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Performance bounds for relative configuration and global transformation in cooperative localization☆

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Abstract

Cooperative localization introduces internode measurements to provide the node relative locations instead of absolute locations. This paper decomposes the absolute locations into relative configuration and global transformation, where the former can be specified by the internode measurements while the latter requires reference information. This decomposition can be used to investigate the relative localization which uses only internode measurements and the absolute localization with the consideration of anchor location uncertainty. After deriving the coordinate representations, error metric, and performance bounds for the global transformation, we evaluate the performance of a node location calibration that uses the measurements from sources in unknown locations.

Keywords: Cramér–Rao lower bound (CRLB); Fisher information matrix (FIM); Procrustes coordinates; Time-difference-of-arrival (TDOA); Unbiased estimate

1. Introduction

Localization problems, such as array localization or sensor network localization, involve a set of labeled nodes whose locations are usually represented by their pointwise absolute coordinates. However, in many applications [1], it is required only the node relative locations or the network (central) location and orientation, which needs other representations of the node locations.

Decomposing the node locations into the relative configuration and the global transformation separates the node relative locations and the network location and orientation [2]. The property of the relative configuration has been investigated in [3], which includes its coordinate representations, error metric, and performance bounds. By using the relative configuration, several problems in cooperative localization have been solved.

This paper further investigates the global transformation, including its coordinate representations, error metric, and performance bounds. The performance bounds are composed of the Cramér–Rao lower bounds (CRLBs) for the coordinate representations and a CRLB-type bound for the error metric. By using the CRLB-type bound, we evaluate the performance of a localization problem that uses the time-difference-of-arrival (TDOA) measurements from sources in known locations. Compared with existing work [4,5], quantifying the error on the relative configuration and the global transformation significantly reduces the complexity of the analysis.

The remaining part of this paper is organized as follows. Section 2 introduces the relative configuration and the global transformation, including their definitions, coordinate representations, and error metrics. For the error metrics, Section 3 derives the CRLB-type bounds through the CRLBs for the coordinate representations. An application of the CRLB-type bounds is given in Section 4. In Section 5, we conclude this paper.

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2. Relative configuration and global transformation

2.1. Definition

Suppose a network composed of n nodes, whose locations are $\mathbf{s}_i = [s_{i,x}, s_{i,y}]^T$, $i = 1, 2, \ldots, n$. The global transformation is defined by the congruence/rigid transformation

$$\mathcal{T}(\mathbf{s}_i) = \boldsymbol{\Gamma}_0 \begin{pmatrix} s_{i,x} \\ s_{i,y} \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}, \quad i = 1, 2, \ldots, n \tag{1}$$

where $\boldsymbol{\Gamma}_0$ is a 2-by-2 orthogonal matrix indicating global rotation/reflection operation, and x and y indicate the translation parameter in x and y directions, respectively. The relative configuration is defined as an object invariant to the congruence/rigid transformation (1), which forms an equivalence class with respect to the global transformation.

For easy derivation, we rewrite (1) in a vector form as

$$\mathcal{T}(\mathbf{s}) = \boldsymbol{\Gamma}\mathbf{s} + x\mathbf{1}_x + y\mathbf{1}_y \tag{2}$$

where $\mathbf{s} = [\mathbf{s}_1^T, \mathbf{s}_2^T, \ldots, \mathbf{s}_n^T]^T \in \mathbb{R}^{2n}$ is the location vector, $\mathbf{1}_x = [1, 0, \ldots, 1, 0]^T \in \mathbb{R}^{2n}$, $\mathbf{1}_y = [0, 1, \ldots, 0, 1]^T \in \mathbb{R}^{2n}$, and total rotation/reflection matrix $\boldsymbol{\Gamma} = \mathrm{diag}\,(\boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_0, \ldots, \boldsymbol{\Gamma}_0)$ is a $2n$-by-$2n$ block diagonal matrix whose 2-by-2 diagonal blocks are $\boldsymbol{\Gamma}_0$.

