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A quantum derivation of a reputational risk premium

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ABSTRACT

Using quantum modeling, we propose a novel approach to reputational risk management arguing that taking care of corporate reputation can be considered a coalitional strategy framed into a quantum game theory schema. Following a stochastic mechanics approach, and assuming that the revenues of a firm can be modeled as an Ornstein–Uhlenbeck process represented by the well-known Langevin equation for the diffusion of a particle with unit mass under non-linear friction, we offer a mathematical derivation for the discount rate of a firm that does not manage its reputational risk and for a company that optimally manages its reputational risk. By comparison, we derive an analytical expression for the reputational risk premium. Our approach provides useful managerial and financial implications, suggesting that the use of the quantum ideology may result useful in enlarging the body of knowledge of corporate management.

1. Introduction

Quantum finance is an interdisciplinary branch of research that uses analogies from quantum mechanics to describe and explain financial issues. It is an emerging part of the prolific field of econophysics, which shows that theories, concepts and methods originally developed for the explanation of physical phenomena can be successfully applied to the understanding of social-economic problems (Chen & Li 2012; Haven & Khrennikov 2013). Some remarkable results in this subject include the proposal of economic analogs for basic physical quantities such as economic mass and economic Plank's constant, or financial applications for the Heisenberg's uncertainty principle (Soloviev & Saptsin 2011) or the quantum anthropic principle (Piotrowski & Sładkowski 2001).

The purpose of this paper is to offer a novel approach to reputational risk management based on quantum modeling, something that to our knowledge has not been done yet. Framing the analysis into a quantum market game schema, and following the Nelson's stochastic mechanics approach (Nelson 1966), we use differential equations, as well as remarkable results from stochastic calculus and dynamic programming, with the aim of acquiring an explicit expression for a reputational risk premium. Ultimately, our contribution enlarges the current knowledge in the branch of corporate social responsibility, encouraging the addition of the quantum ideology and formalism in further investigation about corporate governance and business management.

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The rest of the investigation is structured as follows. The second section provides a literature review about financial applications of stochastic partial differential equations and quantum modeling. The third section relies on the link between the stochastic basis of diffusion processes and quantum mechanics. The forth section argues about the suitability of a quantum approach for the analysis of reputational risk and derives a reputational risk premium. Finally, the fifth section concludes and points out the managerial and financial applications of the results of this research.

2. Literature review

The Black–Scholes formula for the valuation of a European option is one of the most prominent applications of stochastic processes in financial research (Black & Scholes 1973). Further developments have resulted in an generalization of this methodology so that it can be used to value a wide set of financial derivatives (Hull 2006; Poitras 2002; Wilmott 2007), capital structure (Bensoussan, Crouhy, Galai, Wilkie, & Dempster 1994), financial responsibilities (Longstaff & Schwartz 1995; Merton & Samuelson 1990), corporate securities (Black & Cox 1976), corporate liabilities (Brealey, Myers, & Allen 2006; Merton 1974), investment opportunities (Abel, Dixit, Eberly, & Pindyck 1996; Amram & Kulatilaka 1999; Brennan & Schwartz 1985; Copeland & Antikarov 2001; Dixit 1994; McDonald & Siegel 1985; Smit & Trigeorgis 2012; Trigeorgis 1993; Trigeorgis 1996), and even a whole company (Bowman & Hurry 1993; Copeland Thomas, Koller, & Murrin 1994; Koller, Goedhart, & Wessels 2010; McDonald & Siegel 1985; Sanchez 1995) or the issue of business failure (McGrath 1999).

At a more general level, the same approach has been successfully applied in construction and technology projects (Benaroch 2002; Ford, Lander, & Voyer 2002; Kumar 2002; McGrath 1997; Schwartz & Zozaya-Gorostiza 2003; Zou & Zhang 2009), research and development (Eckhause, Hughes, & Gabriel 2009; Huchzermeier & Loch 2001; Paxson 2001; Schneider et al. 2008), production (Gamba & Fusari 2009; Xu, Lu, & Li 2012), manufacturing (Kogut & Kulatilaka 1994), supply chains (Cucchiella & Gastaldi 2006; Tang & Nurmaya Musa 2011; Wong, Potter, & Naim 2011), joint ventures (Chi 2000; Kogut 1991), budgeting (Costa & Paixão 2010; Meier, Christofides, & Salkin 2001), corporate social responsibility (Husted 2005) and risk management (Cruz 2002).

These models are developed under the assumption that a certain financial magnitude can be represented as a stochastic process. Among other examples, the modeling of interest rates offers a wide range of motivating results (Handhika 2012; McLeish & Kolkiewicz 1997; Nowman 1997). In this sense, it is worthwhile mentioning the Merton model (Merton 1973), the Vasicek model (Vasicek 1977), the Cox–Ingersoll–Ross or CIR models (Cox, Ingersoll, & Ross 1980; Cox, Ingersoll, & Ross 1985), the Dothan model (Dothan 1978), the Courtadon model (Courtadon 1982) and the Brennan-Schwartz model (Brennan & Schwartz 1980). In addition, some comparison has been carried out, showing that a general formulation with the appropriate parameters selection can include all of them, leading to the Chan–Karolyi–Longstaff–Sanders or CKLS model (Chan, Karolyi, Longstaff, & Sanders 1992).

