

Basis expansion model for channel estimation in LTE-R communication system



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ABSTRACT

This paper investigates fast time-varying channel estimation in LTE-R communication systems. The Basis Expansion Model (BEM) is adopted to fit the fast time-varying channel in a high-speed railway communication scenario. The channel impulse response is modeled as the sum of basis functions multiplied by different coefficients. The optimal coefficients are obtained by theoretical analysis. Simulation results show that a Generalized Complex-Exponential BEM (GCE-BEM) outperforms a Complex-Exponential BEM (CE-BEM) and a polynomial BEM in terms of Mean Squared Error (MSE). Besides, the MSE of the CE-BEM decreases gradually as the number of basis functions increases. The GCE-BEM has a satisfactory performance with the serious fading channel.

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1. Introduction

The Long Term Evolution for Railway (LTE-R) systems is a promising solution to the high-speed railway communication [1,2]. However, the fast movement of the train and the special geographical environment characteristics, cause the dynamic change of the channel, thus seriously affecting the system performance [3]. As a consequence, it is quite necessary to conduct research on the channel estimation algorithm for the LTE-R communication system to ensure that reliable wireless communication services are provided for users in the high-speed railway scenario.

Recently, channel estimation algorithms have attracted extensive research attentions. According to the usage of auxiliary data [4], the algorithms are classified into three categories that are non-blind, blind, and semi-blind channel estimation algorithms. Based on the training sequences or pilot frequencies, the non-blind channel estimation algorithm possesses the advantages of fast convergence speed, low implementation complexity, and strong anti-interference ability [5]. Whereas, the training sequences or pilot tones inevitably extend the effective bandwidth of the system, which implies the reduction of the overall transmission efficiency. For the blind channel estimation algorithm, it has a higher spectrum efficiency than the non-blind algorithm. The dynamic change of the channel can be analyzed according to the channel statistical

characteristics without using training sequences [6]. Nonetheless, there are also some shortcomings, such as high computational complexity, slow convergence speed, and poor flexibility restrict the application in practice. The semi-blind channel estimation algorithm is a trade-off between the non-blind and the blind channel estimation algorithms [7]. It exploits a small number of pilot symbols and channel statistical characteristics to acquire the channel state information and optimize channel parameters.

However, the traditional channel estimation algorithms are primarily suited to the quasi-static channel. Thus, it is difficult for them to work effectively in the LTE-R communication system, in which user terminals move at a speed of more than 300 km per hour. The basis expansion model (BEM) can fit the channel with dynamic changes by several basis functions, which is a valid tool for fast time-varying (TV) channel estimation. Therefore, there is a strong motivation to investigate the channel estimation algorithm based on the BEM in the high-speed railway scenario in order to improve the system performance.

In this paper, the BEM is employed to explore the fast TV channel estimation in the LTE-R communication system. Firstly, the channel characteristics of the LTE-R system are described according to the Rician fading channel model. Then, the BEM is introduced to approximate a fast TV channel. In other words, the fast TV channel is represented as the sum of several basis functions multiplied by the corresponding coefficients. Besides, optimal coefficients are obtained by theoretical deduction. Finally, the mean square error (MSE) of three channel estimation algorithms based on the BEM is evaluated via simulation.

The rest of this paper is organized as follows. In Section 2, the network architecture of the LTE-R communication system is

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described. The fast TV channel estimation algorithm based on the BEM is analyzed in Section 3. In Section 4, some simulation results and discussions are exhibited. Conclusions and future work are given in Section 5.

2. System model

The network architecture of the LTE-R communication system is shown in Fig. 1. In order to reduce the transmission loss and guarantee reliability of high-speed train communication, the LTE-R communication system based on distributed base stations is composed of a ground subsystem and a vehicle subsystem [8]. The ground subsystem includes the Building Baseband Unit (BBU) and the Radio Remote Unit (RRU). Multiple RRUs are connected to a BBU through optical fiber. The vehicle subsystem consists of the Vehicular Station (VS), Repeaters (R) and User Equipment (UE). UEs are accessed to R by the wireless link, and R is connected to VS by a wire link. What's more, the ground subsystem and the vehicle subsystem are linked via the communication between the RRU and VS.

