International Journal of Industrial Organization 45 (2016) 1–9

Contents lists available at ScienceDirect

International Journal of Industrial Organization

journal homepage: www.elsevier.com/locate/ijio

Fuzzy products

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ARTICLE INFO

Article history:
Received 25 April 2014
Received in revised form 1 December 2015
Accepted 3 December 2015
Available online 10 December 2015

JEL classification:
L11
D81
D86

Keywords:
Contracting
Horizontal differentiation
Monopolistic competition
Experience goods

ABSTRACT

A fuzzy product (FP) has characteristics specified only imprecisely at time of sale. Building fuzziness into its product gives a firm flexibility to exploit favorable supply opportunities that arise between sale and delivery, and so reduce expected costs. While increased competition reduces price, the effect on fuzziness is ambiguous. Socially-optimal fuzziness is characterized. Firms provide goods that are too fuzzy compared to first-best, though entry serves to correct this inefficiency for certain types of goods. Considering competition with a niche good, a FP sells for a lower price, although it captures a larger market share and is more profitable.

1. Introduction

In many markets goods are traded that are only "fuzzily" defined at the moment of purchase—a consumer has imperfect information about some characteristic of the good, with this characteristic only realized after purchase. While the study of *experience goods* is not new, in certain cases a firm may want to deliberately leave some characteristics of a good less than precisely defined *ex-ante* and only define such characteristics *ex-post*, after sale but before delivery.[3] This gives a firm the ability to exploit favorable supply conditions that arise between sale and delivery. In essence, a firm may wish to sell a *lottery* over product characteristics.

To capture this, we (1) introduce the notion of a fuzzy product (FP), and (2) develop a theoretical model to understand the role of product fuzziness in firms' product design decisions. We define a FP as a good or service that (a) is traded in a market (i.e., not something individually contracted between a firm and a buyer) and (b) whose *horizontal* characteristics are imprecisely defined at the moment of sale. Before considering real-world examples, it may be useful to consider an abstract example to make the structure of a FP clear and frame the real world examples. Consider a product that can take on one of three different characteristics: a, b, and c. An example of a FP is then $\{a, c\}$. A consumer can buy a FP in the market today with the understanding that tomorrow they will receive a good of either variety a or variety c. The variety delivered will depend on whichever variety turns out to be cheapest before delivery.

Example 1. *When a condominium is bought "off plan" builders routinely make adjustments during construction. The buyer is entitled to a home of agreed square-footage on a particular lot, but within that envelope the builder has much flexibility. In a default building contract in the state of Florida, for example, "[t]he builder retains the option to re-site the home on the lot, reverse the floor plan, modify room dimensions or exterior elevations, change amenities, substitute appliances and other buyer selections, and add to, modify or delete community features and facilities."* (Bowie, 2014, *para. 3*).

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[1] Heyes holds a Tier 1 Canada Research Chair and acknowledges financial support from the CRC program. We are grateful to Sandeep Kapur, Peth Tuppe, seminar participants at UCL and Oxford, two referees and an editor of this journal for constructive suggestions. Errors are ours.

[2] Martin acknowledges financial support from SSHRC through its Canada Graduate Scholarships program.

[3] This paper does *not* offer a model of experience goods in the usual sense; the distribution of characteristics is known to the buyer before purchase. Models of experience goods typically assume that the seller has perfect (or at least better) information regarding these characteristics and the resulting asymmetry of information causes incentive problems which have been much studied (e.g., Akerlof, 1970). In our paper, the characteristics of the good that is finally delivered are not known by either buyer or seller until after the transaction. Both buyers and sellers do, however, have rational prior distributions over them. Further those prior distributions coincide (there is at no asymmetry of information between buyer and seller).

http://dx.doi.org/10.1016/j.ijindorg.2015.12.003
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Example 2. *Home-delivered organic fruit and vegetable boxes have become popular in many cities. Suppliers offer to send a seasonally appropriate mixture of produce, determined week to week according to what is in good supply in the local market. This means that " … you don't know what you'll get each week and you might end up with veg you've never heard of (or know you don't like) let alone know how to cook."* (Blackmore, 2013, *para. 4*).

Example 3. *Package holidays bundle tickets, hotel nights, tours and other elements but " … tour operators not only allow themselves the right to alter flight times but can also change your hotels, resorts, itinerary and the airport from which you're flying. And that's after your holiday is booked and paid for."* (Butler, 2012, *para. 2*). *The article goes on to observe that the most frequent adjustments are to flight times and departure airports, resulting from " … an operator trying to cut costs through 'consolidation', the term used when passengers on two planes flying to the same destination and departing on the same day, are put on a single flight." In addition, travel operators increasingly sell packages—particularly for discount and last minute bookings—that explicitly leave unspecified the resort, hotel or other details but simply constrain it within broad parameters (3 star, Majorca, family-oriented resort, accommodation allocated on arrival).*

Example 4. *When subscribing to a magazine, the reader knows the general topic of the publication, such as "sports" or "baseball," but the content of any particular issue is only known after delivery. A reader purchasing a baseball magazine knows that the articles within will be about baseball, but for which teams or players the articles relate are not known at the time of subscription. Certain magazines also have a broader scope than others; a sports magazine covers a broader range of stories than a baseball magazine. A sports magazine may have an article about baseball, but a baseball magazine will have one for sure.*

In each example the supplier leaves some "wiggle room" by defining inexactly what characteristics the delivered product will embody. This fuzziness allows the supplier to adjust what is actually delivered in a way that minimizes cost. In each example, a consumer purchasing a FP is purchasing a lottery over the horizontal characteristics of a good. In the context of a model of horizontal product differentiation, an ice cream vendor chooses a neighborhood of the beach to place his stand, selecting the cheapest location in that neighborhood to setup on any particular day.

