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Application of capon method to direct position determination☆,☆☆

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Abstract

The direct position determination (DPD) approach is a single-step method which uses the Maximum Likelihood estimator to localize sources emitting electromagnetic energy using combined data from all available sensors. The DPD is known to outperform the traditional 2-step methods under low Signal to Noise Ratio (SNR) conditions. We propose an improvement to the DPD approach, using the well known minimum-variance-distortionless-response (MVDR) approach. Unlike Maximum Likelihood, the number of sources need not be known before applying the method. The combination of both the direct approach and MVDR yields unprecedented localization accuracy and resolution for weak sources. We demonstrate this approach on the problem of multistatic radar, but the method can easily be extended to general localization problems.

1. Introduction

The localization of sources emitting electromagnetic or acoustic energy is needed in wild-life tracking, radio-astronomy, seismology, medical-diagnosis, communications, and other engineering applications. Common localization methods use two estimation steps. First, intermediate parameters are estimated. Intermediate parameters are usually time of arrival, direction of arrival, Doppler frequency shift or signal strength. These estimated parameters are then used, in a second step, to estimate the actual location of the emitter. The Direct Position Determination (DPD) approach has been recently proposed [1] as a single-step Maximum Likelihood localization technique. A single-step approach is a technique in which the estimator uses exactly the same data as used in two-step methods but estimates the source location directly, skipping the intermediate (first) step. This can be viewed as searching for the emitter location that best explains the collected data. From estimation theory point-of-view the two-step approach is inferior, since in the first step the parameters are measured independently, ignoring the constraint that the measurements relate to the same emitter location. Indeed, this method has been shown to be superior to the two-step methods for low SNR. In addition, the DPD method has been shown to be more robust by inherently selecting reliable observations without the need for a goodness-of-fit test (such as the chi-square test). This method was also extended to radar scenarios in [2], where the Maximum Likelihood target location estimation was developed as well as the Cramer–Rao lower bound for the estimation error.

When there are multiple sources the DPD is no longer equivalent to the Maximum Likelihood Estimator (MLE). The exact MLE can be derived but it requires a multi-dimensional search which is usually impractical. An alternative for the Maximum Likelihood parameter estimator is the Minimum Variance Distortionless Response (MVDR) estimator. It was originally proposed by Capon [3] for frequency–wavenumber power spectral density analysis, but has since been used extensively as a high resolution method. The idea is to adaptively select the weight vector in order to fix the response for the parameter value of interest while minimizing the output power. Unlike the Maximum Likelihood approach, the MVDR approach does not need to know a-priori the number of targets (or model order) and therefore it is a robust approach with good resolution and immunity to jamming and interference.

Our test case is multistatic radar, which is a generalization of the classical mono-static radar system. In a multistatic radar system multiple cooperative receivers are used for target localization. This could be generalized further with the addition of multiple transmitters (a scheme usually termed MIMO radar,

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e.g. [4]), but we limit our demonstration to a single transmitter, thus ignoring difficulties caused by mutual interference of the transmitted signals.

The focus of this paper is the demonstration of the single-step (direct) MVDR concept for source localization. As an example, we use a simplified multi-static radar model, neglecting radar clutter for brevity. Future work includes the full blown radar model. Note that MVDR is applied to the target's location estimation in a single step, and not for intermediate parameters estimation. Some previous publications touch upon our proposed approach but in no way cover the full potential of this method (see for example [5–7]). Other papers use MVDR for Angle of Arrival Estimation but titles that refer to localization [8]. We show that the estimation method proposed here can significantly improve target resolution compared with the single-source MLE. Fine target resolution can prove very useful for target localization within many dense decoys.

2. Problem formulation

Consider a transmitter and L widely separated receiving arrays. Each receiving array consists of M_r elements. The array aperture is typically a few signal wavelengths. The transmitted signal $s(t)$ is confined to the time interval $t \in [0, T]$. It is assumed that the signal $s(t)$ is perfectly known, which is usually the case when there is line of sight from the transmitter to the receiving array. In the following analysis the signal impinges on a single target whose coordinates vector is denoted by $\mathbf{p}_t$, and is reflected by the target towards the receiving arrays. We assume that the transmitter, arrays and target are all confined to a plane, and the transmitter and receiving arrays locations are known while the target location needs to be estimated.