In the high-speed railway communication scenario, the wireless channel between RRU and VS presents fast TV characteristics. Considering that the base stations are always near the railway line, not only multiple indirect Non-Line-of-Sight (NLOS) paths but also a direct Line-of-Sight (LOS) path exists between the transmitter and receiver [9]. In consequence, this paper adopts Rician fading channel to describe the characteristics of the LTE-R communication system. Additionally, the Multiple-Input Multiple-Output (MIMO) technology is employed to boost the overall system performance. Assuming that RRU and VS are equipped with N_T transmit antennas and receive antennas, respectively the channel impulse response between the t th transmit and the t th receive antennas at time may be denoted as

$$h_{p,q}(t) = \sqrt{\frac{K}{1+K}} h_{p,q}^{LOS}(t) + \sqrt{\frac{1}{1+K}} h_{p,q}^{NLOS}(t) \quad (1)$$

where $1 \leq p \leq N_T$, $1 \leq q \leq N_R$, $h_{p,q}^{LOS}(t)$ is the channel impulse

response of LOS path, $h_{p,q}^{NLOS}(t)$ is the channel impulse response of NLOS paths, and K is the Rician factor that is defined as the ratio of power in the LOS component to the total power in the scattered NLOS components. The value of K directly reflects the degree of channel fading. In other words, the smaller K is, the deeper the channel fading will be. For the $h_{p,q}^{LOS}(t)$, it is described as

$$h_{p,q}^{LOS}(t) = \sqrt{G_T(\theta_T)} \exp\left(j\frac{2\pi}{\lambda} d_p \sin(\theta_T)\right) \times \sqrt{G_R(\theta_R)} \exp\left(j\frac{2\pi}{\lambda} d_q \sin(\theta_R) + \psi_{LOS}\right) \times \exp\left(j\frac{2\pi}{\lambda} v \cos(\theta_R - \theta_v) t\right) \quad (2)$$

where θ_T and θ_R indicate the angles of departure and arrival for the LOS path, $G_T(\theta_T)$ and $G_R(\theta_R)$ denote the gains of transmit antenna and receive antenna, λ is the carrier wavelength, d_p and d_q represent the distances from the reference antenna to the p th transmit antenna and the q th receive antenna, ψ_{LOS} stands for the phase of the LOS path, v is the train speed, and θ_v represents the angle of the speed vector. For the $h_{p,q}^{NLOS}(t)$, it is indicated as

$$h_{p,q}^{NLOS}(t) = \sum_{i=1}^{N_i} \sqrt{P_i} \sqrt{G_T(\theta_T^i)} \exp\left(j\left[\frac{2\pi}{\lambda} d_p \sin(\theta_T^i)\right] + \psi_i\right) \times \sqrt{G_R(\theta_R^i)} \exp\left(j\frac{2\pi}{\lambda} d_q \sin(\theta_R^i)\right) \times \exp\left(j\frac{2\pi}{\lambda} v \cos(\theta_R^i - \theta_v) t\right) \quad (3)$$

where N_i indicates the number of paths, P_i is the power of the i th path, θ_T^i and θ_R^i denote the angles of departure and arrival for the i th path, $G_T(\theta_T^i)$ and $G_R(\theta_R^i)$ represent the gains of the transmit antenna and receive antenna, and ψ_i stands for the phase of the i th path.

