



A Numerical Solution for the Shape of Fabric-formed Concrete Structures



Robert M. Foster ^{*,1}, Tim J. Ibell

Department of Architecture and Civil Engineering, University of Bath, Bath BA2 7AY, United Kingdom

ARTICLE INFO

Article history:

Received 19 June 2016

Received in revised form 5 August 2016

Accepted 5 August 2016

Available online 07 August 2016

Keywords:

Concrete

Fabric formwork

Form-finding

Hydrostatic load

ABSTRACT

This paper details a new numerical method for determining the form of a section of flexible, impermeable and inextensible hanging fabric subject to the hydrostatic load imposed by wet concrete. A closed form solution is already known to exist in the form of incomplete elliptic integrals, but this can be difficult to implement in practice. The numerical method presented here was developed by the authors, and the method has been subsequently implemented by researchers investigating the behaviour of fabric-formed concrete beams. It is thought that this method may be of use to other designers and investigators interested in the form of flexible formworks and also of wider interest for the design of flexible scoops or other hydrostatically loaded structures. The method is shown to be applicable to full, part-full and overfull, i.e. surcharged, containers. The method's accuracy is demonstrated by comparison with the predictions of the closed form solution and with reference to the conclusions of previous empirical investigations. Comparison is also made on a graphical basis to a number of reported hydrostatically determined forms, with good agreement being shown. This indicates that the approach has direct application as a form-finding procedure for fabric-formed concrete structures.

© 2016 The Institution of Structural Engineers. Published by Elsevier Ltd. All rights reserved.

1. Introduction

Wet concrete is effectively liquid. In order to form useful structural elements, liquid concrete must be contained by formwork until it is suitably cured. If the container is rigid, the determination of the final form of the concrete is simple, as the concrete takes the shape of the container into which it is poured. If the container is soft, such that it undergoes significant out of plane deformations when filled, the determination of the final form of the concrete is more difficult. This paper provides a numerical solution to the two-dimensional case of this problem. The method was originally developed by the authors for the determination of the sectional profile of a fabric-formed concrete beam. Both this specific application and the more general numerical solution are discussed.

2. Fabric-formed concrete elements

While rigid, impermeable, rectilinear containers constructed from earth, timber, steel or plastic are the principal methods used for the forming of concrete elements, there are alternatives. A design for the use of draped flexible fabrics as formwork for variable section concrete spanning elements was patented by Gustav Lilienthal [10]. Fletcher [2]

explicitly observed in relation to his own textile formwork designs that, despite the obvious potential advantages of variable section concrete elements in achieving optimised profiles, they are “not generally put into practice because of the prohibitive cost of constructing a form in which to mold such a shape”. This continued to be the case for much of the last century, prompting Nervi [11] to note that:

“although reinforced concrete has been used for over a hundred years and with increasing interest during the last few decades, few of its properties and potentialities have been fully exploited so far. Apart from the unconquerable inertia of our minds, which do not seem able to adopt freely any new ideas, the main cause of this delay is a trivial technicality: the need to prepare wooden forms.”

In the latter half of the twentieth century, a number of innovators including Miguel Fisac and Kenzo Unno explored the architectural possibilities of quasi-structural concrete elements and panels formed in fabrics. Exploratory work by Mark West, has led to resurgence in interest in fabric formed concrete as an architectural language and as a means for achieving optimised structural sections. Approaches to the optimisation of concrete elements using variable section profiles, often with fabric formwork, is a growing research area with analytical and experimental work carried out in recent years by Bailiss [1], Garbett [4], Veenendaal [14], Foster [3], Lee [8], Hashemian [6], and Orr [12]. While the sometimes bulbous profile of fabric formed concrete elements is a natural consequence of the flexibility of the concrete filled fabric, various approaches may be adopted to limit formwork deformations for the

* Corresponding author.

E-mail address: rmf41@cam.ac.uk (R.M. Foster).

¹ Present address: Department of Architecture, University of Cambridge, Cambridge CB2 1PX, United Kingdom.

purposes of optimisation, including tensioning of the fabric, stitching or pinching of the web, and the use of rigid web formers. A range of woven materials have been used as fabric formwork including hessian, geotextiles and polyesters [13].

Aside from the architectural possibilities opened up by the curvilinear forms that result from the use of fabric formwork (Fig. 1), there are a number of structural, economic and environmental benefits:

1. Structurally efficient profiles may be obtained without the need for elaborate formwork, leading to reduced concrete volumes compared with 'equivalent' rectangular elements [13]. This material saving has a direct benefit in terms of reduction in embodied carbon and cost.
2. A significant proportion of the load carried by typical concrete structures is due to the self-weight of the structural elements themselves. Profile optimisation thus has a further beneficial effect in terms of reducing element self-weight. This reduction affects not only the design of the element itself, but also the design of the supporting elements. Lighter spanning elements can lead to smaller column sizes and reduced foundations. Smaller columns also lead to greater net internal areas, thereby further increasing the value proposition.
3. Shallower portions of such beams, typically in regions of lower bending moment, can provide natural paths for services (Fig. 2). This may allow ceiling voids to be reduced and commensurate increases in floor to ceiling height, or reductions in overall storey height, to be achieved. Furthermore, the aesthetic quality of the curvilinear beams that result may encourage architectural designers to forego suspended ceilings altogether, enabling the benefits of the exposed concrete thermal mass to be more readily exploited.
4. Improved surface concrete quality, both mechanical and aesthetic, has been observed as a result of the reduced water to cement ratio of the surface concrete due to the permeability of the formwork [5].

