# Pricing the razor: A note on two-part tariffs ${ }^{2}$ 

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## A R T I C L E I N F O

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#### Abstract

The "razor-and-blades" pricing strategy involves setting a low price for a durable basic product (razors) and a high price for a complementary consumable (blades). In a timeless model, Oi (1971) showed that if consumers' demand curves differ and do not cross and unit costs are constant, a monopolist should always price blades above cost. This note studies the optimal razor price. With a uniform distribution of parallel linear demand curves it is never optimal to sell the razor below cost, while with two types of consumers and non-crossing linear demands it is optimal to do so for some parameter values.


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## 1. Introduction

The so-called "razor-and-blades" pricing strategy involves a firm with market power setting a low price for a basic, durable product, like a razor, and to earn all or most of its profits from sales of a complementary consumable, like blades, that is used to produce something the buyer values, like shaves. In a timeless setting this strategy is, in effect, a two-part tariff for shaves. One sometimes hears this strategy summarized as, "Give away the razor and make money on the blades." This note is concerned with whether within the classic timeless framework it is in fact ever optimal for a monopolist to sell the razor at a loss and, if so, when it is optimal to do so.

In some cases the link between the basic product and the consumable is technological, but in others it results from a tying contract. In a pioneering analysis of such contracts, Bowman (1957) discussed an 1895 antitrust case involving the seller of a patented machine for attaching buttons to high button shoes that required users of that machine to purchase the unpatented staples the machine employed from it at a high price relative to available alternatives. ${ }^{1}$ Bowman argued

[^0]that this requirement served as "a counting device" that enabled the seller to earn more from users who valued the machine more, that is, to implement a monopolistic two-part tariff. Bowman did not address the pricing of the machine.

The first formal analysis of monopolistic two-part tariffs was given in Oi's (1971) classic Disneyland Dilemma paper. The basic product was admission to the park and the complementary product was tickets for rides. Considering a finite set of possible buyers with different demand curves, Oi showed that it was always optimal for Disneyland to set the price of ride tickets above the corresponding marginal cost if those demand curves did not cross. He also showed that it was generally optimal for Disneyland to charge a positive price for admission, for which he assumed a zero unit cost.

In the same timeless framework, Schmalensee (1981) considered a monopolist with positive and constant unit costs for both basic and consumable products that faced a continuum of consumers. He retained Oi's assumption that one unit of the consumable product was required to produce one unit of the product ultimately demanded, an assumption retained here for notational simplicity. Following Oi, he showed that if demand curves do not cross, an assumption retained throughout this note, it is always optimal to set the price of the consumable product above cost. ${ }^{2}$

[^1]Schmalensee (1981, Proposition 8) also showed that if $R$ is the price of the basic product, called the razor in all that follows, $F$ is its constant unit cost, and $Q$ is total sales of the consumable product, called blades in all that follows, then at a profit-maximizing point, $(R-F)$ has the sign of
$m=(\bar{q}-\hat{q}) \frac{\partial Q}{\partial R}-\sigma$,
where $\sigma$ is a negative substitution term, $\bar{q}$ is the average demand for blades of those who purchase razors, and $\hat{q}$ is the demand of the marginal buyer of razors, where clearly $\hat{q}<\bar{q}$. Since increases in $R$ reduce the demand for blades, the sign of $m$ is ambiguous when demand curves do not cross. Schmalensee went on to argue that the greater the diversity in potential buyers' demands, the larger would likely be the difference in parentheses in Eq. (1), and thus the likelier it would be that the optimal $R$ would be below $F$.

But since all the terms in Eq. (1) are evaluated at the profitmaximizing point, without solving the profit-maximization problem one cannot generally know their magnitudes. Thus Eq. (1) does not enable one to determine the sign of $m$ from knowledge of costs and demands. Most importantly, as the analysis below demonstrates, the diversity of demands among those who choose to buy the basic product is endogenous; even if there is great diversity in the population of potential buyers, it may be profit-maximizing to serve only a small fraction of them.