3. Fast time-varying channel estimation algorithm

In this section, the fast TV channel estimation algorithm based on the BEM is investigated in the LTE-R communication system. Firstly, we analyze the basic idea of channel estimation using the

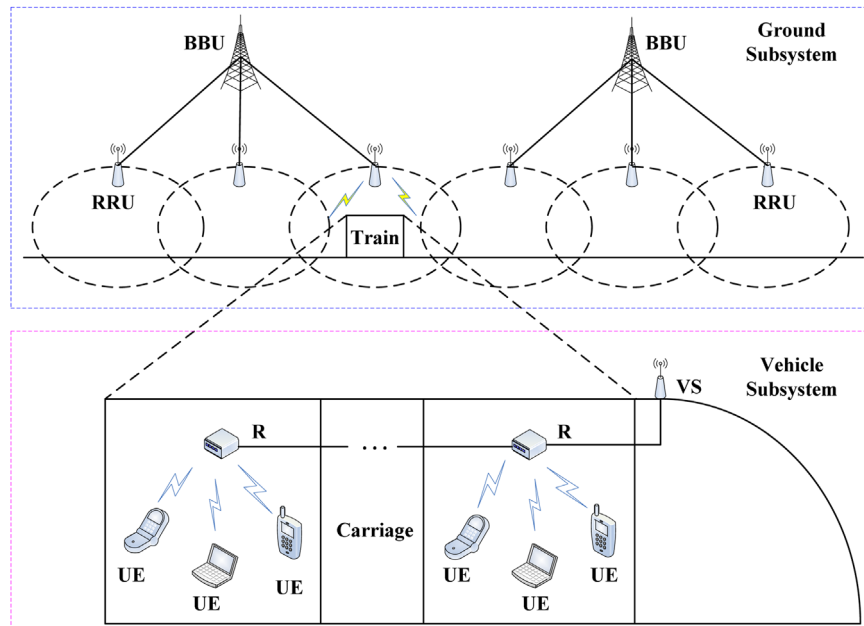


Fig. 1. Network architecture of LTE-R communication system.

BEM. Then, the optimal coefficients of the basis functions are given by theoretical deduction.

3.1. Basis expansion model

The basic idea of channel estimation algorithm based on the BEM is to approximate the fast TV channel by adopting several mutually orthogonal basis functions. Since the basis functions are known, the essence of the algorithm is to estimate the BEM coefficients [10]. In accordance with this idea, if the TV characteristic of each path is captured through M mutually orthogonal basis functions, channel impulse response at time t is expressed as

$$h(t) = \mathbf{B}(t)\mathbf{C}(t) = \sum_{i=1}^M b_{t,i}c_{t,i} \quad (4)$$

where $\mathbf{B}(t) = [b_{t,1} \ b_{t,2} \ \dots \ b_{t,M}]_{1 \times M}$ is a $1 \times M$ matrix that collects M basis functions $b_{t,i}$ as columns, and $\mathbf{C}(t) = [c_{t,1} \ c_{t,2} \ \dots \ c_{t,M}]_{M \times 1}^T$ indicates the matrix of coefficients. Note that the expressions of basis functions vary with the selected BEM. The existing type of BEM mainly includes complex-exponential BEM (CE-BEM), generalized CE-BEM (GCE-BEM), and polynomial BEM (P-BEM). Basis functions of the above BEM are discussed below.

For the CE-BEM, its basis functions at time t are represented as

$$b_{t,m} = e^{j2\pi t \frac{m-M/2}{N}} \quad (5)$$

where $1 \leq m \leq M$, integer M is required to satisfy $M \geq 2 \times \lceil f_{\max} N T_s \rceil$. Here, f_{\max} denotes the maximum Doppler frequency shift:

$$f_{\max} = \frac{v \times f_c}{c} \quad (6)$$

where f_c stands for carrier frequency, c indicates speed of light. Moreover, N stands for the number of subcarriers, T_s is the sampling interval, and $\lceil \cdot \rceil$ indicates the integer ceiling. The CE-BEM gets a great deal of attention thanks to its algebraic ease. However, it will induce a large edge error, and bring about the Gibbs phenomenon [11]. Consequently, its performance is severely affected.

