



## Collusion with costly consumer search<sup>☆</sup>



Vaiva Petrikaitė

*Instituto de Análisis Económico (CSIC), Barcelona GSE, Campus UAB, 08193 Bellaterra, Barcelona, Spain*

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### ABSTRACT

I use standard consumer search models to study how an increase in market transparency (lower search costs or higher share of fully informed consumers) affects cartel stability. When firms sell horizontally differentiated products, cartels become more stable as the search cost increases; with homogeneous products, by contrast, the opposite holds. A higher share of fully informed consumers makes collusion less stable when the market is initially sufficiently transparent, whereas it happens otherwise if the market is originally little transparent.

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## 1. Introduction

Market transparency affects many decisions of market participants, including the choice of sellers whether to collude or not. The economics literature has shown that in markets where firms observe each other actions imperfectly collusion is less likely (e.g. [Abreu et al., 1985](#); [Compte, 2002](#); [Green and Porter, 1984](#); [Kandori and Matsushima, 1998](#); [Stigler, 1964](#)). What happens is that as the market becomes less transparent, the punishment that follows a deviation from collusion is softer.

The aforementioned studies focused on market transparency on the supply side of the market. In this paper I study a different notion of market transparency. In particular, I ask how the stability of collusion relates to market transparency from the point of view of the consumers. To the best of my knowledge, much less is known about how transparency on

the demand side of the market affects the choice of sellers whether to collude or not. This paper contributes to filling this gap by studying cartel stability in consumer search markets.

I start by looking at collusion in the [Wolinsky \(1986\)](#) sequential search framework with horizontally differentiated products. The model features a finite number of symmetric firms, playing a repeated price-setting game with an infinite horizon. Consumers engage in costly sequential search to observe the characteristics of products. I relate the notion of market transparency to the search cost: a market where search costs are higher is said to be less transparent. I consider the stability of collusion when firms play grim-trigger strategies. Higher search costs affect the short-run gains from deviation and the long-run punishment. When search costs increase, a consumer on average chooses to compare fewer alternatives. This gives market power to the sellers and so they earn more in a static Nash equilibrium. From the point of view of colluding firms, this implies that the incentives to deviate increase as search costs go up because the punishment that follows a deviation becomes softer. However, it turns out that the gain from deviating from collusion decreases with search costs. This occurs because as search costs increase, fewer consumers happen to visit a deviating firm. Thus, the increase in profits that is obtained by undercutting the rival firms goes down as search costs increase. I show that the increase in search costs usually makes cartels more stable. What happens is that the deviation profit typically decreases with the search costs

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E-mail address: [asvaiva@gmail.com](mailto:asvaiva@gmail.com).

more than the competitive profit increases. The same qualitative result holds if firms apply stick-and-carrot strategies as in Häckner (1996).<sup>1</sup>

I then move to examine how market transparency affects collusion in Stahl's (1989) consumer search market with homogeneous products. This model has a finite number of symmetric firms that repeatedly compete in prices over an infinite number of periods. All consumers have the same valuation for the product. A fraction of them (shoppers) are fully informed, while the rest of the consumers engage in costly search to learn the prices that are actually charged by the firms. This shopper/non-shopper feature of demand results in a static Nash equilibrium in mixed strategies. As in the case with differentiated products, an increase in search costs gives market power to the firms. Thus, the firms' profits go up in the static Nash equilibrium. This makes the punishment that follows a deviation milder, which tends to strengthen the incentives to deviate from collusion. Differently from the differentiated products case, it turns out that search costs have no influence on the deviation profit and therefore the gains from defecting from the collusive equilibrium are independent of the search costs. Hence, with homogeneous products higher search costs make collusion less easy to sustain.

The model of Stahl (1989) lends itself to model transparency in a different way. Because of the shopper/non-shopper feature of demand, we can alternatively relate market transparency to the share of shoppers. I then regard a market with more shoppers as more transparent and study how cartel stability is related to the share of shoppers in the market. I show that both the deviation gain and the deviation loss increase with the share of shoppers. The net influence of these effects depends on which of these forces is dominant. It turns out that it depends on the initial level of market transparency. Specifically, the critical discount factor above which collusion is stable decreases with the share of shoppers when the share of shoppers is initially small and increases with the share of shoppers when this is initially high.

To the best of my knowledge, only Nilson (1999) and Schultz (2005) study how market transparency on the demand side affects collusive outcomes. Nilson (1999) analyses how the magnitude of consumer search costs affect collusion in a duopoly version of the non-sequential consumer search model of Burdett and Judd (1983). In his set-up, firms sell homogeneous products, some consumers observe the prices of both firms, whereas others have to pay a positive search cost per firm to learn the offers of the sellers. Nilson (1999) shows that cartel become less stable as the search cost increases. This finding is similar to what I find in the Stahl's setting, which suggests that the search protocol does not influence the relation between stability of collusion and search costs.<sup>2</sup>

Schultz (2005) also analyses how market transparency on the demand side affects collusion. He studies this question in a Hotelling market with shoppers and non-shoppers. In his model, there are no search costs. The shoppers are fully informed and the non-shoppers are assumed to visit only one seller. Schultz (2005) finds that when products are almost homogeneous, the share of fully informed consumers does not have any effect on cartel stability. This finding differs from my result in the Stahl setting and it is due to the restriction that non-shoppers are not allowed to search. When products are differentiated, though, Schultz (2005) shows that cartels are more stable in less transparent settings, which is in line with what I find in the Wolinsky's framework.

To the extent that an increase in search costs weakens competition in the marketplace, my findings are also related to those from the literature on cartel stability in horizontally differentiated product settings. In my paper, higher search costs lead to more stable cartels whereas in the papers of Deneckere (1983), Rothschild (1997) and Ross (1992) the degree to which collusion is stable turns out to be non-monotonic

in the degree of product differentiation. The key difference between my work and that on product differentiation is that changes in search costs do not affect willingness to pay whereas changes in product differentiation do.

In order to deter collusion, competition authorities try to identify structural factors that influence the likelihood of collusion. Much is known about how e.g. the number of competitors, product differentiation and the nature of competition affect the stability of collusion. My paper indicates that significant consumer search frictions should be regarded as a sign of risk in environments where products are horizontally differentiated, but not in homogeneous product settings. As a matter of fact, there are instances of detected or suspected cartels in markets where consumer search and product differentiation are prominent features. An example is the market for laundry detergents (The EC, 2011); this is a market where products differ from one another in for example their content of fabric softeners and perfumes and, as shown by Pinna and Seiler (2014), consumers incur non-negligible search costs while shopping. The Danish competition authorities (Borum, 2014) fined several driving schools for coordinating their pricing in 2014; according (Muir et al., 2013) this is a market where search frictions also seem to be important. Finally, collusion has also been detected in the automobile market, which is a market where consumers also experience significant search costs and products are highly differentiated (see Moraga-González et al., 2015; Meikle, 1999; Foggo, 2005 for more details).

The rest of the paper is organized as follows. In Section 2, I analyse collusion in the Wolinsky's consumer search market with differentiated products. Collusion in the Stahl's consumer search market for homogeneous products is studied in Section 3. Some concluding remarks can be found in Section 4. The proofs of the propositions are placed in the Appendix of the paper.

## 2. Differentiated products

### 2.1. Model

On the demand side, there is a unit mass of consumers. A consumer wants to buy one unit of a product and she can buy it from  $n \geq 2$  sellers. Every firm offers one variety, and products are horizontally differentiated. All the firms have the same constant unit production cost that is normalized to zero. Consumer  $i$  who buys product  $j$  gets utility  $u_{ij}$ :

$$u_{ij} = \varepsilon_{ij} - p_j \quad (1)$$

where  $\varepsilon_{ij}$  is the match value between consumer  $i$  and product  $j$ , and  $p_j$  stands for the price of the product. A consumer prefers to buy the product that gives her the highest utility. However, she has to pay a positive search cost  $s$  per firm to observe the price and the match value. The consumer visits firms sequentially with costless recall and can terminate her search after sampling any number of firms.