Section 2 presents a model with constant unit costs of both razors and blades and a continuum of buyers, uniformly distributed with parallel linear demand curves, in which it is never optimal to price razors below cost no matter how diverse potential buyers' demand curves are. This result rests on very strong assumptions, however, that it has not proven possible to relax in a continuum setup without great loss of tractability.

Accordingly, Section 3 considers a model with variable numbers of two types of potential buyers and constant unit costs of both products. Individual demand curves are assumed linear and non-crossing, though not generally parallel. In a number of special cases of this model it is again never optimal to price razors below cost. But we also show that in a relatively small portion of the parameter space it is optimal to sell razors for less than their cost of production.

While this analysis makes it clear that one cannot absolutely rule out a monopolist finding it optimal to sell the basic product below cost in the standard timeless multi-consumer model, it at least suggests that such a policy is unlikely to be optimal. Section 4 provides a few concluding observations.

## 2. A continuum model

Consider a firm with market power that can be treated as a monopolist and that has constant per-unit cost $F$ for razors and $v$ for blades. Consumers have parallel linear demand curves for shaves, the service provided jointly by these products, with one blade providing one shave. The assumption of linearity allows us to set $v=0$ without loss of generality. ${ }^{3}$ By choice of units, the slopes of the individual demand curves and the total mass of consumers can be set equal to unity, so that the demand curve for shaves of a consumer of type $t$ who owns a razor becomes
$q_{t}=t-P$,

[^2]where $P$ is the price of blades. Let $R$ be the price of razors, as above, and $\theta$ be the index of the lowest type that buys a razor. Then $R$ must equal the consumer's surplus of a consumer of type $\theta$ :
$R=\frac{1}{2}(\theta-P)^{2}, \quad$ or $\quad \theta=\sqrt{2 R}+P$.
Finally, we assume that $t$ is uniformly distributed between 0 and $T$, so that higher values of $T$ correspond to more dispersion in the population of potential buyers. For there to be any possibility of positive profit, $F$ must be less than the surplus of the highest type when $P=0$, i.e., $F<T^{2} / 2$. If $N$ is the total number of razors sold and $Q$ is the total number of blades sold, the monopoly's profit function is
\[

$$
\begin{align*}
\Pi & =(R-F) N+P Q \\
& =(R-F) \frac{T-\theta}{T}+\frac{P}{T}\left[\frac{T^{2}}{2}-\frac{\theta^{2}}{2}-P(T-\theta)\right], \tag{4}
\end{align*}
$$
\]

where $\theta$ is given by (3).
Differentiation of (4) yields the two first-order conditions:
$2 T \frac{\partial \Pi}{\partial P}=3 P^{2}-4 P T-4 R+2 F+T^{2}=0, \quad$ and
$T \frac{\partial \Pi}{\partial R}=-\sqrt{\frac{R}{2}}+\frac{F}{\sqrt{2 R}}+T-\sqrt{2 R}-2 P=0$.
It is useful to re-write Eqs. (5a) and (5b) in terms of the following variables:
$X={ }^{P} / T, \quad Z=\sqrt{2 R} / T, \quad$ and $\quad W=\sqrt{2 F} / T$.
Substituting (6) into (5a) and (5b) and combining similar terms, Eq. (5a) and (5b) become
$\frac{2}{T} \frac{\partial \Pi}{\partial P}=3 X^{2}-4 X-2 Z^{2}+W^{2}+1=0, \quad$ and
$2 \frac{\partial \Pi}{\partial R}=\frac{1}{Z}\left[-3 Z^{2}+W^{2}+2 Z-4 Z X\right]=0$.
Note that $F \in\left[0, T^{2} / 2\right)$ is equivalent to $W \in[0,1)$. When $W=F=0$, (7b) is linear in $X$ and $Z$. Substituting for $Z$ in (7a) and, using asterisks to denote optima, solving the resulting quadratic yields $X^{*}=1 / 5$, and (7b) then yields $Z^{*}=2 / 5$. In this case the monopolist could profitably sell to all buyers if it could discriminate perfectly. Because it cannot do so, it optimally excludes some low-type buyers. The fraction of potential buyers who do not buy a razor at the optimum in this case is equal to $\theta / T$, and Eqs. (3) and (6) imply
$\theta / T=X+Z$,
From the values of $X^{*}$ and $Z^{*}$ above, it follows that $3 / 5$ of buyers are excluded in this case. ${ }^{4}$

