Contents lists available at ScienceDirect



International Review of Financial Analysis



CrossMark

Implied volatility index for the Norwegian equity market

Sebastian A. Bugge, Haakon J. Guttormsen, Peter Molnár*, Martin Ringdal

Norwegian University of Science and Technology, Trondheim, Norway

ARTICLE INFO

Article history: Received 6 July 2016 Accepted 30 July 2016 Available online 6 August 2016

Keywords: Implied volatility OBX index VIX Volatility forecasting Leverage effect

ABSTRACT

We introduce and evaluate the NOVIX - an implied volatility index for the Norwegian equity index OBX. NOVIX is created according to the VIX methodology. We compare the NOVIX to the German VDAX-NEW and the U.S. VIX and find that NOVIX has similar properties as these two indices. We also evaluate the VIX, VDAX-NEW and NOVIX in terms of volatility forecasting. As a benchmark model we use a precise HAR model of Corsi (2009) based on high-frequency data. All three implied volatility indices significantly improve daily, weekly and monthly forecasts of volatility of their underlying equity indices. This improvement is largest for the VIX, followed by VDAX-NEW and NOVIX.

© 2016 Elsevier Inc. All rights reserved.

1. Introduction

Implied volatility indices based on equity index options have become immensely popular during the two decades they have existed. Investors use them as an expectation of future volatility, a gauge of market sentiment, and as a way to buy and sell volatility itself. In this paper, we introduce the NOVIX - a volatility index for the Norwegian market based on the VIX methodology. The NOVIX is part of a larger trend, in which more and more exchanges have introduced their own implied volatility indices. In addition to the many official volatility indices, there are several academic studies that construct and evaluate volatility indices for markets without official volatility indices, see e.g. Skiadopoulos (2004) and González and Novales (2009).

Internationally, the interest in implied volatility indices has been growing since the Chicago Board Option Exchange (CBOE) introduced the CBOE Volatility Index (VIX) in 1993. Whaley (1993) proposed that these indices can help the investment community in at least two different ways. First, they provide reliable estimates of expected short-term stock market volatility. Second, they offer a market volatility "standard" upon which derivative contracts may be written. The potential to hedge against volatility risk and for profit trading in volatility has led to successful introductions of markets

* Corresponding author. E-mail address: peto.molnar@gmail.com (P. Molnár). for volatility derivatives and exchange traded products that replicate implied volatility indices.

Today, the combined trading activity in VIX options and futures is over 800,000 contracts per day (Chicago Board Options Exchange, 2015). CBOE alone publishes 28 volatility indices for stock indices, ETFs, interest rates, commodities, currencies and individual stocks. Gradually, other derivatives exchanges have begun offering volatility indices for their respective markets. Some notable examples are Deutsche Börse with the VDAX (1994), later updated to VDAX-NEW (2005), the Marche des Options Negociables de Paris (MONEP) with VX1 and VX6 (1997) and NYSE Euronext with the FTSE 100 Volatility Index (2008).

There is no official implied volatility index for Oslo Børs or the Norwegian market. Oslo Børs is an independent exchange, and the only regulated market for securities trading in Norway (Oslo Børs, 2015). It is internationally recognized as a global leader in the segments energy, shipping and seafood. The main objective of Oslo Børs is to be the central marketplace for listing and trading of financial instruments in the Norwegian market, and nearly all Norwegian companies regard Oslo Børs as the natural place to list.

We construct the NOVIX from options on the OBX Total Return Index (OBX). The OBX is a stock market index composed of the 25 most traded securities on Oslo Børs, and a natural choice as the underlying for an implied volatility index. Oslo Børs has offered options on OBX since 1990, and futures since 1992. On May 22nd 2016, the 25 stocks in OBX had a total market capitalization of NOK 1437bn compared to the total market capitalization of NOK 1810bn for all stocks listed on Oslo Børs, representing over 75% of the total value. Considering the central position of Oslo Børs, we believe the NOVIX can be used as a reference for both practitioners and academic studies about volatility in the Norwegian market. As a way to facilitate for further research, we calculate and provide continuously updated 5-minute intraday values for the NOVIX available at https:// novix.xyz.