The firms never observe individual match values, and consumers do not know how much they like the products without searching them. However, it is common knowledge that the match value  $\varepsilon$  is distributed identically and independently across consumers, products and time according to a continuous and differentiable distribution function  $F(\varepsilon)$  in the interval between zero and  $\bar{\varepsilon}$ .<sup>3</sup> The density function of  $\varepsilon$  is denoted by  $f(\varepsilon)$ , it is positive and log-concave. Additionally, I require that  $f(y) + pf'(y) > 0, \forall p \leq y$ .<sup>4</sup>

In symmetric equilibrium consumers expect all  $n$  sellers to charge the same price  $p^*$ . Thus, the distribution of utilities across the firms is

<sup>3</sup> The assumption about the independent distribution of  $\varepsilon$  over time is reasonable while analysing the markets where the assortment in shops changes between the purchases of a consumer. The examples of such markets can be the markets where consumers shop rather infrequently, and products change rapidly because of technological progress, e.g. home appliances, computers, phones, and cars.

<sup>4</sup> This assumption ensures concavity of pay-off functions.

<sup>1</sup> The details are in the working paper version of this paper.

<sup>2</sup> This fact also holds in the model with differentiated products. In the working paper version of this paper I show that the critical discount factor above which collusion is sustainable also decreases with search costs when consumers search non-sequentially.

identical, and a consumer samples the sellers randomly. Then, by the optimal stopping rule (Kohn and Shavell, 1974), a consumer uses the reservation utility of a firm to decide when to terminate her search. The reservation utility of a seller equals  $\bar{x} - p^*$ , where  $\bar{x}$  is the solution to Eq. (2).

$$\int_{\bar{x}}^{\bar{\varepsilon}} (\varepsilon - \bar{x}) dF(\varepsilon) = s \tag{2}$$

The left hand-side (LHS) of Eq. (2) is the expected gain from searching one more firm when the highest observed match value equals  $\bar{x}$ . Then if the maximum observed utility is greater than  $\bar{x} - p^*$ , then a consumer terminates her search and buys the product that provides her with the highest observed utility. Otherwise, the consumer continues searching further. If a consumer samples all  $n$  firms and the highest observed utility is negative, then the customer does not buy anything. I assume that  $s$  is sufficiently low such that a consumer samples at least one firm in equilibrium. This implies that

$$s \leq \int_{p^m}^1 (\varepsilon - p^m) dF(\varepsilon)$$

where  $p^m = \arg \max_p p(1 - F(p))$ .<sup>5</sup> This implies that  $\bar{x} \geq p^m$ .

### 2.2. Non-cooperative static equilibrium

When products are horizontally differentiated, the competitive symmetric equilibrium is like the one in Wolinsky (1986). Here I provide a summary of it. Let  $p^*$  be the expected price that all the firms charge in the absence of collusion and consider a firm  $j$  that deviates to a price  $p \neq p^*$ . Firm  $j$  may be visited in the first, second and in any other position up to the  $n^{th}$  with probability  $1/n$ . A consumer who reaches firm  $j$  observes the deviation price  $p$  and interprets it as the defection of firm  $j$  only. Therefore, she continues to expect to see  $p^*$  in other shops. As a result, the consumer terminates her search at firm  $j$  if  $\varepsilon_j - p$  is greater than or equal to  $\bar{x} - p^*$ . Otherwise, the customer continues searching. If the consumer arrives at firm  $j$  after having visited other firms, this means that the utilities at the preceding shops must have been less than  $\bar{x} - p^*$  (or  $\max \{\varepsilon_i\}_{i < j} < \bar{x}$ ). The number of consumers who arrive at firm  $j$  and terminate their search there equals what is called the *fresh demand* of firm  $j$ . I label this demand  $f_j$ .<sup>6</sup>

$$f_j = \frac{1}{n} \sum_{i=1}^n F(\bar{x})^{i-1} (1 - F(\bar{x} - p^* + p)) = \frac{1}{n} \frac{1 - F(\bar{x})^n}{1 - F(\bar{x})} (1 - F(\bar{x} - p^* + p))$$

The consumers who continue searching after visiting firm  $j$  may return to the seller and buy there. This happens if the buyers visit all the sellers and find that the utility at firm  $j$  is the highest, and it is greater than zero. The number of consumers who return to firm  $j$  and buy there are called the *returning demand* of firm  $j$ , and it is labelled  $r_j$ .

$$r_j = \Pr \left[ \max \{ \varepsilon_l - p^*, 0 \}_{l \neq j} \leq \varepsilon_j - p < \bar{x} - p^* \right] = \int_p^{\bar{x} - p^* + p} F(\varepsilon - p + p^*)^{n-1} dF(\varepsilon)$$

I label the profit function of firm  $j$   $\pi_j(p)$ .

$$\pi_j(p) = p(f_j + r_j) \tag{3}$$

In a symmetric equilibrium, firm  $j$  sets  $p = p^*$ ,<sup>7</sup> and the first-order condition of firm  $j$  simplifies to Eq. (4).

$$1 - \frac{1 - F(\bar{x})^n}{1 - F(\bar{x})} f(\bar{x}) p^* - F(p^*)^n + p^* n \int_{p^*}^{\bar{x}} F(\varepsilon)^{n-1} f'(\varepsilon) d\varepsilon = 0 \tag{4}$$

In equilibrium only the consumers with valuations above the equilibrium price  $p^*$  buy; therefore, the total market demand is  $1 - F(p^*)^n$ . An individual firm sells one  $n^{th}$  of the total demand so that the profit of one seller is then given in Eq. (5).

$$\pi^* = \frac{p^*}{n} (1 - F(p^*)^n) \tag{5}$$

### 2.3. Collusive pricing

Firms that sell horizontally differentiated products exert negative pricing externalities on each other. Thus, the sellers have incentives to maximize their joint profit by forming a cartel. In the subsequent derivations, I denote the optimal price charged by cartel members by  $p^c$ , the profit of a single colluding firm by  $\pi^c$ , and the joint cartel profit by  $\Pi^c$ .

What price the firms set when they collude depends on the search cost. More concretely, to keep consumers visiting the sellers, the reservation utility of every cartel member must be non-negative, which may in effect result in a cap on the joint profit-maximizing price. In the analysis that follows, I divide the range of the search cost values into two intervals. In the first interval, the search cost is sufficiently low such that the profit-maximizing price  $p^c$  is an interior solution. In the second interval, the search cost is high such that a cartel pricing policy is restricted by consumer participation, and so it chooses to charge the maximum price that makes consumers indifferent between searching and not searching.

I begin with the case when the search cost is sufficiently low. If the firms collude, then consumers expect that all the sellers charge the same price  $p^c$  and they sample the firms randomly.<sup>8</sup> Because of random sampling, the pay-off of one cartel member, let's say firm  $j$ , is similar to the pay-off of a deviating firm in Section 2.2. Specifically, if I replace  $p^*$  and  $p$  by  $p^c$  in the profit function of firm  $j$  in Section 2.2, then I obtain the pay-off of one cartel member, which is given in Eq. (6).

$$\pi_j^c = \frac{p^c}{n} (1 - F(p^c)^n) \tag{6}$$

The joint profit-maximizing price is the unique solution to the first-order condition of the cartel, which is given in Eq. (7).<sup>9</sup>

$$\left. \frac{\partial \Pi}{\partial p} \right|_{p=p^c} = 1 - F(p^c)^n - p^c n F(p^c)^{n-1} f(p^c) = 0 \tag{7}$$

The price that is defined by Eq. (7) is a symmetric price that would be set by a multiproduct monopolist in the market where consumers engage in costly sequential search among the products of the firm. I denote the price of a monopolist selling  $n$  varieties by  $p_n^m$ . This price is

<sup>7</sup> There is a unique value of  $p^*$  that satisfies Eq. (4), and it is the unique symmetric equilibrium price. More details are provided in the Appendix A of the working paper.

<sup>8</sup> In deriving the joint pay-off of a cartel, I assume that consumers know that firms collude. Then if they see a price different from the expected one, they conclude that all cartel members charge that price. This assumption is not very stringent. In the Appendix A of the working paper, I allow colluding firms to charge different prices. In such a case, if consumers observe a deviation in one shop, they do not update their expectations about the prices of not-yet-visited shops. For the case of duopoly, I prove that joint profit-maximization implies equal prices. Unfortunately, because of the large number of search orders consumers can follow, it proves difficult to extend this result to the case of  $n > 2$ . Numerical results for markets with three or four sellers however reveal the same result.

<sup>9</sup> Because  $f$  is log-concave, the pay-off Eq. (7) is quasi-concave and the unique solution to the first-order condition is a maximum.

<sup>5</sup> Because  $f(\varepsilon)$  is log-concave, the maximization problem has a unique solution.