It is worth to note that these financial stochastic models are usually based on variations of the well-known Ornstein–Uhlenbeck process, which has numerous applications in physics, usually cited as the Langevin equation (Caceres & Budini 1997; Lindner 2010; Wilkinson & Pumir 2011). This similarity opens the path for a shared knowledge between researchers in financial theory and physics. Thus, the financial field has benefited from different mathematical tools coming from physics and quantum mechanics, usually referred to the analysis of non-linear dynamics, uncertainty, stochastic calculus and dynamic programming. Therefore, there are notable results concerning financial applications of quantum games (Piotrowski & Sładkowski 2002; Piotrowski & Sladkowski 2004; Piotrowski & Sładkowski 2008) and quantum financial modeling (Baaquie 2013; Ilinski 2001). In particular, a quantum approach for the diffusion of prices and profits has been offered (Piotrowski & Sładkowski 2005) and a quantum model for the stock market has been proposed (Bagarello 2009a; Bagarello 2009b; Cotfas 2013; Zhang & Huang 2010). In addition, the valuation of financial derivatives has been enriched by the use of path integrals (Dash 1989; Linetsky 1997) and Hamiltonian operators (Baaquie 2004; Baaquie 2013). As a consequence, we find some notable results like a derivation of a quantum Black–Scholes equation (Accardi & Boukas 2007) and the proposition of a Black–Scholes–Schrödinger equation (Haven 2002; Haven 2003). Also, a quantum extension of a European option valuation has been derived (Piotrowski, Schroeder, & Zambrzycka 2006) and a quantum harmonic oscillator framework has been considered for option pricing (Cotfas & Cotfas 2013). Finally, a quantum reconsideration of the Cox–Ross–Rubinstein binominal model has also been offered for option valuation (Chen 2001).

3. Quantum modeling and stochastic processes

There is a prolific branch of academic research that establishes deep connections between well-known results of stochastic calculus applied to the theory of diffusion processes and quantum modeling (Davidson 2006). In particular, it is worth mentioning the approach proposed by Nelson (1966), who hypothesizes that any particle can be modeled by a Brownian motion with no friction, using analogies from the Ornstein–Uhlenbeck theory of macroscopic Brownian motion with friction. Under this assumption, he is able to derive the Schrödinger equation into a stochastic mechanics framework, based on stochastic differential equations and remarkable results from stochastic calculus (Bacciagaluppi 2005; Nelson 1985).

The approach proposed by Nelson is typically referred through academic literature as a stochastic version of the pilot-wave theory of de Broglie (1928) and Bohm (1952a, 1952b), which is one of the most famous approximations to the basis of quantum mechanics. In fact, it is remarkable pointing out that the pilot-wave theory has been already suggested as an accurate framework for the analysis of different financial phenomena (Choustova 2004; Khrennikov 2014). In particular, this methodology has been successfully applied for the description of stock markets (Choustova 2008b), as well as the dynamics of stock prices (Choustova 2008a). Also, it has been used for offering a wave-equivalent equation for the Black–Scholes equation (Haven 2004). At a more general level, but with deep implications in the field of behavioral finance, this framework has been also proposed for the explanation of decision processes (Aerts & Aerts 1995).

In this paper we follow an approach similar to the one adopted by Nelson (1966) based on stochastic mechanics. Thus, the mathematical reasoning relies on stochastic partial differential equations, as well as other notable results from stochastic calculus and dynamic optimization.

4. A quantum model for a reputational risk premium

The public relevance acquired by recent reputational scandals, especially in the banking industry, has forced corporate managers to put a special focus in social and reputation related issues (Fiordelisi, Soana, & Schwizer 2013). Thus, in so far as the compromise with reputational risk management and corporate social responsibility is mediated by the attention given to this subject by stakeholders and society as a whole, there is a connection between observer and observed phenomenon that suits well with the quantum ideology and, as a consequence, research on the field of reputational risk can take advantage of the quantum formalism.

