection shapes can contribute to obtaining realistic and trustworthy results. Furthermore, it was shown that the shapes of the initial imperfections that lead to the lowest buckling load and those that result in the strongest bracing forces, are generally not the same.

1. Introduction

Structural imperfections are critical for determining the behavior of slender structural elements and their bracing systems. These imperfections include construction tolerances, geometrical deviations, residual stresses, load eccentricities and material deficiencies. The numerical modeling of some of the aforementioned imperfections can be cumbersome in design. For example, the modeling of residual stresses would likely require the use of either shell or solid elements (resulting in complex models). Moreover, the code does not specify a geometric imperfection to be used if the residual stresses were to be handled separately. Accordingly, the geometrical imperfections used in design are normally larger than the actual (measurable) geometrical deviations, so as to be able to account for the effect of all imperfections.

The important characteristics of a bracing system include its stiffness and its strength properties. Different in-plane bracing methods include discrete (as examined in this study), continuous, relative and lean on (as defined in Galambos et al. [1]), see Fig. 1.

In 1958, Winter [2] presented a simple yet powerful rigid link model employed for calculating the strength and stiffness requirements of bracings. This method can be used in particular for calculating the full-bracing (ideal stiffness) requirement. This requirement represents a conservative limit for the required bracing stiffness that is needed in order to achieve buckling between successive bracings. According to

this model, a column can be braced at one or more points. Winter's rigid link model was later extended by Yura [3] to allow for cases in which less than full bracing is provided. The rigid link model can also account for initial imperfections, making the study of bracing forces and thus the strength requirements of the bracings possible. While simplified approaches such as the rigid link model are possible, analytical solutions can in some cases also be derived; see e.g. Timoshenko et al. [4] regarding the concept of buckling capacity when less than full bracing is provided. For derivation of the full bracing requirement of a sway prevented column with one intermediate bracing, see e.g. Galambos et al. [1]. The bracing force for a varying applied load, was derived by Trahair [5]. Even in simple cases, however, such as that of a column with only one intermediate bracing, closed-form solutions are rather involved and may not be as practical as ones based on a rigid link model. In the case of more complicated systems, closed-form solutions may not even exist. Since the rigid link model usually assumes equally spaced bracings, Plaut et al. [6–8] in several studies analyzed the implications of having unsymmetrically spaced bracings. It was found that no ideal stiffness could be defined if the bracings are spaced unsymmetrically. This was due to the fact that the displacements at the bracing points can be suppressed only if there is perfect symmetry, i.e. equal spans. Theoretically, this means that there will always be an additional elastic buckling capacity if the bracing stiffness would be increased (see for instance Mehri et al. [9] who thouroughly analyzed this case). Practically, however, a "full bracing requirement" can still be said to exist even for the unsymmetrical case; i.e. when the stiffness of the bracing tends to a value that generates a buckling capacity that would be obtained if infinitely rigid bracings were assumed.

* Corresponding author.
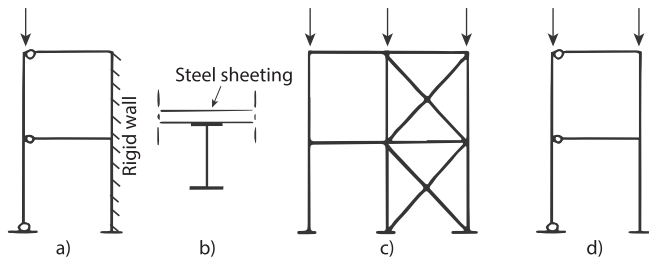*E-mail address:* anders.klasson@kstr.lth.se (A. Klasson).

Fig. 1. a) Discrete bracing. b) Continuous bracing. c) Relative bracing. d) Lean on bracing.

In modern design, simplified methods such as those involving the use of rigid-link models for columns, are uncommon since most engineers have access to advanced finite element (FE) software. Such modeling may be comparatively easy and fast, even for users who lack an adequate physical understanding of the problem involved. It is well known, however, that the validity of results obtained using FE-modeling is strongly dependent upon the accuracy of the input and may completely misguide users who interpret it inaccurately [10]. Thus, as is also demonstrated in the present study, it is important that the effect of modeling assumptions, such as imperfections, are considered when design is based directly on FE-modeling.

For the design of columns aided by nonlinear incremental analysis, it has been shown in numerous studies that the choice of the imperfection shape strongly affects the results obtained. For instance, Wang et al. [11] clearly demonstrated how bracing forces can vary with different choices of the imperfection shape to employ. Giro Coelho et al. [12] studied a non-sway column (lacking intermediate support), and determined that the assumed imperfection shape affects the pre-buckling stiffness and thus the load-bearing capacity of the column. It should be mentioned that the most critical imperfection shape for the column did not always correspond to the first elastic buckling mode of the corresponding perfect (i.e. without imperfections) system.

The Eurocode 3 [13] design code states that the most unfavorable combination of initial imperfections should be used in design without clearly specifying what that combination is. In contrast to what was said in the previous paragraph the Eurocode tacitly suggests, according to the authors' interpretation, that imperfections related to buckling modes of the highest order, i.e. buckling between restraints (bracings) should be used, possibly in combination with the sway imperfections inherent in the structure (tolerances that the structure has). In addition, simplified requirements for bracings are specified by the Eurocode, e.g. a bracing stiffness requirement simply expressed in terms of the design load of the column. Overall, the approaches specified by the code do not adequately describe the true physical nature of the structure in an intuitive manner; something further being needed.

The output obtained in the FE-modeling of columns can be used in basically two different ways:

1. The FE-modeling is used simply for calculating the elastic critical load of a column, P_e. this value then determines the relative slenderness ratio, $\lambda = \sqrt{f_y A / P_e}$, and design with Eurocode 3 [13] for steel can be used. An FE-program can usually calculate the elastic critical load of a column either through an elastic buckling analysis of a perfect column, or by incremental analysis of an imperfect column (with an assumed minor imperfection).
2. Alternatively, an appropriate imperfection can be assigned to the column followed by an incremental non-linear (inelastic) analysis. Design is then based directly on these results.

The present study investigates, by means of nonlinear finite element analysis of discretely braced steel columns, what imperfection shapes to use in order to obtain an over all safe design (2nd alternative above). Attention is directed at the response of the column and of the entire bracing system (i.e. the bracing forces).

1.1. Aims

The specific aims of the present study are the following:

1. Determine the ideal stiffness of the bracing systems considered and the corresponding buckling modes of idealized/perfect columns (i.e. columns without imperfections). This will be mainly analyzed analytically, by means of the energy method, to be described more later on in the method section. The purpose of using the energy method is twofold: (1) to find suitable imperfections shapes (also linear buckling analysis could have been used here) for use in design based on FE-analysis and (2) to serve as a validation of the results from the incremental analysis.
2. Investigate the full bracing requirements, in terms of both the column strength and its elastic buckling limit, and the bracing forces involved when employing different imperfection shapes in an FE-analysis (i.e. a non-linear incremental analysis) of the columns in question.
3. Examine what the most unfavorable imperfection shapes would appear to be for the systems in question with respect to both column strength and the bracing forces.
4. Compare the column strength according to the code, Eurocode 3 chapter 6.3.1.2 [13] (using the relative slenderness ratio (λ)) with the strength predicted by the non-linear FE analysis for different imperfection shapes.
5. Investigate if there are any imperfection shapes that lead to unrealistic results in terms of FE-analysis and should thus be avoided in design.
6. Provide a reference aimed at aiding practicing engineers in the nonlinear design of columns.