In defining a FP, we restrict attention to variety over characteristics that are horizontal rather than vertical in nature. When discussing a FP we are not talking about variants that are generically "better" or "worse" than one another. While this distinction is not strictly necessary, the rationale is twofold. First, horizontal characteristics seem more interesting. The four examples given all relate to *horizontal* characteristics where quality is held fixed. In Example 2, the vegetable box may end up containing aubergine or may end up containing squash. One is not better or worse than the other – they are simply different – and there is no particular reason to suppose consumers will disproportionately favor one over the other.[4] In Example 3, the hotels are of the same quality but differ horizontally – for example by being located at different parts of a holiday island – between which consumers' preferred types can be expected to vary. Second, there is already a literature that considers contracting over quality. The interest here lies in contracting over variety and abstracting from quality allows for a sharper focus on this point.[5]

The literature on moral hazard and quality in bilateral contracts (e.g., Allen and Lueck, 1995; Baiman et al., 2000) is related but distinct to the analysis here. We envision the contract as defined over variety, not quality, and firms do not face a moral hazard problem. The incentive for a firm to deliver a particular variety is aligned for certain types of consumers. Similarly, the literature on experience goods is concerned with asymmetric information, either moral hazard (Riordan, 1986; Bagwell and Riordan, 1991) or adverse selection (Akerlof, 1970). While a FP shares a key feature with an experience good – the consumer only learns the product's type *after* purchase – the incentive for a firm to minimize cost does not affect consumers uniformly. Simply, there is no asymmetry of information with a FP: the firm is in effect selling a lottery over variety, and the consumer is willingly purchasing this lottery. The literature on "confusopoly," both theoretical (e.g., Gabaix and Laibson, 2006; Chioveanu and Zhou, 2013) and empirical (e.g., Simmons and Lynch, 1991; Brown et al., 2010), is again related but distinct. Agents here have no bounded rationality and there is no scope for a firm to confuse consumers with complex products, or to hide product characteristics. The terms of the lottery that is on sale are equally and fully understood by both buyer and seller. Lastly, with respect to product differentiation, the model here can be seen as a generalization of the standard model of horizontal product differentiation (e.g., Salop, 1979) but with a product being defined by an interval rather than a point.

With respect to results, firms charge a mark-up over marginal cost, with prices as strategic complements. Product fuzziness, however, can be either a strategic substitute or complement, depending on the type of good. For goods where a firm directly incurs the cost of production (e.g., the vegetable box example), fuzziness is a strategic complement, whereas for firms that contract supply (e.g., a magazine), fuzziness is a strategic substitute. The competitive equilibrium features products that are fuzzier than socially optimal, although increased competition can increase product fuzziness if price is sufficiently responsive to competition for goods that behave as strategic substitutes in fuzziness. Otherwise competition reduces product fuzziness. Niche products (those that are precisely defined) command a higher price than fuzzy products, but account for a smaller share of the market and are less profitable.

The remainder of the paper proceeds as follows: Section 2 presents the basic model; Section 3 gives normative results; Section 4 provides a number of extension; Section 5 concludes.

2. Basic model

2.1. Products

Consider a good that can be one of a number of horizontally-differentiated types. While different varieties of a good can be of different quality (e.g., both horizontally and vertically differentiated), the model here treats variety as purely horizontal. The product characteristic space $V \in \mathbb{R}^2$ is the circumference of a circle with diameter $1/\pi$. The circumference of a circle was pioneered by Salop (1979) as a space within which to analyze product differentiation and is commonly used by economists to model variety.

[4] To capture this in the model we endow consumers with horizontally-differentiated preferences but assume them to be uniformly distributed over ideal points. In the binary setting here this amounts to an assumption that half of consumers prefer aubergines, half prefer squash. This seems a natural assumption.

[5] A haircut is an example of something that is *not* a FP. When purchasing a haircut, the cut received is (presumably) "centered" on the consumer's preferred specification. Barbers, however, vary in the precision with which they move their scissors so the resulting cut is noisy around the consumer's preferred specification. But the amount of noise is clearly a measure of *vertical* quality: a higher quality barber takes more care and delivers a haircut that is (probabilistically) closer to that desired.
Note that for a FP to make sense in a vertical setting, it must be possible that the *ex-post* cost of production for a high quality variant is lower than the cost for a low quality variant. Otherwise the model reduces to one of contracting over quality.

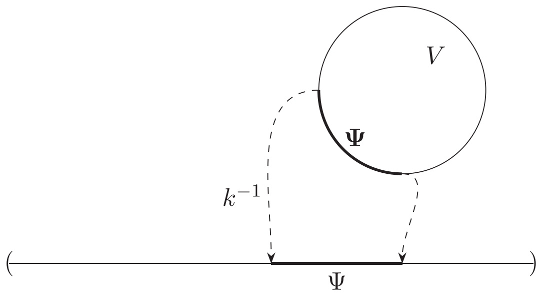

Fig. 1. Mapping the issue space V into the real line.

Definition 1. *A precisely defined product (PDP) $\boldsymbol{\psi} \in V$ is a point in the product space.*

A PDP corresponds to the usual notion of a product in models of horizontal product differentiation.

Definition 2. *A fuzzy product (FP) $\boldsymbol{\Psi} \subset V$ is a compact, connected arc of the product space along with a probability density f_ψ on $\boldsymbol{\Psi}$.*

A FP generalizes the usual notion of a product. A firm that sells a FP *ex-ante* is then selling a contract to deliver a PDP *ex-post*. At the moment of purchase a consumer does not know the particular variety that will be delivered, but rather knows the set of possible characteristics that might be (to continue our earlier example, she may know that the vegetable box will contain either an aubergine or a squash).

To simplify the analysis, it will be useful to "open up" the circle and map V onto the unit interval. Let k: $[0, 1) \rightarrow V$ be given by $k(x) = \frac{1}{2\pi}(\cos(2\pi x), \sin(2\pi x))$. It is straightforward to show that k is a continuous bijection such that k^{-1} maps V onto the unit interval while preserving distance. (To be precise, k^{-1} projects V onto $[0, 1)$.) When working with the unit interval, a PDP will be given by $\psi = k^{-1}(\boldsymbol{\psi})$ and a FP will be given by $\Psi = k^{-1}(\boldsymbol{\Psi}) = [\ell - \frac{w}{2}, \ell + \frac{w}{2}]$, where w is the "width" of the FP centered about a point ℓ.[6] Fig. 1 provides an example.

2.2. Firms

There is a set of n identical firms that can offer a FP. Assume firms operate with a constant returns technology. The unit cost of delivering any particular ψ will vary depending on factors that are realized *ex-post*.[7] Since the contract is made *ex-ante*, the firm delivers the characteristic $\psi_d \in \Psi$ that minimizes unit costs. (An alternative way to think of this is that *ex-post* all but one variety is available and this gives the variety the firm will deliver.) Assume unit costs for all $\psi \in \Psi$ follow the same, independent process so that ψ_d is uniformly distributed on Ψ and that this is common knowledge. Since characteristics are horizontal this is the most natural assumption: no variety is inherently more costly to produce than another variety.