Actually, the aforementioned analogy has been already used to explain several social concerns, with some remarkable applications of open quantum systems to decision making processes, either generally considered (Busemeyer, Wang, & Townsend 2006) or framed into specific scenarios, like a political context (Khrennikova, Haven, & Khrennikov 2014) or a game theory framework (Asano, Ohya, & Khrennikov 2011). That is why we hypothesize that the analysis of reputational risk can be also framed into a quantum market game schema (Piotrowski & Sładkowski 2002). In fact, previous researchers have tackled corporate social responsibility related issues from a game theory point of view (Lambertini & Tampieri 2011; Sacconi 2004; Sacconi 2006; Shaw 2009). So, as an extension of this analysis that benefits from the ideology and formalism of quantum modeling, we argue that reputational risk management can be conceptually framed into a quantum market game. Given that corporate social responsibility and reputational management require taking care of relations with stakeholders, as well as cooperation with the environment (Castelo Branco & Lima Rodrigues 2006), a company that does not intend to manage its reputational risk is assumed to hold a non-coalitional strategy within a quantum game theory scenario. Otherwise, if the company is willing to deploy an active reputational risk management, this attitude can be thought of as a coalitional strategy between the company and its environment.

Adopting an approach based in stochastic mechanics that is close to the one proposed by Nelson (1966), we assume that the revenues of a firm follow an Ornstein–Uhlenbeck type process (Vizcaíno-González 2010). In particular, and focusing in the non-coalitional scenario where a company does not manage its reputational risk, the revenues can be modeled as the following Langevin equation for the diffusion of a particle with unit mass under non-linear friction (Lindner 2010):

$$dR = -\mu Rdt + \sqrt{2\mu k_B T}dW \quad (1)$$

In this equation:

- μ represents the friction coefficient
- R stands for the revenue

- dW is a *Wiener* process
- $\sigma=\sqrt{2\mu k_B T}$ is the volatility term obtained from the Stokes–Einstein equation:

$$D=\frac{\sigma^2}{2}=\mu k_B T \tag{2}$$

In this expression:

- D is the diffusion constant.
- μ represents the mobility.
- k_B is the Boltzmann constant.
- T represents the temperature.

We begin by computing the profit of the company (P), deducting the proportion of varying expenditures (k_e) and the amount of fixed expenditures (E) from the revenue (Apabhai et al. 1997):

$$P=(1-k_e)R-E \tag{3}$$

This profit is invested at an interest rate r, so that the available cash flow (C) at a given time t results:

$$C_t=\int_0^t((1-k_e)R_\tau-E)e^{r(t-\tau)}d\tau \tag{4}$$

Using the Leibniz rule for the derivation of this parametric integral, we get:

$$dC=((1-k_e)R-E+rC)dt \tag{5}$$

We can estimate the value of the profit generation potential associated to the company by calculating the present value of the expectation of the terminal cash flow at time t_N, using the discount factor ρ_A:

$$V_t=V(R,C,t)=E[C_{t_N}]e^{-\rho_A(t_N-t)} \tag{6}$$

Now, we can derive the Fokker Plank equation as follows:

$$\frac{\partial V}{\partial t}-\mu R\frac{\partial V}{\partial R}+((1-k_e)R-E+rC)\frac{\partial V}{\partial C}+\mu k_B T\frac{\partial^2 V}{\partial R^2}-\rho_A V=0 \tag{7}$$

As a result, we can obtain an analytic expression for the discount rate ρ_A:

$$\rho_A=\frac{1}{V}\left(\frac{\partial V}{\partial t}-\mu R\frac{\partial V}{\partial R}+((1-k_e)R-E+rC)\frac{\partial V}{\partial C}+\mu k_B T\frac{\partial^2 V}{\partial R^2}\right) \tag{8}$$

Now, we consider the coalitional scenario of the quantum market game, that is, we contemplate the case of a company that actively manages its reputational risk. In this situation, the revenue of the company is assumed to be expressed as follows:

$$dR=-\mu(R+\alpha(e^{-\beta\gamma}-1))dt+\sqrt{2\mu k_B T}dW \tag{9}$$

In this expression:

- γ represents the reputational effort
- α and β are multipliers that introduce some sort of leverage phenomenon

This negative exponential relation collects the non-linearity inherent to reputational risk management, since the marginal return of reputational effort is assumed to be decreasing.

Now, the available cash flow (C) at a given time t results:

$$C_t=\int_0^t((1-k_e)R_\tau-\gamma_\tau-E)e^{r(t-\tau)}d\tau \tag{10}$$

We can derive this parametric integral as follows:

$$dC=((1-k_e)R-\gamma-E+rC)dt \tag{11}$$

We have to take into account that reputational risk needs to be optimally managed in a continuous basis, that is, the company is dealing with a dynamic optimization problem (Epstein, Mayor, Schonbucher, Whalley, & Wilmott 1998). Using the discount factor ρ_B, the value of the profit generation potential for the firm results:

$$V_t=\max_{\gamma_t}E[C_{t_N}]e^{-\rho_B(t_N-t)} \tag{12}$$

Using the Bellman's principle of optimality (Dixit 1994), the previous expression can be disaggregated by differentiating between the immediate action (ϕ) and the subsequent decision:

$$V_t=\max_{\gamma_t}\left(\phi(\gamma_t,R_t)+E[V_{t+1}](1+\rho_B)^{-1}\right) \tag{13}$$