1.2. Limitations

The current study is limited to the investigation of in-plane buckling of steel columns with symmetrical cross-sections (e.g. I-profiles) and two different statical systems, namely:

1. The first system, referred to as System A and shown in Fig. 2, is a non-sway column with a single intermediate mid-length bracing. Although such a system is uncommon among real structures, it appears to be the most common bracing system referred to in the literature such as in [14,3,5,15,9]. System A could, for example, be a scaffolding strut such as that shown in Fig. 2, which is attached at its top to a very rigid structure.
2. The second system, referred to as System B, is a sway column with two bracings of equal stiffness, placed at the top and at mid-length of the column. According to the authors' perception, this system is more commonly found among real structures than System A. It is hard to imagine many buildings where the top bracing would be stiffer than the lower, intermediate, ones. One example of the application to which System B can be put is shown in Fig. 3.
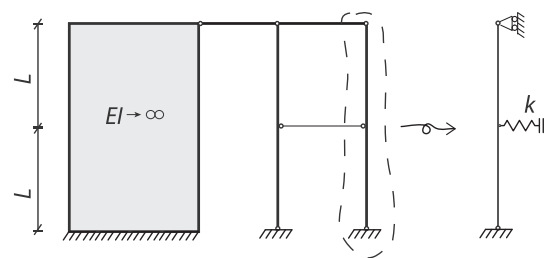

Fig. 2. System A. A column in which the top bracing can be considered to be rigid in relation to the middle bracing. It could be a scaffolding strut, connected at its top to the building.
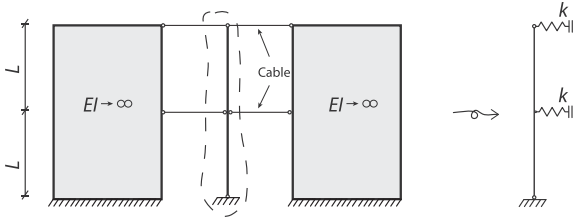
Fig. 3. System B, a column in which the two bracings can be considered to be equally stiff. The bracings, in this case cables, are connected to two very rigid walls.

2. Materials and methods

The methods of investigation applied for study of System A and System B are summarized by the following three steps:

1. Find by means of the energy method an approximate value of the elastic buckling load and the bracing criteria of the systems.
2. Choose on the basis of the results of the energy method one or more imperfection shapes to be used in a large displacement incremental FE-analysis. The imperfection shapes are applied both one-by-one and combined in different ways (as explained in greater detail later on in Section 2.3).
3. Evaluate on the basis of FE-analysis the buckling load with and without regard to the material yield strength. This buckling load (determined irrespective of material strength) was then used for strength prediction, in accordance with Eurocode 3, on the basis of the relative slenderness ratio, as described in Section 1.

2.1. Material properties and geometry

The geometries of the two systems studied are shown in Figs. 2 and 3, respectively. Doubly symmetric cross sections are used in the analysis, i.e. that $I = \frac{h}{2}W$ where I is the moment of inertia (assumed constant) and W the section modulus. The columns all have a relative slenderness of $\bar{\lambda} = \sqrt{\frac{f_y A}{P_e}} = 1$ based on the unbraced length L, where $P_e = \frac{\pi^2 EI}{L^2}$, which is of relevance when the strength of the columns are being discussed. The young's modulus $E = 210GPa$ and yield strength $f_y = 275MPa$ is used in the analyses.

2.2. Energy method

The energy method is based on the concept of equilibrium between the loss of potential energy, ΔT, and the gain of internal (strain)energy, ΔU, at the point of buckling (bifurcation), i.e. that $\Delta T = \Delta U$; see e.g. Timoshenko et al. [4]. In the case of column buckling, the loss of potential energy is derived from the vertical movement (work) of the load, such that $\Delta T = P\delta_x$, where δ_x is the vertical shortening of the column and P is the load applied. The gain in internal energy in the case of braced column buckling consists both of the increase in energy of the bracings and the strain energy required to deform the column into a given shape (usually assumed to be sinusoidal). In order for the energy method to work, a buckled shape of the column must be assumed. Due to this fact, the results obtained may be nonconservative or, at the best, equal to the exact solution dependent on the assumed shape. Generally, the accuracy of the results are determined by the assumed buckling shape.

The buckling loads for both System A and B are derived below. The solution contains a parameter c, see Table 1, in the assumed shape function $\eta(x)$ that will be different for the two systems due to the different boundary conditions these posses.

Table 1

Buckling and imperfection shapes used in the study. $\eta(x) = \delta_\eta \sin(c\frac{\pi x}{2L})$ is the shape of the column where δ_η is the magnitude and c a coefficient defining the shape in question. β is a correction factor used for the iv:th shape (sway imperfection) so that the actual maximum displacement at the top assumes the correct magnitude ($L/100$).

	i	ii	iii	iv	v	
Shape $\eta(x)$						L L
Magnitude δ_η	L/250	L/500	L/750	βL/100	L/250	
c-factor	1	2	3	0.001	0.5	

The first step is to calculate the loss of potential energy, ΔT:

$$\Delta T = P\delta_x = P\frac{1}{2}\int_0^{2L}\eta(x)'^2 dx = \left[\eta(x) = \delta_\eta \sin\left(\frac{c\pi x}{2L}\right)\right]$$
$$= P\pi c\frac{2\pi c + \sin(2\pi c)}{16L}\delta_\eta^2 \qquad (1)$$

where P is the buckling load, δ_x the vertical shortening of the column, $2L$ the column length, $\eta(x)$ the buckling shape, $\delta_\eta(x)$ maximum amplitude of the imperfection and c a shape coefficient.

The second step is to calculate the gain of strain energy in the column, ΔU_{column}:

$$\Delta U_{column} = EI\frac{1}{2}\int_0^{2L}\eta(x)''^2 dx = \left[\eta(x) = \delta_\eta \sin\left(\frac{c\pi x}{2L}\right)\right]$$
$$= EI\pi^3 c^3 \frac{2\pi c - \sin(2\pi c)}{64L^3}\delta_\eta^2 \qquad (2)$$

where EI is the bending stiffness of the column (assumed constant).