2.2. Coordinate representation

Given a reference vector $\mathbf{r} = [\mathbf{r}_1^T, \mathbf{r}_2^T, \ldots, \mathbf{r}_n^T]^T \in \mathbb{R}^{2n}$, the coordinate representation of the global transformation of $\mathbf{s}$ is defined through the partial Procrustes coordinates [6]

$$\mathbf{r}_{\mathbf{s}} = \arg\min_{\mathcal{T}(\mathbf{r})} \|\mathbf{s} - \mathcal{T}(\mathbf{r})\| = \boldsymbol{\Gamma}^\star\mathbf{r} + x^\star\mathbf{1}_x + y^\star\mathbf{1}_y \tag{3}$$

which superimposes a known relative configuration, specified by $\mathbf{r}$, onto $\mathbf{s}$. In (3),

$$\boldsymbol{\Gamma}^\star = \mathrm{diag}\left(\boldsymbol{\Gamma}_0^\star, \boldsymbol{\Gamma}_0^\star, \ldots, \boldsymbol{\Gamma}_0^\star\right) \tag{4}$$

$$\left[x^\star, y^\star\right]^T = \boldsymbol{\mu}_{\mathbf{s}} - \boldsymbol{\Gamma}_0^\star\boldsymbol{\mu}_{\mathbf{r}} \tag{5}$$

where $\boldsymbol{\Gamma}_0^\star = \mathbf{V}\mathbf{W}^T$, $\mathbf{W}\mathbf{D}\mathbf{V}^T$ is a singular value decomposition (SVD) of the covariance matrix $\boldsymbol{\Sigma}_{\mathbf{r},\mathbf{s}} = \frac{1}{n}\sum_{i=1}^n(\mathbf{r}_i - \boldsymbol{\mu}_{\mathbf{r}})(\mathbf{s}_i - \boldsymbol{\mu}_{\mathbf{s}})^T$, and the mean vectors $\boldsymbol{\mu}_{\mathbf{s}} = \frac{1}{n}\sum_{i=1}^n \mathbf{s}_i$, $\boldsymbol{\mu}_{\mathbf{r}} = \frac{1}{n}\sum_{i=1}^n \mathbf{r}_i$.

A coordinate representation of the relative configuration of $\mathbf{s}$ can be derived by superimposing the relative configuration of $\mathbf{s}$ onto the reference $\mathbf{r}$ as

$$\mathbf{s}_{\mathbf{r}} = \arg\min_{\mathcal{T}(\mathbf{s})} \|\mathbf{r} - \mathcal{T}(\mathbf{s})\| \tag{6}$$

where a closed form solution is given in [3].

2.3. Error metric

Let $\hat{\mathbf{s}}$ be an estimate of $\mathbf{s}$, and $\hat{\mathbf{s}}_{\mathbf{r}}$ and $\mathbf{r}_{\hat{\mathbf{s}}}$ be the coordinate representations of the relative configuration and the global transformation estimates. The estimation error of the relative configuration and the global transformation can be evaluated through the squared distances between the coordinate representations, *i.e.*, $\|\hat{\mathbf{s}}_{\mathbf{r}} - \mathbf{s}_{\mathbf{r}}\|^2$ and $\|\mathbf{r}_{\hat{\mathbf{s}}} - \mathbf{r}_{\mathbf{s}}\|^2$, respectively.

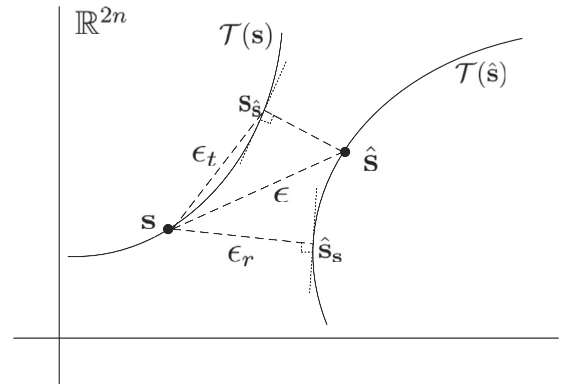


Fig. 1. Relative and transformation error: The relative error ϵ_r is the lowest squared distance from the location vector $\mathbf{s}$ to the trajectory with the same relative configuration of $\hat{\mathbf{s}}$. The transformation error ϵ_t is the squared distance between the location vector $\mathbf{s}$ and its global transformation closest to $\hat{\mathbf{s}}$.