<sup>6</sup> Because the mass of consumers is normalized to one, the probability that a consumer buys equals the number of consumers who buy. The terms "*fresh demand*" and "*returning demand*" that appears later are borrowed from Armstrong et al. (2009).

higher than  $p^m$ . Therefore, the cartel charges  $p^c = p_n^m$  only if the search cost is sufficiently low, i.e.  $\bar{x} \geq p_n^m$ .

If the search cost is higher, then the firms have to set a lower joint profit-maximizing price. The incentives to lower the price are created by two factors. Firstly, if the reservation utilities of all the firms are negative, then no consumer enters the market and the market collapses. Secondly, the colluding firms prefer that consumers search sufficiently among the sellers because this leads to better matching and a higher willingness to pay. The pay-off  $\pi_i^c$  increases with  $p^c$  up to the unique solution to Eq. (7). Thus, if the search cost is higher such that  $\bar{x} < p_n^m$ , the cartel sets its price equal to  $\bar{x}$ , and the profit of one cartel member equals  $\pi^c = \bar{x}(1 - F(\bar{x})^n)/n$ . This profit decreases with the search cost.

2.4. Stability of collusion

As is well known, an individual firm has incentives to undercut the cartel price  $p^c$  if the game lasts for only a finite number of periods. Such a deviation is not necessarily profitable from a long term perspective. If all sellers interact in the market for infinitely periods, then the deviant firm may indefinitely be punished by the other firms in the cartel in the periods that follow a deviation. If the punishment is sufficiently hard, then the seller will not find it profitable to deviate. In this section, I assume that firms interact for an infinite number of periods and apply the standard grim trigger strategy.<sup>10</sup> According to this strategy, a cartel member sets  $p^c$  every period if all coalition members did this in the preceding periods. If at least one shop deviates from the collusive price, then the sellers revert to the one period Nash equilibrium price for the rest of the interaction. I denote the deviation price by  $p^d$  and the deviation profit by  $\pi^d$ .

The expectations of consumers about the prices that firms charge affect their decisions to stop searching. In turn, this has an impact on the deviation profit and the strength of the punishment. When firms collude and consumers expect that all the sellers charge the collusive price, any deviation price is unexpected, and it is attributed to a single seller. Note that because of search costs, not all consumers observe a deviation. The consumers who observe it know that the next period becomes a punishment period. However, the consumers who do not observe it do not know about the start of the punishment. Nevertheless, after an uninformed consumer samples the first firm in the first punishment period, she sees that the seller charges the competitive price  $p^*$ . Then the consumer realizes that the cartel has collapsed and updates her expectations about the prices of other firms.<sup>11</sup>

Because the joint profit-maximizing price depends on the search cost, while deriving the deviation price and the deviation profit, I also consider two intervals of the search cost's values. Suppose that  $s$  is sufficiently small such that  $\bar{x} \geq p_n^m$ . In this case the pay-off function of the deviant is similar to the pay-off function of firm  $j$  in Section 2.2. In fact, replacing  $p^*$  by  $p^c$  and  $p$  by  $p^d$  in  $\pi_j$  gives the pay-off function of the deviant. Thus, the deviation profit-maximizing price  $p^d$  is defined by the first-order condition Eq. (8).

$$\frac{1 - F(\bar{x})^n}{1 - F(\bar{x})} \left( 1 - F(\bar{x} - p^c + p^d) - p^d f(\bar{x} - p^c + p^d) \right) + n \int_0^{\bar{x} - p^c} F(\varepsilon + p^c)^{n-1} \left( f(\varepsilon + p^d) + p^d f'(\varepsilon + p^d) \right) d\varepsilon = 0 \tag{8}$$

In the Appendix A of the working paper, I show that there exists a deviation price  $p^d$  that satisfies Eq. (8) and  $p^* \leq p^d < p^c$ ; moreover, the deviation price increases with  $s$ . The higher the search cost is, the less

<sup>10</sup> In the working paper, I provide the analysis of cartel stability when firms apply the stick-and-carrot strategy with non-negative prices. The qualitative results with this alternative punishment strategy are practically the same.

<sup>11</sup> Another option is to assume that all the prices are observed in the end of every period by everyone.

consumers want to search, which makes the demand of a firm less price-elastic. Therefore, the deviant charges a higher price when search becomes more costly. Nevertheless, the deviation profit decreases with the search cost. This happens because fewer consumers visit the deviant firm when searching becomes costlier.

The one-period non-cooperative equilibrium price also increases with search costs. However, differently from the deviation profit, because the increase in the price of an individual firm is met by its competitors, the competitive profit increases with the search cost.

Next, consider the range of high search cost values for which the cartel sets  $p^c = \bar{x}$  in all its shops. Because the reservation utility of a colluding firm equals zero, the profit maximization problem of a deviant is similar to the profit maximization problem of a single-product monopolist. Thus, the deviant sets  $p^m$  and earns

$$\pi^d = \frac{1 - F(\bar{x})^n}{n(1 - F(\bar{x}))} p^m (1 - F(p^m)). \tag{9}$$

Similarly to the case of low search costs discussed above, because of lower search intensity, the profit  $\pi^d$  in Eq. (9) decreases when  $s$  increases.

By deviating from collusion, a firm gets a deviation gain  $\pi^d - \pi^c$ . However, after all sellers turn to the punishment, the profit of the firm decreases by  $\pi^c - \pi^*$  for every punishment period. Therefore, by deviating the firm knows that it will experience a deviation loss that equals  $\sum_{t=1}^{\infty} \delta^t (\pi^c - \pi^*)$ , where  $\delta$  is the discount factor. The discount factor  $\delta^*$  that makes the deviation gain equal to the deviation loss equals  $\delta^* = (\pi^d - \pi^c) / (\pi^d - \pi^*)^{-1}$ . For discount factors above this threshold value, a cartel is sustainable. Otherwise, firms never start colluding.

If the search cost is low, then  $\pi^c$  does not depend on  $s$ , the deviation profit decreases with  $s$ , and the competitive profit increases with  $s$ .<sup>12</sup> As a result, both the deviation gain  $\pi^d - \pi^c$  and the difference  $\pi^c - \pi^*$  decrease with the search cost. If the search cost is sufficiently high such that  $p^c = \bar{x}$ , then the collusive profit is also a decreasing function of the search cost. In that case, numerical simulations suggest that it decreases less than the deviation profit and I again obtain that  $\pi^d - \pi^c$  and the difference  $\pi^c - \pi^*$  decrease with the search cost. As a result, whether the critical discount factor  $\delta^*$  increases or decreases with the search cost depends on the magnitude of the slopes of all three pay-offs. More particularly, the sign of the derivative of  $\delta^*$  with respect to  $s$  is the same as the sign of Eq. (10)

$$\frac{\partial \pi^d}{\partial s} (\pi^c - \pi^*) - \frac{\partial \pi^c}{\partial s} (\pi^d - \pi^*) + \frac{\partial \pi^*}{\partial s} (\pi^d - \pi^c). \tag{10}$$

As discussed above in detail, the first derivative in Eq. (10) is negative, the second is zero for low search costs and negative for high search costs, and the third derivative is positive. Because these derivatives are multiplied by quantities that represent differences in profits, it is very difficult to discern whether the critical discount factor  $\delta^*$  increases or decreases with  $s$  without computing the actual values of the derivatives and the differences in profits multiplying them. An additional difficulty is that it is impossible to compute prices in closed form for general distributions of the match values. In spite of this, Proposition 1 provides a number of theoretical results showing that the overall sign of Eq. (10) is negative. Numerical simulations for more complicated distributions of match values, some of which are plotted in Fig. 1, confirm that the result of Proposition 1 is more general.<sup>13</sup>

<sup>12</sup> Details about the derivatives of  $\pi^d$ ,  $\pi^*$  and  $\pi^c$  with respect to  $s$  are in the Appendix A of the working paper.

<sup>13</sup> The numerical simulations were performed with a variety of distributions including power, exponential, Normal, Extreme value type I, and  $F(\varepsilon) = e^\varepsilon$  distributions (all distributions were truncated for the interval  $[0, 1]$ ).