Given that reputational risk management does not produce an immediate outcome, we get:

$$V_t=\max_{\gamma_t}\left(E[V_{t+1}](1+\rho_B)^{-1}\right) \tag{14}$$

Generalizing for any time interval dt:

$$V_t=\max_{\gamma_t}\left(E[V_t+dV](1+\rho_B dt)^{-1}\right)=\left(V_t+\max_{\gamma_t}E[dV]\right)(1+\rho_B dt)^{-1} \tag{15}$$

It follows that:

$$\max_{\gamma}E[dV]=V\rho_B dt \tag{16}$$

Using *Ito's Lemma*, we get:

$$\max_{\gamma}\left(\frac{\partial V}{\partial t}-\mu(R+\alpha(e^{-\beta\gamma}-1))\frac{\partial V}{\partial R}+((1-k_e)R-\gamma-E+rC)\frac{\partial V}{\partial C}+\mu k_B T\frac{\partial^2 V}{\partial R^2}\right)=V\rho_B \tag{17}$$

Given that reputational effort cannot be negative, we are located into an optimization problem with non-negative restrictions. In this situation, the following conditions need to be satisfied (Chiang & Wainwright 1987):

$$\gamma\geq 0$$
$$\Pi=\frac{\partial}{\partial\gamma}\left(\frac{\partial V}{\partial t}-\mu(R+\alpha(e^{-\beta\gamma}-1))\frac{\partial V}{\partial R}+((1-k_e)R-\gamma-E+rC)\frac{\partial V}{\partial C}+\mu k_B T\frac{\partial^2 V}{\partial R^2}\right)\leq 0$$
$$\gamma\cdot\Pi=0$$

These conditions require that $\gamma=0$ or $\Pi=0$. In the first case, the reputational effort is null. In the second case, the following holds:

$$\gamma=-\frac{1}{\beta}\ln\left(\frac{\frac{\partial V}{\partial C}}{\left(\frac{\partial V}{\partial R}\right)\mu\alpha\beta}\right) \tag{18}$$

So, as a consequence the optimal value for reputational effort is:

$$\gamma^* = \max\left(0; -\frac{1}{\beta}\ln\left(\frac{\frac{\partial V}{\partial C}}{\left(\frac{\partial V}{\partial R}\right)\mu\alpha\beta}\right)\right) \tag{19}$$

Using this result in expression (17) it follows that:

$$\frac{\partial V}{\partial C} - \mu\left(R - \alpha\left(e^{-\beta\max\left(0;-\frac{1}{\beta}ln\left(\frac{\frac{\partial V}{\partial C}}{\left(\frac{\partial V}{\partial R}\right)\mu\alpha\beta}\right)\right)} - 1\right)\right)\frac{\partial V}{\partial R} + \left((1-ke)R - \max\left(0; -\frac{1}{\beta}ln\left(\frac{\frac{\partial V}{\partial C}}{\left(\frac{\partial V}{\partial R}\right)\mu\alpha\beta}\right)\right) - E + rC\right)\frac{\partial V}{\partial C} + \mu k_B T\frac{\partial^2 V}{\partial R^2} - \rho_B V = 0 \tag{20}$$

As a consequence, we can get an analytic expression for ρ_B:

$$\rho_B = \frac{1}{V}\left(\frac{\partial V}{\partial t} - \mu\left(R + \alpha\left(e^{-\beta\max\left(0;-\frac{1}{\beta}ln\left(\frac{\frac{\partial V}{\partial C}}{\left(\frac{\partial V}{\partial R}\right)\mu\alpha\beta}\right)\right)} - 1\right)\right)\frac{\partial V}{\partial R} + \left((1-ke)R - \max\left(0; -\frac{1}{\beta}ln\left(\frac{\frac{\partial V}{\partial C}}{\left(\frac{\partial V}{\partial R}\right)\mu\alpha\beta}\right)\right) - E + rC\right)\frac{\partial V}{\partial C} + \mu k_B T\frac{\partial^2 V}{\partial R^2}\right) \tag{21}$$

So, we have derived a discount rate for a company that does not manage its reputational risk in expression (8). In addition, we have obtained the discount rate for a firm that optimally manages its reputational risk in expression (21). Ultimately, we can conquer the analytical expression for the reputational risk premium as follows:

$$\rho_A - \rho_B = \frac{1}{V}\left(\frac{\partial V}{\partial C}\max\left(0; -\frac{1}{\beta}\ln\left(\frac{\frac{\partial V}{\partial C}}{\left(\frac{\partial V}{\partial R}\right)\mu\alpha\beta}\right)\right) + \mu\alpha\frac{\partial V}{\partial R}\left(e^{-\beta\max\left(0;-\frac{1}{\beta}\ln\left(\frac{\frac{\partial V}{\partial C}}{\left(\frac{\partial V}{\partial R}\right)\mu\alpha\beta}\right)\right)} - 1\right)\right) \tag{22}$$

In this expression, it is shown how the reputational risk premium is inversely affected by the value of the company. Consequently, an inadequate reputational risk management strategy will result in a lower valuation of the firm, which will be instantly reflected in a higher reputational risk premium.