Particularly, when the reference $\mathbf{r}$ is set at the true location $\mathbf{s}$, $\|\hat{\mathbf{s}}_{\mathbf{r}} - \mathbf{s}_{\mathbf{r}}\|^2$ and $\|\mathbf{r}_{\hat{\mathbf{s}}} - \mathbf{r}_{\mathbf{s}}\|^2$ can be simplified as $\|\hat{\mathbf{s}}_{\mathbf{s}} - \mathbf{s}\|^2$ and $\|\mathbf{s}_{\hat{\mathbf{s}}} - \mathbf{s}\|^2$. For $\hat{\mathbf{s}}_{\mathbf{s}}$, we have $\|\hat{\mathbf{s}}_{\mathbf{s}} - \mathbf{s}\|^2 \leq \|\hat{\mathbf{s}} - \mathbf{s}\|^2$, and the coordinate representation $\hat{\mathbf{s}}_{\mathbf{s}}$ owns the lowest squared distance to $\mathbf{s}$ compared with other choices of the reference $\mathbf{r}$ [3]. For convenience, $\epsilon_t \triangleq \|\mathbf{s}_{\hat{\mathbf{s}}} - \mathbf{s}\|^2$ is named transformation error in this paper,[1] $\epsilon_r \triangleq \|\hat{\mathbf{s}}_{\mathbf{s}} - \mathbf{s}\|^2$ is named relative error [2], and $\epsilon \triangleq \|\hat{\mathbf{s}} - \mathbf{s}\|^2$ refers to the location error. The relationship among ϵ_t, ϵ_r, and ϵ can be found in Fig. 1.

3. Performance bounds

3.1. CRLBs for coordinate representations

Proposition 1 gives the CRLB for the coordinate representation of the relative configuration.

Proposition 1 (*CRLB for the Coordinate Representation of the Relative Configuration [3]*). *Suppose $\hat{\mathbf{s}}_{\mathbf{r}}$ is an unbiased estimate of $\mathbf{s}_{\mathbf{r}}$, where $\mathbf{s}_{\mathbf{r}}$ and $\hat{\mathbf{s}}_{\mathbf{r}}$ are the coordinate representations of the relative configuration and its estimate, then*

$$\mathrm{E}\left[(\hat{\mathbf{s}}_{\mathbf{r}} - \mathbf{s}_{\mathbf{r}})(\hat{\mathbf{s}}_{\mathbf{r}}^T - \mathbf{s}_{\mathbf{r}}^T)\right] \geq \mathbf{U}_{\mathbf{r}}\left(\mathbf{U}_{\mathbf{r}}^T\mathbf{J}_{\mathbf{r}_{\mathbf{s}}}\mathbf{U}_{\mathbf{r}}\right)^{-1}\mathbf{U}_{\mathbf{r}}^T \tag{7}$$

where $\mathrm{E}[\cdot]$ denotes the expectation operation, $\mathbf{J}_{\mathbf{s}_{\mathbf{r}}}$ is a Fisher information matrix (FIM) at $\mathbf{s} = \mathbf{s}_{\mathbf{r}}$, and $\mathbf{U}_{\mathbf{r}}$ is a $2n$-by-$(2n-3)$ matrix whose columns form an orthonormal basis of the null space of $[\mathbf{1}_x, \mathbf{1}_y, \mathbf{v}_{\mathbf{r}}]^T$ with

$$\mathbf{v}_{\mathbf{r}} = [r_{1,y}, -r_{1,x}, r_{2,y}, \ldots, -r_{n,x}]^T \in \mathbb{R}^{2n}. \tag{8}$$

Proposition 2 gives the CRLB for the coordinate representation of the global transformation.

Proposition 2 (*CRLB for the Coordinate Representation of the Global Transformation*). *Suppose $\mathbf{r}_{\hat{\mathbf{s}}}$ is an unbiased estimate of*

[1] The term transformation error is also used in [2], which refers to $\epsilon - \epsilon_r$.