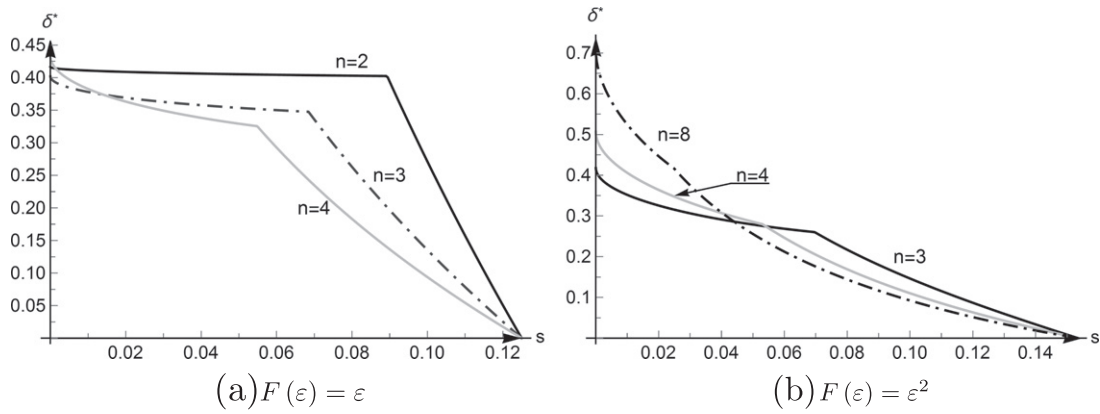


Fig. 1. How  $\delta^*$  varies with  $s$  for different  $F(\varepsilon)$ .

**Proposition 1.** *In the model with sequential search for differentiated products (Wolinsky, 1986), the critical discount factor  $\delta^*$  above which collusion is sustainable decreases with the search cost*

- if  $n = 2$  and the match value  $\varepsilon$  is distributed uniformly in the interval  $[0, 1]$ ,
- for any  $n$  if the match value  $\varepsilon$  is distributed uniformly in the interval  $[0, 1]$  and  $s \leq \frac{1}{2} \left(1 - \frac{1}{(n+1)^{1/n}}\right)^2$  and
- for any  $F(\varepsilon)$  if  $n \rightarrow \infty$  and  $s \rightarrow 0$ .

To gain some intuition behind this result, consider the case in which the search cost is low. In this case, the second derivative in Eq. (10) is equal to zero and there are only two terms to compare in Eq. (10). The deviation profit falls with the search cost while the competitive profit increases with the search cost. The competitive profit increases with the search cost because of the positive effect of  $s$  that comes indirectly via the prices of other firms. This positive indirect effect is weakened by the negative direct effect of the search cost. Namely, less consumers visit a firm when it is costlier to search, and therefore, the firm sells less. Then, the competitive profit increases with the search cost relatively little. Meanwhile, the deviation profit suffers only the negative effect of the search cost. This effect is very strong because the deviation profit is made from the consumers who visit and buy and the number of these consumers depends directly on the search costs. As a result, the fall in deviation profits has a dominating influence.

When the search cost goes up, consumers search less and thereby compare fewer alternatives. This, in a sense, is similar to the case in which the number of firms decreases. In such a case, the economics literature has shown that cartels become more stable.<sup>14</sup> In my analysis, the effect of a decrease in the number of firms on the stability of collusion is however more subtle. When search costs are low, a decrease in the number of firms makes collusion more stable, which confirms the results in earlier work; however, when search cost is high the reverse result holds. This observation, which can be seen in Fig. 1a and b, is due to the fact that an increase in search cost  $s$  weakens the positive effect of a higher collusive price on the deviation gain less than on the deviation loss.

The sensitiveness of the critical discount factor  $\delta^*$  to the search cost  $s$  depends on both the number of firms  $n$  and the density function  $f(\varepsilon)$ . The effect of an increase in  $s$  on  $\delta^*$  is weaker as the number of firms increases. In fact, in the extreme situation where there are just two firms in the market,  $\delta^*$  decreases very slowly as  $s$  goes up. This is because a firm that deviates from collusion is visited by more than a half of the consumers irrespective of the search cost and therefore the deviation

profit is relatively insensitive to the search cost. The same observation applies to the competitive profit  $\pi^*$ .<sup>15</sup>

The magnitude of the effect of an increase in  $s$  on  $\delta^*$  also depends on the density function  $f(\varepsilon)$ . Consider the power distribution  $F(\varepsilon) = \varepsilon^\lambda$ ,  $\lambda > 1$ . As  $\lambda$  increases, high match values become more and more frequent, which may be interpreted as a decrease in horizontal product differentiation. Then the critical discount factor  $\delta^*$  turns out to decrease more sharply with  $s$  if  $\lambda$  is higher.

### 3. Homogeneous products

To analyse how market transparency on the consumer side affects cartel stability with homogeneous products, I use the unit-demand version of the standard sequential consumer search set-up of Stahl (1989). The unit-demand version, analysed by Janssen et al. (2005), has the advantage that the equilibrium can be computed in closed form. The model features a finite number of firms  $n$  competing in prices. Marginal costs are normalized to zero. Products are exactly identical and consumers buy from the cheapest firm they know. All consumers have the same willingness to pay for the product, denoted by  $\nu$ . There is a positive fraction of consumers  $\lambda$  who are fully informed; these buyers are called shoppers. The rest of the consumers observe one price freely and have to engage in costly search to learn other prices; they are often referred to as non-shoppers.

The one-period Nash equilibrium is like the one in Janssen et al. (2005). I briefly sketch the necessary derivations of it. In a competitive equilibrium, firms mix by choosing their prices from an interval  $[p, p_r]$  according to a distribution function  $G(p)$ . The upper bound of the distribution of  $p$  is the reservation price  $p_r$  that is defined by equating the gain from search and the search cost, which gives Eq. (11).<sup>16</sup>

$$\int_p^{p_r} (p_r - p) dG(p) = s \tag{11}$$

The reservation price increases with the search cost,<sup>17</sup> and, to avoid situations when the reservation price exceeds the valuation  $\nu$ , I assume that the search cost is such that  $p_r \leq \nu$ , i.e.

$$s \leq \int_{p(\nu)}^\nu (\nu - p) dG(p)$$

<sup>15</sup> Because both the deviation gain and the deviation loss are relatively insensitive to the search cost when there are very few firms, the result that  $\delta^*$  decreases with the search cost may be violated. More particularly, while performing simulations with different distribution functions, I have observed that if  $n = 2$  and  $f(\varepsilon) = 2\varepsilon$  or  $f(\varepsilon) = e^\varepsilon/(e - 1)$ , then the critical discount factor is non-monotonic in  $s$ . However, the range of search cost values for which  $\delta^*$  increases with  $s$  is relatively narrow and the rate of increase is very small.

<sup>16</sup> See Stahl (1989) Section 1 and Janssen et al. (2005) Section 3 for more details.

<sup>17</sup> See Janssen et al. (2005) for more details.

<sup>14</sup> See e.g. Ivaldi et al. (2003) and Jacquemin and Slade (1989).

I now move to the derivation of the equilibrium distribution of prices. Consider a firm  $i$  charging price  $p_i$ . Firm  $i$  receives demand from both shoppers and non-shoppers. Suppose that a consumer has zero search costs. Because she observes the prices of all  $n$  sellers and, firm  $i$  only sells to this customer if its price is the lowest in the market. The probability of this event equals  $(1 - G(p_i))^{n-1}$ .

Now consider a non-shopper who obtains the first price quotation for free and randomly, and has to pay a positive search cost in order to learn the price of a second, third, and so forth seller. It turns out that in equilibrium such a consumer does not search any further. This happens because in equilibrium no seller charges a price above  $p_r$  and, therefore, the gain from sampling one more firm is lower than the search cost. Thus, firm  $i$  sells to a non-shopper with probability  $1/n$ . The expected profit of firm  $i$  then equals

$$\pi_i = p_i \left( \lambda (1 - G(p_i))^{n-1} + \frac{1-\lambda}{n} \right).$$

In equilibrium, the expected profit of a firm  $i$  charging  $p_i < p_r$  must be equal to the profit of the seller when it sets  $p_r$ . In the latter case, the firm sells only to consumers who have positive search costs, and the profit equals  $p_r(1 - \lambda)/n$ . By equating the two profit levels and solving for  $G(p)$ , I obtain that the equilibrium distribution of prices:

$$G(p) = 1 - \left( \frac{(p_r - p)(1 - \lambda)}{np\lambda} \right)^{\frac{1}{n-1}}.$$

The lower bound of the support of the price distribution is  $\underline{p} = p_r(1 - \lambda)/(1 + (n-1)\lambda)$ . Denote the equilibrium profit by  $\pi^* = p_r(1 - \lambda)/n$ , and note that this profit increases with search cost (because the reservation price does).

Consider now that firms interact in this market for infinitely many periods, and they collude by using grim-trigger strategies. It is straightforward to see that a collusive equilibrium has firms setting a price equal to the maximum willingness to pay of consumers, i.e.  $p^c = v$ . In such a case, firms will obtain a profit equal to  $\pi^c = v/n$ .