5. Conclusions

We offer an innovative methodological approach to the analysis of reputational risk based on quantum modeling. Previous researchers have contextualized corporate social responsibility related issues into a game theory scenario. As an extension of these analyses, we propose that reputational risk management can be framed into a quantum game theory schema, taking advantage of the ideology and formalism from quantum modeling. In this sense, it is argued that taking care of corporate reputation can be assimilated to a coalitional strategy into a quantum market game.

In addition, and following a stochastic mechanics approach, we model the revenue of a company as a stochastic process using the Ornstein–Uhlenbeck approach represented by the Langevin equation for the diffusion of a particle with unit mass under non-linear friction. Exploiting stochastic calculus, partial differential equations and dynamic optimization, we supply a mathematical derivation of the discount rate for a company that does not manage reputational risk management. In addition, by incorporating a measure of the reputational effort, we derive the discount rate for a company that optimally manages its reputational risk. Finally, we calculate an analytic expression for a reputational risk premium.

This research adopts a fresh approach to reputational management based on econophysics, providing some promising insights on how quantum ideology can become helpful in assessing reputational risk, something that our knowledge has not been done yet. In this sense, the novelty of our approach relies on pointing out that the relation between the observer and the observed phenomenon is important when managing reputational risk and, as a consequence, it should be carefully considered by means of a quantum analogy. So, this idea contributes to advance the existent knowledge in the field of corporate social responsibility, opening the path for the incorporation of the quantum ideology and formalism in future research regarding corporate governance and business management.

Finally, our approach permits conquering an explicit expression for a reputational risk premium, and this result has immediate managerial and financial implications and applications. First, from a managerial perspective, it provides a receipt for the quantification of the reputational risk exposure of any firm, which is one of the major tasks for academics and researchers, as well as for practitioners and boards of directors. And second, adopting a financial point of view, it is worthwhile signaling that an explicit expression for the reputational risk premium can be used as any other risk premium, in so far as it can be incorporated to a discount factor in order to get a fair valuation of an investment, a project or a whole company.

References

Abel, A. B., Dixit, A. K., Eberly, J. C., & Pindyck, R. S. (1996). Options, the value of capital, and investment. *The Quarterly Journal of Economics*, *111*(3), 753–777.

Accardi, L., & Boukas, A. (2007). The quantum Black–Scholes equation. *Global Journal of Pure and Applied Mathematics*, *2*(2), 155–170.

Aerts, D., & Aerts, S. (1995). Applications of quantum statistics in psychological studies of decision processes. *Foundations of Science*, *1*(1), 85–97.

Amram, M., & Kulatilaka, N. (1999). *Real Options: Managing Strategic Investment in an Uncertain World*. Boston: Harvard Business School Press.

Apabhai, M., Georgikopoulos, N., Hasnip, D., Jamie, R., Kim, M., & Wilmott, P. (1997). A model for the value of a business, some optimization problems in its operating procedures and the valuation of its debt. *IMA Journal of Applied Mathematics*, *59*(3), 273–285.

Asano, M., Ohya, M., & Khrennikov, A. (2011). Quantum-like model for decision making process in two players game. *Foundations of Physics*, *41*(3), 538–548.

Baaquie, B. E. (2004). *Quantum Finance: Path Integrals and Hamiltonians for Options and Interest Rates*. Cambridge University Press.

Baaquie, B. E. (2013). Financial modeling and quantum mathematics. *Computers & Mathematics with Applications*, *65*(10), 1665–1673.

Bacciagaluppi, G. (2005). A Conceptual Introduction to Nelson's Mechanics. In R. Buccheri, E. A., & S. M. (Eds.), *Endophysics, Time, Quantum and the Subjective* (pp. 367–388). Singapore: World Scientific.

Bagarello, F. (2009a). Simplified stock markets described by number operators. *Reports on Mathematical Physics*, *63*(3), 381–398.

Bagarello, F. (2009b). A quantum statistical approach to simplified stock markets. *Physica A: Statistical Mechanics and its Applications*, *388*(20), 4397–4406.

Benaroch, M. (2002). Managing information technology investment risk: A real options perspective. *Journal of Management Information Systems*, *19*(2), 43–84.

Bensoussan, A., Crouhy, M., Galai, D., Wilkie, A., & Dempster, M. (1994). Stochastic equity volatility and the capital structure of the firm [and discussion], philosophical transactions of the royal society of london. *Series A: Physical and Engineering Sciences*, *347*(1684), 531–541.

Black, F., & Cox, J. C. (1976). Valuing corporate securities: Some effects of bond indenture provisions. *The Journal of Finance*, *31*(2), 351–367.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *The Journal of Political Economy*, *81*, 637–654.

Bohm, D. (1952a). A suggested interpretation of the quantum theory in terms of "hidden" variables, I. *Physical Review*, *85*(2), 166–179.