Considering only costs, a firm would prefer greater width as it allows for greater opportunities to choose lower cost varieties: *ex-ante* unit cost $c(w)$ is decreasing in w. In particular, assume $c'(w) < 0$ and $c''(w) > 0$ for $w \in (0, 1)$, $\lim_{w \rightarrow 1} c(w) = \underline{c}$, $c(0) = \overline{c}$, with $\overline{c} > \underline{c}$, $\lim_{w \rightarrow 0} c'(w) < 0$, and $\lim_{w \rightarrow 1} c'(w) = 0$.[8] The technology of production for fuzziness then exhibits decreasing returns to scale.

2.3. Consumers

Consumers have preferences over characteristics for the good, a consumer of type $\theta \in k^{-1}(V)$ having a preferred PDP $\psi = \theta$. Consumer types are uniformly distributed on the characteristic space and are risk neutral.

Consumers have unit demand. The willingness to pay for a PDP ψ by consumer type θ is

$$u(\psi; \theta) = \overline{u} - (\psi - \theta)^2;$$

willingness to pay for the consumer's preferred variety is $\bar{u}$ and the value of a variety further from the consumer preferred variety decreases at an increasing rate.[9] Assume $\overline{u} > \overline{c} > \underline{c}$.

A consumer effectively buys a lottery over the horizontal characteristic of the good, or a lottery over PDPs. When buying a FP Ψ, with price p, expected utility for consumer type θ is

$$\mathbb{E}[u(w, \ell, p; \theta)] = \int_{\ell - \frac{w}{2}}^{\ell + \frac{w}{2}} \left[\overline{u} - (x - \theta)^2\right] f_\psi(x) dx + v(y - p),$$

where f_ψ is the (continuous) probability density defined on Ψ. The function v: $\mathbb{R}_{++} \rightarrow \mathbb{R}_+$ is utility from consuming the numeraire. Assume $v \in C^2$, $v(y) > 0$, $\lim_{p \rightarrow y} v(y - p) = -\infty$, $v'(z) > 0$ and $v''(z) < 0$ for all $z \in \mathbb{R}_{++}$, and $\lim_{p \rightarrow y} v'(y - p) = \infty$.

2.4. Sequence of events

The sequence of events is the following.

1. Firms simultaneously design a FP Ψ and offer it for sale at price p.
2. Consumers decide which (if any) FP to purchase.
3. Unit costs for all $\psi \in \Psi$ are realized and observed by firms. Firms deliver ψ_d to consumers who purchased the FP.

The solution concept is sub-game perfect Nash and only symmetric equilibria are considered.

[6] It should be clear that $\ell \in V$ can always be chosen so that k^{-1} is continuous at ℓ.

[7] Returning to the vegetable box example, in a certain week a particular mix of vegetables may be more expensive than another. When the contract to provide the box is defined, the price of a particular mix is not known. Instead, prices are updated week to week depending on local conditions.

[8] Consider w as analogous to the sample size (sampling with replacement) n of a random variable X distributed uniformly on the support $[0, \alpha]$. Define $M(n)$ as the expected minimum value of X for a given sample size n. For any realization x, the probability density of the minimum value is $\frac{n}{\alpha}(1 - \frac{x}{\alpha})^{n-1}$. Integrating over all realizations of X gives

$$M(n) = \int_0^\alpha \frac{nx}{\alpha}\left(1 - \frac{x}{\alpha}\right)^{n-1} dx = \frac{\alpha}{(n+1)}.$$

Extending M to $\mathbb{R}_+$, it follows that $M'(n) = \frac{-\alpha}{(n+1)^2} < 0$ and $M''(n) = \frac{2\alpha}{(n+1)^3} > 0$, with $\lim_{n \rightarrow 1} M'(n) < 0$ and $\lim_{n \rightarrow \infty} M'(n) = 0$. The map $l(x) = (\frac{1+x}{1-x})^2$ is a strictly increasing surjection from $[0, 1)$ to $[1, \infty)$, and so l^{-1} is suitable for interpreting n in terms of w.

To show that X uniform is not necessary, suppose that X is exponential with decay λ on $(0, \infty)$. Reasoning as before, the density for the minimum value is $\lambda n \exp(-\lambda n x)$. The expected minimum value is then

$$M(n) = \lambda n \int_0^\infty x \exp(-\lambda n x) dx = \frac{1}{\lambda n}.$$

Again $M'(n) < 0$ and $M''(n) > 0$.

[9] An alternate approach, although not common in the product differentiation literature, employs a spatial discount function. Analogous to temporal discount, willingness to pay could be given by

$$\tilde{u}(\psi, \theta) = \overline{u} \exp(|\psi - \theta|),$$

similar to Baron (2010). Taking such an approach gives an ambiguous relationship between demand and product fuzziness. Incorporating competition between firms, however, becomes intractable and so only a monopolist can be considered. The qualitative results of this model with respect to product design do not depend on the particular form of $\tilde{u}(\psi, \theta)$ and remain unchanged with quadratic loss. In any case, using quadratic loss is standard and keeps our model comparable to existing literature on product differentiation.

2.5. Stages 2 and 3

As already mentioned, since *ex-post* unit costs for each variety follow an independent and identically distributed process, both firms and – importantly – consumers, take ψ_d to be uniformly distributed on Ψ[10]; consumers are rational and forward looking. Expected utility to a consumer of type θ from purchase of the FP Ψ at price p is then

$$\begin{aligned}\mathbb{E}[u(w,\ell,p;\theta)] &= \frac{1}{w}\int_{\ell-\frac{w}{2}}^{\ell+\frac{w}{2}} \bar{u}-(x-\theta)^2 dx + v(y-p)\\ &= \bar{u}-(\ell-\theta)^2 - aw^2 + v(y-p),\end{aligned}$$

with $a = 1/12$ to simplify notation. Suppose there are n firms that offer a FP, each spaced $1/n$ apart on V. To focus on the competitive implications of fuzziness, assume that $\bar{u}$ is sufficiently large to ensure that a consumer will always make a purchase. This last assumption ensures that the market is covered so that firms are competitors and do not act as monopolists. (See Hinloopen and van Marrewijk (1999) for relaxing this assumption.)