If a firm deviates from collusion, it does not attract any additional non-shoppers just because these consumers do not observe price deviations. However, by setting a deviation price  $p^d = v - \epsilon$ , where  $\epsilon$  is a very small positive number, the seller attracts all fully informed consumers. As a result, the total demand of the deviant equals  $(1 - \lambda)/(n + \lambda)$  and the deviation profit is then

$$\pi^d = v \left( \frac{1-\lambda}{n} + \lambda \right).$$

**Proposition 2.** *In the model with sequential search for homogeneous products (Stahl, 1989) the critical discount factor  $\delta^*$  above which collusion is sustainable*

- increases with the search cost;
- is non-monotonic in the share of shoppers  $\lambda$  (first decreasing and then increasing) if  $n = 2$ .

Neither the deviation profit nor the collusive profit depend on the search cost. Therefore, the only effect of  $s$  on the stability of collusion follows from its effect on the punishment, i.e. on the static equilibrium profit  $\pi^*$ . If the search cost increases, the expected competitive profit increases and the punishment for the deviation becomes softer. Then the critical discount factor  $\delta^*$  above which collusion is stable increases when search becomes costlier. Thus, a cartel becomes less stable when the market becomes less transparent.

This model lends itself to model transparency in a different way. Because of the shopper/non-shopper feature of demand, we can alternatively relate market transparency to the share of shoppers. If the share of shoppers goes up, then an average consumer observes more prices,

which implies higher market transparency. The share of shoppers affects both the deviation profit and the expected competitive profit. In Proposition 2, I show that the critical discount factor is non-monotonic in  $\lambda$  when there are two firms in the market: it first decreases and later increases. Unfortunately for higher values of  $n$  the derivative of the critical discount factor with respect to  $\lambda$  is difficult to study analytically. This happens because the derivative of  $p_r$  with respect to  $\lambda$  is quite complicated, and this makes it difficult to compare the derivatives  $\partial \pi_d / \partial \lambda$  and  $\partial \pi^* / \partial \lambda$ . Therefore, I resort to numerical results for cases when the number of firms is larger. In Fig. 2a, I plot the values of  $\delta^*$  for different values of  $n$ . Similarly to the case when  $n = 2$ , if  $\lambda$  is high, then the critical discount factor increases with  $\lambda$ , and it decreases if  $\lambda$  is small.<sup>18</sup>

The deviation profit increases with the share of fully informed consumers linearly. This is because the deviant gets all shoppers; meanwhile the collusive profit does not change with  $\lambda$ . As a result, the deviation gain increases with  $\lambda$  linearly. The share of fully informed consumers has a negative effect on the competitive profit via two channels. First of all, if  $\lambda$  increases, then more consumers are lured away by the firms that set lower prices. Secondly, the interval of prices over which sellers mix in the competitive equilibrium moves towards lower prices when  $\lambda$  increases. Thus, the competitive profit decreases with  $\lambda$ , and the punishment becomes harder when there are more shoppers in the market. The negative effect of  $\lambda$  on  $\pi^*$  is not equally strong for all values of  $\lambda$ : it becomes weaker when  $\lambda$  is bigger. Thus, although the punishment from deviation increases with  $\lambda$ , it does it decreasingly. Therefore, when  $\lambda$  is small, the deviation gain increases with  $\lambda$  less than the punishment, and  $\delta^*$  decreases with  $\lambda$ . However, if  $\lambda$  is high, then the deviation gain increases with  $\lambda$  more than the punishment, and  $\delta^*$  increases with  $\lambda$ .

The number of firms has a negative effect on all three profits. Because by deviating a firm attracts all fully informed consumers, the negative effect of  $n$  on the collusive profit is stronger than on the deviation profit and the deviation gain increases with  $n$ . Meanwhile, the competitive profit is less sensitive to the change of  $n$  than the collusive profit. Thus, the difference  $\pi^c - \pi^*$  decreases with  $n$ . As a result,  $\delta^*$  increases with  $n$  (Fig. 2b).

Nilson (1999) finds that the critical discount factor above which collusion is sustainable does not decrease with the search cost.<sup>19</sup> By comparing his findings to the ones in Proposition 2, I conclude that the effect of market transparency on the stability of collusion is the same when consumers search sequentially or non-sequentially. However, the search protocol matters when it is analysed how the stability of collusion varies with the share of shoppers. Nilson (1999) shows that the critical discount factor either does not depend on the share of shoppers or increases with the share of shoppers. Meanwhile, according to Proposition 2 the critical discount factor is non-monotonic in  $\lambda$ . To understand the difference in result, note that in the set-up of Stahl (1989) the reservation price  $p_r$  decreases with  $\lambda$ , and a consumer who has positive search costs never visits more than one firm. On the contrary, in the duopoly model of Nilson (1999) the reservation price does not depend on  $\lambda$  and some consumers who have positive search costs visit both sellers and become fully informed. This difference, leads to a different outcome when it comes to the sign of  $\partial \delta^* / \partial \lambda$ .

#### 4. Conclusion

According to the International Competition Network (2005) “the prohibition against cartels is now an almost universal component of

<sup>18</sup> Note that for a fixed search cost  $s$  when  $\lambda$  is sufficiently small it happens that  $p_r > v$ . In such situations, the reservation price does not restrict firm pricing and then the sequential search model converges to the model of Varian (1980). For those cases  $\delta^* = (n-1)/n$ , which does not depend on  $\lambda$  (horizontal lines in Fig. 2).

<sup>19</sup> In the model of Nilson (1999), the critical discount factor decreases with  $\beta$  that is the share of consumers who observe both prices, and  $\beta$  does not increase with the search cost.

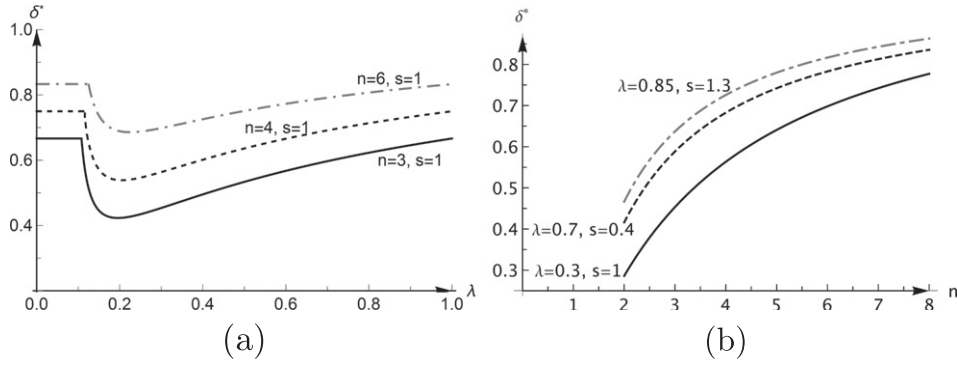


Fig. 2. The values of  $\delta^*$  for different  $n$ ,  $s$  and  $\lambda$ ,  $\nu = 10$ .

competition laws” because competition between sellers is beneficial for consumers and the “competitive process only works when competitors set prices independently.” On the contrary, firms prefer to engage in secret price (quantity) coordination because then they earn higher profits. The objectives of firms and competition authorities contradict each other. Hence, it is important for policy makers to determine the conditions under which collusion is more likely to arise in order to implement proper consumer protection policy measures.

This paper studies how notions of market transparency from the consumer point of view affect the likelihood of collusion. The analysis reveals that the answer to this question depends on two factors. One is the nature of the product market, in particular whether firms sell homogeneous or horizontally differentiated products. The other is the measurement of market transparency. When I relate market transparency to search costs, it turns out that cartel stability increases as search costs go up if products are horizontally differentiated, and it decreases if products are homogeneous. When I relate market transparency to the share of consumers who are fully informed, then in a homogeneous product market I find that cartel stability is non-monotonic in the share of shoppers. Hence, there is no unique answer to the question whether market transparency on the demand side hinders or facilitates collusion.

My results suggest that collusion preventing measures in frictional markets should be carefully fine-tuned. In doing so, competition authorities trade off the costs of taking measures and the benefits from lowering the risk of collusion. Because competition becomes softer as the search cost rises, so does the damage from collusion. As a result, if reducing search costs is rather costly, it may be inefficient to implement this type of policies just to decrease the risk of collusion.