The ℓ-th array output is given by the $M_r \times 1$ vector,

$$\mathbf{r}_\ell(t) = \mathbf{a}_\ell(\mathbf{p}_t)\alpha_\ell s(t - \tau_\ell(\mathbf{p}_t))e^{i2\pi f_{D,\ell}t} + \mathbf{n}_\ell(t) \tag{1}$$

where α_ℓ is the signal attenuation at the ℓ-th array, τ_ℓ is the signal delay associated with the propagation from the transmitter to the target and then to the ℓ-th array, which satisfies,

$$\tau_\ell(\mathbf{p}_t) = \frac{\|\mathbf{p}_t - \mathbf{p}_\ell\|}{c} + \frac{\|\mathbf{p}_t - \mathbf{p}_{\mathrm{Tx}}\|}{c} \tag{2}$$

where $\mathbf{p}_{\mathrm{Tx}}$ is the transmitter's location, $\mathbf{p}_\ell$ is the ℓ-th array location, and c is the speed of propagation. Further, $\mathbf{n}_\ell(t)$ is a $M_r \times 1$ wide-sense stationary, white, zero mean, complex Gaussian noise, $\mathbf{a}_\ell(\mathbf{p}_t)$ is a $M_r \times 1$ vector representing the ℓ-th array response to a target at $\mathbf{p}_t$, and $f_{D,\ell}$ is the Doppler frequency shift. We note that without loss of generality we can impose the constraint $\|\mathbf{a}_\ell(\mathbf{p}_t)\|^2 = 1$. Since the transmitter and arrays cooperate the signal transmission time is perfectly known. This can be accomplished by direct interception of the transmitted signal or by synchronization of the transmitter and receiving arrays. Finally, we assume the target is illuminated by M_p consecutive pulses. To simplify the exhibition, it is assumed that the target speed is small enough to neglect the Doppler effect. Note that this formulation can be modified to describe somewhat different localization problem. For example, in source localization, such as smart phones localization, assuming the transmitted signal is known, the only change is in the dependence of τ_ℓ on the target position. A similar derivation was performed in [9]. Further, if the signal is not known, it could be incorporated into the estimation problem, but this is beyond the scope of this work.

The DFT of the received jth pulse is given by

$$\bar{\mathbf{r}}_{\ell,k}(j) = \mathbf{a}_\ell(\mathbf{p}_t)\,\alpha_\ell(j)\,\bar{s}_k e^{-i2\pi f_k \tau_\ell(\mathbf{p}_t)} + \bar{\mathbf{n}}_{\ell,k}(j) \tag{3}$$

where $f_k = \frac{k}{K} f_s$ is the frequency associated with the kth coefficient, K is the number of samples, f_s is the sampling frequency, and $\bar{\mathbf{r}}_{\ell,k}$, $\bar{s}_k$ and $\bar{\mathbf{n}}_{\ell,k}$ are the kth Fourier coefficients of $\mathbf{r}_\ell(t)$, $s(t)$ and $\mathbf{n}_\ell(t)$, respectively, and where it is assumed that the observation time is longer than the received signal interval plus its delays at all sensors.

Define

$$\begin{aligned}
\bar{\mathbf{r}}_\ell(j) &\triangleq [\bar{\mathbf{r}}_{\ell,1}^T(j), \bar{\mathbf{r}}_{\ell,2}^T(j), \ldots, \bar{\mathbf{r}}_{\ell,K}^T(j)]^T \\
\bar{\mathbf{n}}_\ell(j) &\triangleq [\bar{\mathbf{n}}_{\ell,1}^T(j), \bar{\mathbf{n}}_{\ell,2}^T(j), \ldots, \bar{\mathbf{n}}_{\ell,K}^T(j)]^T \\
\mathbf{A}_\ell(\mathbf{p}_t) &\triangleq \mathrm{diag}(e^{-j2\pi f_1 \tau_\ell}, \ldots, e^{-j2\pi f_K \tau_\ell}) \otimes \mathbf{a}_\ell(\mathbf{p}_t) \\
\bar{\mathbf{s}} &\triangleq [\bar{s}_1, \ldots, \bar{s}_K]^T
\end{aligned} \tag{4}$$

where the dependence of τ_ℓ on $\mathbf{p}_t$ is suppressed and where $\otimes$ denotes the Kronecker product. We can now write (3) in a vector form

$$\bar{\mathbf{r}}_\ell(j) = \alpha_\ell(j)\,\mathbf{A}_\ell(\mathbf{p}_t)\,\bar{\mathbf{s}} + \bar{\mathbf{n}}_\ell(j). \tag{5}$$

In the next section we derive the (single-source) Maximum Likelihood estimator for $\mathbf{p}_t$, where $\{\alpha_\ell(j)\}$ are treated as unknown parameters. As explained in the introduction, it is possible to derive an exact Maximum Likelihood estimator. However, such an estimator needs to know a-priori the number of targets, and it requires a multi-dimensional search. In Section 2.2 we use the single-source Maximum Likelihood estimator to obtain the multi-source MVDR estimator, which is our main goal.