At the other extreme, as $W \rightarrow 1$, the set of potentially profitable buyers shrinks to the highest type. In the limit, with no buyer heterogeneity, the best the monopolist can do is to set $X^{*}=P^{*}=0$ and just break even by setting $Z^{*}=W=1$, giving away blades and capturing all available surplus via the razor price. We now show that $Z^{*}>W$ for all $W \in[0,1)$, which establishes

[^3]Proposition 1. In the continuum model of this section, as long as it is profitable for the firm to produce (i.e., for all $F \in\left[0, T^{2} / 2\right.$ )), the profitmaximizing value of $R$ exceeds $F$.

Because the functions in (7a) and (7b) are smooth, $Z^{*}$ is a continuous function of $W$. Thus since $Z^{*}(0)=2 / 5>0$, as shown above, for there to exist a $W<1$ such that $Z^{*}(W)<W$, there must be some $\hat{W}<1$ such that $Z^{*}(\hat{W})=\hat{W}$. Setting $Z=W$ in (7b) yields a linear equation in $W$ and $X$. Substituting from (7b) for $X$ in (7a) yields a quadratic of which $W=1$ is the only root. Thus for all $W<1$, i.e., for all $F<T^{2} / 2, Z^{*}(W)>W$, so that $R^{*}>F$, and it is never optimal to sell the razor below cost no matter how diverse are potential buyers' tastes.

Some comparative statics of this model are straightforward. As F increases from 0 to $T^{2} / 2$ for fixed $T, W$ increases, and it follows from the analysis above that $X^{*}$ falls from $1 / 5$ to zero, while $Z^{*}$ rises from $2 / 5$ to 1 . As the cost of razors increases, all else equal, the optimal price of blades falls, and the optimal price of razors rises. Holding $F$ constant and increasing $T$ decreases $W$, so that $X^{*}$ rises and $Z^{*}$ falls. For $X^{*}$ to rise when $T$ rises, $P^{*}$ must also rise. Increasing the dispersion of potential buyers increases the optimal price of blades. Unfortunately, the fact that $Z^{*}$ falls when $T$ rises says nothing about whether $R$ rises or falls.

This model is special in many respects. While the assumptions of linear demands and constant unit costs are at least familiar, the assumptions that demand curves are exactly parallel and that there are the same numbers of consumers of each type (the natural interpretation of the assumption of a uniform distribution) seem particularly strong. Unfortunately, it has not proven possible to relax either of these assumptions with a continuum of buyers and obtain insights about the general relation between $R^{*}$ and $F$. Restricting the number of types to two does make this possible, however, as the next section demonstrates.

## 3. A two-type model

Consider a market consisting of high and low types of potential buyers. There are $N_{L}$ low-type buyers. By choice of units, their linear demand curves can be written as
$q_{L}=1-P$,
where, as above, $P$ is the price of blades. There are $N_{H}$ high-type buyers, and each has demand curve
$q_{H}=T-\alpha P, \quad$ with $T \geq 1$ and $T / \alpha \geq 1$.

We assume that at least one of the inequalities in (10) is strict so that the two groups have non-identical, non-intersecting demands. Costs are the same as in Section 2: a zero unit cost of blades and a constant unit cost of razors equal to $F$.