In order to solve the problem of large edge error that exists in the CE-BEM, the GCE-BEM adopts a higher sampling frequency, which improves the frequency resolution and promotes the fitting performance, but fails to diminish the influence of the Gibbs effect on the channel estimation [12]. Basis functions of the GCE-BEM at time t are denoted as

$$b_{t,m} = e^{j2\pi t \frac{m-M/2}{DN}} \quad (7)$$

where $1 \leq m \leq M$, integer M is required to satisfy $M \geq 2 \times \lceil D f_{\max} N T_s \rceil$, and D is a positive integer. When $D = 1$, the GCE-BEM turns into the CE-BEM.

The P-BEM based on the theory of a Taylor series uses a set of polynomials to fit the channel. Its basis functions at time t are indicated as

$$b_{t,m} = (t - \lfloor N/2 \rfloor)^m \quad (8)$$

where $1 \leq m \leq M$, $\lfloor \cdot \rfloor$ indicates the integer floor. Compared with the CE-BEM, the edge error of the P-BEM is smaller. Nevertheless, it is rather sensitive to the Doppler spread, and the order of the polynomials affects its performance [13].

3.2. Optimal coefficients of basis functions

In this subsection, the fast TV channel of the LTE-R communication system is modeled by utilizing the aforementioned BEM. Thus, the channel impulse response between the p th transmit and

the q th receive antennas at time t is represented as

$$h_{p,q}(t) = \mathbf{B}_{p,q}(t)\mathbf{C}_{p,q}(t) = \sum_{i=1}^M b_{t,i}^{p,q} c_{t,i}^{p,q} \quad (9)$$

where $\mathbf{B}_{p,q}(t) = [b_{t,1}^{p,q} \ b_{t,2}^{p,q} \ \dots \ b_{t,M}^{p,q}]_{1 \times M}$ and $\mathbf{C}_{p,q}(t) = [c_{t,1}^{p,q} \ c_{t,2}^{p,q} \ \dots \ c_{t,M}^{p,q}]_{M \times 1}^T$ are the matrixes of the known basis functions and the unknown coefficients, respectively. In general, at any given time, the channel between different transmission and receiving antennas is fitted by the same basis functions. As a result, we define $\mathbf{B}_{p,q}(t) = \mathbf{B}(t)$, then (9) is described as

$$h_{p,q}(t) = \mathbf{B}(t)\mathbf{C}_{p,q}(t) = \sum_{i=1}^M b_{t,i}c_{t,i}^{p,q} \quad (10)$$

If the estimation of $\mathbf{C}_{p,q}(t)$ is indicated as (11), the estimation of $h_{p,q}(t)$ is expressed as (12):

$$\tilde{\mathbf{C}}_{p,q}(t) = [\tilde{c}_{t,1}^{p,q} \ \tilde{c}_{t,2}^{p,q} \ \dots \ \tilde{c}_{t,M}^{p,q}]_{M \times 1}^T \quad (11)$$

$$\tilde{h}_{p,q}(t) = \mathbf{B}(t)\tilde{\mathbf{C}}_{p,q}(t) = \sum_{i=1}^M b_{t,i}\tilde{c}_{t,i}^{p,q} \quad (12)$$

The mean squared error (MSE) of the channel impulse response is represented as

$$\begin{aligned} MSE &= (h_{p,q}(t) - \tilde{h}_{p,q}(t))^H (h_{p,q}(t) - \tilde{h}_{p,q}(t)) \\ &= (h_{p,q}(t) - \mathbf{B}\tilde{\mathbf{C}}_{p,q}(t))^H (h_{p,q}(t) - \mathbf{B}\tilde{\mathbf{C}}_{p,q}(t)) \end{aligned} \quad (13)$$

where $[\cdot]^H$ stands for complex conjugate transpose.

Obviously, Eq. (13) indicates that the value of MSE is relevant to the real channel, the selected BEM and the corresponding coefficients of the basis functions. In case one real channel is fitted by a certain BEM, only the coefficients of the basis functions directly affect the value of the MSE . Therefore, the optimal coefficients of the basis functions should be calculated to minimize the MSE .