In stage 1, firms choose (p, w) simultaneously. If all firms except the ith choose $\overline{p}$ and $\overline{w}$, demand for the ith firm is

$$q_i(p_i, w_i; \overline{p}, \overline{w}) = an\left[\overline{w}^2 - w_i^2\right] + n[v(y-p_i) - v(y-\overline{p})] + \frac{1}{n}$$

if and only if

$$v(y-p_i) - aw_i^2 \geq v(y-\overline{p}) - a\overline{w}^2 - \frac{1}{n^2},$$

and

$$v(y-p_i) - aw_i^2 \leq v(y-\overline{p}) - a\overline{w}^2 + \frac{1}{n^2}. \quad (1)$$

Given the form of demand, product width can be seen as a form of *ex-ante* quality. The ability of a firm to offer a FP then turns a model of horizontal product differentiation into a model of horizontal and vertical product differentiation. Care must be taken, however, in seeing w as completely analogous to quality. First, the effect of increasing w on utility for a consumer type θ depends on θ's location relative to ℓ. To see this, write expected utility as

$$\begin{aligned}\mathbb{E}[u(w,\ell,p;\theta)] &= \bar{u} - \frac{1}{w}\int_{\ell-\frac{w}{2}}^{\ell}(x-\theta)^2 dx - \frac{1}{w}\int_{\ell}^{\ell+\frac{w}{2}}(x-\theta)^2 dx + v(y-p)\\ &= \bar{u} - \frac{aw^2}{2} + \frac{(\theta-\ell)w}{4} - \frac{(\theta-\ell)^2}{2}\\ &\quad - \frac{aw^2}{2} + \frac{(\ell-\theta)w}{4} - \frac{(\theta-\ell)^2}{2} + v(y-p).\end{aligned}$$

Consider some $\theta > \ell$. It follows that

$$D_w\mathbb{E}[u(w,\ell,p;\theta)] = \underbrace{-aw + \frac{\theta-\ell}{4}}_{\text{effect on right}}\underbrace{-aw + \frac{\ell-\theta}{4}}_{\text{effect on left}}.$$

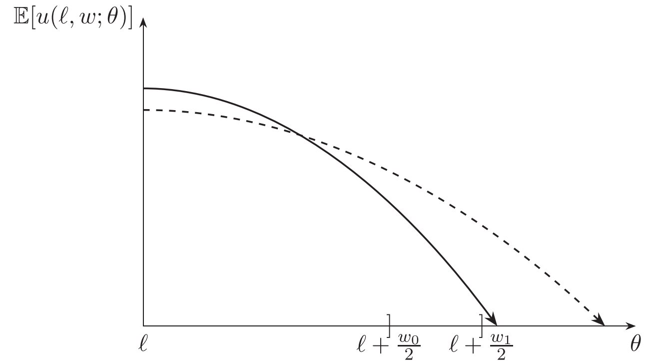

Fig. 2. Effect on utility for consumer types with is $\theta \geq \ell$ from increasing w from w_0 to w_1 while holding domain left of ℓ fixed.

The effect of introducing new varieties on the right of the FP's domain is positive for a consumer type θ if and only if $\theta \geq \ell + \frac{w}{3}$. Only consumer types sufficiently close to ℓ dislike extra width on the right; all other types welcome these new potential varieties. This effect, however, is balanced out exactly by new potential varieties on the left because of quadratic loss, and so the net effect of extra width is to decrease utility. See Fig. 2.

Second, consumers differ in their *ex-post* valuation of w because what is delivered is a horizontally differentiated variety. If w were a measure of quality, all consumers would have the same *ex-post* valuation of w. Lastly, consumers do not vary in their preference for width; in models of vertical quality, it is standard for consumers to vary in their willingness to pay for quality (e.g., Economides, 1993; Motta, 1993).

2.6. Stage 1

Firm i's problem is

$$\max_{p_i, w_i} \pi(p_i, w_i; \overline{p}, \overline{w}) = [p_i - c(w_i)]q_i(p_i, w_i; \overline{p}, \overline{w}).$$

Let

$$S = \left\{(w,p) \in [0,1] \times [\underline{c}, y) : v(y-p_i) - aw_i^2 \geq v(y-\overline{p}) - a\overline{w}^2 - \frac{1}{n^2}\right\}.$$

Lemma 1. *The set S has the following properties: i) non-empty and not a singleton, ii) compact and convex.*

Proof. See Appendix A. □

The set S simply gives the combinations of w and p so that demand for a firm is positive; clearly a firm will design its product so that $(w, p) \in S$. The lemma establishes that this set has desirable properties, making the study of a firm's problem straightforward. (See Fig. 3.)

Ignoring (1) for the moment, a firm's problem is

$$\max_{p_i, w_i \in S} \pi(p_i, w_i; \overline{p}, \overline{w}) = [p_i - c(w_i)]\left[an\left[\overline{w}^2 - w_i^2\right] + n[v(y-p_i) - v(y-\overline{p})] + \frac{1}{n}\right].$$

Since π is continuous and S is compact, there is a solution (p_i^*, w_i^*) to this problem. Note that $(p_i^*, w_i^*) \in \text{int } S$. To see this, observe that

$$\lim_{w_i \to 0} D_{w_i}\pi(p_i, w_i; \overline{p}, \overline{w}) = -\lim_{w_i \to 0} c'(w_i)q(p_i, w_i; \overline{p}, \overline{w}) > 0$$

[10] If a FP were defined over quality the analogous utility function would be

$$\mathbb{E}[u(w,\ell,p;\theta)] = \int_{\ell-\frac{w}{2}}^{\ell+\frac{w}{2}}[\bar{u} + \theta x]f_q(x)dx + v(y-p),$$

where q is quality. While f_ψ uniform is the most reasonable assumption for purely horizontal characteristics, a higher quality PDP should be more costly to produce than a lower quality variant and so f_q uniform is a tenuous assumption. In the case of f_q uniform, $\mathbb{E}[u(w,\ell,p;\theta)] = \bar{u} + \theta\ell + v(y-p)$, and so the model is a standard one of contracting over quality. Similarly, if *ex-post* unit cost is increasing in q, then $f_q(x) = \delta(x-\ell+\frac{w}{2})$, where δ is the delta function. It follows that $\mathbb{E}[u(w,\ell,p;\theta)] = \bar{u} + \theta(\ell-\frac{w}{2}) + v(y-p)$; again the model is a standard one of contracting over quality.

and

$$\lim_{w_i \to 1} D_{w_i}\pi(p_i, w_i; \overline{p}, \overline{w}) = -\frac{4an}{q(p_i, w_i; \overline{p}, \overline{w})} < 0,$$

implying that $w_i^* \in (0, 1)$. (If $w = 1$ does not belong to S, then only the first limit needs to be considerEd.) It follows that it is never optimal to choose (p_i^*, w_i^*) in the boundary on S.