Bohm, D. (1952b). A suggested interpretation of the quantum theory in terms of "hidden" variables, II. *Physical Review*, *85*(2), 180–193.

Bowman, E. H., & Hurry, D. (1993). Strategy through the option lens: An integrated view of resource investments and the incremental-choice process. *Academy of Management Review*, *18*(4), 760–782.

Brealey, R. A., Myers, S. C., & Allen, F. (2006). *Corporate Finance*. New York: McGraw-Hill.

Brennan, M. J., & Schwartz, E. S. (1980). Analyzing convertible bonds. *Journal of Financial and Quantitative Analysis*, *15*(04), 907–929.

Brennan, M. J., & Schwartz, E. S. (1985). Evaluating natural resource investments. *Journal of Business*, *58*, 135–157.

Busemeyer, J. R., Wang, Z., & Townsend, J. T. (2006). Quantum dynamics of human decision-making. *Journal of Mathematical Psychology*, *50*(3), 220–241.

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Caceres, M. O., & Budini, A. A. (1997). The generalized Ornstein–Uhlenbeck process. *Journal of Physics A: Mathematical and General*, *30*(24), 8427.

Castelo Branco, M., & Lima Rodrigues, L. (2006). Corporate social responsibility and resource-based perspectives. *Journal of Business Ethics*, *69*(2), 111–132.

Chan, K. C., Karolyi, G. A., Longstaff, F. A., & Sanders, A. B. (1992). An empirical comparison of alternative models of the short-term interest rate. *The Journal of Finance*, *47*(3), 1209–1227.

Chen, S., & Li, S. (2012). Econophysics: Bridges over a turbulent current. *International Review of Financial Analysis*, *23*, 1–10.

Chen, Z. (2001). *Quantum Finance: The Finite Dimensional Case, arXiv preprint quant-ph/0112158.*

Chi, T. (2000). Option to acquire or divest a joint venture. *Strategic Management Journal*, *21*(6), 665–687.

Chiang, A. C., & Wainwright, K. (1987). *Métodos Fundamentales de Economía Matemática, Mc Graw Hill, 3.*

Choustova, O. (2004). Bohmian mechanics for financial processes. *Journal of Modern Optics*, *51*(6-7), 1111.

Choustova, O. (2008a). Application of Bohmian mechanics to dynamics of prices of shares: Stochastic model of Bohm–Vigier from properties of price trajectories. *International Journal of Theoretical Physics*, *47*(1), 252–260.

Choustova, O. (2008b). *Pilot Wave Quantum Model for the Stock Market, arXiv preprint quant-ph/0109122.*

Copeland, T., & Antikarov, V. (2001). *Real Options.* Texere New York.

Costa, A., & Paixão, J. M. (2010). An approximate solution approach for a scenario-based capital budgeting model. *Computational Management Science*, *7*(3), 337–353.

Cotfas, L. (2013). A finite-dimensional quantum model for the stock market. *Physica A: Statistical Mechanics and its Applications*, *392*(2), 371–380.

Cotfas, L., & Cotfas, N. (2013). *Quantum Harmonic Oscillator in Option Pricing, arXiv preprint arXiv:1310.4142.*

Courtadon, G. (1982). The pricing of options on default-free bonds. *Journal of Financial and Quantitative Analysis*, *17*(01), 75–100.

Cox, J. C., Ingersoll, J. E., & Ross, S. A. (1980). An analysis of variable rate loan contracts. *The Journal of Finance*, *35*(2), 389–403.

Cox, J. C., Ingersoll, J. E., & Ross, S. A. (1985). *A Theory of the Term Structure of Interest Rates.* Econometrica: Journal of the Econometric Society, 385–407.

Cruz, M. G. (2002). *Modeling, Measuring and Hedging Operational Risk.* New York: John Wiley & Sons.

Cucchiella, F., & Gastaldi, M. (2006). Risk management in supply chain: A real option approach. *Journal of Manufacturing Technology Management*, *17*(6), 700–720.

Dash, J. (1989). *Path Integrals and Options – I.* II, Centre National de la Recherche Scientifique: Parts I.

Davidson, M. P. (2006). *Stochastic Models of Quantum Mechanics – A Perspective, arXiv preprint quant-ph/0610046.*

De Broglie, L. (1928). La Nouvelle Dynamique des Quanta. In A. Lorentz (Ed.), *Electrons Et Photons: Rapports Et Discussions Du Cinquieme Conseil De Physique Solvay. Vol. 24.* (pp. 105–132). Paris: Gauthier–Villars.

Dixit, A. K. (1994). *Investment Under Uncertainty.* New Jersey: Princeton university press.

Dothan, L. U. (1978). On the term structure of interest rates. *Journal of Financial Economics*, *6*(1), 59–69.

Eckhause, J. M., Hughes, D. R., & Gabriel, S. A. (2009). Evaluating real options for mitigating technical risk in public sector R&D acquisitions. *International Journal of Project Management*, *27*(4), 365–377.