Market transparency on the demand side is likely to be related to market transparency on the supply side. If this is so, measures to increase transparency on the demand side may backfire because of the increased transparency on the supply side. Acknowledging this double-edged nature of the problem of increasing market transparency, is an interesting avenue for further inquiry into the relation between market transparency and collusion.

**Appendix**

**Proof of proposition 1.** Firstly, I prove that the critical discount factor  $\delta^*$  decreases with the search cost when  $\varepsilon$  is distributed uniformly and  $\bar{x} \geq p_n^m$ . Later I prove that  $\partial\delta^*/\partial s < 0$  when  $\varepsilon$  is distributed uniformly and a cartel charges  $p^c = \bar{x}$ . In the last part of the proof I tackle the case when  $n \rightarrow \infty$  and  $\bar{x} \rightarrow \bar{\varepsilon}$ .

**Part I.**  $\varepsilon \sim U(0, 1)$  and  $\bar{x} \geq p_n^c$ .

In this case the collusive price  $p^c$  equals  $(n + 1)^{-1/n}$ , and the maximum value of  $s$  is

$$s_{max} = \int_{p^c}^1 (\varepsilon - p^c) d\varepsilon = \frac{1}{2}(1 - p^c)^2 = \frac{1}{2} \left( 1 - \frac{1}{(n + 1)^{1/n}} \right)^2.$$

After solving the first order condition for an optimal deviation price, I get that  $p^d$  equals

$$p^d = \frac{1}{2} \left( \frac{1 - \bar{x}}{1 - \bar{x}^n} (1 - (p^c)^n) + p^c \right).$$

Firstly, I observe that  $sign [\partial\delta^*/\partial s] = - sign [\partial\delta^*/\partial \bar{x}]$ . Secondly, the profit of a coalition member does not depend on the search cost. Thus, the derivative of the critical discount factor with respect to  $\bar{x}$  depends only on the changes of  $\pi^d$  and  $\pi^*$ .

$$\begin{aligned} \frac{d\delta^*}{d\bar{x}} &= \frac{1}{(\pi^d - \pi^*)^2} \left[ \frac{d\pi^d}{d\bar{x}} (\pi^d - \pi^*) - \left( \frac{d\pi^d}{d\bar{x}} - \frac{d\pi^*}{d\bar{x}} \right) (\pi^d - \pi^c) \right] \\ &= \frac{1}{(\pi^d - \pi^*)^2} \left[ \frac{d\pi^d}{d\bar{x}} (\pi^c - \pi^*) + \frac{d\pi^*}{d\bar{x}} (\pi^d - \pi^c) \right] \end{aligned} \tag{12}$$

The denominator of the first fraction in Eq. (12) is positive. Hence,

$$sgn \left[ \frac{d\delta^*}{d\bar{x}} \right] = sgn \left[ \frac{d\pi^d}{d\bar{x}} (\pi^c - \pi^*) + \frac{d\pi^*}{d\bar{x}} (\pi^d - \pi^c) \right]. \tag{13}$$

As  $\partial p^c / \partial \bar{x} = 0$  and  $\partial \pi^d / \partial p^d = 0$  then

$$\frac{d\pi^d}{d\bar{x}} = \frac{p^d}{n} \frac{1 + (n-1)\bar{x}^n - n\bar{x}^{n-1}}{(1-\bar{x})^2} (p^c - p^d).$$

The competitive equilibrium profit may be written as follows

$$\pi^* = (p^*)^2 \frac{\partial q_j}{\partial p_j} \Big|_{p_j=p^*} = \frac{(p^*)^2}{n} \frac{1 - \bar{x}^n}{1 - \bar{x}}.$$

Then, by using the Implicit function theorem for  $\partial p^* / \partial \bar{x}$ , the derivative of the competitive profit with respect to  $\bar{x}$  may be written as

$$\frac{\partial \pi^*}{\partial \bar{x}} = \frac{(p^*)^2}{n} \frac{1 + (n-1)\bar{x}^n - n\bar{x}^{n-1}}{(1-\bar{x})^2} \left( 1 - \frac{2 \frac{1 - \bar{x}^n}{1 - \bar{x}}}{\frac{1 - \bar{x}^n}{1 - \bar{x}} + n(p^*)^{n-1}} \right).$$

Note that both the derivative of  $\pi^d$  with respect to  $\bar{x}$  and the derivative  $\pi^*$  with respect to  $\bar{x}$  have the same positive element  $\frac{1}{n} \frac{1 + (n-1)\bar{x}^n - n\bar{x}^{n-1}}{(1-\bar{x})^2}$ . The sign of the RHS of Eq. (13) does not change if I divide it by this expression. Therefore, I explore the sign of the expression  $\phi(\bar{x}, p^*, n)$  further on.

$$\begin{aligned} \phi(\bar{x}, p^*, n) &\equiv p^d (p^c - p^d) \left( p^c (1 - (p^c)^n) - (p^*)^2 \frac{1 - \bar{x}^n}{1 - \bar{x}} \right) \\ &\quad + (p^*)^2 \left( 1 - \frac{2 \frac{1 - \bar{x}^n}{1 - \bar{x}}}{\frac{1 - \bar{x}^n}{1 - \bar{x}} + n(p^*)^{n-1}} \right) \left( (p^d)^2 \frac{1 - \bar{x}^n}{1 - \bar{x}} - p^c (1 - (p^c)^n) \right) \end{aligned}$$

By using the expression of  $p^d$  and the fact that  $1 - (p^c)^n = n(p^c)^{n-1}$  I get

$$\begin{aligned} n(\pi^d - \pi^c) &= \frac{1}{4} \left( (p^c)^2 + 2 \frac{1-\bar{x}}{1-\bar{x}^n} n(p^c)^{n+1} + \left( \frac{1-\bar{x}}{1-\bar{x}^n} \right)^2 n^2 (p^c)^{2n} \right) \frac{1-\bar{x}^n}{1-\bar{x}} - n(p^c)^{n+1} \\ &= \frac{1}{4} \frac{1-\bar{x}^n}{1-\bar{x}} \left( (p^c)^2 - 2 \frac{1-\bar{x}}{1-\bar{x}^n} n(p^c)^{n+1} + \left( \frac{1-\bar{x}}{1-\bar{x}^n} \right)^2 n^2 (p^c)^{2n} \right) \\ &= \frac{1}{4} \frac{1-\bar{x}^n}{1-\bar{x}} \left( p^c - \frac{1-\bar{x}}{1-\bar{x}^n} n(p^c)^n \right)^2 = \frac{1-\bar{x}^n}{1-\bar{x}} (p^c - p^d)^2. \end{aligned}$$

The collusive price is higher than the deviation price. Therefore, if I divide  $\phi(\bar{x}, p^*, n)$  by  $p^c - p^d$  then the new expression will have the same sign as  $\phi(\bar{x}, p^*, n)$ . I denote this new expression by  $\phi_1(\bar{x}, p^*, n)$

$$\begin{aligned} \phi_1(\bar{x}, p^*, n) &= p^d \left( n(p^c)^{n+1} - (p^*)^2 \frac{1-\bar{x}^n}{1-\bar{x}} \right) \\ &\quad + (p^*)^2 \left( 1 - \frac{2 \frac{1-\bar{x}^n}{1-\bar{x}}}{\frac{1-\bar{x}^n}{1-\bar{x}} + n(p^*)^{n-1}} \right) (p^c - p^d) \frac{1-\bar{x}^n}{1-\bar{x}}. \end{aligned}$$

From the first order condition (4) it is true that

$$(p^*)^{n-1} = \frac{1}{p^*} - \frac{1-\bar{x}^n}{1-\bar{x}} = \frac{1}{p^*} - \gamma.$$

Then I can rewrite  $\phi_1(\bar{x}, p^*, n)$  as

$$\phi_1(\bar{x}, p^*, n) \equiv p^d \left( n(p^c)^{n+1} - (p^*)^2 \gamma \right) + (p^*)^2 \left( \frac{n-p^* \gamma(n+1)}{n-\gamma(n-1)p^*} \right) (p^c - p^d) \gamma.$$

Now note that

$$\begin{aligned} \frac{1}{(p^*)^2} \frac{\partial(\phi_1(\bar{x}, p^*, n))}{\partial \gamma} &= -p^d - \left( \frac{2np^* \gamma}{(n-\gamma(n-1)p^*)^2} - \frac{n-p^* \gamma(n+1)}{n-\gamma(n-1)p^*} \right) \\ &\quad \times (p^c - p^d) < 0. \end{aligned}$$

The inequality has been obtained because  $\frac{\partial \pi^c}{\partial \bar{x}} < 0$  implies that  $\frac{n-p^* \gamma(n+1)}{n-\gamma(n-1)p^*} < 0$ .

