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Sparsity-aware target localization using TDOA/AOA measurements in distributed MIMO radars☆

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Abstract

In this paper, a sparsity-aware hybrid target localization method in multiple-input-multiple-output (MIMO) radars from time difference of arrival (TDOA) and angle of arrival (AOA) measurements is proposed. This method provides a maximum likelihood estimate of target position by employing compressive sensing techniques. A blockwise approach is addressed in order to achieve better accuracy for a constant computational complexity. The mismatch problem due to grid discretization is also tackled by a dictionary learning technique. The Cramer–Rao lower bound for this model is derived as a benchmark. Numerical simulations are included to corroborate the theoretical developments.

Keywords: Multiple-input-multiple-output (MIMO) radar; Target localization; Compressed sensing (CS); Time difference of arrival (TDOA); Angle of arrival (AOA)

1. Introduction

The problem of determining the location of a radiating or reflecting target has drawn considerable attention in various applications, such as radar [1], sonar [2], communications [3] and wireless sensor networks [4]. Many different approaches have been proposed in the literature to estimate the target location based on time of arrival (TOA) [5], time difference of arrival (TDOA) [3], angle of arrival (AOA) [6] or frequency difference of arrival (FDOA) [7]. Recently, this topic has gained popularity in multiple-input-multiple-output (MIMO) radar systems [8–13]. MIMO radar with colocated antennas provides waveform diversity and that with widely-separated antennas provides spatial diversity [14,15]. The aforementioned diversity makes the targets to be better detected and localized.

There is a rapidly growing literature on the localization in MIMO radars. In [8], the performance of TDOA based localization has been studied. Godrich et al. have proposed a best linear unbiased estimator (BLUE) by linearizing the elliptic equations using Taylor series [9]. A closed-form method has been proposed in [12] for TDOA based localization in MIMO radar, while a two-step weighted least square approach is derived in [13]. Rossi et al. have proposed an AOA estimate employing the sparse representation of spatial angles [6]. In [10], the centralized direct positioning algorithm is derived for localizing the stationary targets which do not require data association.

Generally speaking, accuracy of target localization can be improved by jointly utilizing different types of measurements. The authors of [11] have proposed a compressive sensing (CS) based algorithm by exploiting joint TDOAs and FDOAs. In [16,17], two hybrid algorithm based on TDOA/AOA measurements has been derived for single-receiver scheme and has been shown to outperform TDOA only methods. In addition, data association with TDOA/AOA measurements is also more reliable in comparison with use of TDOA only measurements. As a matter of fact, two targets located on a single transmitter–receiver ellipsoid cannot be resolved with TDOAs while they can be easily separated by adding the angular information. Thus, hybrid TDOA/AOA is found to be a good choice for target localization in realistic MIMO radar systems.

In this paper, we model the problem of target localization in MIMO radars with use of TDOA and AOA measurements in a sparse representation framework. The proposed method

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solves the sparsity-aware maximum likelihood (ML) estimation problem of target position and incorporates an iterative trick to enhance accuracy while keeping low complexity. We will address a dictionary learning based method in order to mitigate the off-grid mismatch due to discretization.

The rest of paper is organized as follows. The Section 2 describes the system model. In Section 3, we elaborate our proposed method by formulating the problem as a sparsity-aware ML estimation. Section 4 is devoted to investigate the performance of the proposed method. Finally, Section 5 concludes the paper.

The notation is introduced as follows. Matrices, vectors and scalars are denoted by bold upper-case, bold lower-case and italic, respectively. The ℓ_p norm of $\mathbf{x}$ is denoted by $\|\mathbf{x}\|_p$. The $(.)^T$ and $(.)^{-1}$ denote matrix transpose and inverse. The ith element of $\mathbf{x}$ and the (i, j)th element of $\mathbf{A}$ are denoted by $[\mathbf{x}]_i$, and $[\mathbf{A}]_{(i,j)}$, respectively.