If only the high-type buyers are served, it is clear that it is optimal to set $P$ equal to zero and to set the price of razors, $R$, equal to the corresponding consumers surplus. Letting $\mu=N_{H} / N_{L}$ and letting $\Pi$ be profit divided by $N_{L}$, it is easy to show that in this case the optimal profit is given by
$\Pi_{H}^{*}=\mu R_{H}^{*}-\mu F=\frac{\mu T^{2}}{2 \alpha}-\mu F$.
If it is more profitable to serve only the high-type buyers than to serve both types, it is clearly never optimal to sell razors below cost. The interesting question here is whether it can be better to serve both types and to sell razors below cost. If both types are served, the optimal
value of $R$ for any price of blades is the consumer's surplus of a low-type buyer at that price. Thus the (scaled) profit function if both types are served is

$$
\begin{align*}
\Pi_{B}= & (1+\mu) R_{B}+V_{B}-(1+\mu) F \equiv(1+\mu) \frac{(1-P)^{2}}{2} \\
& +P[(1+\mu T)-(1+\mu \alpha) P]-(1+\mu) F . \tag{12}
\end{align*}
$$

Differentiating yields the optimal value of $P$ in this case:
$P_{B}^{*}=\frac{\mu(T-1)}{2 \alpha \mu-\mu+1} \equiv \frac{A}{B}$.
The numerator, $A$, is non-negative, so for this problem to have a sensible solution, $B$ must be positive, with $A<B$ so that $P_{B}^{*}$ is less than the low type's choke price of unity. Substitution into (12) yields the optimal values of $R_{B}$ and $V_{B}$.

The question of whether it is optimal to serve both types and to price razors below cost reduces to whether for allowable values of the other parameters there exist values of $F$ satisfying
$R_{B}^{*} \equiv F_{\min }<F, \quad$ and
$\Pi_{B}^{*}=(1+\mu) R_{B}^{*}+V_{B}^{*}-(1+\mu) F>\Pi_{H}^{*}=\mu R_{H}^{*}-\mu F, \quad$ or
$(1+\mu) R_{B}^{*}+V_{B}^{*}-\mu R_{H}^{*} \equiv F_{\max }>F$.
If $F$ is below $F_{\min }$, it is not optimal to price the razor below cost if both types are served, while if $F$ is above $F_{\text {max }}$ it is optimal to sell only to the high types. If and only if $F_{\text {min }}$ is less than $F_{\max }$ there exist values of $F$ such that it is optimal to serve both types and to sell the razor below cost.

A very simple special case is instructive. If $\mu=\alpha=1$, Eqs. (11)-(14a) and (14b) imply directly ${ }^{5}$
$P_{B}^{*}=\frac{T-1}{2}, \quad F_{\min }=\frac{9-6 T+T^{2}}{8}, \quad F_{\max }=\frac{5-2 T-T^{2}}{4}, \quad$ and
$F_{\max }-F_{\min }=\frac{1+2 T-3 T^{2}}{8}$.
Both $F_{\text {max }}$ and $F_{\text {min }}$ are decreasing functions of $T$ for $T$ in the relevant range. The right-hand side of ( 15 b ) is zero at $T=1$ and decreasing in $T$ for $T>1 / 3$. It is thus negative for all $T>1$, implying that there do not exist values of $F$ satisfying both (14a) and (14b), so that it is never optimal to sell the razor below cost in this very special case.

In the general case, substituting from (11)-(13) and multiplying the inequality $F_{\text {max }}>F_{\text {min }}$ by $2 \alpha B^{2}$ yields
$\Psi \equiv \mu B^{2}\left(\alpha-T^{2}\right)+2 \alpha A B(\mu T+1-\mu)+\alpha A^{2}(\mu-2-2 \alpha \mu)>0$,
where $A$ and $B$ are defined by (13).
This expression is a cubic in $\alpha$ and $\mu$ and a quadratic in $T$. Despite its complexity, it can be relatively easily signed in a few special cases:

Proposition 2. In the two-type model of this section, if (a) the two types have the same maximum demand ( $T=1$ ) or (b) the two types have parallel demand curves ( $\alpha=1$ ) or (c) there are equal numbers of the two types ( $\mu=1$ ), then $\Psi<0$, so that it is never optimal to serve both types with $R^{*}<F$.