2.1. Target localization using the maximum likelihood estimator

Using (5) and the fact that $\{\bar{\mathbf{n}}_\ell(j)\}$ are statistically independent complex Gaussian vectors, the Maximum Likelihood cost function is

$$Q(\mathbf{p}_t) = \sum_{\ell=1}^{L}\sum_{j=1}^{M_p} \|\bar{\mathbf{r}}_\ell(j) - \alpha_\ell(j)\,\mathbf{A}_\ell(\mathbf{p}_t)\,\bar{\mathbf{s}}\|^2. \tag{6}$$

The signal attenuation $\{\alpha_\ell(j)\}$ is assumed to be independent from pulse to pulse, as is suggested by the well known Swerling II and IV target models, which assume the target radar-cross-section (RCS) is independent from pulse to pulse (see [10]). The attenuation coefficient that minimizes the cost function (6) is given by

$$\begin{aligned}
\hat{\alpha}_\ell(j) &= \left(\bar{\mathbf{s}}^H \mathbf{A}_\ell^H(\mathbf{p}_t)\,\mathbf{A}_\ell(\mathbf{p}_t)\,\bar{\mathbf{s}}\right)^{-1} \bar{\mathbf{s}}^H \mathbf{A}_\ell^H(\mathbf{p}_t)\,\bar{\mathbf{r}}_\ell(j) \\
&= (\bar{\mathbf{s}}^H\bar{\mathbf{s}})^{-1}\bar{\mathbf{s}}^H \mathbf{A}_\ell^H(\mathbf{p}_t)\,\bar{\mathbf{r}}_\ell(j)
\end{aligned} \tag{7}$$

where we used

$$\mathbf{A}_\ell^H(\mathbf{p}_t)\,\mathbf{A}_\ell(\mathbf{p}_t) = \mathbf{I}_{M_r K}\|\mathbf{a}_\ell\|^2 = \mathbf{I}_{M_r K}. \tag{8}$$

Substituting (7) back into (6) we get

$$\begin{aligned} Q_{\mathrm{ML}}(\mathbf{p}_t) &= \sum_{\ell=1}^{L}\sum_{j=1}^{M_p}\left\|\bar{\mathbf{r}}_\ell(j) - \epsilon^{-1}\mathbf{A}_\ell(\mathbf{p}_t)\bar{\mathbf{s}}\bar{\mathbf{s}}^H\mathbf{A}_\ell^H(\mathbf{p}_t)\bar{\mathbf{r}}_\ell(j)\right\|^2 \\ &= \sum_{\ell=1}^{L}\sum_{j=1}^{M_p}\left(\|\bar{\mathbf{r}}_\ell(j)\|^2 - \epsilon^{-1}\left|\bar{\mathbf{s}}^H\mathbf{A}_\ell^H(\mathbf{p}_t)\bar{\mathbf{r}}_\ell(j)\right|^2\right) \end{aligned} \tag{9}$$

where

$$\epsilon \triangleq \bar{\mathbf{s}}^H\bar{\mathbf{s}}. \tag{10}$$

Since $\|\bar{\mathbf{r}}_\ell(j)\|^2$ is constant, minimization of this expression is equivalent to the maximization of

$$\begin{aligned} \widetilde{Q}_{\mathrm{ML}}(\mathbf{p}_t) &= \frac{1}{M_p}\sum_{\ell=1}^{L}\sum_{j=1}^{M_p}\epsilon^{-1}\left|\bar{\mathbf{s}}^H\mathbf{A}_\ell^H(\mathbf{p}_t)\bar{\mathbf{r}}_\ell(j)\right|^2 \\ &= \sum_{\ell=1}^{L}\epsilon^{-1}\bar{\mathbf{s}}^H\mathbf{A}_\ell^H(\mathbf{p}_t)\hat{\mathbf{R}}_\ell\mathbf{A}_\ell(\mathbf{p}_t)\bar{\mathbf{s}} \end{aligned} \tag{11}$$

where the sample covariance matrix is defined as

$$\hat{\mathbf{R}}_\ell \triangleq \frac{1}{M_p}\sum_{j=1}^{M_p}\bar{\mathbf{r}}_\ell(j)\bar{\mathbf{r}}_\ell^H(j). \tag{12}$$

Note that it is always possible, through proper normalization of the received signals, to set $\epsilon = 1$.