This has particularly important implications in situations where concrete grade is governed by durability requirements, with enhanced surface quality potentially allowing a reduction in concrete grade [13]. The higher quality of the surface concrete further enhances the aesthetic case for the foregoing of suspended ceilings and the exposure of the structural mass.

5. Suitable fabric selection can allow for a reusable, compact, lightweight formwork system that reduces transportation costs and construction waste.

Despite the potential benefits, designing for flexible formwork presents a number of challenges – not least of which has been the historical difficulty of determining the final shape of the fabric container when filled with concrete. This was the challenge that led the authors to develop the numerical solution presented here.

3. Form finding

Flexible, inextensible elements may take a variety of forms dependent on loading condition:

- the *parabola* – uniform vertical pressure by plan length,
- the *catenary* – uniform vertical pressure by arc length,
- the *lintearia* – hydrostatic normal pressure,
- the *valeria* – uniform normal pressure.

These forms have long been the subject of enquiry and their solutions have been closely related to the development of elastic bending theory [9]. A comparison of these forms is shown in Fig. 3 for profiles of equal sag and varying span. While the forms of the parabola, catenary and valeria are relatively easy to describe mathematically, the form of

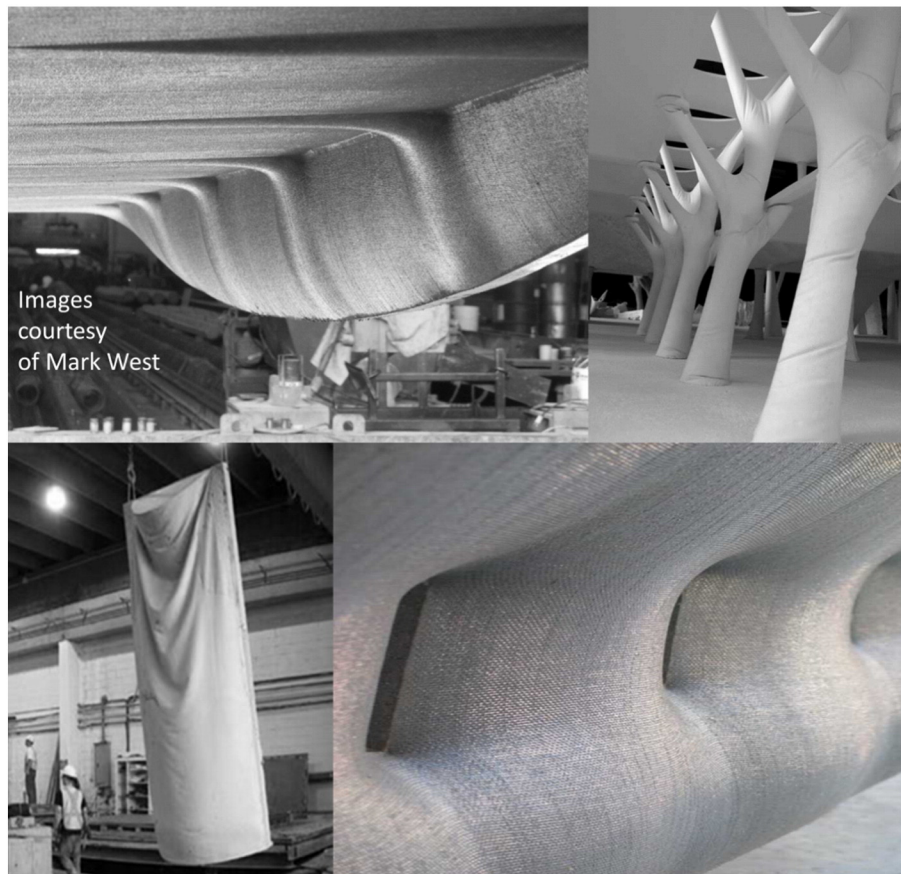


Fig. 1. Architectural possibilities afforded by fabric-formed concrete. Images courtesy of Mark West.

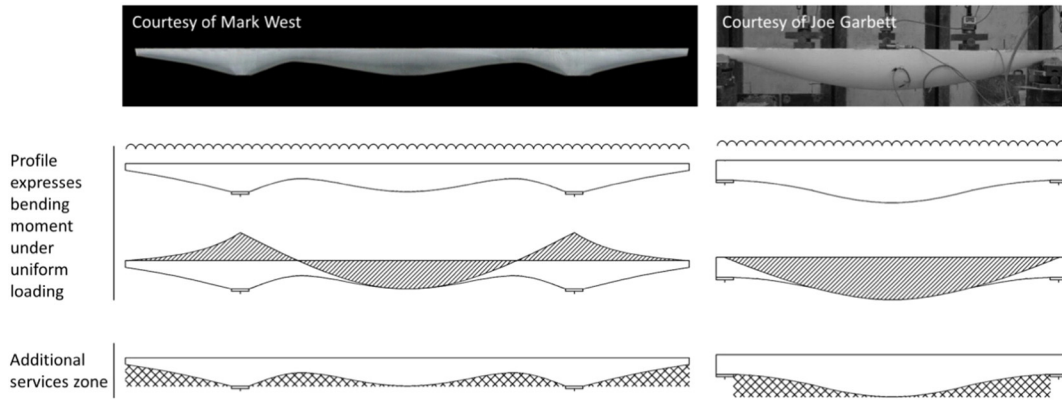


Fig. 2. Indicative relationships between profile, bending moment and resulting additional service zone for typical fabric-formed flexural members. Photographic images courtesy of Mark West, and Garbett [4].