The third step is to calculate the energy gain in the bracings, $\Delta U_{bracings}$. This step will be different for the two systems in question since System A has only one bracing (at mid-length for $x = L$) whereas System B has two (one at mid-length and one at the top).

$$\Delta U_{bracings}^{SystemA} = \frac{k}{2}\left(\eta(L)^2\right) = \left[\eta(x) = \delta_\eta \sin\left(\frac{c\pi x}{2L}\right)\right] = \frac{k}{2}\sin^2\left(\frac{c\pi}{2}\right)\delta_\eta^2 \qquad (3)$$

$$\Delta U_{bracings}^{SystemB} = \frac{k}{2}\left(\eta(L)^2 + \eta(2L)^2\right) = \left[\eta(x) = \delta_\eta \sin\left(\frac{c\pi x}{2L}\right)\right]$$
$$= \frac{k}{2}\left(\sin^2\left(\frac{c\pi}{2}\right) + \sin^2(c\pi)\right)\delta_\eta^2 \qquad (4)$$

Where k is the bracing stiffness.

The fourth step is to equate the gained energy to the lost energy such that $\Delta T = \Delta U_{column} + \Delta U_{bracings}$ and solve for the buckling load, P, which gives:

$$P^A = \frac{1}{4}\frac{EI\pi^3\left(2\pi c^4 - \sin(2c\pi)c^3\right) + 32L^3 k\sin^2\left(\frac{c\pi}{2}\right)}{L^2\pi c(2c\pi + \sin(2c\pi))} \qquad (5)$$

and

$$P^B = \frac{1}{4}\frac{EI\pi^3\left(2\pi c^4 - \sin(2c\pi)c^3\right) + 32L^3 k\left(\sin^2\left(\frac{c\pi}{2}\right) + \sin^2(c\pi)\right)}{L^2\pi c(2c\pi + \sin(2c\pi))} \qquad (6)$$

where P^A is the buckling load for System A and P^B the buckling load for System B.

Eqs. (5) and (6) are plotted in Figs. 4 and 5, respectively. For System A, due to its end conditions (no displacement can occur here), only integer values of the c coefficient are allowed whereas any (positive) values of the c coefficient are allowed for System B. It should be emphasized
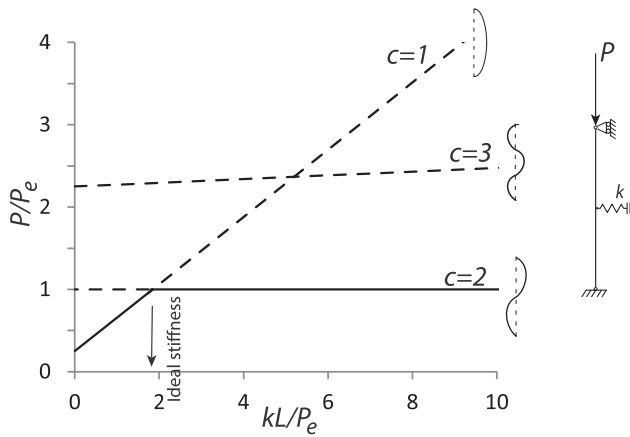
Fig. 4. The critical load for System A (Eq. (5))for varying bracing stiffness k and different values of the c-coefficient. P is the current buckling load and $P_e = \frac{\pi^2 EI}{L^2}$, i.e. that when buckling between successive bracings would occur. The ideal stiffness is indicated by an arrow.

that in order to find the lowest anticipated buckling load for varying bracing stiffness, several different curves for different values of the c-coefficient have to be plotted.

As can be seen in Fig. 4, the maximum buckling load of System A can never exceed the value given by the curve marked $c = 2$, corresponding to the buckling mode of two half sine waves (or exactly the Euler load). This is the buckling load for a normalized bracing stiffness of greater than approximately 2, i.e. $kL/P_e > 2$. This value thus defines the ideal stiffness of System A (as indicated by the arrow in Fig. 4). This stiffness can also be estimated by the rigid link method, as described by Winter [2]. For the simple system involved here, it is possible to derive the ideal stiffness analytically. Galambos [1] derived the normalized stiffness as being exactly $kL/P_e = 2$. The ideal stiffness predicted by use of the energy method (indicated by the arrow in Fig. 4) is slightly less than the exact value due to the approximate nature of the method. For a normalized stiffness of <2 ($kL/P_e < 2$), the maximum buckling load of the column is given by the curve marked $c = 1$, which is a buckling of half a sine wave over the entire length of the column. It is also clear that buckling in three half sine waves according to the curve marked $c = 3$ can hardly be of practical interest since it always predicts a higher buckling capacity than the two previous cases; i.e. $c = 1$ and $c = 2$.

As for System A, the maximum possible buckling load of System B (Fig. 5) is defined by the curved marked $c = 2$. The normalized ideal stiffness that enables the column to buckle at this value was found to
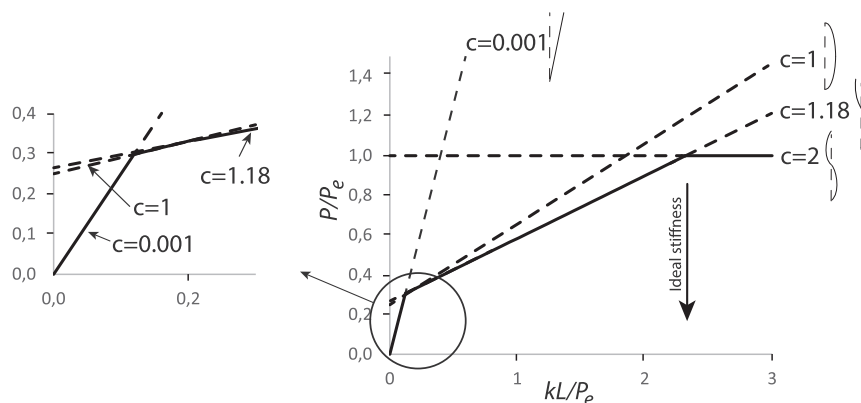
be approximately $kL/P_e = 2.3$ which is the point where the curve marked $c = 1.18$ intersects the curve $c = 2$. It should be emphasized that the curve c = 1.18 which constituted the highest demand on the bracing stiffness prior to reaching the buckling mode of $c = 2$, was determined parametrically by considering many values of c.

2.3. Geometrical imperfections

One or more buckling shapes of a structural element are commonly used to define its initial (imperfect) shape (in deviating from straightness). In engineering applications, sine-functions are commonly used for the modeling of the initial shape of a column.

The geometrical imperfections used in this study (i.e. the numerical modeling) were based on the buckling shapes used to estimate the elastic buckling loads by means of the energy method. These shapes are used both one-by-one and combined in different ways. When these shapes were combined, one is selected as being a major shape (large magnitude) whereas the others (one or more) are considered as being secondary (small magnitude).