2. System model

Consider that there are M transmitters and N receivers distributed over a $2D$ surface which is discretized into K grid points, whose positions are denoted by $\mathbf{x}_m^t = \left[x_m^t, y_m^t\right]^T, m = 1, \ldots, M$ and $\mathbf{x}_n^r = \left[x_n^r, y_n^r\right]^T, n = 1, \ldots, N$, respectively. We aim to find the position of a target, which is located at $\mathbf{x} = [x, y]^T$. Note that although we assume $2D$ case, extension to the $3D$ localization is straightforward. Denoting the wave propagation speed by c, the noisy TDOA and AOA measurements can be modeled as (1) and (2), respectively.

$$\tau_{m,n} = \tau_m^t(\mathbf{x}) + \tau_n^r(\mathbf{x}) + \epsilon_{m,n}^{\tau}, \tag{1}$$

$$\alpha_n = \alpha_n^r(\mathbf{x}) + \epsilon_n^{\alpha}, \tag{2}$$

where $\tau_m^t(\mathbf{x}) = \frac{1}{c}\left\|\mathbf{x}_m^t - \mathbf{x}\right\|_2, \tau_n^r(\mathbf{x}) = \frac{1}{c}\left\|\mathbf{x}_n^r - \mathbf{x}\right\|_2, m = 1, \ldots, M, n = 1, \ldots, N$ denote the delay equations between target and the mth transmitter and the nth receiver, respectively and $\alpha_n^r(\mathbf{x}) = \tan^{-1}\left(\frac{y - y_n^r}{x - x_n^r}\right)$ denotes the angle of arrival equation in the nth receiver. TDOA and AOA measurements are perturbed by independent zero mean Gaussian noises of $\epsilon_{m,n}^{\tau}$ and ϵ_n^{α} with the standard deviation of $\sigma_{m,n}^{\tau}$ and σ_n^{α}, respectively.

By multiplying $\tau_{m,n}$ by c, we can obtain the bistatic range (BR) measurements, denoted by $d_{m,n}$ as follows.

$$d_{m,n} = R_{m,n}(\mathbf{x}) + \epsilon_{m,n}^R, \quad m = 1, \ldots, M, \; n = 1, \ldots, N$$

where $R_{m,n}(\mathbf{x}) = \left\|\mathbf{x}_m^t - \mathbf{x}\right\|_2 + \left\|\mathbf{x}_n^r - \mathbf{x}\right\|_2$ and $\epsilon_{m,n}^R$ is zero mean Gaussian measurement noise with standard deviation of $\sigma_{m,n}^R$.

We combine BR and AOA measurements in a vector as $\mathbf{b} = [\mathbf{b}_R^T, \mathbf{b}_\alpha^T]^T$, in which $\mathbf{b}_R = [d_{1,1}, d_{1,2}, \ldots, d_{M,N}]^T$ and $\mathbf{b}_\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_N]^T$. In a similar way, we form vector $\mathbf{h}(\mathbf{x}) = [\mathbf{h}_R^T(\mathbf{x}), \mathbf{h}_\alpha^T(\mathbf{x})]^T$ containing the corresponding BR and AOA equations, in which $\mathbf{h}_R(\mathbf{x}) = [R_{1,1}(\mathbf{x}), R_{1,2}(\mathbf{x}), \ldots, R_{M,N}(\mathbf{x})]^T$ and $\mathbf{h}_\alpha(\mathbf{x}) = [\alpha_1^r(\mathbf{x}), \alpha_2^r(\mathbf{x}), \ldots, \alpha_N^r(\mathbf{x})]^T$.

Now we form the sensing matrix $\mathbf{A}$ by computing $\mathbf{h}(.)$ in all grid points $\left\{\mathrm{g}^{(i)}\right\}_{i=1}^K$ in spatial domain as

$$\mathbf{A} = \left[\mathbf{h}(\mathrm{g}^{(1)}), \mathbf{h}(\mathrm{g}^{(2)}), \ldots, \mathbf{h}(\mathrm{g}^{(K)})\right].$$