Based on the Cramer–Rao Lower Bound–Vector Parameter theorem proposed by Steven M. Kay [14], since $h_{p,q}(t) = \mathbf{B}(t)\mathbf{C}_{p,q}(t)$ is a linear model, we can assume that the probability density function $f(h_{p,q}(t); \mathbf{C}_{p,q}(t))$ satisfies the following regularity conditions:

$$E \left[\frac{\partial \ln f(h_{p,q}(t); \mathbf{C}_{p,q}(t))}{\partial \mathbf{C}_{p,q}(t)} \right] = \mathbf{0} \quad (14)$$

where $E[\cdot]$ stands for mathematical expectation, $\partial(\cdot)/\partial(\cdot)$ denotes partial derivative. Then a certain function g exists to make:

$$\frac{\partial \ln f(h_{p,q}(t); \mathbf{C}_{p,q}(t))}{\partial \mathbf{C}_{p,q}(t)} = \mathbf{I}(\mathbf{C}_{p,q}(t)) [g(h_{p,q}(t)) - \mathbf{C}_{p,q}(t)] \quad (15)$$

where \mathbf{I} indicates unit matrix. The minimum variance unbiased (MVU) estimator expressed as $\tilde{\mathbf{C}}_{p,q}^{opt}(t) = g(h_{p,q}(t))$, which is also the optimal coefficients of the basis functions, will be obtained when Eq. (15) equals $\mathbf{0}$. The expression of $\tilde{\mathbf{C}}_{p,q}^{opt}(t)$ is indicated as

$$\begin{aligned} \tilde{\mathbf{C}}_{p,q}^{opt}(t) &= \arg \min_{\mathbf{C}_{p,q}(t)} \left\{ (h_{p,q}(t) - \tilde{h}_{p,q}(t))^H (h_{p,q}(t) - \tilde{h}_{p,q}(t)) \right\} \\ &= \arg \min_{\mathbf{C}_{p,q}(t)} \left\{ (h_{p,q}(t) - \mathbf{B}\tilde{\mathbf{C}}_{p,q}(t))^H (h_{p,q}(t) - \mathbf{B}\tilde{\mathbf{C}}_{p,q}(t)) \right\} \\ &= (\mathbf{B}^H \mathbf{B})^{-1} \mathbf{B}^H h_{p,q}(t) \end{aligned} \quad (16)$$

Finally, the least MSE of channel estimation algorithm is acquired through plugging (16) into (13), and its expression is shown as follows:

$$\begin{aligned}
MSE_{min} &= \left(h_{p,q}(t) - \mathbf{B}\tilde{\mathbf{C}}_{p,q}^{opt}(t) \right)^H \left(h_{p,q}(t) - \mathbf{B}\tilde{\mathbf{C}}_{p,q}^{opt}(t) \right) \\
&= \left[h_{p,q}(t) - \mathbf{B}(\mathbf{B}^H\mathbf{B})^{-1}\mathbf{B}^H h_{p,q}(t) \right]^H \left[h_{p,q}(t) - \mathbf{B}(\mathbf{B}^H\mathbf{B})^{-1}\mathbf{B}^H h_{p,q}(t) \right] \\
&= \left\{ h_{p,q}(t) \left[1 - \mathbf{B}(\mathbf{B}^H\mathbf{B})^{-1}\mathbf{B}^H \right] \right\}^H \left\{ h_{p,q}(t) \left[1 - \mathbf{B}(\mathbf{B}^H\mathbf{B})^{-1}\mathbf{B}^H \right] \right\} \\
&= \left[1 - \mathbf{B}(\mathbf{B}^H\mathbf{B})^{-1}\mathbf{B}^H \right]^H h_{p,q}^H(t) h_{p,q}(t) \left[1 - \mathbf{B}(\mathbf{B}^H\mathbf{B})^{-1}\mathbf{B}^H \right] \quad (17)
\end{aligned}$$