Epstein, D., Mayor, N., Schonbucher, P., Whalley, A. E., & Wilmott, P. (1998). The valuation of a firm advertising optimally. *The Quarterly Review of Economics and Finance*, *38*(2), 149–166.

Fiordelisi, F., Soana, M., & Schwizer, P. (2013). The determinants of reputational risk in the banking sector. *Journal of Banking & Finance*, *37*(5), 1359–1371.

Ford, D. N., Lander, D. M., & Voyer, J. J. (2002). A real options approach to valuing strategic flexibility in uncertain construction projects. *Construction Management & Economics*, *20*(4), 343–351.

Gamba, A., & Fusari, N. (2009). Valuing modularity as a real option. *Management Science*, *55*(11), 1877–1896.

Handhika, T. (2012). Market price of risk analysis from three major industrial countries on the stability of the Brennan–Schwartz model. *Indonesian Capital Market Review*, *4*(1).

Haven, E. (2002). A discussion on embedding the Black–Scholes option pricing model in a quantum physics setting. *Physica A: Statistical Mechanics and its Applications*, *304*(3), 507–524.

Haven, E. (2003). A Black–Scholes Schrödinger option price:'Bit'versus 'qubit'. *Physica A: Statistical Mechanics and its Applications*, *324*(1), 201–206.

Haven, E. (2004). The wave-equivalent of the Black–Scholes option price: An interpretation. *Physica A: Statistical Mechanics and its Applications*, *344*(1), 142–145.

Haven, E., & Khrennikov, A. (2013). *Quantum Social Science.* Cambridge University Press.

Huchzermeier, A., & Loch, C. H. (2001). Project management under risk: Using the real options approach to evaluate flexibility in R… D. *Management Science*, *47*(1), 85–101.

Hull, J. C. (2006). *Options, Futures, and Other Derivatives.* Pearson Education India.

Husted, B. W. (2005). Risk management, real options, corporate social responsibility. *Journal of Business Ethics*, *60*(2), 175–183.

Ilinski, K. (2001). *Physics of Finance: Gauge Modelling in Non-Equilibrium Pricing.* J. Wiley.

Khrennikov, A. Y. (2014). *Ubiquitous Quantum Structure.* Berlin: Springer.

Khrennikova, P., Haven, E., & Khrennikov, A. (2014). An application of the theory of open quantum systems to model the dynamics of party governance in the US political system. *International Journal of Theoretical Physics*, *53*(4), 1346–1360.

Kogut, B. (1991). Joint ventures and the option to expand and acquire. *Management Science*, *37*(1), 19–33.

Kogut, B., & Kulatilaka, N. (1994). Operating flexibility, global manufacturing, and the option value of a multinational network. *Management Science*, *40*(1), 123–139.

Koller, T., Goedhart, M., & Wessels, D. (2010). *Valuation: Measuring and Managing the Value of Companies.* New Jersey: John Wiley & Sons.

Kumar, R. L. (2002). Managing risks in IT projects: An options perspective. *Information Management*, *40*(1), 63–74.

Lambertini, L., & Tampieri, A. (2011). *Corporate Social Responsibility and Firms Ability to Collude.*

Lindner, B. (2010). Diffusion of particles subject to nonlinear friction and a colored noise. *New Journal of Physics*, *12*(6), 063026.

Linetsky, V. (1997). The path integral approach to financial modeling and options pricing. *Computational Economics*, *11*(1–2), 129–163.

Longstaff, F. A., & Schwartz, E. S. (1995). A simple approach to valuing risky fixed and floating rate debt. *The Journal of Finance*, *50*(3), 789–819.

McDonald, R. L., & Siegel, D. R. (1985). Investment and the valuation of firms when there is an option to shut down. *International Economic Review*, *26*(2), 331–349.

McGrath, R. G. (1997). A real options logic for initiating technology positioning investments. *Academy of Management Review*, *22*(4), 974–996.

McGrath, R. G. (1999). Falling forward: Real options reasoning and entrepreneurial failure. *Academy of Management Review*, *24*(1), 13–30.

McLeish, D. L., & Kolkiewicz, A. W. (1997). Fitting diffusion models in finance. *Lecture Notes-Monograph Series*, 327–350.

Meier, H., Christofides, N., & Salkin, G. (2001). Capital budgeting under uncertainty – an integrated approach using contingent claims analysis and integer programming. *Operations Research*, *49*(2), 196–206.

Merton, R. C. (1973). Theory of rational option pricing. *The Bell Journal of economics and management science*, *141-183*.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance*, *29*(2), 449–470.

Merton, R. C., & Samuelson, P. A. (1990). *Continuous-Time Finance.* Cambridge: Blackwell.

Nelson, E. (1966). Derivation of the Schrödinger equation from Newtonian mechanics. *Physical Review*, *150*(4), 1079–1085.