Finally, we can express the ML estimator as

$$\widehat{\mathbf{p}}_t^{\mathrm{ML}} = \arg\max_{\mathbf{p}_t}\widetilde{Q}_{\mathrm{ML}}(\mathbf{p}_t) = \arg\max_{\mathbf{p}_t}\sum_{\ell=1}^{L}Q_\ell^{\mathrm{ML}}(\mathbf{p}_t) \tag{13}$$

where the individual cost functions are given by

$$Q_\ell^{\mathrm{ML}}(\mathbf{p}_t) \triangleq \left(\mathbf{w}_\ell^{\mathrm{ML}}(\mathbf{p}_t)\right)^H\hat{\mathbf{R}}_\ell\mathbf{w}_\ell^{\mathrm{ML}}(\mathbf{p}_t) \tag{14}$$

and

$$\mathbf{w}_\ell^{\mathrm{ML}}(\mathbf{p}_t) \triangleq \mathbf{A}_\ell(\mathbf{p}_t)\bar{\mathbf{s}}. \tag{15}$$

This ends the derivation for the Maximum Likelihood estimator of the target position assuming a single target and no other interference. This expression can be seen as focusing on the hypothesized target location using the weight vectors $\left\{\mathbf{w}_\ell^{\mathrm{ML}}(\mathbf{p}_t)\right\}$, then combining the responses.

2.2. *Target localization using MVDR*

We now turn to the derivation of the MVDR estimator for this case. As discussed in the introduction, high resolution DOA (Direction of Arrival) or frequency–wavenumber estimation can be achieved by replacing the Maximum Likelihood weight vector with a weight vector designed to minimize the array output power while maintaining a constant response for the hypothesized parameter value. The same idea is implemented here for the target's position estimation.

First, note that the Maximum Likelihood cost function (11) can be expressed as

$$\widetilde{Q}_{\mathrm{ML}}(\mathbf{p}_t) = \mathbf{v}_{\mathrm{ML}}^H(\mathbf{p}_t)\hat{\mathbf{\Gamma}}\mathbf{v}_{\mathrm{ML}}(\mathbf{p}_t) \tag{16}$$

where

$$\mathbf{v}_{\mathrm{ML}}(\mathbf{p}_t) \triangleq \left[\left(\mathbf{w}_1^{\mathrm{ML}}\right)^T, \ldots, \left(\mathbf{w}_L^{\mathrm{ML}}\right)^T\right]^T \tag{17}$$

where the dependence of $\mathbf{w}_\ell^{\mathrm{ML}}$ on $\mathbf{p}_t$ is suppressed, and where $\hat{\mathbf{\Gamma}}$ is a block diagonal matrix given by

$$\hat{\mathbf{\Gamma}} \triangleq \begin{bmatrix}\hat{\mathbf{R}}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \hat{\mathbf{R}}_L\end{bmatrix}. \tag{18}$$

Note that

$$\mathbf{v}_{\mathrm{ML}}^H(\mathbf{p}_t)\mathbf{v}_{\mathrm{ML}}(\mathbf{p}_t) = \sum_{\ell=1}^{L}\left(\mathbf{w}_\ell^{\mathrm{ML}}\right)^H\mathbf{w}_\ell^{\mathrm{ML}} = L \tag{19}$$

where we used

$$\left(\mathbf{w}_\ell^{\mathrm{ML}}\right)^H\mathbf{w}_\ell^{\mathrm{ML}} = \left(\mathbf{A}_\ell(\mathbf{p}_t)\bar{\mathbf{s}}\right)^H\mathbf{A}_\ell(\mathbf{p}_t)\bar{\mathbf{s}} = 1. \tag{20}$$

The proposed MVDR weight vectors should satisfy

$$\mathbf{v}_{\mathrm{MVDR}} = \arg\min_{\mathbf{v}}\mathbf{v}^H\hat{\mathbf{\Gamma}}\mathbf{v}, \tag{21}$$

subject to

$$\mathbf{v}^H\mathbf{v}_{\mathrm{ML}}(\mathbf{p}_t) = L. \tag{22}$$

This is a minimization of a quadratic function under a linear constraint, which can be solved using the complex gradient operator [11]. The solution is given by

$$\mathbf{v}_{\mathrm{MVDR}}(\mathbf{p}_t) = L\frac{\hat{\mathbf{\Gamma}}^{-1}\mathbf{v}_{\mathrm{ML}}(\mathbf{p}_t)}{\mathbf{v}_{\mathrm{ML}}^H(\mathbf{p}_t)\hat{\mathbf{\Gamma}}^{-1}\mathbf{v}_{\mathrm{ML}}(\mathbf{p}_t)}. \tag{23}$$