3. Proposed method

In this section, we aim to find the position of a single target by comparing the received measurements with the values of their equations computed in all grid points. By this way, the problem of target localization can be expressed in the sparse representation framework given by $\mathbf{b} = \mathbf{A}\mathbf{z} + \boldsymbol{\epsilon}$, in which $\boldsymbol{\epsilon}$ is a $(M + 1)N \times 1$ vector containing the TDOA and AOA measurement noises, i.e. $\boldsymbol{\epsilon} = \left[\boldsymbol{\epsilon}_R^T, \boldsymbol{\epsilon}_\alpha^T\right]^T$, where $\boldsymbol{\epsilon}_R = \left[\boldsymbol{\epsilon}_{1,1}^R, \boldsymbol{\epsilon}_{1,2}^R, \ldots, \boldsymbol{\epsilon}_{M,N}^R\right]^T$ and $\boldsymbol{\epsilon}_\alpha = \left[\boldsymbol{\epsilon}_1^\alpha, \boldsymbol{\epsilon}_1^\alpha, \ldots, \boldsymbol{\epsilon}_N^\alpha\right]^T$. Vector $\mathbf{z}$ is a $K \times 1$ vector with $K - 1$ zero elements and a one element which is corresponding to the index of the grid point where the target is located.

3.1. Sparsity-Aware maximum likelihood estimator

Since $\mathbf{b} = \mathbf{A}\mathbf{z} + \epsilon$ has an underdetermined nature, accurate positioning with the conventional maximum likelihood (ML) estimator is not possible. A simple solution for this problem is to compute the objective function of the ML estimation for all grid points and select the minimum one (brute force). This trivial method has high complexity and limited positioning accuracy according to grid size. Instead, compressed sensing techniques allow us to reconstruct $\mathbf{z}$ from $\mathbf{b}$ with much lower complexity, by considering the target spatial sparsity. Thus, the target localization problem can be formulated using the following ℓ_1 minimization.

$$\hat{\mathbf{z}} = \underset{\mathbf{z}}{\operatorname{argmin}} \, (\mathbf{A}\mathbf{z} - \mathbf{b})^T \, \mathbf{C}_\epsilon^{-1} \, (\mathbf{A}\mathbf{z} - \mathbf{b}) + \lambda \, \|\mathbf{z}\|_1 \tag{3}$$

where λ is a regularization parameter that controls the sparsity of $\mathbf{z}$ and $\mathbf{C}_\epsilon$ denotes the covariance matrix of ϵ.

In order to employ classical reconstruction methods, such as orthogonal matching pursuit (OMP) [18], a whitening technique is applied. To this end, we modify the first term in (3) to the ℓ_2 norm $\left\|\tilde{\mathbf{A}}\mathbf{z} - \tilde{\mathbf{b}}\right\|_2^2$ by multiplying $\mathbf{b}$ and $\mathbf{A}$ by the weighting matrix $\mathbf{W}_{(M+1)N \times (M+1)N}$. This matrix is selected so that $\mathbf{W}^T\mathbf{W} = \mathbf{C}_\epsilon^{-1}$ or equivalently $\mathbf{W} = \sqrt{\mathbf{C}_\epsilon^{-1}}$ where $\sqrt{.}$ denotes the square root operation. By applying this transformation, (3) can be rewritten as

$$\hat{\mathbf{z}} = \underset{\mathbf{z}}{\operatorname{argmin}} \left\|\tilde{\mathbf{A}}\mathbf{z} - \tilde{\mathbf{b}}\right\|_2^2 + \lambda \, \|z\|_1 \tag{4}$$

where $\tilde{\mathbf{A}} = \mathbf{W}\mathbf{A}$ and $\tilde{\mathbf{b}} = \mathbf{W}\mathbf{b}$.

3.2. Blockwise complexity reduction

In classical localization methods, as long as the condition of $N_{TDOA} + N_{AOA} = (M + 1)N \geq 4$ was held, the target position in $2D$ can be uniquely estimated [19]. Furthermore, compressive sensing imposes another necessary condition $(M+1)N > 2L$ for reconstruction in noise-free scenarios. In other words, in order to reconstruct a L-sparse vector $\mathbf{z}$ (i.e. $\mathbf{z}$ has L non zero elements, in our case $L = 1$), every $2L$ column subset of $\mathbf{A}$ should be full column rank. Note that this is theoretical lower bound and in the presence of noise, its value is more than $2L$.

When the problem tends to a determined problem (assuming $\mathbf{z}$ remains sparse) the position estimation will be more precise. Furthermore, accuracy of grid-based methods is limited to the grid size. On the other hand, decreasing the grid size for a specific area increases the computational complexity and makes the problem more under-determined.