We use the VDAX-NEW and the VIX from the German and US markets as reference indices to evaluate the properties and behaviour of the NOVIX. In general, we find that the NOVIX exhibits many of the same characteristics as the reference indices, and that its relevance has increased in the most recent years. The degree of negative correlation between OBX returns and NOVIX returns has increased consistently over the last decade, and approaches the level seen in the other markets. This increases the potential of NOVIX derivatives as a tool for risk management. Further, we find that the NOVIX exhibits an asymmetric leverage effect, which is in line with the findings for VDAX-NEW and VIX. However, the asymmetric effect is more pronounced in our reference indices.

Finally, we study how useful the NOVIX is for predicting future volatility in the Norwegian market. For this, we use realized volatility from high-frequency OBX data as a proxy for the true volatility, and include the NOVIX in the Heterogeneous Autoregressive model (HAR-RV) of Corsi (2009). Our out-of-sample results show that the NOVIX adds information beyond the information that is captured by past realized volatility.

The rest of the paper is organized as follows. In Section 2, we describe the methodology. Section 3 presents the data. Section 4 is an analysis of the relationship between the NOVIX and OBX returns. Section 5 investigates the potential of the NOVIX for the volatility forecasting, before we give a conclusion in Section 6.

2. Methodology

The Chicago Board Option Exchange (CBOE) introduced the VIX volatility index in 1993 as a measure of the expected 30-day future market volatility (Whaley, 1993). This original VIX index was based on the Black-Scholes (BS) pricing model (Black & Scholes, 1973), and calculated as the average BS implied volatility from S&P 100 put and call options. In total, this method uses eight near-the-money puts and calls for the nearby and second most nearby maturity. The original VIX depends on the assumptions of the BS model, and is therefore a model-based implied volatility index. Although it captures more info than the implied volatility of a single strike, it does not capture all the information in the wide range of strikes available.

A decade after its introduction, the VIX was revised in a collaboration with Goldman Sachs. The purpose was to provide exchangetraded volatility derivatives. Still a measure of the expected 30-day future market volatility, the underlying index changed from the S&P 100 to the S&P 500. More importantly, the method for calculating the index was replaced by a model-free approach. The concept of model-free implied variance was first coined by Britten-Jones and Neuberger (2000), and is based on work by Derman and Kani (1994), Dupire (1994, 1997) and Rubinstein and Neuberger (1994). They use no-arbitrage conditions to extract common features of all stochastic processes that are consistent with observed option prices. This has the advantage of not depending on any particular optionpricing model, and extracts information from all relevant option prices (Jiang & Tian, 2005). Demeterfi and Zou (1999) show theoretically how a portfolio of standard options can replicate a variance swap and that the cost of this replicating portfolio is the fair price of a variance swap. The VIX methodology is essentially a discretization of the formula for the fair value of a variance swap. Other exchanges have followed CBOE, and like the VIX, the VDAX was updated with a similar model-free approach in 2005 and renamed VDAX-NEW.

2.1. Creating the NOVIX

The NOVIX is constructed with the model-free VIX methodology, according to the formula (Chicago Board Options Exchange, 2015)

$$\sigma^{2} = \frac{2}{T} \sum_{i} \frac{\Delta K_{i}}{K_{i}^{2}} e^{RT} Q(K_{i}) - \frac{1}{T} \left(\frac{F}{K_{0}} - 1\right)^{2},$$
(1)

where

σ	NOVIX/100
Т	Time to expiration in years
F	Forward level of underlying
K ₀	First strike below F
K _i	Strike price of the <i>i</i> -th out-of-money option
ΔK_i	$\frac{1}{2} \times (K_{i+1} - K_{i-1})$
R	Řisk-free rate

 $Q(K_i)$ Midpoint of bid-ask spread for option with strike K_i

Following Chicago Board Options Exchange (2015), we compute the implied volatility estimate σ for two selected maturities, a *near-term* and *next-term* maturity, that represents the options expiring before and after the desired 30-day horizon. For each maturity, we select a subset of options to include in the calculation by the procedure below.

We determine the forward level from the option prices by first identifying the strike with the smallest absolute difference in put-call price and then applying the formula

$$F =$$
Strike + $e^{RT} \times |$ Call Price – Put Price $|$.