The MVDR location estimator is therefore given by

$$\begin{aligned} \widehat{\mathbf{p}}_t^{\mathrm{MVDR}} &= \arg\max_{\mathbf{p}_t}\left[\mathbf{v}_{\mathrm{MVDR}}^H(\mathbf{p}_t)\hat{\mathbf{\Gamma}}\mathbf{v}_{\mathrm{MVDR}}(\mathbf{p}_t)\right] \\ &= \arg\max_{\mathbf{p}_t}\frac{L^2}{\mathbf{v}_{\mathrm{ML}}^H(\mathbf{p}_t)\hat{\mathbf{\Gamma}}^{-1}\mathbf{v}_{\mathrm{ML}}(\mathbf{p}_t)} \\ &= \arg\min_{\mathbf{p}_t}\tilde{Q}_{\mathrm{MVDR}}(\mathbf{p}_t) \end{aligned} \tag{24}$$

where

$$\tilde{Q}_{\mathrm{MVDR}}(\mathbf{p}_t) \triangleq \mathbf{v}_{\mathrm{ML}}^H(\mathbf{p}_t)\hat{\mathbf{\Gamma}}^{-1}\mathbf{v}_{\mathrm{ML}}(\mathbf{p}_t). \tag{25}$$

This expression can be further simplified, using the fact that the inverse of $\hat{\mathbf{\Gamma}}$ is a block diagonal matrix as well, given by

$$\hat{\mathbf{\Gamma}}^{-1} \triangleq \begin{bmatrix}\hat{\mathbf{R}}_1^{-1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \hat{\mathbf{R}}_L^{-1}\end{bmatrix}. \tag{26}$$

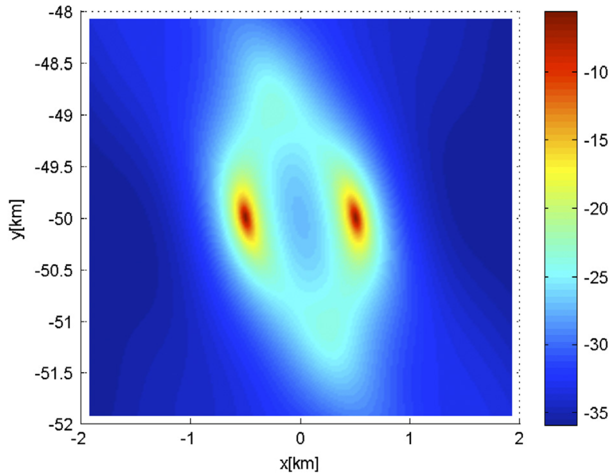

Fig. 1. MVDR target position cost function in dB.

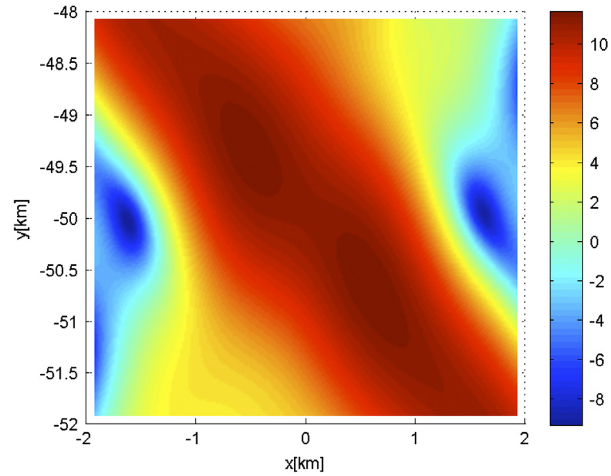

Fig. 2. ML target position cost function in dB.

Substituting (26) and (17) back in (25) we get

$$\tilde{Q}_{\mathrm{MVDR}}(\mathbf{p}_t) = \sum_{\ell=1}^{L} Q_\ell(\mathbf{p}_t) \tag{27}$$

where

$$Q_\ell(\mathbf{p}_t) \triangleq \bar{\mathbf{s}}^H \mathbf{A}_\ell^H(\mathbf{p}_t)\,\bar{\mathbf{R}}_\ell^{-1}\mathbf{A}_\ell(\mathbf{p}_t)\,\bar{\mathbf{s}}. \tag{28}$$

This ends the derivation of the MVDR target location estimator. To summarize, the proposed estimator uses the raw data gathered at the sensors to evaluate the exact MVDR cost function of the target's position in a single step. Thus, MVDR is used for direct localization and not the estimation of the Direction of Arrival (DOA). It is important to note that no assumption was made on the number of targets during the derivation of the MVDR estimator.