Nelson, E. (1985). *Quantum Fluctuations.* Princeton: Princeton University Press.

Nowman, K. B. (1997). Gaussian estimation of single-factor continuous time models of the term structure of interest rates. *The Journal of Finance*, *52*(4), 1695–1706.

Paxson, D. A. (2001). Introduction to real R&D options. *R and D Management*, *31*(2), 109–114.

Piotrowski, E. W., & Sładkowski, J. (2001). *Quantum-like Approach to Financial Risk: Quantum Anthropic Principle, arXiv preprint quant-ph/0110046.*

Piotrowski, E. W., & Sładkowski, J. (2002). Quantum market games. *Physica A: Statistical Mechanics and its Applications*, *312*(1), 208–216.

Piotrowski, E. W., & Sladkowski, J. (2004). *Quantum Game Theory in Finance, arXiv preprint quant-ph/0406129.*

Piotrowski, E. W., & Sładkowski, J. (2005). Quantum diffusion of prices and profits. *Physica A: Statistical Mechanics and its Applications*, *345*(1), 185–195.

Piotrowski, E. W., & Sładkowski, J. (2008). Quantum auctions: Facts and myths. *Physica A: Statistical Mechanics and its Applications*, *387*(15), 3949–3953.

Piotrowski, E. W., Schroeder, M., & Zambrzycka, A. (2006). Quantum extension of European option pricing based on the Ornstein–Uhlenbeck process. *Physica A: Statistical Mechanics and its Applications*, *368*(1), 176–182.

Poitras, G. (2002). *Risk Management, Speculation, and Derivative Securities.* USA: Elsevier.

Sacconi, L. (2004). *Corporate Social Responsibility (CSR) as a Model of'Extended'Corporate Governance: An Explanation Based on the Economic Theories of Social Contract, Reputation and Reciprocal Conformism, LIUC, Ethics, Law and Economics Paper, (142).*

Sacconi, L. (2006). A social contract account for CSR as an extended model of corporate governance (I): Rational bargaining and justification. *Journal of Business Ethics*, *68*(3), 259–281.

Sanchez, R. (1995). Strategic flexibility in product competition. *Strategic Management Journal*, *16*(S1), 135–159.

Schneider, M., Tejeda, M., Dondi, G., Herzog, F., Keel, S., & Geering, H. (2008). Making real options work for practitioners: A generic model for valuing R&D projects. *R&D Management*, *38*(1), 85–106.

Schwartz, E. S., & Zozaya-Gorostiza, C. (2003). Investment under uncertainty in information technology: acquisition and development projects. *Management Science*, *49*(1), 57–70.

Shaw, W. H. (2009). Marxism, business ethics, and corporate social responsibility. *Journal of Business Ethics*, *84*(4), 565–576.

Smit, H. T., & Trigeorgis, L. (2012). *Strategic Investment: Real Options and Games.* Princeton: Princeton University Press.

Soloviev, V., & Saptsin, V. (2011). *Heisenberg Uncertainty Principle and Economic Analogues of Basic Physical Quantities, arXiv preprint arXiv:1111.5289.*

Tang, O., & Nurmaya Musa, S. (2011). Identifying risk issues and research advancements in supply chain risk management. *International Journal of Production Economics*, *133*(1), 25–34.

Thomas, C., Koller, T., & Murrin, J. (1994). *Valuation: Measuring and Managing the Value of Companies.* Wiley frontiers in finance).

Trigeorgis, L. (1993). Real options and interactions with financial flexibility. *Financial Management*, *22*(3), 202–224.

Trigeorgis, L. (1996). *Real Options: Managerial Flexibility and Strategy Inresource Allocation.* USA: The MIT Press.

Vasicek, O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics*, *5*(2), 177–188.

Vizcaíno-González, M. (2010). Riesgo de reputación: revision teórica y aproximación a su valoración. *Revista Galega de Economía*, *19*(1), 199–216.

Wilkinson, M., & Pumir, A. (2011). Spherical Ornstein–Uhlenbeck processes. *Journal of Statistical Physics*, *145*(1), 113–142.

Wilmott, P. (2007). *Paul Wilmott Introduces Quantitative Finance*. Chichester: John Wiley & Sons.

Wong, H., Potter, A., & Naim, M. (2011). Evaluation of postponement in the soluble coffee supply chain: A case study. *International Journal of Production Economics*, *131*(1), 355–364.

Xu, S. X., Lu, Q., & Li, Z. (2012). Optimal modular production strategies under market uncertainty: A real options perspective. *International Journal of Production Economics*, *139*(1), 266–274.

Zhang, C., & Huang, L. (2010). A quantum model for the stock market. *Physica A: Statistical Mechanics and its Applications*, *389*(24), 5769–5775.

Zou, P. X., & Zhang, G. (2009). Managing risks in construction projects: life cycle and stakeholder perspectives. *International Journal of Construction Management*, *9*(1), 61–77.

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