$\mathbf{r_s}$, *where* $\mathbf{r_s}$ *and* $\mathbf{r_{\hat{s}}}$ *are the coordinate representations of the global transformation and its estimate, then*

$$\mathrm{E}\left[(\mathbf{r_{\hat{s}}}-\mathbf{r_s})(\mathbf{r_{\hat{s}}}^T-\mathbf{r_s}^T)\right] \geq \mathbf{V_r}\left(\mathbf{V_r}^T\mathbf{J_{r_s}}\mathbf{V_r}\right)^{-1}\mathbf{V_r}^T \tag{9}$$

where $\mathbf{J_{r_s}}$ *is an FIM at* $\mathbf{s}=\mathbf{r_s}$, *and* $\mathbf{V_r}$ *is a column orthonormalized version of* $[\mathbf{1}_x, \mathbf{1}_y, \mathbf{u_r}]$ *with*

$$\mathbf{u_r} = \det(\boldsymbol{\Gamma}_0^\star)\left[\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\boldsymbol{\Gamma}_0^\star\mathbf{r}_i\right]_{i=1,2,\ldots,n} \in \mathbb{R}^{2n}. \tag{10}$$

The proof can be found in the Appendix.

3.2. CRLB-type bound

After setting the reference $\mathbf{r}$ at $\mathbf{s}$, the CRLB-type bounds for the relative and the transformation errors can be derived directly from Propositions 1 and 2, respectively.

Proposition 3 (*CRLB-Type Bound for Relative Error [3]*)**.** *Suppose* $\hat{\mathbf{s}}_\mathbf{s}$ *is an unbiased estimate of* $\mathbf{s}$, *where* $\hat{\mathbf{s}}_\mathbf{s}$ *is the coordinate representation of the relative configuration estimate at the reference* $\mathbf{r}=\mathbf{s}$, *then*

$$\mathrm{E}\left[\epsilon_r\right] \geq \mathrm{tr}\left(\left(\mathbf{U_s}^T\mathbf{J_s}\mathbf{U_s}\right)^{-1}\right) \tag{11}$$

where $\mathrm{tr}(\cdot)$ *denotes the trace operation.*

Proposition 4 (*CRLB-Type Bound for Transformation Error*)**.** *Suppose* $\mathbf{s_{\hat{s}}}$ *is an unbiased estimate of* $\mathbf{s}$, *where* $\mathbf{s_{\hat{s}}}$ *is the coordinate representation of the global transformation estimate at the reference* $\mathbf{r}=\mathbf{s}$, *then*

$$\mathrm{E}\left[\epsilon_t\right] \geq \mathrm{tr}\left(\left(\mathbf{V_s}^T\mathbf{J_s}\mathbf{V_s}\right)^{-1}\right). \tag{12}$$

Notably, when $\mathbf{r}=\mathbf{s}$, the columns of $\mathbf{U_s}$ and $\mathbf{V_s}$ form an orthonormal basis of $\mathbb{R}^{2n}$.

4. An application

The CRLB-type bounds for the relative configuration and the global transformation errors are applied to investigate a node location calibration problem.

In this problem, the nominal locations of n nodes are calibrated by the TDOA measurements from m sources in unknown locations. The nominal locations $\tilde{\mathbf{s}}_i = [\tilde{s}_{i,x}, \tilde{s}_{i,y}]^T$, $i=1,2,\ldots,n$, follow n independent Gaussian distributions with mean $\mathbf{s}_i$, $i=1,2,\ldots,n$, and covariance $\tau^2\mathbf{I}_2$. The TDOA measurements from the sources located at $\mathbf{u}_j=[u_{j,x}, u_{j,y}]^T$, $j=1,2,\ldots,m$, are modeled as

$$r_{i,j} = d_{i,j} + \xi_{i,j}, \quad i=2,3,\ldots,n,\ j=1,2,\ldots,m \tag{13}$$

where $d_{i,j} = \|\mathbf{s}_i-\mathbf{u}_j\| - \|\mathbf{s}_1-\mathbf{u}_j\|$, the noise vector $\boldsymbol{\xi} = [\xi_{i,j}]_{i=2,3,\ldots,n,j=1,2,\ldots,m}$ is a Gaussian stochastic vector with zero mean and covariance matrix $\mathbf{Q}$. Since there exists correlation among the measurements, the covariance matrix $\mathbf{Q}$ is set as a block diagonal matrix with $(n-1)$-by-$(n-1)$ diagonal block $\frac{\sigma^2}{2}\left(\mathbf{I}_{n-1}+\mathbf{1}_{n-1}\mathbf{1}_{n-1}^T\right)$ [7].