In this subsection an efficient blockwise technique is described for reducing the complexity and increasing the localization's accuracy simultaneously. In this method the number of grid points in all steps is considered to be an appropriate constant value K, which is chosen in a way to provide sufficient sparsity and accuracy. As shown in Fig. 1(a), we first solve (4) in the coarse grid points, which are located in the center of each block. Then, the solution block is divided into K smaller blocks and (4) is applied again. This procedure is repeated until a certain accuracy is provided.

3.3. Off-Grid mitigation

In grid-based localizations, the targets which are not located on the grids (off-grid) are not accurately localized. In order to resolve the aforementioned problem, an algorithm based on dictionary learning (DL) techniques is proposed which is designed to minimize the following cost function.

$$F(\mathbf{x}) = (\mathbf{b} - \mathbf{h}(\mathbf{x}))^T \mathbf{C}_\epsilon^{-1} (\mathbf{b} - \mathbf{h}(\mathbf{x})) = \left\| \tilde{\mathbf{b}} - \tilde{\mathbf{h}}(\mathbf{x}) \right\|_2^2 \qquad (5)$$

where $\tilde{\mathbf{h}}$ denotes the column of $\tilde{\mathbf{A}}$ corresponding to the estimated target position on the grids.

The cost function is convex with respect to $\mathbf{x}$. Therefore, a simple steepest decent can iteratively estimate the true target's location by learning from the matrix $\tilde{\mathbf{A}}$. Note that, in the following method we just employ TDOAs in order to satisfy convexity condition. Thus $\mathbf{b}, \mathbf{h}$ and $\mathbf{C}_\epsilon^{-1}$ are considered to be the TDOA part of them. The steepest decent iteration equation is

$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} - \mu^{(i)} \nabla_{\mathbf{x}} F(\mathbf{x}^{(i)}). \qquad (6)$$

In Appendix A, it is proved that the final recursion for updating the estimated location vector $\mathbf{x}$ at the $(i+1)$th iteration can be written as

$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} + 2\mu^{(i)} \mathbf{e}^T \mathbf{C}_\epsilon^{-1} \boldsymbol{\Psi} \qquad (7)$$

where $\mathbf{e} = \mathbf{b} - \mathbf{h}(\mathbf{x}^{(i)})$ and $\boldsymbol{\Psi} = [\boldsymbol{\psi}_{1,1}, \boldsymbol{\psi}_{1,2}, \ldots, \boldsymbol{\psi}_{M,N}]$ with $\boldsymbol{\psi}_{m,n} = \frac{\mathbf{x}^{(i)} - \mathbf{x}_m^t}{\|\mathbf{x}^{(i)} - \mathbf{x}_m^t\|_2} + \frac{\mathbf{x}^{(i)} - \mathbf{x}_n^r}{\|\mathbf{x}^{(i)} - \mathbf{x}_n^r\|_2}$.

The initial value of $\mathbf{x}$, $\mathbf{x}^{(0)}$ is chosen from the estimate of $\mathbf{x}$ in the previous subsection. The value of $\mu^{(i)}$ is selected according to convergence issues, i.e. $0 < \mu^{(i)} < \frac{2}{\lambda_{\max}^{(i)}}$, in which $\lambda_{\max}$ denotes the maximum eigen value of $\tilde{\mathbf{h}}(\mathbf{x})\tilde{\mathbf{h}}(\mathbf{x})^T$ [20]. To further clarify the localization procedure, in Algorithm 1 we provide a pseudo-code that describes the individual phases of the localization scheme.

The Cramer–Rao lower bound (CRLB) on the estimation error is summarized in Appendix B.

Algorithm 1 Pseudo-Code of the Proposed Method

for n = 1:Block level **do**
1. Divide the nth block to K sub-blocks.
2. Solve ML estimation (4), find the solution sub-block and use it for the $(n+1)$th iteration.

end for
3. Set $\mathbf{x}^{(0)}$ from the last estimated target's grid point in step 2.

$i = 0$, $\mathbf{x}^{(1)} = inf$.
while $|\mathbf{x}^{(i+1)} - \mathbf{x}^{(i)}| < \epsilon$ **do**
4. Find $\mathbf{x}^{(i+1)}$ from (7).

end while

4. Simulation results

In this section, we aim to assess the performance of the proposed method and compare it with other existing algorithms and the CRLB. To this end, we consider the area of interest with size of 10×10 km^2 is divided into $K = 10$ grid points and 5 transmitters and 5 receivers are located at $\{(1, 1), (1, 9), (5, 5), (6, 1), (6, 9)\}$ and $\{(5, 2), (2, 5), (5, 8), (7, 7), (7, 3)\}$ in km, respectively. The target is located at $(3.54, 6.23)$ km.