The magnitude of the major imperfection shape is *Length*/500 for bow imperfections and the *Length*/200 for sway imperfections; these values are based on imperfection magnitudes suggested by Eurocode 3 [13]. The magnitude of the secondary imperfections are considered as being 1/10 of the magnitude of the major imperfection in the combination considered. It is assumed that the major imperfection should define the appearance and the magnitude of the intended imperfection shape while the secondary imperfection shape should be regarded as a minor disturbance. The assumed value of 1/10 turned out to work rather well here, i.e. it did not change the magnitude of the intended imperfection shape significantly, nor was it so small as to be "neglected" numerically by the FE-software. It should be emphasized that the value could just as well be 1/50, 1/100 or any other value substantially smaller in size than the major imperfection shape value, as long as the FE-software is capable of "detecting" it. The principle for how imperfection shapes were combined is shown by Expression (7). Table 1 presents all the imperfection shapes and magnitudes used for System A and B.

$$\eta(x) = \sum_{n=i}^{v} a_n \delta_n \sin\left(C_n \frac{\pi x}{2L}\right) \tag{7}$$

where $\eta(x)$ is the resulting combined imperfection shape, a a constant which specifies whether the different shapes in the combination in question are either used as a major imperfection shape ($a = 1$), a secondary one ($a = 1/10$) or not used at all ($a = 0$), and c the shape factor as defined in Table 1.

Fig. 5. The critical load for System B (Eq. (6)) for varying bracing stiffness k and different values of the c-coefficient. P is the current buckling load and $P_e = \frac{\pi^2 EI}{L^2}$, i.e. that when buckling between successive bracings would occur. At the left is a zoom of the beginning of the plot, the three modes being defined as $c = 0.001$, $c = 1$ and $c = 1.18$, these competing so as to generate the lowest buckling capacity in the region. The ideal stiffness is indicated by an arrow.

2.4. FE modeling and evaluation

The commercial FE software Abaqus 6.13.1 is used for the investigations here. In order to vary the parameters, such as those of bracing stiffness and of imperfection shapes, use is made of Matlab R2014a for generating input files for Abaqus. The analyses are carried out in 2D, use being made of type B21 beam elements. A subdivision of 4000 (element mesh) is employed. This was found to be more than adequate for obtaining convergence in the model, although its use is primarily motivated in terms of its generating a smooth shape when the imperfections are applied. The bracings are modeled as linear elastic springs connected to the ground with one degree of freedom, which is an engineering feature in Abaqus.

The incremental analyses is based on 3rd order theory (i.e. large deformations) with load control. The typical appearance of the curves obtained in an analysis is shown in Fig. 6.

The definition of elastic "failure", $P_{el-fail}$, used in the study is also indicated in Fig. 6. This is the value that is later presented for the different analyses in the result section as the "elastic limit".

Expression (8), known as Navier's formula, is used as failure criterion taking material strength into consideration. This is a simplified failure criterion defining failure as occurring when the outermost surface of the cross section reaches a yielding point. Real steel columns, to be sure, can develop larger plastic zones prior to failure. However, as soon as yielding in any part of a column's cross section starts, its stiffness will be reduced dramatically. Accordingly, the additional load that can be applied after yielding is usually negligible for slender columns. Thus Eq. (8) is assumed to provide a reasonable failure criterion.

$$\sigma_{failure} = f_y = \frac{N}{A} + \frac{M}{W} \tag{8}$$

where f_y is the yield stress of the material, N the axial force, M the bending moment and W the elastic section modulus of the cross section of the column in question.

3. Results and discussion

3.1. System A

The results of the nonlinear incremental analyses of System A, using different imperfection shapes one-by-one, are presented in Fig. 7 with the normalized buckling load, P/P_e, on the vertical axis and the normalized bracing stiffness, kL/P_e, on the horizontal one. The solid lines constitute the elastic limit of the maximum capacity irrespective of the material yield-
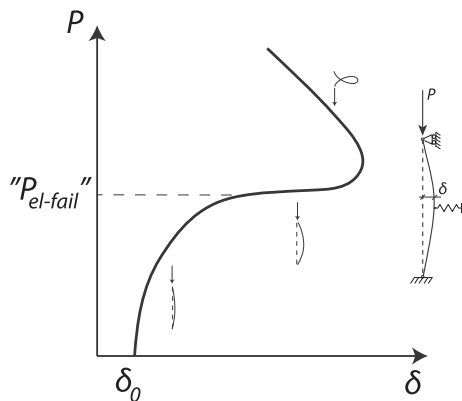

Fig. 6. The typical appearance of the load displacement curves from the FE analyses performed. The elastic limit as defined in this study is the point where the first plateau is reached. This since larger displacements than that would be unrealistic for a real column.
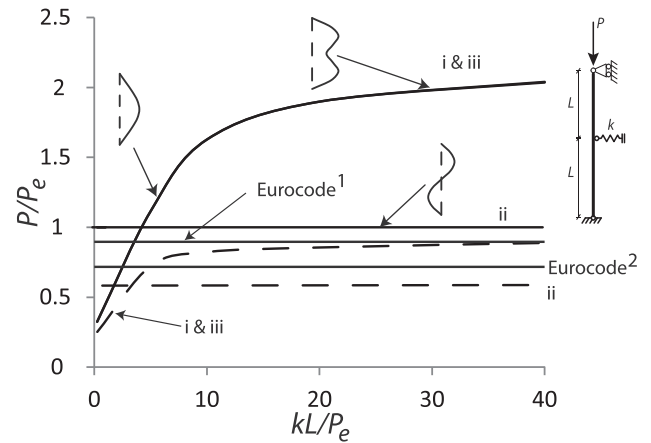

Fig. 7. Nonlinear incremental analysis of System A based on the imperfection shapes of (i),(ii) and (iii) (see Table 1) when used one-by-one. Eurocode[1] refers to the design load that is obtained in accordance to Eurocode 3 if the relative slenderness ratio was specified by the elastic capacity predicted by the use of either of imperfection shapes (i) or (iii). Eurocode[2] is the corresponding value if the elastic capacity predicted by the use of imperfection shape (ii) is used. Otherwise, the solid lines are the buckling loads irrespective of the material yield strength and the dashed ones the buckling loads when the material yield strength is accounted for directly in the FE-analysis with Eq. (8) as Failure criterion.

strength, i.e. when the instability ($P_{el-fail}$ as described in Section 2) for a given bracing stiffness has been reached. The dashed lines are the buckling loads obtained when taking the material yield-strength into account (using Eq. (8) as the failure criterion as explained in Section 2.4). The capacity determined in accordance with Eurocode 3 on the basis of the relative slenderness ratio (as explained in the method section), full bracing being assumed, is also indicated in the figure. The line marked Eurocode[1] ($P/P_e \approx 0.9$), is the design buckling load obtained in accordance with Eurocode 3 if the theoretical buckling capacity (neglecting the yield strength) of $P/P_e \approx 2$ (as predicted by the imperfections (i) and (iii)) had been used to define the relative slenderness ratio. The line marked Eurocode[2] ($P/P_e \approx 0.7$) is the Eurocode 3 design load if the elastic capacity (neglecting the yield strength) of $P/P_e = 1$ had been used to define the relative slenderness ratio. P is the "actual" buckling load and $P_e = \frac{\pi^2 EI}{L^2}$ (used for the normalization) is the Euler load obtained if the column should buckle between successive restraints.