The nominal node locations and the TDOA measurements determine the logarithm of the likelihood function of node location vector $\mathbf{s}$ and source location vector $\mathbf{u}=[\mathbf{u}_1^T, \mathbf{u}_2^T, \ldots, \mathbf{u}_m^T]^T \in \mathbb{R}^{2m}$ as

$$l(\mathbf{s},\mathbf{u}) = -\frac{1}{2}(\mathbf{r}-\mathbf{d})^T\mathbf{Q}^{-1}(\mathbf{r}-\mathbf{d}) - \frac{1}{2\tau^2}\|\tilde{\mathbf{s}}-\mathbf{s}\|^2 + c \tag{14}$$

where the measurement vector $\mathbf{r}=[r_{i,j}]_{i=2,3,\ldots,n,j=1,2,\ldots,m}$, the noise-free TDOA vector $\mathbf{d}=[d_{i,j}]_{i=2,3,\ldots,n,j=1,2,\ldots,m}$, the nominal node location vector $\tilde{\mathbf{s}}=[\tilde{\mathbf{s}}_1^T, \tilde{\mathbf{s}}_2^T, \ldots, \tilde{\mathbf{s}}_n^T]^T \in \mathbb{R}^{2n}$, and c is a constant independent of $\mathbf{s}$ and $\mathbf{u}$.

After taking the negative expectation of the second derivation of (14), we get the FIM of both $\mathbf{u}$ and $\mathbf{s}$ [8]

$$\begin{aligned}\mathbf{J_{u,s}} &= -\begin{bmatrix}\mathrm{E}\left[\dfrac{\partial^2 l(\mathbf{u},\mathbf{s})}{\partial\mathbf{u}\partial\mathbf{u}^T}\right] & \mathrm{E}\left[\dfrac{\partial^2 l(\mathbf{u},\mathbf{s})}{\partial\mathbf{u}\partial\mathbf{s}^T}\right]\\ \mathrm{E}\left[\dfrac{\partial^2 l(\mathbf{u},\mathbf{s})}{\partial\mathbf{s}\partial\mathbf{u}^T}\right] & \mathrm{E}\left[\dfrac{\partial^2 l(\mathbf{u},\mathbf{s})}{\partial\mathbf{s}\partial\mathbf{s}^T}\right]\end{bmatrix}\\ &= \begin{bmatrix}\mathbf{F_u}^T\mathbf{Q}^{-1}\mathbf{F_u} & \mathbf{F_u}^T\mathbf{Q}^{-1}\mathbf{F_s}\\ \mathbf{F_s}^T\mathbf{Q}^{-1}\mathbf{F_u} & \mathbf{F_s}^T\mathbf{Q}^{-1}\mathbf{F_s}+\tau^{-2}\mathbf{I}_{2n}\end{bmatrix}\end{aligned} \tag{15}$$

and thus the FIM of $\mathbf{s}$

$$\mathbf{J_s} = \tau^{-2}\mathbf{I}_{2n} + \widetilde{\mathbf{J}}_\mathbf{s} \tag{16}$$

where

$$\widetilde{\mathbf{J}}_\mathbf{s} = \mathbf{F_s}^T\mathbf{Q}^{-1}\mathbf{F_s} - \mathbf{F_s}^T\mathbf{Q}^{-1}\mathbf{F_u}\left(\mathbf{F_u}^T\mathbf{Q}^{-1}\mathbf{F_u}\right)^{-1}\mathbf{F_u}^T\mathbf{Q}^{-1}\mathbf{F_s}. \tag{17}$$

Note that $\mathbf{J_s}$ is invertible, the CRLB of $\mathbf{s}$ can be derived as

$$\mathbf{C_s} = \mathbf{J_s}^{-1}. \tag{18}$$

Here, $\mathbf{F_u}$ is a $(n-1)m$-by-$2m$ matrix stacked by

$$\left[\mathbf{0}_{1\times 2(j-1)}, \frac{\mathbf{u}_j^T-\mathbf{s}_i^T}{\|\mathbf{u}_j-\mathbf{s}_i\|} - \frac{\mathbf{u}_j^T-\mathbf{s}_1^T}{\|\mathbf{u}_j-\mathbf{s}_1\|}, \mathbf{0}_{1\times 2(m-j)}\right] \tag{19}$$

in row, and $\mathbf{F_s}$ is a $(n-1)m$-by-$2n$ matrix stacked by

$$\left[-\frac{\mathbf{s}_1^T-\mathbf{u}_j^T}{\|\mathbf{s}_1-\mathbf{u}_j\|}, \mathbf{0}_{1\times 2(i-2)}, \frac{\mathbf{s}_i^T-\mathbf{u}_j^T}{\|\mathbf{s}_i-\mathbf{u}_j\|}, \mathbf{0}_{1\times 2(n-i)}\right] \tag{20}$$

in row.