the *linteria* or hydrostatically loaded profile is not. This is problematic because precise description of the form taken by a flexible fabric or container subject to hydrostatic load is thought to be of potential interest in a wide variety of engineering applications. A rigorous closed form solution is provided by Iosilevskii [7] in the form of incomplete elliptic

integrals, but the mathematics involved is likely to be unfamiliar to the non-specialist and may be difficult to implement. In the context of fabric-formed concrete elements, experimental investigations have determined some approximate empirical relationships between characteristic dimensions [1], but have been unable to provide comprehensive descriptions of the fabric profile. The numerical approach presented here was developed in order to provide such a description [3] and thereby open up possibilities for the design of fabric-formed concrete elements.

- parabola: uniform vertical pressure by plan length
- catenary: uniform vertical pressure by arc length
- - - linteria: hydrostatic normal pressure
- valeria: uniform normal pressure

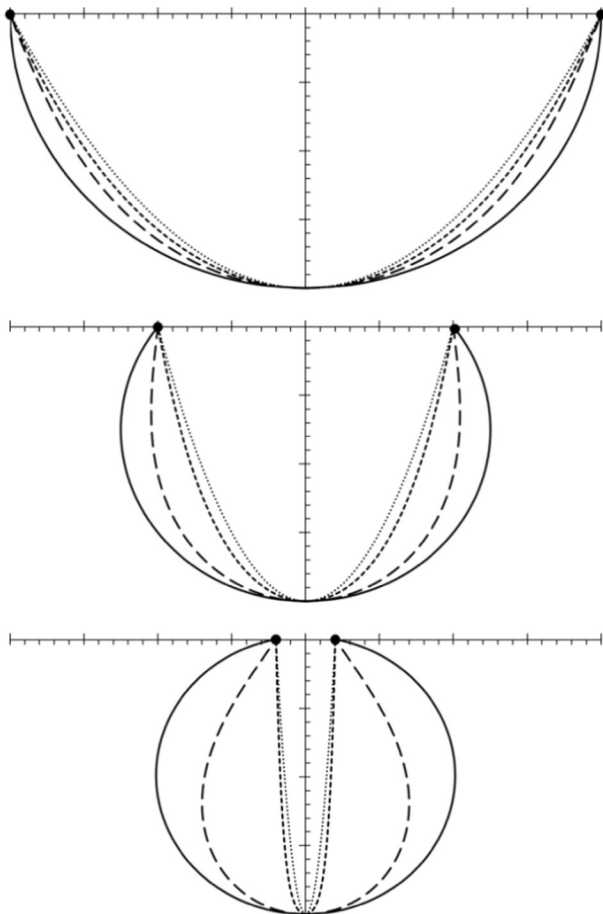


Fig. 3. Comparison of parabola, catenary, linteria and valeria profiles of equal sag and varying span.

4. Formulation

Consider a flexible, inextensible fabric having negligible out of plane stiffness and self-weight. The fabric is hung between two nodes positioned such that a line connecting them is horizontal. The fabric supports a fluid of density ρ . The nodes are separated by an opening breadth b ; the fabric is of length p and forms the hanging perimeter of a section of maximum depth d . It is taken as axiomatic that $p \geq b$. A coordinate system of mutually orthogonal axes can be defined such that the origin is located at the lowest point of the hanging fabric (Fig. 4). The x -axis is horizontal and the y -axis is vertical. Thus the coordinates of the hanging nodes are $(\pm b/2, d)$ and the coordinates of the lowest point of the hanging perimeter are $(0, 0)$.

The fabric will freely deform under any applied out-of-plane forces, and static equilibrium can only be maintained by resisting forces carried entirely in-plane. By definition, all hydrostatic forces must act perpendicular to the fabric and the fabric will freely deform to carry the resisting forces as a tension in plane. It follows that the tension T_0 in the fabric is constant around the perimeter of any given hydrostatically determined profile [7]. Thus, the hanging perimeter can be thought of as forming a two dimensional surface minimizing surface tension. An alternative approach to the same problem might be to think in terms of minimizing the height in y of the centre of mass of the enclosed fluid.

Since the only applied forces acting on the fabric are the hydrostatic forces due to weight of the fluid, it can be deduced that the curvature of the profile at any given point is solely determined by the hydrostatic pressure at that point. Given that hydrostatic pressure $\rho g z$ varies linearly with hydraulic height z in y , a simple model for the variation in curvature κ and thus the hydrostatically determined profile of the perimeter can be defined:

$$T_0 = \frac{\rho g z}{\kappa} \tag{1}$$

Rather than proceeding towards a closed-form analytical characterisation cf. Iosilevskii [7], the hydrostatically determined form can be investigated by starting from the point $(0, 0)$ and ‘walking out’ a perimeter in accordance with the relation between the curvature and z . The starting point of $(0, 0)$ was chosen here for the simplification available