From Fig. 7 it can be seen that all of the imperfection shapes used for System A (see Table 1), when used one-by-one, yielded unreasonable results when employed in a nonlinear incremental analysis. Imperfections (i) and (iii), respectively, made the column buckle in a shape similar to 3 half sine waves, corresponding to $P/P_e \approx 2$, for full bracing. At the same time, it is clear from the results of the energy study (shown in Fig. 4) that the maximum capacity has to be <1, i.e. that $P/P_e \leq 1$. The "unrealistic" behavior in the FE-model can be explained in terms of the lack of a disturbance (e.g. an additional imperfection shape) that would trigger the column to snap into the more intuitive and lower buckling mode, i.e. into the shape of two half sine waves.

The imperfection shape (ii), on the other hand, when the column is considered fully braced, can be said to yield more accurate results in terms of the maximum buckling load; although it was found to be independent of the bracing stiffness, which is obviously wrong. This can also be explained by a lack of a disturbance, in this case at the bracing points.

The column strength calculated by means of the Eurocode using the elastic buckling load predicted by the choice of either imperfection shape (i) or (iii) is $P/P_e = 0.9$ for full bracing as indicated in (7). This is an overestimation by almost 30% as compared with that obtained by the use of the buckling load as predicted by the use of imperfection shape (ii), i.e. involving a capacity of $P/P_e = 0.6$. The failure criterion of

Eq. (8)) which takes material yield strength directly into account generally provides a conservative estimate as compared with Eurocode values based on the relative slenderness ratio. This may be due to the fact that the empirical formulas in the Eurocode are correlated with the results of laboratory tests [16] that can permit some degree of plasticity to occur prior to failure or simply that the imperfection magnitudes are conservative.

The results of the nonlinear modeling of a selection of combined imperfection shapes applied to System A are presented in Fig. 8 in the same manner as the results in Fig. 7. The imperfection shape (see Table 1) that was selected as major in a specific combination is marked with a * which means that a in Eq. (7) is set to 1 for that imperfection. For the secondary imperfection shapes a is set to 1/10 and for those that are not considered a is naturally set to 0.

In Fig. 8 it is clearly shown that most of the analyses of System A that made use of combined imperfection shapes yielded reasonable results in terms of elastic buckling capacity. However, the combination of imperfection shapes (i) + (iii) yielded unrealistic results, similar to those obtained in using the imperfection shapes one-by-one. This indicates that, even if combined imperfection shapes are employed, unrealistic results may still be obtained. As shown in Fig. 8, all the other combinations that were tested gave the same elastic response; i.e. a maximum elastic capacity of $P/P_e = 1$ corresponding to the case of full bracing. The full bracing stiffness, corresponding to the ideal stiffness of a perfect system, is $kL/P_e \approx 8$, which is greater than $kL/P_e = 2$, which was obtained for the corresponding perfect system (i.e. without imperfections, see Fig. 4). These results demonstrates clearly how imperfections increase the bracing requirement.

Although most of the analyses using combined imperfection shapes yielded the same elastic response, there was an apparent difference in expected design capacity according to Eq. (8) (the dashed curves in Fig. 8). It is clear that if the maximum initial displacement of the total imperfection shapes is located between the restraints (e.g. with imperfection shape (ii) as the major one), a lower capacity was predicted. This is due to the larger bending moments found in the column as compared with the case of the maximum initial displacement of the major imperfection shape being located at or close to the bracing point (e.g. imperfection shape (i)). This also means that a combination of initial imperfection shapes that results in a maximum initial displacement between the restraints is needed in order for a conservative estimate of the strength of the column to be obtained.

For the two different imperfection combinations i* + ii + iii and i + ii* + iii, respectively, the bracing force was recorded in the nonlinear analyses; the results are presented in Fig. 9. The curves shown are a function of the varying bracing stiffness at different load levels, as
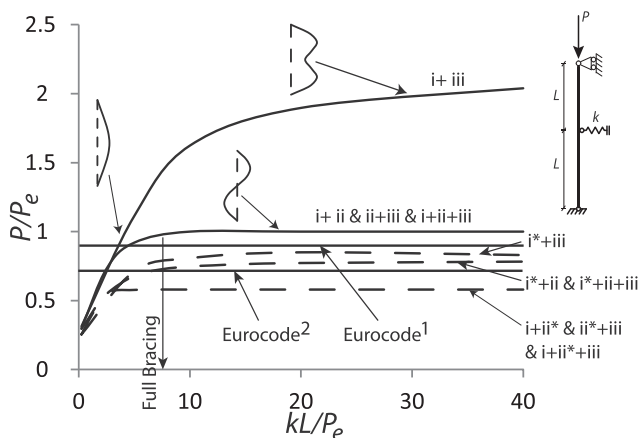
expressed by P/P_e, where P_e is the Euler load for buckling between the restraints.

From Fig. 9 it is evident that the choice of imperfection shapes affects the bracing force obtained. When the imperfection shape (i) is considered as being major (Fig. 9a), the maximum initial displacement is located close to the bracing. This yields a stronger bracing force than when mode (ii) is considered as being major (Fig. 9b), in which there is a small initial imperfection displacement at the bracing point. In fact, the bracing forces are 6–10 times larger. This indicates clearly that if a conservative estimate of the bracing requirements is to be made, a combination of imperfection shapes that results in a large initial displacement at the bracing point should best be employed. In addition, as indicated in Fig. 9, the bracing forces tend to be very large in the case of high level of load (i.e. for $P/P_e = 1$ and $P/P_e = 0.7$ in the figure), when the stiffness is close to or less than the full bracing requirement. This confirms statements by Winter [2] and Yura [3] that a bracing stiffness substantially greater than the full bracing requirement (sometimes called ideal stiffness) is required in order to keep the bracing forces at reasonable levels.

To summarize the investigation of System A, the results indicate that different combinations of imperfection shapes yields conservative results both in terms of the bracing system requirements and of the strength of the column. An imperfection generating an initial displacement that has its maximum between the bracings yielded conservative results in terms of predicting the strength of the column, whereas an imperfection shape generating a large displacement at the bracing point yielded conservative results in terms of the strength required of the bracings. This suggests that if nonlinear modeling is used for designing purposes, more than one combination of imperfection shapes ought to be considered in order to most effectively obtain an overall safe design.

3.2. System B

The results of the nonlinear incremental modeling of System B, using the different imperfection shapes (defined in Table 1) one-by-one, are presented in Fig. 10. The solid lines represent the elastic buckling load, disregarding the material yield strength. The dashed lines are the buckling loads accounting for the material yield strength according to Eq. (8). The line marked as Eurocode[1] ($P/P_e \approx 0.9$) is the design load predicted by Eurocode 3 if the elastic capacity predicted by the use of either imperfection shape (i), (iii) or (iv) (from the FE-analysis) would be used to define the relative slenderness ratio. The line marked Eurocode[2] ($P/P_e \approx 0.7$) is the design load in accordance with Eurocode 3 if the elastic capacity of $P/P_e = 1$ would be used instead. P is the actual load and $P_e = \frac{\pi^2 EI}{L^2}$ is the Euler load if the column buckles between the restraints.