It is worth pointing out that the columns of $\mathbf{V_s}$ belong to the null space $\widetilde{\mathbf{J}}_\mathbf{s}$. Therefore, there exists an SVD

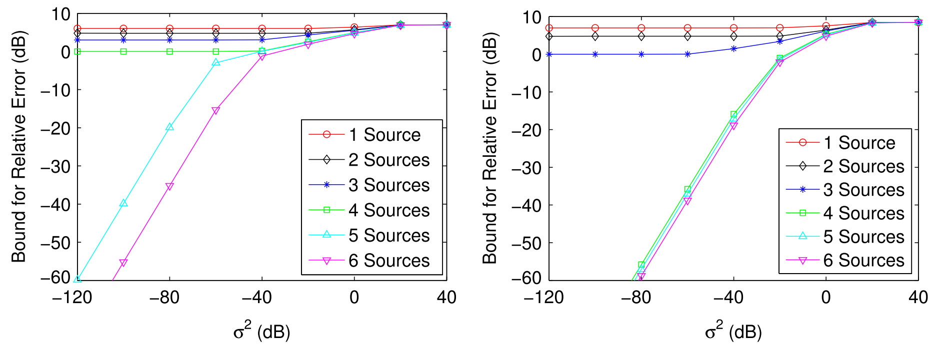
$$\widetilde{\mathbf{J}}_\mathbf{s} = \mathbf{U_s}\boldsymbol{\Lambda}\mathbf{U_s}^T \tag{21}$$

where $\boldsymbol{\Lambda}$ is a $(2n-3)$-by-$(2n-3)$ diagonal matrix whose diagonal elements are the eigenvalues for the eigenvectors composed of the columns of $\mathbf{U_s}$. From (16) and (21), the CRLB-type bounds for the relative and transformation errors can be represented as

$$c_r = \mathrm{tr}\left(\left(\tau^{-2}\mathbf{I}_{2n-3}+\boldsymbol{\Lambda}\right)^{-1}\right), \qquad c_t = 3\tau^2 \tag{22}$$

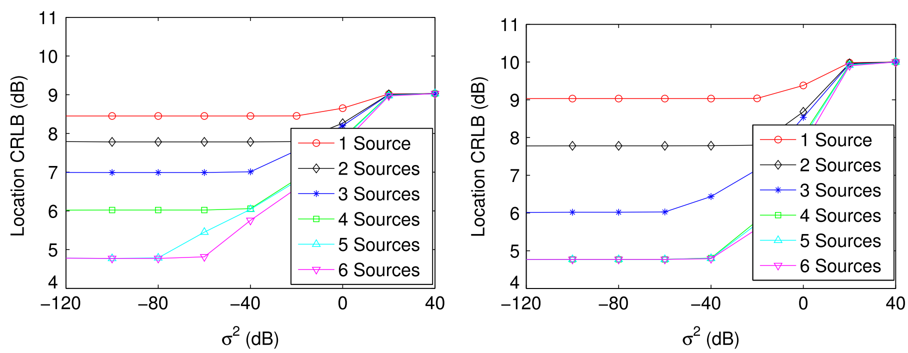
by using Propositions 3 and 4, respectively.

The CRLB-type bound c_r indicates that the network relative configuration can be calibrated by using the TDOAs from



(a) Four nodes. (b) Five nodes.

Fig. 2. CRLB-type bound for the relative configuration versus the measurement variance σ^2, under different node–source numbers.



(a) Four nodes. (b) Five nodes.

Fig. 3. Trace of the location CRLB versus the measurement variance σ^2, under different node–source numbers.

Table 1
Node&source locations.