The estimate of target's position is shown in Fig. 1(b). As shown in Fig. 1(c), the mean square error (MSE) of the target's position is reduced in each iteration of the off-grid mitigation.

In the following, performance of the proposed method is evaluated in the presence of TDOA and AOA noises. The MSE of the target's position versus the standard deviation of TDOA and AOA noises was calculated using 1000-trial Monte-Carlo runs. The proposed method is applied in 4 block-levels and in each level a margin is added to the solution block to handle the possible reconstruction error for the next level. For DL step, the parameter μ is considered to be $\mu = 10^{-4}$ for all iterations. As shown in Fig. 2(a), (b), the proposed method works close to the CRLB. Moreover, it is shown that the off-grid mitigation step enhances the performance of localization.

Now, we aim to compare the proposed method with prior state of the arts. To the best of our knowledge, there is not any TDOA/AOA localization algorithm in context of MIMO radar. Thus, we compared TDOA-based version of the algorithm with the existing methods. To this end, the combined linear least squares method (CLLS) [12] and the two-step weighted least squares method (WLS) [13] are utilized for comparison. As shown in Fig. 2(c), the proposed method outperforms the others in terms of MSE.

5. Conclusion

In this paper, we formulated the problem of target localization in MIMO radars in sparse representation framework.

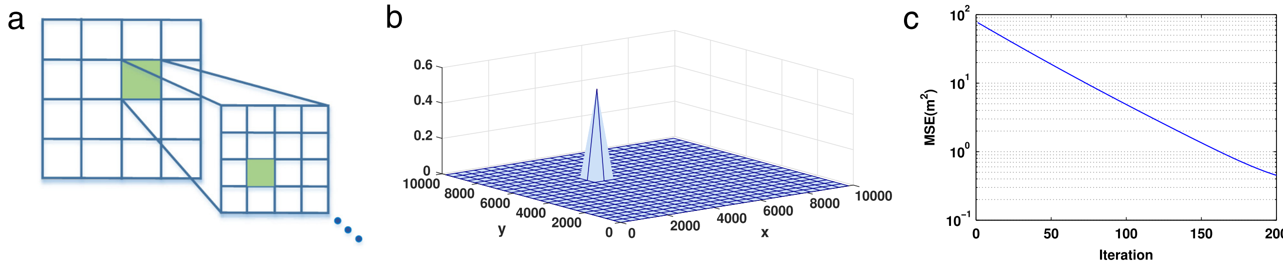

Fig. 1. (a) Block search approach, (b) Estimate of target's position, (c) Off-grid mismatch elimination by applying the proposed method.

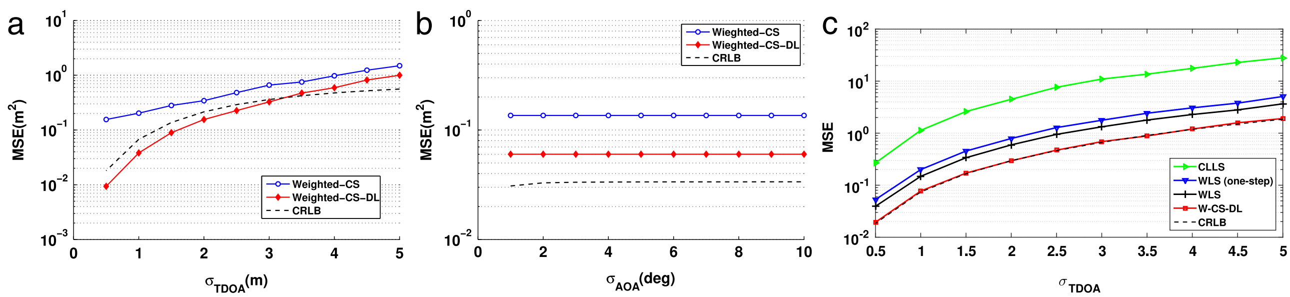

Fig. 2. (a) MSE of target position versus the standard deviation of noise of TDOA measurements, (b) MSE of target position versus the standard deviation of noise of AOA measurements, (c) Comparison with the existing algorithms.