Fig. 10 can be interpreted as indicating that imperfection shapes (i), (ii),(iii) and (iv) for System B all yielded unreasonable results when used one-by-one in the nonlinear incremental analysis. Imperfections (i), (iii) and (iv), respectively, made the column buckle in a shape similar to 3 half sine waves and yielded a buckling load of $P/P_e \approx 2$ for the case of a fully braced system. This result is unrealistic since it should be <1, i.e. that $P/P_e \leq 1$ (as was found and explained for System A). It was also found that imperfection shape (ii) can yield reasonable results in terms of the maximum buckling load when it is fully braced. However, this load was independent of the bracing stiffness, which as previously statet, is unrealistic. The imperfection shape (v), on the other hand, appears to generate reasonable results. In this case since the maximum capacity of $P/P_e = 1$ is reached for a bracing stiffness that is greater than the ideal stiffness predicted for the corresponding perfect system; see Fig. 5. This can probably be explained by the non-symmetrical nature of the initial imperfection shape in question.

The results of the nonlinear incremental modeling of System B for a selection of combined imperfection shapes are presented in Fig. 11. It should be pointed out that there are a large number of possible combinations beyond that which is presented here. Therefore, the results presented and discussed here, should be regarded as examples of possible
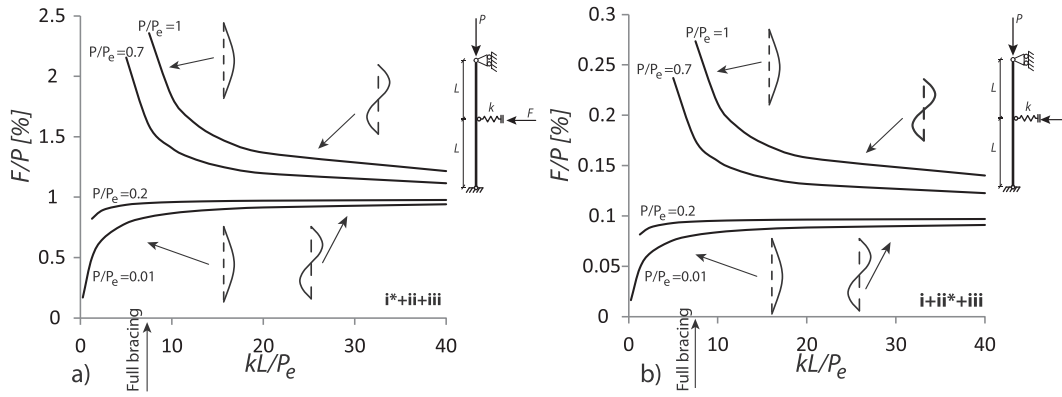

Fig. 8. Nonlinear analysis of System A for a selection of combined imperfection shapes, where the * indicates which shape is considered as being the major imperfection.

Fig. 9. The bracing forces of System A when a combination of imperfections (i) + (ii) + (iii) (defined in Table (7)) was employed, where in a) the imperfection shape (i) is considered being the major imperfection, whereas in b) the imperfection shape (ii) is considered being the major imperfection.

output when using the non-linear incremental analysis in design, not necessarily representative of the best design assumptions. As for System A, the imperfection(s) considered major in a specific combination is/are marked by a $*$.

Fig. 11 indicates clearly that most of the System B analyses using combined imperfection shapes yielded reasonable results in terms of elastic buckling capacity. In contrast, the combination of imperfection shapes (i) + (iii) yielded unrealistic results that were similar to those obtained in using the imperfections separately. This indicates that, even when combined imperfection shapes are employed, unrealistic results may still be obtained. All other combinations that were tested gave the same elastic response as was indicated in Fig. 11, there being a maximum elastic capacity of $P/P_e = 1$ for full bracing. The full bracing stiffness is $kL/P_e \approx 8$, which is greater than $kL/P_e \approx 2.3$ for the corresponding perfect system (shown in Fig. 5). This demonstrates clearly how imperfections increase the bracing requirement level, as was shown for System A. Also, as was likewise shown for System A, it is evident that an imperfection shape generating a maximum initial displacement between the bracings, such as the combination of (ii)$*$ + (iv)$*$ in Fig. 11, predicts a lesser degree of strength than combinations in which the maximum initial imperfection displacement is located at (or close) to the bracing point, such as the combination (iv)$*$ + (v)$*$.

For the two different imperfection shape combinations of (i)$*$ + (ii) + (iii) + (v) and (i) + (ii)$*$ + (iii) + (v) the forces in the bracings were recorded in the nonlinear incremental analyses; see Fig. 12. The

different curves correspond to varying load levels being applied to the column, expressed as a fraction of the Euler load, P_e (defined as buckling between the bracings); values of $P/P_e = 1, 0.7, 0.2, 0.1$ were considered.

In Fig. 12 it is evident that the choice of imperfection shape affects the bracing forces generated in the bracings. When the imperfection shape (i) is considered as being the major imperfection shape, such as in Fig. 12a, the maximum initial imperfection displacement is located close to the middle bracing. This yielded a greater bracing force than shape (ii) as major did, as shown in Fig. 12b. The resulting bracing forces were 7.5–10 times larger for the top bracing, and 9–10 times larger for the middle bracing (in the case of a high level of bracing stiffness). This indicates clearly that if a conservative estimate of the bracing requirement in terms of strength were to be made, a combination of imperfection shapes resulting in a large initial displacement at the bracing points should be employed. Also, as seen in Fig. 12, the bracing force tends to take on a very large value for large loads of P (i.e. for $P/P_e = 1$ and $P/P_e = 0.7$ in the figure) when the level of stiffness is low (close to the full bracing requirement), in the same manner as was found for System A.