Node No. i	$s_{i,x}$	$s_{i,y}$	Source No. j	$u_{j,x}$	$u_{j,y}$
1	8.4872	6.6526	1	2.3338	3.6111
2	9.1316	4.5014	2	6.3346	9.8610
3	7.5480	0.3519	3	2.0716	7.5708
4	9.0272	3.8899	4	8.8633	4.7223
5	9.9812	9.4447	5	1.5891	8.1092
			6	4.7651	1.1629

sources in known locations. Particularly, when $\boldsymbol{\Lambda}$ is nonsingular, the network relative configuration can be accurately calibrated through improving measurement accuracy. To guarantee the nonsingularity, it is required that the rank of $\widetilde{\mathbf{J}}_{\mathbf{s}}$ is $2n-3$, and thus the node–source number should fulfill $(n-3)(m-2) \geq 3$ when no collinearity exists. For the global transformation, the CRLB-type bound c_t shows that there is no improvement.

A numerical example exhibits the calibration performance on the relative configuration, the global transformation, and the locations. In this example, the node locations are chosen from the first four and five node locations in Table 1, respectively, and the source locations are selected successively from the six source locations in Table 1. The variance of the nominal node locations is set as $\tau^2 = 1$.

Fig. 2 displays the calibration performance on the relative configuration. Through the CRLB-type bound for the relative configuration, it can be found that the error on the relative configuration could be reduced. Especially, when the node–source number fulfills $(n-3)(m-2) \geq 3$, *i.e.*, at least five for a four node network (seen in Fig. 2(a)) and four for a five node network (seen in Fig. 2(b)), the relative configuration can be calibrated to any desired accuracy through improving the TDOA measurement quality. This indicates the possibility of the accurate calibration of the relative configuration.

Fig. 3 displays the calibration performance on the node locations. From the figure, it can be seen that the trace of the location CRLB is lower bounded by the CRLB-type bound for the global transformation, which can be asymptotically achieved by improving TDOA measurement quality when node–source number fulfills $(n-3)(m-2) \geq 3$.

5. Conclusion

Decomposing the locations of labeled nodes into the relative configuration and the global transformation provides another perspective on investigating some localization or calibration problems that involve the measurements between the unknowns. This paper presents the coordinate representations, error metric, and performance bounds for the global transformation, which completes the work initiated in [3]. We hope the tools given in [3] and this paper could facilitate the study of more problems related to the locations of labeled nodes.

Acknowledgments

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Appendix. Proof of Proposition 2

Given the reference $\mathbf{r}$, the coordinate representation of the global transformation can be viewed as a vector function of three continuous variables $\boldsymbol{\eta} = \left[x^\star, y^\star, \theta^\star\right]^T$ where $\theta^\star$ parameterizes $\boldsymbol{\Gamma}_0^\star$.

Using the reparameterization $\mathbf{r_s} = \mathbf{r_s}(\boldsymbol{\eta})$, we get the FIM of $\boldsymbol{\eta}$

$$\mathbf{J}_{\boldsymbol{\eta}} = \frac{\partial \mathbf{r_s}^T}{\partial \boldsymbol{\eta}} \mathbf{J}_{\mathbf{r_s}} \frac{\partial \mathbf{r_s}}{\partial \boldsymbol{\eta}^T} \tag{23}$$

and then the CRLB of $\mathbf{r_s}$

$$\mathrm{E}\left[(\mathbf{r}_{\hat{\mathbf{s}}} - \mathbf{r_s})(\mathbf{r}_{\hat{\mathbf{s}}}^T - \mathbf{r_s}^T)\right] \geq \frac{\partial \mathbf{r_s}}{\partial \boldsymbol{\eta}^T} \left(\frac{\partial \mathbf{r_s}^T}{\partial \boldsymbol{\eta}} \mathbf{J}_{\mathbf{r_s}} \frac{\partial \mathbf{r_s}}{\partial \boldsymbol{\eta}^T}\right)^{-1} \frac{\partial \mathbf{r_s}^T}{\partial \boldsymbol{\eta}} \tag{24}$$

where $\frac{\partial \mathbf{r_s}}{\partial \boldsymbol{\eta}^T} = \left[\mathbf{1}_x, \mathbf{1}_y, \mathbf{u_r}\right]$ with $\mathbf{u_r}$ given in (10).

Note that $\mathbf{V_r}$ is a column orthonormalized version of $\frac{\partial \mathbf{r_s}}{\partial \boldsymbol{\eta}^T}$, (9) is proved. □

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