Additionally,

$$\begin{aligned} \frac{\partial(\phi_1(\bar{x}, p^*, n))}{\partial p^*} &= -2p^d p^* \gamma + 2p^* \left( \frac{n-p^* \gamma(n+1)}{n-\gamma(n-1)p^*} \right) (p^c - p^d) \gamma \\ &\quad - 2(p^*)^2 (p^c - p^d) \frac{\gamma^2 n}{(n-\gamma(n-1)p^*)^2} < 0 \end{aligned}$$

and

$$\frac{\partial(\phi_1(\bar{x}, p^*, n))}{\partial p^d} = n(\pi^c - \pi^*) - (p^*)^2 \gamma \left( \frac{n-p^* \gamma(n+1)}{n-\gamma(n-1)p^*} \right) > 0,$$

and the fact  $\partial p^d / \partial \bar{x} < 0$  implies that  $\partial p^d / \partial \gamma < 0$  Therefore,

$$\begin{aligned} \phi_1(\bar{x}, p^*, n) &> \phi_1 \left( 1, \frac{1}{2}, n \right) = \frac{1}{2} \left( p^c + \frac{1}{n+1} \right) \left( \frac{p^c n}{n+1} - \frac{n}{4} \right) \\ &\quad + \frac{1}{8} \left( \frac{n-\frac{n}{2}(n+1)}{n-\frac{n}{2}(n-1)} \right) \left( p^c - \frac{1}{n+1} \right) n \\ &= \frac{n(2(n-3) + (n-1)(5+n)(n+1)^{\frac{1}{n}-1} - (1+n)^{\frac{2}{n}}(n-2))}{4(n-3)(n+1)^{1+2/n}} \end{aligned} \tag{14}$$

$\lim_{n \rightarrow 3} \phi_1(1, \frac{1}{2}, n) = \infty$  and the expression is positive for all  $n > 3$  (see Fig. 3a).

Now I tackle the case  $n = 2$  by using the fact that  $\gamma = (1 - (p^*)^2)/p^*$ . If  $n = 2$  then

$$\begin{aligned} \phi_1(\bar{x}, p^*, 2) &= \frac{1-3(p^*)^2}{27((p^*)^4-1)} \left( -3 + 7\sqrt{3}p^* - 9(p^*)^2 \right. \\ &\quad \left. - 8\sqrt{3}(p^*)^3 + 12(p^*)^4 + 3\sqrt{3}(p^*)^5 \right). \end{aligned}$$

The fraction  $\frac{1-3(p^*)^2}{27((p^*)^4-1)}$  is negative. The polynomial in parenthesis is also negative (See Fig. 3b).

As  $\phi_1 > 0$  then  $\delta^*$  increases in  $\bar{x}$  or decreases with  $s$ .

**Part II.**  $\varepsilon \sim U(0, 1)$  and  $\bar{x} < p_n^m$ .

The sign of the derivative of  $\delta^*$  with respect to  $\bar{x}$  is the same as the sign of Eq. (15)

$$\frac{\partial \pi^d}{\partial \bar{x}} (\pi^c - \pi^*) + \frac{\partial \pi^*}{\partial \bar{x}} (\pi^d - \pi^c) - \frac{\partial \pi^c}{\partial \bar{x}} (\pi^d - \pi^*). \tag{15}$$

After plugging the values of profits and their derivatives, I multiply Eq. (15) by  $(\frac{1-\bar{x}}{1-\bar{x}^n})^2$  and obtain Eq. (16).

$$\begin{aligned} (p^*)^2 \frac{1 + (n-1)\bar{x}^n - n\bar{x}^{n-1}}{(1-\bar{x})} \frac{2}{\frac{1-\bar{x}^n}{1-\bar{x}} + np^{*n-1}} \left( \frac{1}{1-\bar{x}} - \frac{1}{4} - p^{*2} \right) - \frac{(2\bar{x}-1)}{1-\bar{x}} \left( \frac{1}{4} - p^{*2} \right) \end{aligned} \tag{16}$$

I analyse the sign of Eq. (16) for two different values of  $n$ .

If  $n \rightarrow \infty$ , then  $p^* \rightarrow 1-\bar{x}$  and the second line of Eq. (16) equals zero. Additionally,  $1/4 - (1-\bar{x})^2 \geq 1/4 - (1-1/2)^2 = 0$ .

If  $n = 2$ , then  $p^* = \frac{1}{2}(\sqrt{\bar{x}^2 + 2\bar{x} + 5} - \bar{x} - 1)$  and Eq. (16) simplifies to

$$\frac{(2\bar{x}-1)(\bar{x}(3\bar{x}-3\sqrt{\bar{x}(\bar{x}+2)+5}+10)-6\sqrt{\bar{x}(\bar{x}+2)+5}+13)}{4\sqrt{\bar{x}(\bar{x}+2)+5}}. \tag{17}$$

Because the denominator of Eq. (17) is positive, the whole expression is positive if

$$g(x) \equiv \bar{x}(3\bar{x} + 10) - 3(2 + \bar{x})\sqrt{\bar{x}(\bar{x} + 2) + 5} + 13 > 0.$$

I observe that  $g(1/2) = 0, g(1/\sqrt{3}) = -\sqrt{2(8+\sqrt{3})} - 2\sqrt{6(8+\sqrt{3})} + 10/\sqrt{3} + 14 \approx 0.078 > 0$ . Additionally, because  $g(\bar{x})$  is continuous, I check the value of  $\bar{x}$  when  $\partial g(\bar{x}) / \partial \bar{x} = 0$ .

$$\frac{\partial g(\bar{x})}{\partial \bar{x}} = 6\bar{x} + 10 - 3\sqrt{\bar{x}(\bar{x} + 2) + 5} - \frac{3(2 + \bar{x})(\bar{x} + 1)}{\sqrt{\bar{x}(\bar{x} + 2) + 5}}$$

The derivative equals 0 if  $\bar{x} = \frac{1}{36} \left( \sqrt[3]{200537 + 5472\sqrt{3945}} - \frac{4271}{\sqrt[3]{200537 + 5472\sqrt{3945}}} - 43 \right) \approx -0.34 < 0$ . Thus,  $g(\bar{x})$  is always positive.

Hence, I conclude that if  $n = 2$  then Eq. (17) is positive. As a result, Eq. (15) is positive and  $\delta^*$  decreases with  $s$ .

**Part III.**  $\bar{x} \rightarrow \bar{\varepsilon}$  and  $n \rightarrow \infty$ . If  $n \rightarrow \infty$ , then  $\lim_{n \rightarrow \infty} \frac{1-F(\bar{x})^n}{(1-F(\bar{x}))} = \frac{1}{1-F(\bar{x})}$  and  $\lim_{n \rightarrow \infty} p^* = \bar{\varepsilon}$ . Thus, by assuming there is a positive ratio of consumers to firms  $L$ , I obtain, that the profits  $\pi^*, \pi^c$  and  $\pi^d$  are

$$\pi^* = L \frac{1-F(\bar{x})}{f(\bar{x})}, \pi^c = L\bar{x}, \pi^d = L \frac{p^m(1-p^m)}{1-F(\bar{x})}$$

and

$$\delta^* = \frac{\frac{p^m(1-F(p^m))}{1-F(\bar{x})} - \bar{x}}{\frac{p^m(1-F(p^m))}{1-F(\bar{x})} - \frac{1-F(\bar{x})}{f(\bar{x})}} = \frac{p^m(1-F(p^m)) - \bar{x}(1-F(\bar{x}))}{p^m(1-F(p^m)) - \frac{(1-F(\bar{x}))^2}{f(\bar{x})}}$$



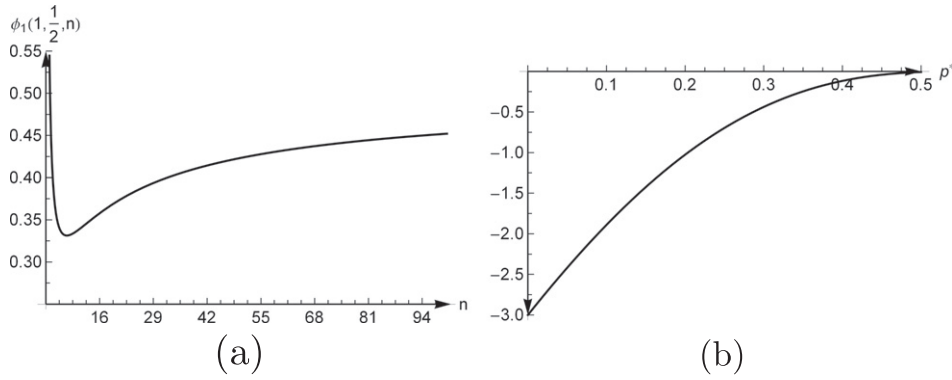


Fig. 3. The values of  $\phi_1$  for different values of  $p^*$  and  $n$ .