In designing according to the Eurocode, the sway imperfection (shape (iv) in Table 1) should likely always be regarded as a major imperfection when present in combination with a bow mode; since this is an imperfection shape related to the erection of the structure rather than to the distortion of the column itself. Fig. 13 was thus prepared so as to include the two imperfection combinations of
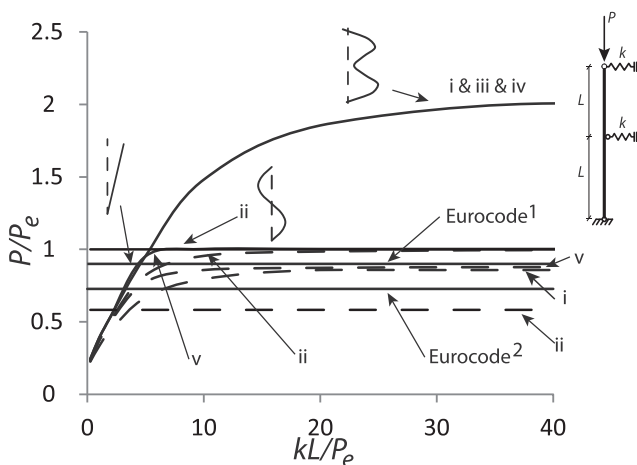

Fig. 10. Nonlinear incremental analyses of System B, using the different imperfection shapes defined in Table 1 one-by-one.
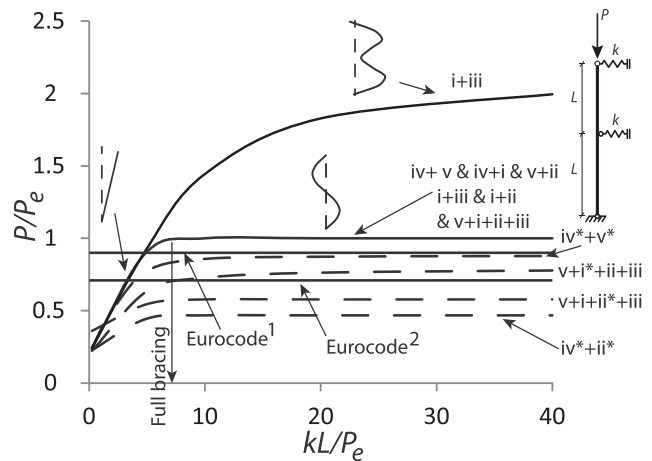

Fig. 11. Nonlinear incremental analyses of System B for a selection of combined imperfection shapes, where the $*$ indicates which shape/s that was/were considered as being the major one/s in a specific combination.
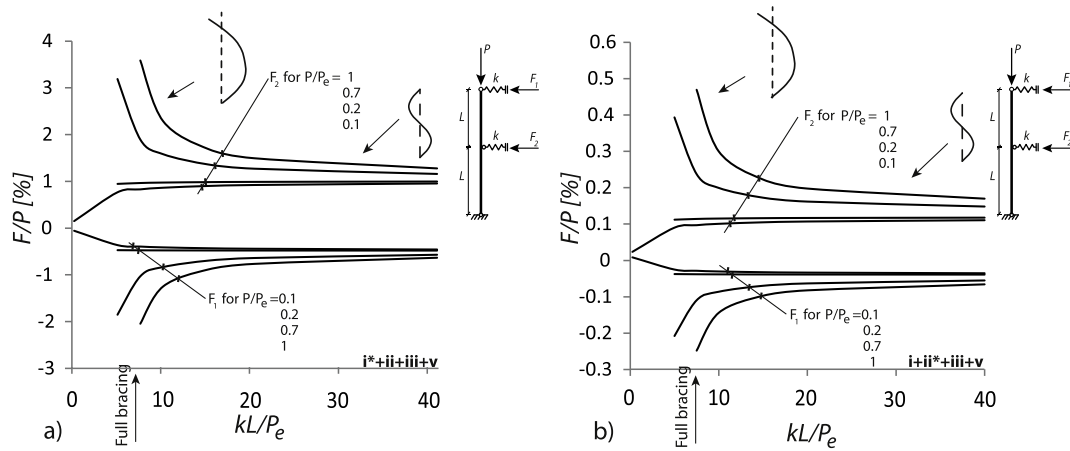
Fig. 12. Bracing forces of System B. The * indicates which imperfection shape was considered as being major, where in a) imperfection shape (iii) is the major one, whereas in b) the imperfection shape (iv) is the major one.

$(i)^* + (ii) + (iii) + (iv)^* + (v)$ and $(i) + (ii)^* + (iii) + (iv)^* + (v)$, respectively.

In Fig. 13 it is shown that the top bracing force is affected more strongly by the sway imperfection than the middle bracing force. When the bow shape (i) is employed as the major imperfection shape, provided it is present together with the sway shape, the top bracing forcec dropped to 0–0.4%; whereas for the same case the bracing force in the middle bracing remained unchanged within the interval of 1–1.4% of the applied load. Shape (ii) as the major bow mode results in a top bracing force in the range of 0.54–0.56% (an increase), whereas the bracing force in the middle bracing increases to 0.32–0.38% of the applied load. This means that a sway imperfection generates a larger, and more reasonable, bracing force for the middle bracing if mode (ii) is considered as being the major (bow) imperfection shape. However, it is still the bow imperfection shape with its maximum initial displacement at the middle bracing point (mode (i)), that resulted in the largest bracing forces.

To summarize the results obtained for System B, there is a strong indication that different combinations of imperfection shapes should be used for conservative design of the bracing system and the column itself. An imperfection shape that generated an initial displacement, the maximum of which was located between the bracings resulted in conservative results in terms of predicting the strength of the column. Whereas an imperfection shape that generated a large displacement at the bracing point resulted in conservative results in terms of the strength of the bracings. This suggests that if nonlinear modeling is used

for designing purposes, more than one combination of imperfections should be considered in order to be as certain as possible of obtaining safe results.

4. Conclusions

The effects of the imperfection shapes on the strength and stiffness requirements of two different column systems and their bracings were investigated. An energy solution to the initially straight and elastic column was derived for both cases and was used successfully for defining the imperfection shapes and the elastic limits of the column. The first column system investigated was that of a sway-prevented column having one intermediate bracing (System A), the second one being a sway-permitted column having one bracing at the top and one at mid-length (System B). The general conclusions of the study, which were found to hold for both systems, are as follows:

1. A poor choice of an imperfection shape can lead to unrealistic results when nonlinear FE analyses on slender columns are performed. For the systems analyzed in the present study, an imperfection choice predicting an elastic capacity twice as high as the Euler load for the buckling between the restraints was presented. At least one solution to this problem was to superimpose different imperfection shapes.

2. A higher degree of stiffness of the bracings was found to be required in order to force the imperfect column to buckle between the restraints than was determined to be the case for initially straight
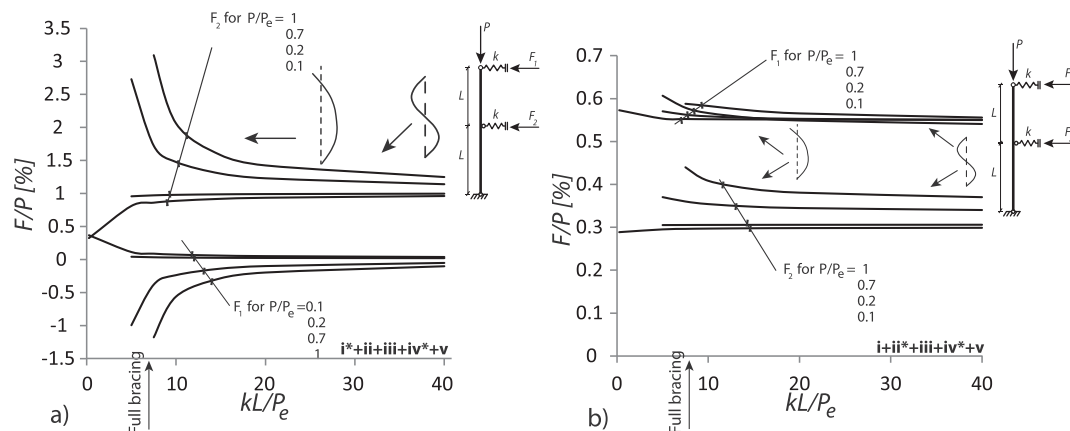

Fig. 13. The bracing forces of System B when imperfection shape (iv), the sway imperfection, is considered as being the major one together with either shape (i) or (ii), where in a) (i) and (iv) are the major imperfection shapes, whereas in b) (ii) and (iv) are the major imperfection shapes.

elastic column. This was not possible to detect in the FE-modeling that was conducted when only an imperfection mode having pivot points at the bracing points was employed, i.e. an imperfection mode associated with the highest possible buckling mode (Euler buckling between the restraints). In order to solve this problem, the imperfection mode in question would need to be complemented by at least one secondary imperfection shape of another mode.