4. Simulation results and analysis

In this section, we evaluate the performance of the channel estimation algorithms for the LTE-R communication system based on the CE-BEM, GCE-BEM, and P-BEM. The related simulation parameters are set to be $N_T = N_R = 2$, $v = 100\text{--}130$ m/s, $N = 128$, $T_s = 1/75,000$ s, $f_c = 2.64$ GHz, $c = 3 \times 10^8$ m/s and $D = 2$. The parameters for the Rician fading channel in (2) and (3) are given as follows: $\theta_T = 20^\circ$, $\theta_R = 157.5^\circ$, $G_T(\theta_T) = 4$ dB, $G_R(\theta_R) = 4$ dB, $\Psi_{LOS} = 0$, and $\theta_v = 120^\circ$. Moreover, without loss of generality, we assume that $N_L = 6$, $\theta_L^i = 50^\circ$, $\theta_R^i = 112.5^\circ$, $\Psi_i = 0$, $P_i = 0.01$ W, $G_T(\theta_T^i) = 4$ dB and $G_R(\theta_R^i) = 4$ dB. Besides, for the transmit antenna, we suppose that the first antenna is the reference antenna. Thus, we have $d_p = 0$, $p = 1$ and $d_p = 10\lambda$, $p = 2$. Here, λ denotes the wavelength, which can be calculated by $\lambda = c/f_c$. Similarly, for the receiving antenna, we assume that the first antenna is the reference antenna. Then, we have $d_q = 0$, $q = 1$ and $d_q = 0.5\lambda$, $q = 2$.

Fig. 2 presents the MSE of the three BEMs versus the moving speed with $K = 6$ dB. In this simulation, we suppose that the numbers of the basis functions of the CE-BEM, GCE-BEM, and P-BEM are 4, 8, and 4, respectively. The simulation result indicates that the MSE increases gradually with the growth of the moving speed under three BEMs. That is because the fast TV characteristic of the channel becomes more and more obvious due to the serious Doppler effect [15,16]. What's more, the GCE-BEM outperforms the P-BEM and the CE-BEM. The reason is that the GCE-BEM can greatly reduce the edge error by adopting the over sampling technology. In addition, the P-BEM has a lower MSE than the CE-BEM. This is because the P-BEM is able to degrade the edge error to some extent by using a series of polynomials to approximate the TV channel.

Fig. 3 depicts the relationship between the number of basis functions and the performance of the CE-BEM estimation algorithm. In this simulation, we assume that $K = 6$ dB. The numbers of the basis functions M are set to be 4, 5, and 7. The simulation

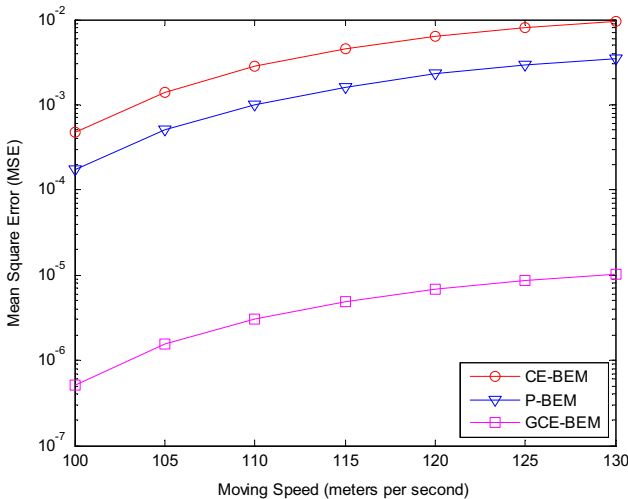


Fig. 2. MSE of three different BEM versus moving speed with $K = 6$ dB.