The proposed method solves the sparsity-aware ML estimation of target's position by utilizing TDOA and AOA measurements and employs an iterative blockwise technique to reduce complexity and enhance accuracy, simultaneously. Then we proposed a dictionary learning based method to mitigate the off-grid mismatch due to discretization. The effectiveness of the proposed method was verified by simulation results.

Appendix A

By some mathematical manipulations, the cost function $F(\mathbf{x})$ can be formed as:

$$\begin{aligned} F(\mathbf{x}) &= \left(\tilde{\mathbf{b}} - \tilde{\mathbf{h}}(\mathbf{x})\right)^T \left(\tilde{\mathbf{b}} - \tilde{\mathbf{h}}(\mathbf{x})\right) \\ &= \tilde{\mathbf{b}}^T\tilde{\mathbf{b}} - 2\tilde{\mathbf{b}}^T\tilde{\mathbf{h}}(\mathbf{x}) + \tilde{\mathbf{h}}(\mathbf{x})^T\tilde{\mathbf{h}}(\mathbf{x}). \end{aligned}$$

The derivation of the cost function with respect to $[\mathbf{x}]_k$, $k = \{1, 2\}$ is as follows:

$$\begin{aligned} \frac{\partial F(\mathbf{x})}{\partial\, [\mathbf{x}]_k} &= -2\frac{\partial}{\partial[\mathbf{x}]_k}\left(\tilde{\mathbf{b}}^T\tilde{\mathbf{h}}(\mathbf{x})\right) + \frac{\partial}{\partial[\mathbf{x}]_k}\left(\tilde{\mathbf{h}}(\mathbf{x})^T\tilde{\mathbf{h}}(\mathbf{x})\right) \\ &= -2\tilde{\mathbf{b}}^T\frac{\partial\tilde{\mathbf{h}}(\mathbf{x})}{\partial[\mathbf{x}]_k} + \left(\frac{\partial\tilde{\mathbf{h}}(\mathbf{x})}{\partial[\mathbf{x}]_k}\right)^T\tilde{\mathbf{h}}(\mathbf{x}) + \tilde{\mathbf{h}}(\mathbf{x})^T\frac{\partial\tilde{\mathbf{h}}(\mathbf{x})}{\partial[\mathbf{x}]_k} \\ &= -2\tilde{\mathbf{b}}^T\frac{\partial\tilde{\mathbf{h}}(\mathbf{x})}{\partial[\mathbf{x}]_k} + \left(\tilde{\mathbf{h}}(\mathbf{x})^T\frac{\partial\tilde{\mathbf{h}}(\mathbf{x})}{\partial[\mathbf{x}]_k}\right)^T + \tilde{\mathbf{h}}(\mathbf{x})^T\frac{\partial\tilde{\mathbf{h}}(\mathbf{x})}{\partial[\mathbf{x}]_k} \\ &= -2\tilde{\mathbf{b}}^T\frac{\partial\tilde{\mathbf{h}}(\mathbf{x})}{\partial[\mathbf{x}]_k} + 2\tilde{\mathbf{h}}(\mathbf{x})^T\frac{\partial\tilde{\mathbf{h}}(\mathbf{x})}{\partial[\mathbf{x}]_k} \\ &= 2\left(\tilde{\mathbf{h}}(\mathbf{x})^T - \tilde{\mathbf{b}}^T\right)\frac{\partial\tilde{\mathbf{h}}(\mathbf{x})}{\partial[\mathbf{x}]_k} \\ &= 2\Big(\underbrace{\mathbf{h}(\mathbf{x}) - \mathbf{b}}_{\mathbf{e}}\Big)^T \mathbf{C}_\epsilon^{-1}\frac{\partial\mathbf{h}(\mathbf{x})}{\partial[\mathbf{x}]_k}. \end{aligned}$$

Also the derivation of $\mathbf{h}(\mathbf{x})$ with respect to $[\mathbf{x}]_k$ can be obtained as:

$$\frac{\partial\mathbf{h}(\mathbf{x})}{\partial[\mathbf{x}]_k} = \left[\frac{[\mathbf{x} - \mathbf{x}_m^t]_k}{\|\mathbf{x} - \mathbf{x}_m^t\|_2} + \frac{[\mathbf{x} - \mathbf{x}_n^r]_k}{\|\mathbf{x} - \mathbf{x}_n^r\|_2}\right]_{\substack{m=1,\dots,M\\ n=1,\dots,N}}.$$