The first-order conditions for an interior solution are

$$\frac{1}{p_i^* - c(w_i^*)} = \frac{nv'(y-p_i^*)}{an\left[\overline{w}^2 - w_i^{*2}\right] + n\left[v(y-p_i^*) - v(y-\overline{p})\right] + \frac{1}{n}} \tag{2}$$

$$\frac{-c'(w_i^*)}{p_i^* - c(w_i^*)} = \frac{2anw_i^*}{an\left[\overline{w}^2 - w_i^{*2}\right] + n\left[v(y-p_i^*) - v(y-\overline{p})\right] + \frac{1}{n}}. \tag{3}$$

Since $\log \circ \pi$ is strictly concave in (p, w), the first-order conditions are also sufficient for a unique maximum. Combining (2) and (3) gives

$$\frac{-c'(w_i^*)}{w_i^*} = \frac{2a}{v'(y-p_i^*)}. \tag{4}$$

Remark 1. *There is a C^1 function $w_i^*: (\underline{c}, y) \to (0, 1)$ such that $Dw_i^*(p_i^*) > 0$.*

Proof. Let $g(w) = \frac{-c'(w)}{w}$. First note that g is one-to-one and onto $\mathbb{R}_{++}$. To see this, note that

$$g'(w) = \frac{-c''(w)w + c'(w)}{w^2} < 0,$$

$\lim_{w \to 0} g(w) = \infty$, and $\lim_{w \to 1} g(w) = 0$. For any $p \in (\underline{c}, y)$ there is then a unique w such that (4) holds. Furthermore, since g is a continuous bijection on an interval, it is a homeomorphism. It follows that $w_i^* \in C^1$ and

$$Dw_i^*(p_i^*) = -\frac{2av''(y-p_i^*)}{c''(w_i^*)w_i^* - c'(w_i^*)} \frac{w_i^{*2}}{v'(y-p_i^*)^2} > 0.$$

□

The interpretation of Remark 1 is simple: the optimal choice of w is positively related to the optimal choice of p. If a firm charges a higher price for a FP, then it also designs a fuzzier product.

From (4), if there is a symmetric Nash equilibrium in p then there is a symmetric Nash equilibrium in (p, w). Furthermore, for any symmetric equilibrium, the constraint (1) does not bind. In looking for a symmetric equilibrium, it is sufficient to consider the reaction function given by (2) and (3).

Proposition 1. *There is a unique, symmetric Nash equilibrium (p^*, w^*) such that*

$$p^* = c(w^*) + \frac{1}{n^2 v'(y-p^*)}$$

and

$$\frac{-c'(w^*)}{w^*} = \frac{2a}{v'(y-p^*)}$$

with each firm location equidistant on V. Furthermore, $p_i^*(\overline{p})$ and $w_i^*(\overline{w})$ are both strictly increasing in a neighborhood of the Nash equilibrium; p and w are strategic complements.

Proof. See Appendix A. □

As in models of horizontal product differentiation, prices are strategic complements and a firm charges a mark-up over marginal cost

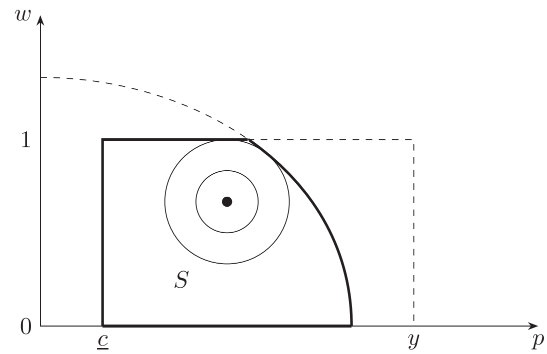

Fig. 3. The set S along with iso-profit contours.

(Salop, 1979). Note also that increasing competition (i.e., increasing n) does not directly affect w^*; instead competition influences fuzziness indirectly through price.

Proposition 2. *Both price and product fuzziness are strictly decreasing in n. That is, there are C^1 functions $p^*: \mathbb{R}_+ \to (\underline{c}, y)$ such that $Dp^*(n) < 0$ and $w^*: \mathbb{R}_+ \to (0, 1)$ such that $Dw^*(n) < 0$.*

Proof. From Proposition 1 and the implicit function theorem,

$$Dp^*(n) = \frac{-\frac{2v'(y-p^*)}{n^3}}{1 - c'(w^*)Dw^*(p^*) - \frac{v''(y-p^*)}{n^2 v'(y-p^*)^2}} < 0.$$

From Remark 1,

$$Dw^*(n) = Dw^*(p^*)Dp^*(n) < 0.$$

□

Proposition 2 establishes that increasing competition acts to decrease price, as expected, and product fuzziness. Intuitively, as competition increases, a firm can attract customers by decreasing price and designing a less fuzzy product, although the former effect must be balanced against the increasing unit cost from reducing fuzziness.

Fig. 4 gives an example of an equilibrium with four firms. Note that in this case a firm may end up delivering a variety that is not the ideal variety of *any* of its customers (since the interval of purchasing consumer types is wider than the FP width). In general, however, this need not

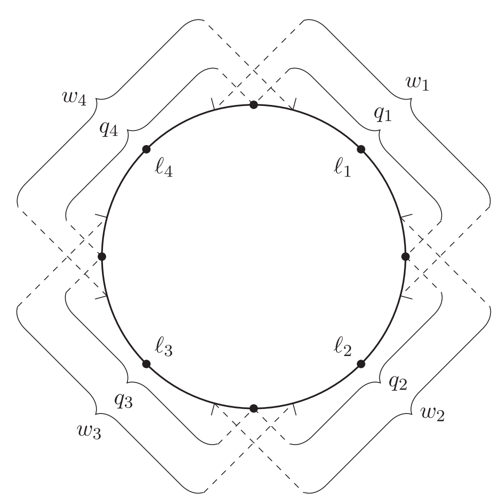

Fig. 4. Equilibrium with $n = 4$ and $w > \frac{1}{n}$.

be true in equilibrium and depends on the particular form of c and the number of firms competing.