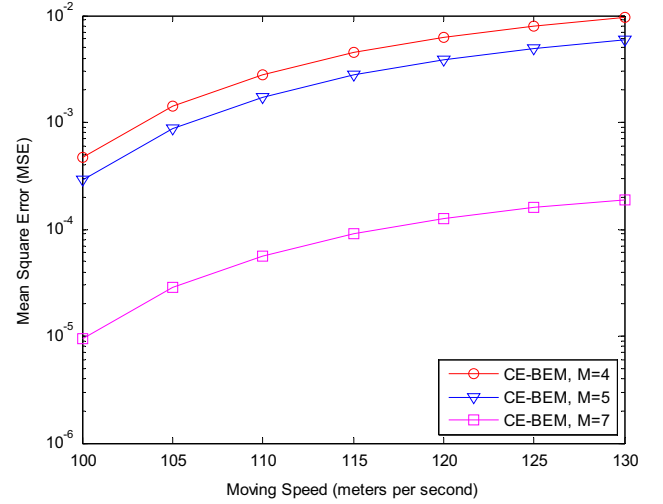


Fig. 3. MSE of the CE-BEM versus moving speed with $K = 6$ dB.

result shows that the MSE of the CE-BEM decreases gradually as the number of basis functions increases. The reason is that the bigger M indicates more mutually orthogonal basis functions are utilized to fit the channel. As a result, the smaller MSE is obtained. However, the larger M is, the higher the computational complexity is. Therefore, we should choose an appropriate value for M to balance the transmission efficiency and reliability of the communication system.

Fig. 4 shows the performance of the GCE-BEM estimation algorithm in different Rician fading channels. As mentioned previously, the Rician factor K directly reflects the degree of channel fading. Thus, K is set to be -12 dB, -6 dB, 0 dB, and 10 dB to represent four different channel fading degrees in this simulation. The simulation results show that the smaller K , which means the deeper channel fading degree, corresponds to the larger MSE. Furthermore, we observe that the MSE is about 10^{-5} when $K = -12$ dB. Therefore, the GCE-BEM exhibits good performance even in conditions of deep fading channel.

5. Conclusion

In this paper, the fast TV channel estimation algorithms based on the CE-BEM, GCE-BEM, and P-BEM was investigated. The simulation revealed that the performance of the GCE-BEM was the

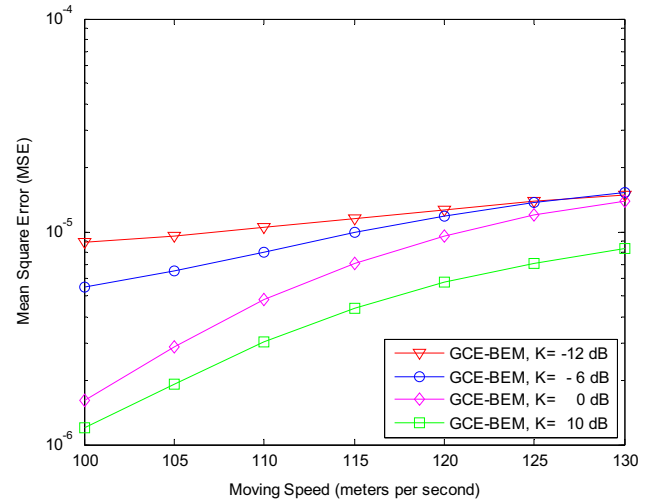


Fig. 4. MSE of the GCE-BEM versus moving speed with $M = 8$.

best, the P-BEM was the second, and the CE-BEM was the worst. For the CE-BEM, its performance was improved by increasing the number of basis functions. In addition, the MSE of the GCE-BEM is small even with deep fading channel. For the coefficients of the basis functions, this paper employs the Cramer–Rao Lower Bound-Vector Parameter theorem to obtain the optimum value. In the further work, other algorithms, such as Least Squares, Minimum Mean Square Error, and Linear Minimum Mean Square Error will be adopted to calculate the coefficients, and compare their performance.

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