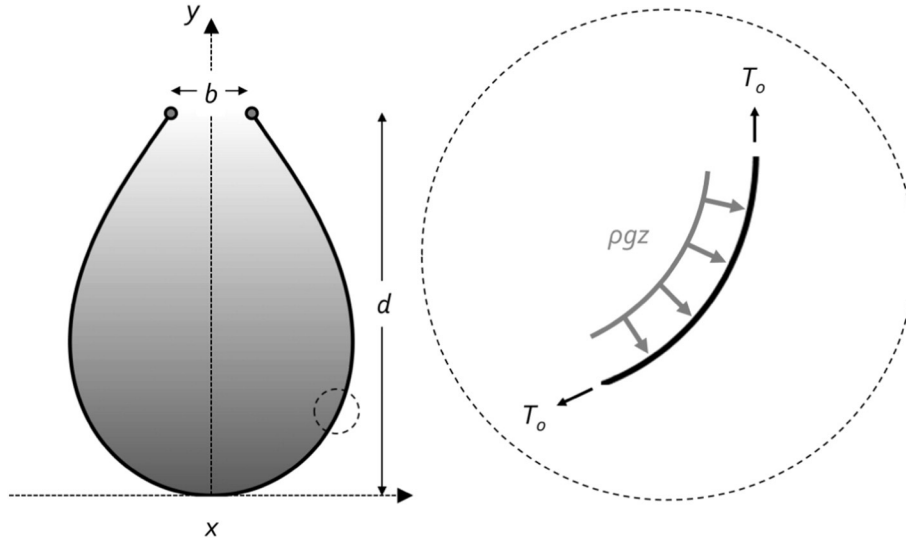


Fig. 4. Coordinate system and forces on free element of fabric.

by symmetry and because section depth is often a determining structural and architectural characteristic. It should be noted that ‘walking out’ the perimeter from (0, 0) simply generates a curve that conforms to a chosen relation between curvature and depth. Altering this relation by varying a coefficient k allows a family of curves to be generated. These are, in effect, the curves of the *elastica*.

5. Procedure

‘Walk out’ from (0, 0) by small increment δL_1 (Fig. 5), arriving at coordinates:

$$(x_1, y_1) = (\delta L_1, 0) \quad (2)$$

Where y_n is the y coordinate of the n th increment, define z_n by:

$$z_n = z - y_n \quad (3)$$

Define a small angle $\delta\theta_n$ by:

$$\delta\theta_n = kz_n \quad (4)$$

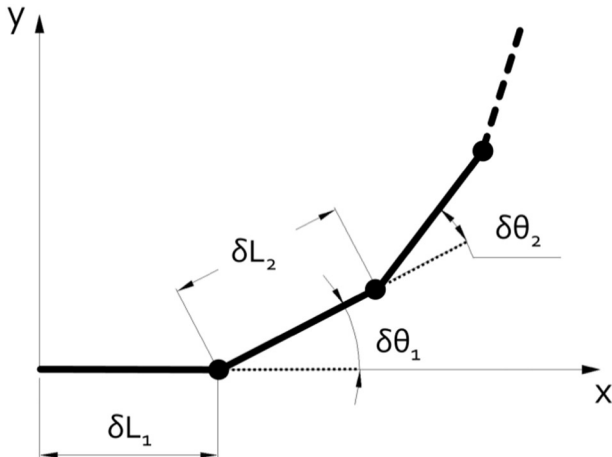


Fig. 5. Characterisation of the ‘walking out’ procedure, beginning at the lowest level of the hanging fabric profile. The perimeter is thus described by a finite number of discrete steps.

Proceed a further small increment δL_2 , at $\delta\theta_1$ to δL_1 , arriving at coordinates:

$$(x_2, y_2) = (\delta L_1 + \delta L_2 \cos(\delta\theta_1), \delta L_2 \sin(\delta\theta_1)) \quad (5)$$

Proceed a further small increment δL_3 , at $\delta\theta_2$ to δL_2 , arriving at coordinates:

$$(x_3, y_3) = \left(\begin{array}{c} \delta L_1 + \delta L_2 \cos(\delta\theta_1) + \delta L_3 \cos(\delta\theta_1 + \delta\theta_2) \\ \delta L_2 \sin(\delta\theta_1) + \delta L_3 \sin(\delta\theta_1 + \delta\theta_2) \end{array} \right) \quad (6)$$

Thus, for the n th small increment δL_n , at $\delta\theta_{n-1}$ to δL_{n-1} , arrive at coordinates:

$$(x_n, y_n) = \left(\sum_{i=1}^n \delta L_i \cos \left(\sum_{j=1}^{i-1} \delta\theta_j \right), \sum_{i=2}^n \delta L_i \sin \left(\sum_{j=1}^{i-1} \delta\theta_j \right) \right) \quad (7)$$

Choosing a constant value for δL , the expression can be further simplified to:

$$(x_n, y_n) = \left(\delta L \sum_{i=1}^n \cos \left(\sum_{j=1}^{i-1} \delta\theta_j \right), \delta L \sum_{i=2}^n \sin \left(\sum_{j=1}^{i-1} \delta\theta_j \right) \right) \quad (8)$$

The summations to determine these coordinates are easily tabulated or programmed, and an iterative ‘solver’ or similar algorithm can then be applied to converge on a value of k such that a set of coordinates $(x_n, y_n) \approx (b/2, d)$ is obtained to a desired accuracy.

Small angle theory is not used for determining coordinates as a global coordinate system is adopted and coordinates at each step determined directly in reference to this. The progressive summation of small angles would quickly render assumptions about the behaviour of small angles inappropriate. Alternatively, a local coordinate system might have been adopted that ‘walks out’ with the fabric. For the purposes of the original research it was considered more convenient to refer directly to the global coordinate system.