3. Normative implications of competition

Ex-ante expected social welfare from designing a FP with n firms is

$$W(w;n)=2n\int_0^{\frac{1}{2n}}\left[\left(\frac{1}{w}\int_{-\frac{w}{2}}^{\frac{w}{2}}\left[\bar{u}-(x-\tau)^2\right]dx\right)+v(y-c(w))\right]d\tau$$
$$=\bar{u}-aw^2-\frac{a}{n^2}+v(y-c(w)).$$

A planner's problem is then

$$\max_w \bar{u}-aw^2-\frac{a}{n^2}+v(y-c(w)).$$

Since W is strictly concave and there must be a point $w\in(0,1)$ that solves this problem, the optimal width for a FP w^{**} is given by

$$\frac{-c'(w^{**})}{w^{**}}=\frac{2a}{v'(y-c(w^{**}))}. \quad (5)$$

Note that the first-best allocation of goods is not precisely defined. Even though consumers do not benefit *ex-ante* from fuzzy products, reducing the unit cost of production produces some socially optimal benefit to fuzziness.

Proposition 3. *The competitive width is greater than first-best: $w^{**}<w^*$.*

Proof. From (4) and (5), $w^{**}\leq w^*$ if and only if

$$\frac{2a}{v'(y-c(w^{**}))}\geq\frac{2a}{v'(y-p^*)}$$
$$p^*\geq c(w^{**}).$$

Now let $p^*=c(w^*)$. It follows that $c(w^*)=c(w^{**})$ since $w^*<w^{**}$ if $c(w^*)>c(w^{**})$, a contradiction. Since $p^*>c(w^*)$, this implies that $w^*>w^{**}$ from Remark 1. □

The intuition for this point is simple: firms have too much incentive to design fuzzy products. By designing a fuzzier product, a firm makes its product less desirable for consumers. Even though expected production costs decrease, firms have *too much* incentive to trade-off reduced desirability from consumers for reduced cost. Interpreting product fuzziness as an inverse measure of quality, this result is analogous to that in the product differentiation literature where goods are differentiated with respect to both location and quality (Brekke et al., 2010). Interestingly, this is also analogous to the equilibrium that emerges from a model of experience goods when consumers have perfect information (Riordan, 1986).

4. Extensions

There are several interesting extensions to consider for this model. Endogenous entry, fixed cost of fuzziness, linear marginal utility of income, and competition between a FP and a PDP are considered in turn.

4.1. Endogenous entry

Consider an entry stage for the game so that the sequence of events becomes the following.

1. Firms decide whether or not to enter the market.
2. Firms that have entered simultaneously design a FP Ψ and offers it for sale at price p.
3. Consumers decide which (if any) FP to purchase.
4. Unit costs for all $\psi\in\Psi$ are realized and observed by firms. Firms deliver ψ_d to consumers who purchased the FP.

A firm will enter and design a FP provided doing so generates positive profit. Let F be a fixed cost to entry. Firms will enter until profit is exhausted so that the free entry condition is

$$p^*(n)=c(w^*(n))+nF.$$

Lemma 2. *Assume $F<\frac{p^*(2)-c(w^*(2))}{2}$. There is a unique $n^*>2$ such that $p^*(n^*)=c(w^*(n^*))+n^*F$.*

Proof. Let $k(n)=p^*(n)-c(w^*(n))-nF$. By assumption $k(2)>0$. From Proposition 2, $k'(n)<0$ with $\lim_{n\to\infty}k(n)=-\infty$. Note that k is also continuous in n on $[2,\infty)$ since its extension to $\mathbb{R}_+$ is C^1. Therefore k is one-to-one and onto $(-\infty,k(2)]$ and so there is a unique $n^*>2$ such that $k(n^*)=0$. □

In terms of normative implications, first-best width conditional on the free entry number of firms is given by

$$\frac{-c'(w^{**})}{w^{**}}=\frac{2a}{v'(y-c(w^{**})-n^*F)}.$$

Proposition 4. *With free entry of firms fuzziness is chosen optimally: $w^*=w^{**}$.*

Proof. Both w^* and w^{**} are the solutions to

$$\frac{-c'(w)}{w}v'(y-c(w)-n^*F)=2a.$$

The function $\frac{-c'(w)}{w}v'(y-c(w)-n^*F)$ is strictly decreasing and onto $(0,\infty)$ and so there is a unique solution with $w^*=w^{**}$. □

Although competition does not generally produce a first-best allocation of goods, free entry of firms corrects this so that the decentralized solution is first-best. Intuitively, increasing competition forces firms to design more precise products and corrects the incentive to design products that are too wide (fuzzy).

It should be noted that competition produces the first-best allocation of goods conditional on the number of firms. In looking for the optimal number of firms and optimal product fuzziness, this result need not hold. In general the relation between w^* and w^{**} and n^* and n^{**} is ambiguous without more structure on v and c.

4.2. Fixed cost of fuzziness

An alternate approach to thinking about FPs sees firms engaging in a contract with a supplier to supply a variety *ex-post*. A firm does not produce a good *per se* but instead packages the product of some primary producer. Let there be a continuum of producers, each making a single variety and charging a fixed cost to supply the demand of a firm. In searching for the cheapest variety, a firm no longer supplies the variety with the smallest unit cost, but rather supplies the variety with the cheapest contract. An example of such a good is a magazine: the cost of producing a single magazine does not depend on the type of story within, but there is a fixed cost to purchasing the story. In designing a magazine with a broader interest, the firm makes available topics that *ex-post* may have cheaper stories than other topics.

In terms of the model, the cost function c is now a fixed rather than marginal cost, with marginal cost set to zero without loss of generality; everything else from Section 2 remains the same. A firm's problem is now

$$\max_{p_i, w_i} \pi(p_i, w_i; \overline{p}, \overline{w}) = p_i\left[an\left[\overline{w}^2 - w_i^2\right] + n[v(y-p_i) - v(y-\overline{p})] + \frac{1}{n}\right] - c(w).$$

Lemma 1 (with $\underline{c} = 0$ in the definition of S) again ensures there is a solution (p_i°, w_i°) to this problem. As before, $(p_i^\circ, w_i^\circ) \in \text{int } S$. The first-order conditions for an interior solution are

$$p_i^\circ nv'(y-p_i^\circ) = an\left[\overline{w}^2 - w_i^{\circ 2}\right] + n\left[v(y-p_i^\circ) - v(y-\overline{p})\right] + \frac{1}{n} \tag{6}$$

$$\frac{-c'(w_i^\circ)}{w_i^\circ} = 2anp_i^\circ. \tag{7}$$

Since π is concave in (p, w), these conditions are sufficient for a unique maximum.

Proposition 5. *There is a unique, symmetric Nash equilibrium in (p°, w°) with*

$$p^\circ = \frac{1}{n^2 v'(y-p^\circ)}$$

and

$$\frac{-c'(w^\circ)}{w^\circ} = 2anp^\circ.$$

with each firm location equidistant on V. Furthermore, $p_i^\circ(\overline{p})$ and $w_i^\circ(\overline{w})$ are strictly increasing and strictly decreasing in a neighborhood of the Nash equilibrium; p is a strategic complement and w is a strategic substitute.