[^4]To prove (a), note that $P_{B}^{*}=0$ when $T=1$. In this case blades are given away and all profits are earned on razors, so it can never be optimal to price razors below cost. In case (b), (16) implies
$\Psi=\mu(T-1)\left[\left(1-2 \mu^{2}\right)-(1+2 \mu) T\right]$.
The expression in square brackets is negative at $T=1$ and decreasing in $T$. Thus $\Psi$ is zero at $T=1$ and negative for $T>1$; it is never positive and (16) cannot be satisfied. In case (c), (16) implies
$\Psi / \alpha=-(1+2 \alpha) T^{2}+2 T+4 \alpha^{2}-2 \alpha-1$.
The expression on the right of (18) is convex in $\alpha$, so it is maximized at a boundary point of the feasible set. In this case, that set is $[(T-1) / 2, T]$, where the lower bound corresponds to the constraint $P_{B}^{*}<1$. When $\alpha=T$, (18) becomes
${ }^{\Psi} / \alpha=-2 T^{3}+3 T^{2}-1$.
The expression on the right of (19) is zero when $T=1$ and decreasing in $T$ for $T>1$, so $\Psi$ is never positive at this boundary point. When $\alpha=(T-1) / 2$, (18) becomes
$\Psi / \alpha=-T^{3}+T^{2}-T+1$.
The expression on the right hand side of (20) is zero at $T=1$, and its derivative with respect to $T$ is a quadratic that is everywhere negative. Thus $\Psi$ is never positive at this boundary point either, and (16) cannot be satisfied in case (c).

It is interesting that either of the conditions highlighted at the end of Section 2 is sufficient to rule out $R^{*}<F$ here: either parallel demand curves or equal numbers of the two do the job. Propositions 1 and 2 might lead one to conjecture that (16) can never be satisfied in models of this sort, but that conjecture would be false. Numerical experimentation reveals that (16) is sometimes satisfied when $\mu$ is less than one, $T / \alpha$ is near one, and $T$ is large. A fairly tractable limiting case is $T / \alpha=1$ :

Proposition 3. In the two-type model of this section, when the two types have the same choke price, so that $q_{H}=T(1-p)$ with $T>1$, there exist values of $F$ such that it is optimal to serve both types with $R^{*}<F$, if and only if $\mu<1 / \sqrt{T(2 T-1)}$.

Substitution for $\alpha$ in (16) yields, after a good bit of algebra,
$\Psi=\mu T(T-1)\left[1-\mu^{2} T(2 T-1)\right]$,
which is positive if and only if $\mu<1 / \sqrt{T(2 T-1)}$. Note that $\mu<1$ is necessary for this to hold.

The intuition here is fairly straightforward. If $\mu$ is small enough, serving only the relatively few high types will be unattractive even if $T$ is large, so that their individual demands for blades greatly exceed those of the low types. If it is optimal to serve both types, the larger is $T$, the stronger is the incentive to raise the blade price to capture surplus from the high types: when $T=\alpha$, differentiation of (13) shows that the optimal blade price is increasing in $T$. But a high $P$ reduces the consumer surplus of the low types and thus maximum value of $R$ that they will pay. For some values of $F$, that maximum value is optimally below the cost of producing razors. Note that the condition that it is better to serve both types than only the high type implies that even though razors are sold below cost to the low types, serving them is profitable because of the high price they pay for blades.

It is important to recognize that Proposition 3 is only an existence proof: it says nothing about how common the relevant parameter values are. Numerical exploration of the size of the ( $F_{\text {max }}-F_{\text {min }}$ ) gap when $T=\alpha$ sheds some light on this. In general this gap is larger for smaller values of $\mu$. For $\mu=0.01$, for instance, this gap is maximized at around $T=26.9$. At that point $F_{\max }=0.3925$ and $F_{\min }=0.3449$, for a
gap of 0.0476 . At $\mu=0.5$, in contrast, this gap is maximized at around $T=1.45$, where it equals 0.0162 . While there is no natural probability distribution over this parameter space, these relatively small gaps at least suggest that it is unlikely in practice that it will be optimal for a timeless monopolist to sell razors at a loss.