We define K_0 as the first strike below F, and consider the option pair with strike K_0 as at-the-money. Then, we discard all in-themoney options. That is, we only consider the at-the-money options, the call options with strikes $K_i > K_0$ and the put options with strikes $K_i < K_0$. Intuitively, the demand for out-of-the-money options can be interpreted as a need for insurance by investors, which in turn reflects the market volatility. Further, we exclude all out-of-themoney options with a zero bid price, and all options following two zero bid prices in a row, when the options are ordered by type as increasingly out-of-the-money.

We obtain the final desired 30-day volatility estimate from a linear interpolation between the near-term and next-term results,

$$NOVIX = 100 \times \sqrt{\left(T_1 \sigma_1^2 \left[\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}}\right] + T_2 \sigma_2^2 \left[\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}\right]\right) \times \frac{N_{365}}{N_{30}}},$$

where the subscripts 1 and 2 represent the *near-term* and *next-term* options, respectively. The NOVIX is calculated using a time precision of a minute, where *T* is the time to expiration in years quoted as minutes to expiration over minutes in a year. The remaining time terms are

 N_{T_1} = Number of minutes to settlement of *near-term* options

 N_{T_2} = Number of minutes to settlement of *near-term* options

 N_{30} = Number of minutes in 30 days

 N_{365} = Number of minutes in a year (365 days).

For more details on the VIX method, we refer to the CBOE White Paper (Chicago Board Options Exchange, 2015).

Our implementation of the VIX-methodology has been tested on the examples in the CBOE White Paper (Chicago Board Options Exchange, 2015). In addition, it has been more comprehensively



Fig. 1. NOVIX levels (left axis) and OBX levels (right axis) from January 3rd 2000 to February 22nd 2016. In addition, we highlight a selection of financial and geopolitical events in the figure.

tested by calculating equity implied volatility indices on the five stocks provided by CBOE. We use daily close option data for this, gathered from the OptionMetrics database, and compare the results to the actual indices published by CBOE. The comparison shows that our implementation is correct. Code and documentation can be provided upon request.

3. Data

The NOVIX is calculated from OBX options traded on Oslo Børs. All the data was provided by Oslo Børs, and consist of daily close data on all available call and put options for the period January 3rd 2000 to February 22nd 2016. We use the Norwegian Interbank Offered Rate (NIBOR) as the risk-free interest rate, and interpolate yield to maturity from the two closest values around each term expiration.

There are some potential problems with the VIX methodology that are addressed in Jiang and Tian (2007). The VIX methodology introduces truncation and discretization errors due to the limited number of strike prices available. They suggest that interpolation and extrapolation over strikes can improve the accuracy of the model-free approach. This was implemented by Ting et al. (2007) for the Korean stock market, where there were especially large steps between strikes (8%). In the period used, the relevant OBX options have a strike interval of 3%. Based on this, we consider these problems as less critical for the calculation of NOVIX. Since we use the standard VIX methodology, NOVIX is directly comparable to implied volatility indices from other markets. However, we shortened the data period from initially starting from January 3rd 2000 to starting from January 3rd 2006 instead. This was done due to low volumes in option trading during the first years.

To understand NOVIX better, we compare it to the VIX and VDAX-NEW. The VIX index is the most popular volatility index, and financial products based on the VIX are by far the most traded among those based on volatility indices. VDAX-NEW is a recognized volatility index in a major market with strong relations to the Norwegian market. Daily close prices for the VDAX-NEW were downloaded from the Frankfurt Stock Exchange, and daily VIX close prices from CBOE.

Fig. 1 displays the OBX and the NOVIX for the period from January 2000 to February 2016. There is a clear negative relationship between the OBX index and the NOVIX: when the OBX trends downward the NOVIX rises, and vice versa. The largest spikes in the NOVIX correspond to negative geopolitical events, such as the terrorist attack in 2001, the financial crisis in 2008 and the EU debt crisis in August 2011. The largest movements are all responses to global events, and not specific for the Norwegian market.