Proof. See Appendix A. □

As before, prices are strategic complements, although fuzziness is now a strategic substitute. Intuitively, if another firm increases its product fuzziness, this increases a firm's price since the other firm is less competitive, thus decreasing product fuzziness to make its good more appealing to consumers.

Proposition 6. *Price is strictly decreasing in n, but fuzziness is increasing in n if and only if price is elastic in competition. That is, there are C^1 functions $p^\circ: \mathbb{R}_+ \to (0, \infty)$ such that $Dp^\circ(n) < 0$ and $w^\circ: \mathbb{R}_+ \to (0, 1)$ such that $Dw^\circ(n) > 0$ if and only if $-Dp^\circ(n)\frac{n}{p^\circ} > 1$, or equivalently $\frac{-v''(y-p^\circ)p^\circ}{v'(y-p^\circ)} < 1$.*

Proof. From the implicit function theorem and (6) and (7),

$$Dp^\circ(n) = -\frac{2}{n^3[v'(y-p^\circ) - p^\circ v''(y-p^\circ)]} < 0$$

and

$$Dw^\circ(n) = -\frac{2aw^\circ[p^\circ + nDp^\circ(n)]}{2anp^\circ + c''(w^\circ)}.$$

It follows that $Dw^\circ(n) > 0$ if and only if

$$-Dp^\circ(n)\frac{n}{p^\circ} > 1$$
$$-p^\circ v''(y-p^\circ) < v'(y-p^\circ).$$
□

Contrasting Proposition 6 with Proposition 2, increasing competition between firms can now increase the fuzziness of products. Intuitively, competition now has a direct, negative effect on product fuzziness. For competition to increase fuzziness, the positive effect from decreasing price must be sufficiently strong to outweigh the direct effect of competition.

Proposition 7. *$p^\circ < p^*$ and $w^\circ > w^*$ if $v''(y-p)$ is sufficiently small in absolute value for any $p \in (p^\circ, p^*)$.*

Proof. See Appendix A. □

In terms of interpretation, $v''(y-p)$ is small when y is large relative to p so that the marginal utility of income is insensitive to price. For goods that have a small price relative to income, contracting over a supplier produces fuzzier products than contracting over inputs. Keeping with the examples in the introduction, the model implies that a vegetable box is more precisely defined than a magazine.

With respect to normative implications, the first-best fuzziness $w^{\circ\circ}$ is given by

$$-\frac{c'(w^{\circ\circ})}{w^{\circ\circ}} = \frac{2a}{nv'(y-nc(w^{\circ\circ}))}.$$

Proposition 8. *The first-best width is smaller than the competitive width: $w^{\circ\circ} < w^\circ$.*

Proof. The proof is analogous to that of Proposition 3.

As before, competition produces products that are too fuzzy. From Proposition 6, however, increasing competition need not correct the problem. When price is sensitive to competition, increased competition can lead to a worse allocation of goods.

4.3. Linear utility of income

As a special case, consider $v(y-p) = y-p$. It is straightforward to modify Lemma 1 so that, following the previous argument, the unique Nash equilibrium when c is a marginal cost is given by

$$p^* = c(w^*) + \frac{1}{n^2}$$
$$\frac{-c'(w^*)}{w^*} = 2a.$$

From (5), $w^* = w^{**}$: firms choose the optimal product fuzziness.

When c is a fixed cost,

$$p^\circ = \frac{1}{n^2}$$
$$\frac{-c'(w^\circ)}{w^\circ} = \frac{2a}{n}.$$

Again, w is chosen optimally. Consistent with the previous section, $p^* > p^\circ$ and $w^* < w^\circ$, with w° increasing with competition.

4.4. Competition with a PDP

Consider the case of competition between two firms, with one offering a FP (firm 0) and the other a PDP (firm 1). An example of this would be a sports magazine competing with, say, as baseball magazine. This case can be seen when c is a marginal cost by setting $\overline{w} = 0$ in (2) and (3).[11] To keep the problem tractable, assume $v(y-p) = y-p$. For the firm offering the FP,

$$-\frac{c'(w_0^*)}{w_0^*} = 2a; \tag{8}$$

competition with PDP does not alter product fuzziness. (This does not depend on whether fuzziness is chosen before price or

[11] The case when c is a fixed cost is difficult to solve without placing explicit structure on c.

not.) For price,

$$p_0^* = \frac{c(w_0^*) + p_1 - a w_0^{*2}}{2} + \frac{1}{4}.$$

For the firm offering the PDP,

$$p_1^* = \frac{\bar{c} + p_0 + a w_0^2}{2} + \frac{1}{4}.$$

The Nash equilibrium prices (p_0^e, p_1^e) are then given by

$$p_0^e = \frac{2c(w_0^*) + \bar{c} - a w_0^{*2}}{3} + \frac{1}{2}$$
$$p_1^e = \frac{c(w_0^*) + 2\bar{c} + a w_0^{*2}}{3} + \frac{1}{2}.$$

Proposition 9. *The firm offering the PDP charges a higher price than the firm offering a FP but has a smaller market share and profit.*

Proof. See Appendix A. □

The above gives what is expected from a niche good: it appeals to a smaller set of consumers and is less profitable to provide, even if it sells at a higher price.

5. Conclusion

In this paper we develop a model of fuzzy products and analyze its positive and normative implications. While the model shares a similar structure to models of horizontal and vertical product differentiation (e.g., Economides, 1993; Brekke et al., 2010) and experience goods (e.g., Riordan, 1986), this structure comes out of a model of only horizontal differentiation. By having firms sell lotteries over product characteristics, the model can be seen as generalizing the standard model of horizontal differentiation (Salop, 1979).

While firms provide goods that are too vague in equilibrium relative to first-best, whether increased competition reduces or further increases fuzziness depends on the nature of the good. For goods produced by the seller, competition reduces fuzziness because of the impact of competition on price, and with free entry into the market first-best fuzziness can be achieved. When the seller contracts provision of the good from a producer, increased competition now exerts a direct influence on a firm's decision to design a fuzzy product and acts to increase product fuzziness when market price is insensitive to increased competition. In this latter case, at least for goods that represent a small fraction of consumer income, the good is fuzzier in equilibrium than if the seller produced the good itself. An integrated firm has less incentive to produce vague products than does a firm with a separate supply chain. Comparing a fuzzy product with a niche good, firms that offer a fuzzy product are more profitable, reaching a wide set of consumers, even though the fuzzy product sells for less. If a firm is able to sell a fuzzy product, it will wish to do so, even in a competitive setting.