Previous work [1,7] considered a fabric container where the free surface of the fluid is level with the opening i.e., $z = d$ (see Fig. 6). An advantage of the procedure detailed above is that it applies for the over-full case where the fluid is placed under an additional surcharge pressure expressed in head, i.e., $z > d$. This is a useful result because this is the condition that occurs in the bottom flange of a fabric-formed I or T-beam, where stiffened web formers are used to create a more efficient web-profile.

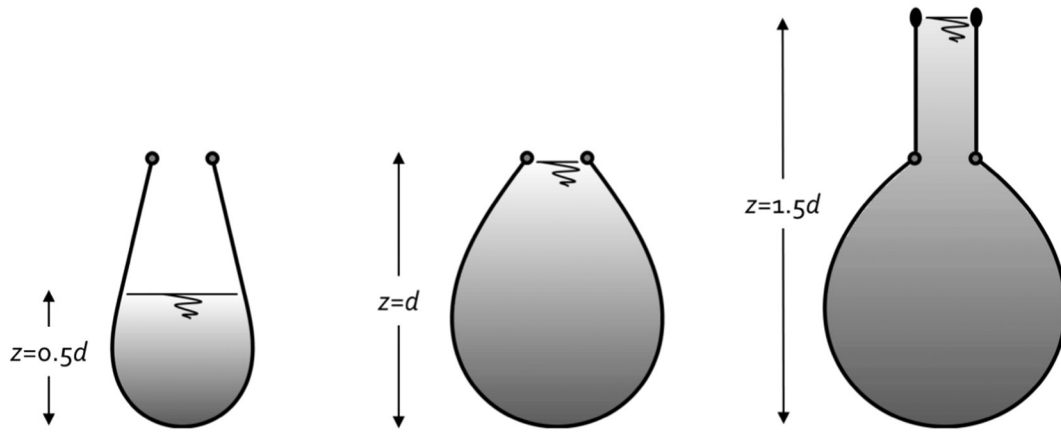


Fig. 6. Part-full; full; and overfull (surcharged) fabric profiles for constant b and d .

Further, the procedure is also readily adapted to the profile of a partially-full container i.e., $z < d$ (see Fig. 6). It is only necessary to set curvatures to zero once $y_n \geq z$ i.e., for $y_n \geq z: \delta\theta_n = 0$. In terms of modelling, the profile is allowed to “walk out” in a straight line above the level of the fluid. This is understood physically by the absence of further out of plane applied loading above the level of the fluid. This is also a potentially useful result for the design of supporting falsework because the partially-full case is the condition that occurs during the casting process.

6. Analysis

Various non-dimensional values for the principal parameters for the b , d , z and p can be calculated and their values plotted. The resulting family of curves define a notional surface on which any simple hydraulic sectional profile can be located (Fig. 7).

The surface provides a scheme in which to relate the key dimensions of the possible simple hydrostatic geometries. The surface also illustrates the relation between some analytically determined boundary conditions and the numerically obtained curves.

Notably:

- Regardless of the relation z/d :
 - When $d/p \rightarrow 0: b/p \rightarrow 1$ because the fabric is flat.
- As $z/d \rightarrow 0$:
 - When $b/p \rightarrow 0: d/p \rightarrow 1/2$ because the fabric is folded in half vertically.

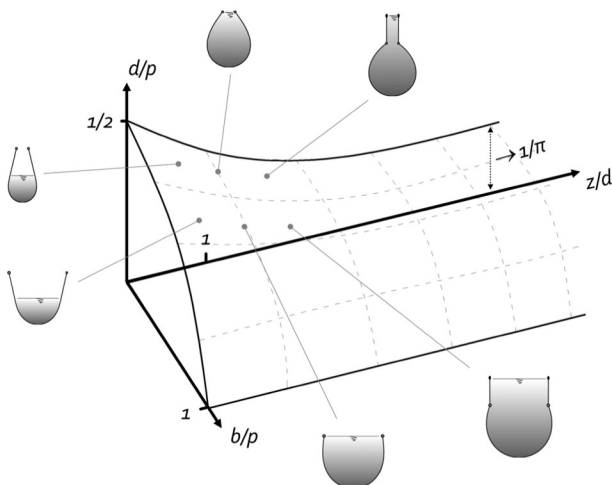


Fig. 7. Sketch of non-dimensional surface and some indicative physical interpretations.

- As $z/d \rightarrow \infty$:
 - The form of the *lintearia* tends to that of the *valeria* (cf. Fig. 3)
 - When $b/p \rightarrow 0: d/p \rightarrow 1/\pi$ because the bulb is circular in section.

7. Validation

Fig. 8 compares the results of the numerical procedure described above with Bailiss' [1] empirical equation for some normalised characteristic dimensions of an arbitrary section; and with losilevskii's [7] analytical results obtained by the evaluation of elliptic integrals. It is apparent that there is good agreement with Bailiss' physical models and very good agreement with the analytical results. Bailiss' [1] best-fit approximations can thus be revisited and their accuracy improved. However, this approach has little further value in the sense that detailed profiling rather than estimations of characteristic dimensions is required for further progress in optimised design [4].