$$\begin{aligned} & \left( p^m(1-F(p^m)) - \frac{(1-F(\bar{x}))^2}{f(\bar{x})} \right)^2 \frac{\partial \delta^*}{\partial \bar{x}} = \\ & -(1-F(\bar{x}) - \bar{x} f(\bar{x})) \left( p^m(1-F(p^m)) - \frac{(1-F(\bar{x}))^2}{f(\bar{x})} \right) - \\ & (p^m(1-F(p^m)) - \bar{x}(1-F(\bar{x}))) \left( 2(1-F(\bar{x})) + \frac{(1-F(\bar{x}))^2 f'(\bar{x})}{f(\bar{x})^2} \right). \end{aligned} \tag{18}$$

If  $\bar{x} \rightarrow \bar{e}$ , Eq. (18) simplifies to  $\bar{e}f(\bar{e})p^m(1-F(p^m)) > 0$ .  $\square$

**Proof of proposition 2.** To determine the sign of the derivative of the critical discount factor with respect to  $\lambda$  I look at the following expression

$$\begin{aligned} & \frac{\partial \pi^d}{\partial \lambda} (\pi^c - \pi^*) + \frac{\partial \pi^*}{\partial \lambda} (\pi^d - \pi^c) = \\ & \nu \left( 1 - \frac{1}{n} \right) \left( \frac{\nu}{n} - \frac{p_r(1-\lambda)}{n} \right) + \left( \frac{\partial p_r}{\partial \lambda} \frac{1-\lambda}{n} - \frac{p_r}{n} \right) \nu \left( \lambda - \frac{\lambda}{n} \right) = \\ & \frac{\nu}{n^2} (n-1) \left( \nu - p_r + \frac{\partial p_r}{\partial \lambda} (1-\lambda) \lambda \right). \end{aligned} \tag{19}$$

In equilibrium Eq. (11) simplifies to

$$\begin{aligned} & \frac{1}{n-1} \left( \frac{1-\lambda}{n\lambda} \right)^{\frac{1}{n-1}} \int_{\frac{1-\lambda}{1+(n-1)\lambda}}^{p_r} \left( \frac{p_r}{p} - 1 \right)^{\frac{1}{n-1}} \frac{p_r}{p} dp \\ & = \frac{1}{n-1} \left( \frac{1-\lambda}{n\lambda} \right)^{\frac{1}{n-1}} p_r \int_{\frac{1-\lambda}{1+(n-1)\lambda}}^1 \left( \frac{1}{t} - 1 \right)^{\frac{1}{n-1}} \frac{1}{t} dt. \end{aligned} \tag{20}$$

If  $n = 2$  then Eq. (20) becomes,

$$\frac{1-\lambda}{2\lambda} p_r \int_{\frac{1-\lambda}{1+\lambda}}^1 \left( \frac{1}{t} - 1 \right) \frac{1}{t} dt = p_r \left( \frac{1-\lambda}{2\lambda} \ln \left( \frac{1-\lambda}{\lambda+1} \right) + 1 \right) = s.$$

Thus,

$$p_r = s \left( \frac{1-\lambda}{2\lambda} \ln \left( \frac{1-\lambda}{\lambda+1} \right) + 1 \right)^{-1}.$$

Then

$$\frac{\partial p_r}{\partial \lambda} = s \left( \frac{1-\lambda}{2\lambda} \ln \left( \frac{1-\lambda}{\lambda+1} \right) + 1 \right)^{-2} \times \left( \frac{2\lambda + (\lambda+1) \ln \left( \frac{1-\lambda}{\lambda+1} \right)}{2\lambda^2(\lambda+1)} \right).$$

The derivative of  $2\lambda + (\lambda+1) \ln \left( \frac{1-\lambda}{\lambda+1} \right)$  with respect to  $\lambda$  is negative:

$$-\frac{2\lambda - (1-\lambda) \ln \left( \frac{1-\lambda}{\lambda+1} \right)}{1-\lambda} < 0.$$

Thus,

$$2\lambda + (\lambda+1) \ln \left( \frac{1-\lambda}{\lambda+1} \right) \leq \lim_{\lambda \rightarrow 0} \left( 2\lambda + (\lambda+1) \ln \left( \frac{1-\lambda}{\lambda+1} \right) \right) = 0.$$

As a result  $\partial p_r / \partial \lambda < 0$ .

I plug the value of  $\partial p_r / \partial \lambda$  and  $s$  in the RHS of Eq. (19), set  $n = 2$  and multiply by  $4/\nu$ . Then I obtain the following expression:

$$\begin{aligned} & \nu - p_r + \left( \frac{2\lambda + (\lambda+1) \ln \left( \frac{1-\lambda}{\lambda+1} \right)}{\frac{2\lambda^2(\lambda+1)}{\frac{1-\lambda}{2\lambda} \ln \left( \frac{1-\lambda}{\lambda+1} \right) + 1}} \right) (1-\lambda) \lambda p_r \\ & = \nu - p_r \frac{4\lambda^2}{(1+\lambda) \left( (1-\lambda) \ln \left( \frac{1-\lambda}{\lambda+1} \right) + 2\lambda \right)}. \end{aligned} \tag{21}$$

It has been shown that  $p_r$  decreases with  $\lambda$ . Now I show that the expression next to  $p_r$  decreases with  $\lambda$ .

$$\begin{aligned} & \frac{\partial}{\partial \lambda} \left( \frac{4\lambda^2}{(1+\lambda) \left( (1-\lambda) \ln \left( \frac{1-\lambda}{\lambda+1} \right) + 2\lambda \right)} \right) \\ & = \frac{8\lambda \left( 2\lambda + \ln \left( \frac{1-\lambda}{\lambda+1} \right) \right)}{(1+\lambda)^2 \left( (1-\lambda) \ln \left( \frac{1-\lambda}{\lambda+1} \right) + 2\lambda \right)^2} < 0 \end{aligned}$$

where the inequality has been obtained because

$$\frac{\partial}{\partial \lambda} \left( 2\lambda + \ln \left( \frac{1-\lambda}{\lambda+1} \right) \right) = 2 - \frac{2}{1-\lambda^2} < 0$$

and  $2\lambda + \ln \left( \frac{1-\lambda}{\lambda+1} \right) \leq 0 + \ln 1 = 0$ . Therefore, I conclude that Eq. (21) increases with  $\lambda$ . Additionally by replacing  $p_r$  with its value, I obtain that

$$\begin{aligned} & \lim_{\lambda \rightarrow 0} \left( p_r \frac{4\lambda^2}{(1+\lambda) \left( (1-\lambda) \ln \left( \frac{1-\lambda}{\lambda+1} \right) + 2\lambda \right)} \right) = \infty \\ & \lim_{\lambda \rightarrow 1} \left( p_r \frac{4\lambda^2}{(1+\lambda) \left( (1-\lambda) \ln \left( \frac{1-\lambda}{\lambda+1} \right) + 2\lambda \right)} \right) = s. \end{aligned}$$

From the limits on the maximum search cost, I have

$$s \leq \nu \frac{1}{n-1} \left( \frac{1-\lambda}{n\lambda} \right)^{n-1} \int_{\frac{1-\lambda}{1+(n-1)\lambda}}^1 \left( \frac{1}{t} - 1 \right)^{\frac{1}{n-1}} \frac{1}{t} dt.$$

When  $n = 2$ , the inequality simplifies to

$$s \leq \nu \left( \frac{1-\lambda}{2\lambda} \ln \left( \frac{1-\lambda}{\lambda+1} \right) + 1 \right) < \nu.$$

Because  $s < \nu$ , I conclude that for any  $s$  and  $\nu$ , there is  $\bar{\lambda}$  such that if  $\lambda < \bar{\lambda}$ , then  $\partial \delta^* / \partial \lambda < 0$  and if  $\lambda > \bar{\lambda}$ , then  $\partial \delta^* / \partial \lambda > 0$ .  $\square$

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