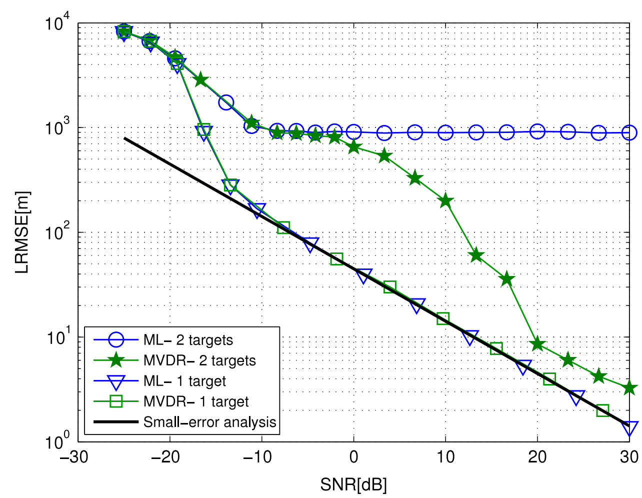

Fig. 3. Single element arrays layout: LRMSE as a function of SNR.

3. Simulation and analysis

Fig. 1 demonstrates the resolution capabilities of the proposed algorithm, by showing the heat map for the inverse of the MVDR cost function defined in (27) for the case of two closely located targets at $\{-0.5, -50\}$ and $\{0.5, -50\}$. The simulated layout consists of a transmitter at $\{300, 0\}$ and four single element arrays ($M_r = 1$) distributed evenly from $\{100, 150\}$ to $\{100, -150\}$. All dimensions are in kilometers. The simulated signal sampling frequency is $f_s = 488$ (kHz), the number of samples is $K = 128$, the signal spectrum is flat with bandwidth of 150 (kHz), and the number of pulses is $M_p = 2$. It is clearly seen that the MVDR method succeeds in separating the targets. For comparison, Fig. 2 shows the heat map for the (single-target) Maximum Likelihood cost function defined in (11), which fails to resolve the two targets.

Fig. 3 shows the location-root-mean-square-error (LRMSE) as a function of SNR for both estimation techniques. The estimated targets' positions are the peaks closest to the actual targets' position of the corresponding cost functions. Thus, we ignore the problem of target detection, which could be addressed in future work. The LRMSE is evaluated separately for each target. However, performance for both targets is equivalent so only the LRMSE of one target is shown. Clearly, the single-target Maximum Likelihood does not improve with SNR while the MVDR shows excellent performance. We note that for high SNR the LRMSE in logarithmic scale depends linearly on the SNR in dB units. Also shown in Fig. 3 is the performance of the two proposed estimation methods when only a single target is present at $\{0, -50\}$, as well as the theoretical small-error analysis for the MVDR estimator for this case. It is expected that in such a scenario the performance of the MVDR estimator would be inferior to the Maximum Likelihood estimator, which is designed with the purpose of minimizing the RMSE. We note that both estimation techniques converge to the theoretical analysis and to one another at high SNR for a single target. This is a pleasing result, since the DPD method was shown in [2] to converge to the Cramer–Rao-Lower-Bound for this estimation problem. We also note that even for moderate SNR, both estimation methods yield similar results. Finally, note that the LRMSE for two close targets does not converge to the LRMSE for one target. This is because the two targets interfere with each other, causing an estimation bias.

4. Conclusion

In this paper we proposed a single-step direct position determination (DPD) using the MVDR approach rather than the single target Maximum Likelihood approach. The proposed method is an adaptive method, using the returns from several consecutive pulses for computation of the target's location directly without first estimation of direction of arrival and delay. We presented DPD based on single target Maximum Likelihood and DPD based on MVDR for multistatic radar, but our approach may easily be extended to other localization problems. We demonstrated that the MVDR approach achieves superior resolution with respect to the Maximum Likelihood by analyzing multiple near targets localization. The targets used in the analysis can represent a single target with multiple, resolvable scatterers or represent separate, small targets. The fine resolution can be employed to resolving real targets from many decoys.

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