Empirical validation is more challenging, as records of as-built versus as-designed fabric-formed sections have proved difficult to obtain. However, it has been possible to graphically compare the profiles against the hydrostatically loaded profiles obtained experimentally by Bailiss [1]. Bailiss physical model consisted of a fabric strip, restrained in plane but free to deform out of plane. The strip was then hydrostatically loaded with a viscous liquid. Various opening breadths of opening were modelled. Fig. 9 compares the profiles of Bailiss' [1] physical models with those predicted by the numerical method based on opening breadth and hanging depth as indicated by the images. Although the initial information is less than ideal, the predicted profiles are a good fit.

8. Application to fabric-formed concrete

The numerical procedure proposed can be applied to generate a range of example profiles, enabling the production of simple design charts. The particular advantage of such charts is that they are accessible to non-engineering professionals and might therefore facilitate conceptual design and initial form generation within interdisciplinary design teams. Charts can be generated to suit a variety of design situations. A useful example, expressing breadth and fabric perimeter in relation to section depth is shown in Fig. 10.

The approach detailed above may also be applied to non-prismatic elements by discretizing the longitudinal profile into a series of sectional profiles. As an example, Fig. 11 illustrates the generation of sectional profiles for non-prismatic beams fabricated by Garbett [4], the first using free hanging fabric, the second using stiff web formers to force a

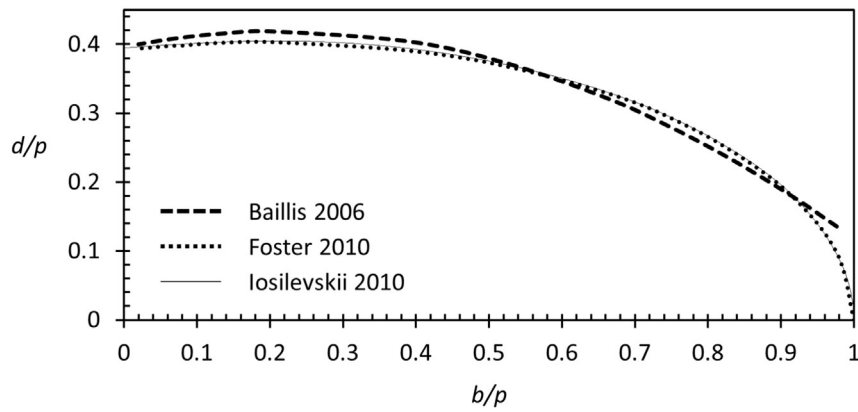


Fig. 8. Comparison of Baillis' [1] non-dimensional relation obtained experimentally, that obtained by 'walking out' ⁶ and Iosilevskii's [7] plot of opening breadth against depth normalised by perimeter.

more slender web geometry through the mid-span. While Garbett [4] was only able to obtain indicative profiles based on Baillis' [1] equations for characteristic dimensions, detailed sectional profiles have been obtained here by the numerical method. Note that the profile of the bulb at the base of the T-section is obtained as for the over-full case.

The number of sections required to suitably model a non-prismatic element will depend on the longitudinal curvatures and the fitness of any interpolation functions used to join the sections. However, longitudinal curvatures in a fabric-formed beam are typically small compared with the transverse curvatures meaning that the number of sections need not be great. Orr [12] has successfully applied this sectional method to the generation of optimised non-prismatic fabric-formed concrete beams. More complex three-dimensional analysis for fabric formed

concrete has been carried out combining dynamic relaxation with finite element analysis by Veenendaal [14].

A further practical benefit of the approach is that if 'real' values of ρg are used, then the tension T_0 in the fabric at a section is known, along with the profile of the fabric. This allows the vertical and horizontal forces F_x and F_y exerted by the hanging fabric formwork to be determined (Fig. 12), allowing the supporting structure to be designed appropriately.