One point abstracted from in the analysis of fuzzy products is the possibility of bargaining between parties. In the context of an off-plan condominium (Example 1), the buyer may be able to bargain with the seller over the set of possible product characteristics. While not treated here, this is an interesting point for future research.

Appendix A

Proof of Lemma 1. To simplify the notation, let $f(w, p) = v(y - p) - aw^2$.

i) For any $(\overline{w}, \overline{p}) \in [0,1] \times [\underline{c}, y)$, $(w, p) = (\overline{w}, \overline{p})$ belongs to S. Since f is continuous, there is a neighborhood N about $(\overline{w}, \overline{p})$ such that $f(x, y) > f(\overline{w}, \overline{n}) - \frac{1}{n^2}$ for all $(x, y) \in N$.

(ii) Let

$$T = \left\{ (w, p) \in \mathbb{R}_+^2 \colon v(y - p_i) - a w_i^2 \geq v(y - \overline{p}) - a\overline{w}^2 - \frac{1}{n^2} \right\}.$$

Since f is continuous, every upper contour set is closed. Let $t = f(\overline{w}, \overline{p}) - \frac{1}{n^2}$. Since v is one-to-one and maps onto $(-\infty, v(y)]$, $v(y) > t$, there is a unique $z > 0$ such that $v(z) = t$. Now $S = T \cap [0,1] \times [\underline{c}, z]$ is compact since T is closed and $[0,1] \times [\underline{c}, z]$ is compact. Since f is concave, every upper contour set is convex, and so S is convex. □

Proof of Proposition 1. Begin with existence and uniqueness. From (2), the goal is to find a unique fixed point for

$$p = c(w) + \frac{1}{n^2 v'(y - p)},$$

where

$$w = g^{-1}\left(\frac{2a}{v'(y - p)} \right).$$

Let

$$h(p) = c(w(p)) + \frac{1}{n^2 v'(y - p)}.$$

From Remark 1, $h'(p) < 0$ for all $p \in (\underline{c}, y)$. Clearly $w(\underline{c}) < 1$ so that $h(\underline{c}) > \underline{c}$. Since

$$\lim_{p \to y} \frac{2a}{v'(y - p)} = 0,$$

$w(p) \to 1$ as $p \to y$. Therefore

$$\lim_{p \to y} h(p) = \underline{c}.$$

It follows that $h(p) - p$ is one-to-one and onto $(\underline{c} - y, h(\underline{c}) - \underline{c})$. Therefore there is a unique p such that $p = h(p)$. From (4)

$$\frac{-c'(w^*)}{w^*} = \frac{2a}{v'(y - p^*)}.$$

With respect to strategic complementarity, conditions (2) and (3) along with the implicit function theorem give that

$$Dp_i^*(p^*) = \frac{2an(p^* - c(w^*)) + \frac{c''(w^*)}{n}}{|J|} > 0$$

and

$$Dw_i^*(w^*) = \frac{(p^* - c(w^*)) v''(y - p^*) c'(w^*)}{|J|} > 0,$$

nothing that $|J| > 0$ since the Jacobian for (2) and (3) is negative definite. From the mean value theorem, there is a neighborhood N about (p^*, w^*) such that p_i^* and w_i^* are strictly increasing for all $(\overline{p}, \overline{w}) \in N$. □

Proof of Proposition 6. Existence and uniqueness follows directly from setting $\overline{p} = p_i^\circ$ and $\overline{w} = w_i^\circ$ in (6) and (7), and noting that $pv'(y - p)$ is a bijection from $(0, y)$ to $(0, \infty)$.

From (6) and (7), the implicit function theorem gives that

$$Dp_i^\circ(p^\circ) = \frac{v'(y - p^\circ) n (2anp^\circ + c'(w^\circ))}{|J|} > 0$$

and

$$Dw_i^{\circ}(w^{\circ}) = \frac{-(2anw^{\circ})^2}{|J|} < 0,$$

noting that $|J| > 0$ since the Jacobian of (6) and (7) is negative definite. □

Proof of Proposition 7. Fix w^*. Then p^* is the solution to

$$[p - c(w^*)]v'(y-p) = \frac{1}{n^2}$$

and p° is the solution to

$$pv'(y-p) = \frac{1}{n^2}.$$

Now both $pv'(y-p)$ and $[p-c(w^*)]v'(y-p)$ are strictly increasing with $pv'(y-p) > [p-c(w^*)]v'(y-p)$ for any p. It follows that $p^* > p^{\circ}$.

Now w^* is the solution to

$$g(w) = \frac{2a}{v'(y-p^*)}$$

and w° is the solution to

$$g(w) = \frac{2a}{nv'(y-p^{\circ})}.$$

Now $w^* < w^{\circ}$ if and only if

$$\frac{2a}{nv'(y-p^{\circ})} < \frac{2a}{v'(y-p^{\circ})}$$
$$\frac{v'(y-p^*)}{v'(y-p^{\circ})} < n. \quad (9)$$

From the mean value theorem, there is an ε such that (9) is true if $v''(y-p) < \varepsilon$ for some $p \in (p^{\circ}, p^*)$. □

Proof of Proposition 9. Clearly

$$p_1^e - p_0^e = \frac{2aw_0^{*2} + \bar{c} - c(w_0^*)}{3} > 0.$$

Demand for firm 0 is

$$q_0(p_0^e, w_0^*; p_1^e) = p_1^e - p_0^e - aw_0^* + \frac{1}{2} = \frac{-aw_0^{*2} + \bar{c} - c(w_0^*)}{3} + \frac{1}{2}.$$

Since c is strictly convex,

$$\bar{c} - c(w) > -c'(w)w$$

for any w. From (8), $-c'(w_0^*)w_0^* = 2aw_0^{*2}$. Therefore

$$\bar{c} - c(w_0^*) - 2aw_0^{*2} > 0,$$

implying that $q_0(p_0, w_0^*; p_1^e) > \frac{1}{2}$.

Now $\pi_i = q_i^2$ so that

$$\pi_0(p_0^e, w_0^*; p_1^e) = q_0(p_0^e, w_0^*; p_1^e)^2 > q_q(p_1^e; w_0^*, p_0^e)^2 = \pi_1(p_1^e; w_0^*, p_0^e).$$

□

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