3. The approach of classifying one imperfection shape as being the major one and assuming one or more other shapes to be secondary turned out to yield reliable results in most cases. However, it was also demonstrated that there could be combinations of different two imperfection shapes that yielded unrealistic results. Accordingly, use of a combination of at least three different imperfection shapes is recommended.

4. An imperfection shape that generates a large initial displacement between the bracings led to higher bending moments in the column at loading than an imperfection shape for which the maximum initial displacement was at the bracing points. This means that an imperfection shape of this type is needed in order to obtain a conservative estimate of the column strength required.

5. An imperfection shape generating a large initial displacement at the bracing points was found to predict a higher level of forces in the bracings at loading. Accordingly, such an imperfection shape should be used so as to obtain a conservative estimate of the required bracing strength and stiffness.

6. Points 4 and 5 suggest that at least two different sets of combinations (each consisting of at least 3 sub-shapes according to paragraph 3 above) of imperfection shapes should be used in design so as to obtain safe results both in terms of the strength of the column and the stiffness and strength requirements of the bracings.

5. Discussion

The conclusions of this investigation are probably valid for other column configurations as-well.

It should be emphasized that if larger models containing several columns with multiple bracings are analyzed, an evaluation of the structural behavior in an intuitive manner may be difficult. The possible number of combinations of different imperfections shapes will simply be too large. However, qualified model assumptions on any structures initial imperfection shapes can hopefully be made, the results of this study being known. The advise for engineers designing slender structures by means of nonlinear analysis are therefore simply to 1) make sure to try many different possible imperfection shapes based on engineering judgement and experience and 2) to use simplified methods to validate the results from the modeling.

Finally, it should be stated that this investigation does not prove which the worst imperfection shape is for any structures, further studies being needed. A case specific evaluation of different imperfection shapes is always recommended when design is based directly on non-linear FE-methods.

References

[1] T. V. Galambos, A. Surovek, Structural Stability OF Steel: Concepts and Applications for Structural Engineers., Hoboken, N.J.: John Wiley & Sons, 2008. [cop. 2008, URL http://ludwig.lub.lu.se/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=cat01310a&AN=lovisa.001993311&site=eds-live&scope=site]

[2] Winter G. Lateral bracing of columns and beams. Proc ASCE 1958;84(ST2) [1561–1–1561–22].

[3] Yura J. Winter's bracing approach revisited. Eng Struct 1996;18(10):821–5 [URL http://ludwig.lub.lu.se/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=edselc&AN=edselc.2-52.0-0030269617&site=eds-live&scope=site].

[4] Timoshenko S, Gere JM. Theory of Elastic Stability, Engineering Societies Monographs. New York: McGraw-Hill; 1961[1961. URL http://ludwig.lub.lu.se/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=cat01310a&AN=lovisa.001623441&site=eds-live&scope=site].

[5] Trahair N. Column Bracing Forces. University of Sydney, Department of Civil Engineering, Centre for Advanced Structural Engineering; 1999[URL https://books.google.se/books?id=x9k6HQAACAAJ].

[6] Plaut R. Requirements for lateral bracing of columns with two spans. J Struct Eng (United States) 1993;119(10):2913–31 [URL http://ludwig.lub.lu.se/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=edselc&AN=edselc.2-52.0-0027675698&site=eds-live scope=site].

[7] Plaut RH, Yang Y-W. Behavior of three-span braced columns with equal or unequal spans. J Struct Eng 1995;121(6):986 [URL http://ludwig.lub.lu.se/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=a9h&AN=6784243&site=eds-live&scope=site].

[8] Plaut RH, Yang J. Lateral bracing forces in columns with two unequal spans. J Struct Eng 1993;119(10):2896 [URL http://ludwig.lub.lu.se/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=edo&AN=ejs14302873&site=eds-live&scope=site].

[9] Mehri H, Crocetti R, Gustafsson PJ. Unequally spaced lateral bracings on compression flanges of steel girders. Structures 2015;3:236–43.

[10] Schlune H. Safety Evaluation of Concrete Structures with Nonlinear Analysis. Thesis for the degree of doctor of philosophy Ny serie: 3232. Gothenburg: Chalmers University of Technology; 2011[2011. URL http://ludwig.lub.lu.se/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=cat01310a&AN=lovisa.001994827&site=eds-live&scope=site].

[11] Wang L, Helwig TA. Critical imperfections for beam bracing systems. J Struct Eng 2005;131(6):933–40 [URL http://ludwig.lub.lu.se/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=a9h&AN=17070134&site=eds-live&scope=site].

[12] Girão Coelho AM, Simão PD, Wadee MA. Imperfection sensitivity of column instability revisited. J Constr Steel Res 2013;90:265–82 [URL http://ludwig.lub.lu.se/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=edselp&AN=S014397 4X13002447&site=eds-live&scope=site].

[13] EN 1993-1-1:2005 Eurocode 3: design of steel structures - part 1-1: general rules and rules for buildings. [URL] http://ludwig.lub.lu.se/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=cat02271a&AN=atoz.ebs1035047059e&site=eds-live&scope=site; 2008. [n.d.].

[14] Gil H, Yura JA. Bracing requirements of inelastic columns. J Constr Steel Res 1999;51:1–19 [URL http://ludwig.lub.lu.se/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=edselp&AN=S0143974X99000024&site=eds-live&scope=site].

[15] Mutton B, Trahair N. Design requirements for column braces. Inst Eng Aust Civ Eng Trans 1975;CE-17(1):30–6 [cited By 10. URL http://www.scopus.com/inward/record.url?eid=2-s2.0-0016444291&partnerID=40&md5=f2ff05a52d2632b4edbda0f62e2a5eae].

[16] Szalai J, Papp F. On the theoretical background of the generalization of ayrton–perry type resistance formulas. J Constr Steel Res 2010;66:670–9 [URL http://ludwig.lub.lu.se/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=edselp&AN=S0143974X09003010&site=eds-live&scope=site].