$$F_x = \frac{T_0(x_n - x_{n-1})}{\delta L} \quad (9)$$

$$F_y = \frac{T_0(y_n - y_{n-1})}{\delta L} \quad (10)$$

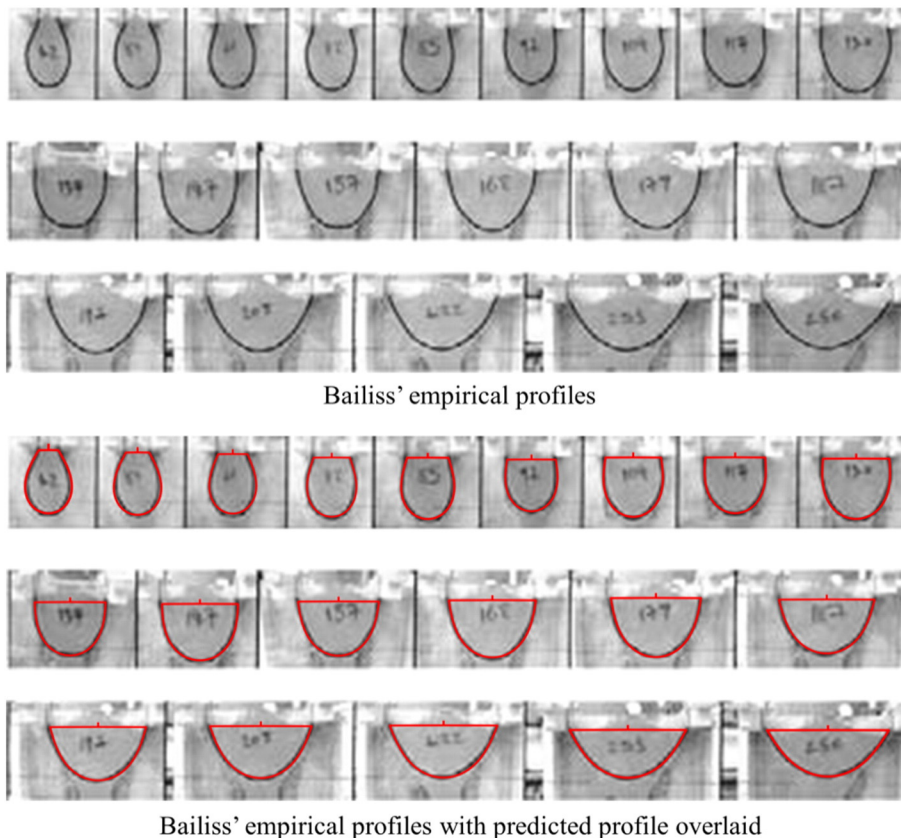


Fig. 9. Comparison with the profiles of Baillis' physical models (data from [1]). Predicted profile in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

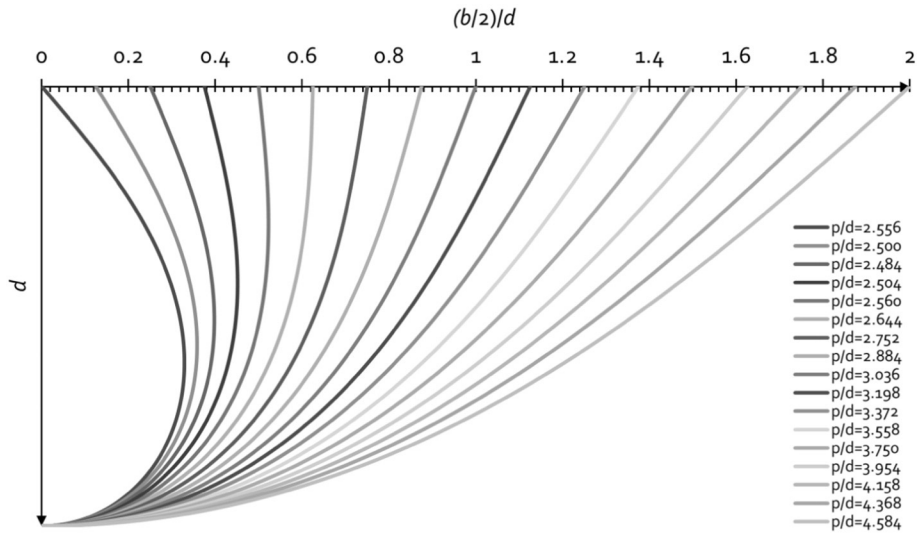


Fig. 10. Design chart for hydrostatic profiles expressing b and p in relation to d for the typical case $z = d$.

A limitation of the presented method is the assumption of inextensibility of the fabric formwork. Real fabrics are not inextensible and axial deformation of the fabric will increase the length of the hanging perimeter p and hence the cross-sectional profile will deviate from that predicted. However, for the sizes of structure normally considered,

the extension of the geotextile fabrics typically used can in most cases be treated as negligible. The expected extension of the fabric can be verified approximately since T_0 for an inextensible fabric is in any case known. The assumption of inextensibility also limits the application of this method to tensioned fabrics.