## 4. Concluding observations

Consistent with the analysis above, cases of below-cost sales of the basic product by monopolies or near-monopolies do not seem common in practice. Gillette is often given credit for originating the "razor-andblades" pricing strategy, but this seems to be a myth. ${ }^{6}$ During its 1904-21 patent monopoly, Gillette charged a high price for its razor, likely well above cost. It only cut that price (but not the blade price) when its patent expired and competitors with low-priced razors appeared. Low razor prices would have encouraged trial, and, as Schmalensee (1982) argued, a satisfactory experience with one razor-blade combination would discourage consumers from trying other combinations.

Similarly, Blackstone (1975) reports that manufacturers of Electrofax copying machines made most of their profit on the special paper those machines required until those manufacturers were barred from requiring that their own high-priced paper be used. But there were many competing manufacturers, and they apparently sold copying machines above cost.

Video game consoles provide a final example. These are typically sold below unit cost when they are launched, and the bulk of console makers' earnings come from royalties on games, which are roughly analogous to blade markups. ${ }^{7}$ However, since console unit costs typically fall with experience, whether expected overall profits on consoles are typically negative is less clear. Moreover, console makers compete for the attention of game developers, who must decide whether or not to develop for a new console well before it is launched. Committing to a low console price provides a positive signal about console sales and thus the audience for games that run on that console to game developers. This is far from classic seconddegree price discrimination.

The lesson of this analysis for firms that could practice razor-andblades pricing is clear: if the main goal of pricing is second-degree price discrimination, it is likely to be optimal to set positive markups on both products. For antitrust authorities, while it may not be commonly optimal to price a basic product below cost purely to facilitate price discrimination, theory does not rule it out, and such pricing should not be automatically treated as predatory.

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[^5]
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    ${ }^{1}$ Bowman (1957, pp. 23-4). The case was Heaton-Peninsular Button-Fastener Co. v. Eureka Specialty Co., 65 Fed 619 (C.C.W.D. Mich. 1895).

[^1]:    ${ }^{2}$ Schmalensee's "direct case" involves a slightly weaker assumption than non-crossing demand curves, but the latter assumption is made here for simplicity.

[^2]:    ${ }^{3}$ That is, in the notation introduced below, if $v>0$, one can define $P^{\prime}=P-v, t^{\prime}=t-v$, $T^{\prime}=T-v$, and $\theta^{\prime}=\theta-v$. Substituting for $P, t, T$, and $\theta$ in the profit function and recognizing that the support of $t^{\prime}$ is $\left[-v, T^{\prime}\right]$, one obtains a profit function of the form of (4), in which $v$ does not appear. This argument also justifies setting $v=0$ in the model of Section 3.

[^3]:    ${ }^{4}$ Interestingly, if $R$ is constrained to be zero, setting $Z=W=0$ in (7a) and solving yields $X^{*}=1 / 3$. Constraining the razor price to be zero makes a higher blade price optimal, but a larger fraction of buyers is served. This constraint reduces profits only slightly, from $2 T^{2} / 25$ to $2 T^{2} / 27$.

[^4]:    ${ }^{5}$ Note that $F_{\max }<0$ for $T>\sqrt{6}-1$. That is, for larger values of $T$ it is optimal to serve only the high types even if $F=0$.

[^5]:    ${ }^{6}$ The discussion of Gillette is based on Picker (2011).
    7 This paragraph is based on Evans et al (2006, chs. 8 and 10) and Hagiu (2006). As Evans et al. stress, video game platforms differ from other software platforms in earning most of their profits from usage fees (game royalties) rather than access fees (console revenues).