Appendix B

We aim to derive the CRLB for estimating x from the TDOA and AOA observations. The CRLB is calculated using the trace of the inverse of corresponding Fisher information matrix, denoted by $\mathbf{I}$ in the case of Gaussian observations, with mean vector $\boldsymbol{\mu}$ and covariance matrix $\mathbf{C}$, as given by [21]

$$\begin{aligned} [\mathbf{I}(\mathbf{x})]_{(i,j)} &= \left[\frac{\partial\boldsymbol{\mu}}{\partial[\mathbf{x}]_i}\right]^T \mathbf{C}^{-1}\left[\frac{\partial\boldsymbol{\mu}}{\partial[\mathbf{x}]_j}\right] \\ &\quad + \frac{1}{2}\mathrm{tr}\left[\mathbf{C}^{-1}\left[\frac{\partial\mathbf{C}}{\partial[\mathbf{x}]_i}\right]\mathbf{C}^{-1}\left[\frac{\partial\mathbf{C}}{\partial[\mathbf{x}]_j}\right]\right]. \end{aligned} \quad \text{(B.1)}$$

In the present study, $\boldsymbol{\mu} = \mathbf{h}(\mathbf{x})$ and $\mathbf{C}$ is independent of $\mathbf{x}$. Thus the second term in (B.1) is equal to zero and the first term yields:

$$\mathrm{cov}(\hat{\mathbf{x}}) \geq [\mathbf{I}(\mathbf{x})]^{-1} = \left[\left[\frac{\partial\mathbf{h}}{\partial\mathbf{x}}\right]^T \mathbf{C}_\epsilon^{-1}\left[\frac{\partial\mathbf{h}}{\partial\mathbf{x}}\right]\right]^{-1}. \quad \text{(B.2)}$$

Since $\mathbf{h} = [\mathbf{h}_\mathbf{R}^\mathbf{T}, \mathbf{h}_\boldsymbol{\alpha}^\mathbf{T}]$, and $\frac{\partial\mathbf{h}}{\partial\mathbf{x}} = \left[\left[\frac{\partial\mathbf{h}_R}{\partial\mathbf{x}}\right]^T, \left[\frac{\partial\mathbf{h}_\alpha}{\partial\mathbf{x}}\right]^T\right]^T$. The partial derivative of $\frac{\partial\mathbf{h}_R}{\partial\mathbf{x}}$ and $\frac{\partial\mathbf{h}_\alpha}{\partial\mathbf{x}}$ as (B.3) and (B.4) respectively.

$$\frac{\partial\mathbf{h}_R}{\partial\mathbf{x}} = \left[\dot{\mathbf{h}}_R^{(1,1)}, \dot{\mathbf{h}}_R^{(1,2)}, \dots, \dot{\mathbf{h}}_R^{(M,N)}\right]^T \quad \text{(B.3)}$$

$$\frac{\partial\mathbf{h}_\alpha}{\partial\mathbf{x}} = \left[\dot{\mathbf{h}}_\alpha^{(1)}, \dot{\mathbf{h}}_\alpha^{(2)}, \dots, \dot{\mathbf{h}}_\alpha^{(N)}\right]^T \quad \text{(B.4)}$$

where $\dot{\mathbf{h}}_R^{(m,n)} = \frac{\mathbf{x}-\mathbf{x}_m^t}{\|\mathbf{x}-\mathbf{x}_m^t\|_2} + \frac{\mathbf{x}-\mathbf{x}_n^r}{\|\mathbf{x}-\mathbf{x}_n^r\|_2}$ and $\dot{\mathbf{h}}_\alpha^{(n)} = \left[\frac{-(y-y_n^t)}{\|\mathbf{x}-\mathbf{x}_n^r\|_2^2}, \frac{x-x_n^r}{\|\mathbf{x}-\mathbf{x}_n^r\|_2^2}\right]^T$. Then CRLB is determined from $\mathrm{tr}[\mathbf{I}^{-1}]$ in (B.2).

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