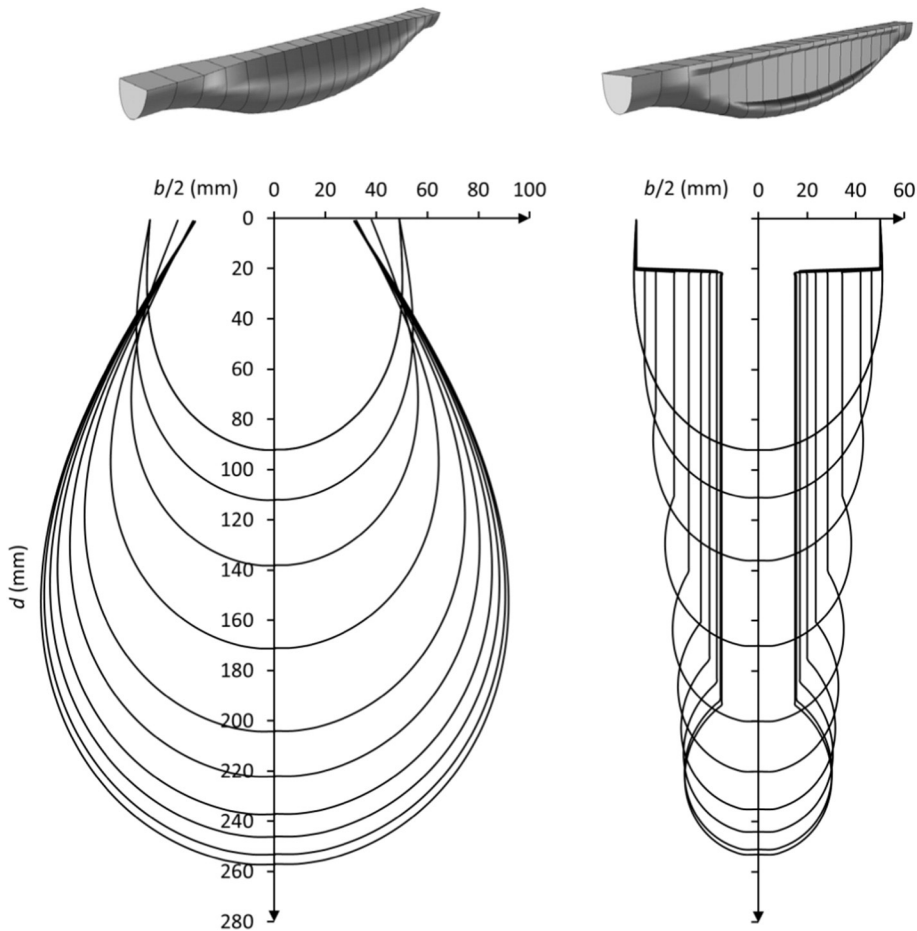


Fig. 11. Detailed sectional profiles generated by the method presented here, for beams fabricated by Garbett [4]. Original isometric approximations (top) courtesy of Joe Garbett.

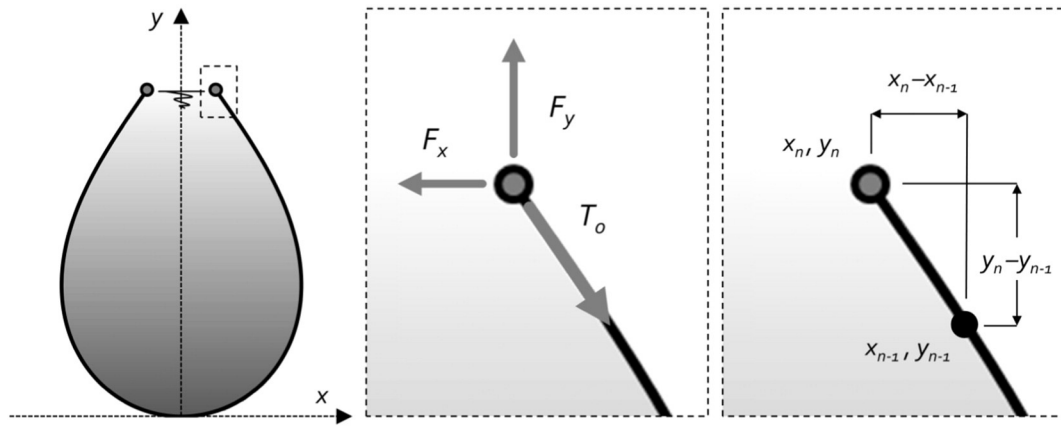


Fig. 12. Forces F_x and F_y at fabric supports can be determined from T_0 .

9. Conclusion

A numerical method has been presented for determining the form of flexible, impermeable and inextensible hanging fabric subject to hydrostatic load. This method is shown to predict the profiles of part-filled, filled and over-filled, i.e. surcharged, and flat-bottomed fabrics, such as those used for moulding fabric-formed concrete beams. The method shows good agreement against analytical and empirical results reported in the literature, thereby allowing engineers and architects to accurately predict the geometry of fabric-formed concrete structures.

Acknowledgements

The first author would like to acknowledge the helpful comments of Chris Williams at the University of Bath in the initial framing of this problem.

References

- [1] Bailiss J. Fabric-formed concrete beams: design and analysis. [MEng Thesis] Department of Architecture and Civil Engineering, University of Bath; 2006.
- [2] Fletcher, M. (1917). Method and means for forming concrete structural members. Patent No. 1241945, United States Patent Office.
- [3] Foster R. Form finding and analysis of fabric-formed concrete beams. [MEng Thesis] Department for Architecture and Civil Engineering, University of Bath; 2010.
- [4] Garbett J. Bone growth analogy for optimising flexibly formed concrete beams. [MEng Thesis] Department of Architecture and Civil Engineering, University of Bath; 2008.
- [5] Ghaib MAA, Gorski J. Mechanical properties of concrete cast in fabric formworks. *Cem Concr Res* 2001;31:1459–65.
- [6] Hashemian FK. Structural behaviour and optimization of moment-shaped reinforced concrete beams. [PhD Thesis] Winnipeg: Department of Civil Engineering, Faculty of Engineering, University of Manitoba; 2012.
- [7] Iosilevskii G. Shape of a soft container under hydrostatic load. *J Appl Mech* 2010;77: 1–3.
- [8] Lee DS-H. Study of construction methodology and structural behaviour of fabric-formed form-efficient reinforced concrete beam. [PhD Thesis] Department of Architecture, University of Edinburgh; 2011.
- [9] Levien R. The elastica: a mathematical history, a report adapted from work done as part of Levien's 2009. [PhD Thesis], From spiral to spline: optimal techniques in interactive curve design. Berkeley: University of California; 2008.
- [10] Lilienthal, W. G. (1897). Decke. Patent No. 100194, Kaiserlich Patentamt, Germany.
- [11] Nervi PL. Structures. New York: FW Dodge; 1956.
- [12] Orr JJ. Flexible formwork for concrete structures. [PhD Thesis] Department of Architecture and Civil Engineering, University of Bath; 2012.
- [13] Orr JJ, Darby AP, Ibell TJ, Evernden MC, Otlet M. Concrete structures using fabric formwork. *Struct Eng* 2011;89(8):20–6 [ISSN 1466-5123].
- [14] Veenendaal D. Evolutionary optimization of fabric formed structural elements. [Master's Thesis] Faculty of Civil Engineering and Geosciences, Delft University of Technology; 2008.