Over the whole period, the correlation between NOVIX and OBX returns is -0.30. In comparison, the correlations between the VDAX-NEW and VIX and their underlying stock indices are 0.70 and 0.65. The difference between the NOVIX and the other indices may appear large, but it has changed considerably over time. As we see in Fig. 2, the VDAX-NEW and VIX have a stable, large negative correlation with their underlying index returns, while the negative correlation between NOVIX and OBX returns has increased in magnitude over the sample. The higher degree of correlation between NOVIX and OBX returns indicates that the NOVIX absorbs information better, and increases the relevance of the NOVIX in risk management.

Table 1 displays descriptive statistics of the implied volatility indices (IV) for both levels and log returns. The NOVIX has similar properties as VDAX-NEW and VIX. The average level of NOVIX is 24.45. This is close to VDAX-NEW (23.43), and somehow higher than VIX (20.55). The highest value for NOVIX of 84.25 was recorded on September 20th 2008 during the financial crisis, and is of similar magnitude to the highest values of the other implied volatility indices. The average returns are close to zero for all IV indices. NOVIX has a similar magnitude of largest and lowest observed returns as VIX, while the maximum observed return of VDAX-NEW is higher.

We use the Augmented Dickey-Fuller (ADF) unit root test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) stationarity test to test for stationarity in the implied volatility indices. Table 2 shows the results for the entire time period, as well as for two non-overlapping subsamples. As common for volatility, we assume no time trend in the long run, and present the test results without any deterministic trend¹.

Both tests conclude that NOVIX levels are non-stationary for all samples. The results for VDAX-NEW and VIX are inconclusive. For the entire-sample, the ADF test rejects non-stationarity at the 1% significance level for VDAX-NEW, whereas the KPSS test rejects the null hypothesis of stationarity at the 5% level. For entire-sample of the

¹ Results are not sensitive to the number of lags used, and do not change significantly with an added time trend.

Fig. 2. Evolution of correlation between log returns of implied volatility indices and underlying stock indices, using a 1000day moving window.

VIX, the ADF test rejects non-stationarity at the 5% level, and the KPSS test rejects stationarity at the 1% level.

Intuitively, we would expect the implied volatility indices to be stationary, maybe a mean-reverting processes. However, the sample may be too short to capture long cycles and their variance is not necessarily constant. From the stationarity tests, it is not obvious whether we should use levels or returns in the subsequent analysis. Therefore, for the sake of robustness, we proceed in the further analysis with both.

4. Leverage effect

The leverage effect refers to the well-established relationship between volatility and equity returns — volatility increases as stock prices fall. We observe this negative relationship in the plot of NOVIX and OBX in Fig. 1, and in Fig. 2 where we display the correlation between NOVIX and OBX returns. In this section, we study this

Table 1

Descriptive statistics for the implied volatility indices NOVIX, VDAX-NEW and VIX, for levels and returns. The period from January 3rd 2006 to February 22nd 2016 is used, with 2439 observation for each series.

Index	Mean (%)	Std. (%)	Min (%)	Max (%)	Skew	Ex. kurt.
NOVIX	24.45	10.01	11.19	84.25	1.86	5.22
VDAX-NEW	23.43	9.06	12.13	83.23	2.31	7.60
VIX	20.55	9.85	9.89	80.86	2.31	7.04
RNOVIX	0.00	5.34	-27.34	29.22	0.37	3.66
R ^{VDAX-NEW}	0.01	7.24	-35.06	49.60	0.65	3.23
R ^{VIX}	0.01	5.68	-26.65	30.57	0.49	2.02

R^{IV} denotes the IV returns, computed as log differences.

leverage effect explicitly, and compare the NOVIX with our reference indices.

The leverage effect was first discussed in Black (1976) and Christie (1982). The term *leverage* refers to the economic interpretation that when asset prices decline, companies become more leveraged as their debt-to-equity ratio increases. As a result, one expects their stock to become more risky, and hence more volatile. However, the magnitude of the effect seems too large to be attributable solely to an increase in financial leverage. Figlewski and Wang (2000) noted among other findings that there is no apparent effect on volatility when leverage changes because of a change in debt or number of shares, only when stock prices change. This questions whether the effect is linked to financial leverage at all.

In previous literature it has been documented that the effect is generally asymmetric, meaning that the increase in volatility is higher for negative returns than the reduction in volatility for positive returns of the same magnitude. The degree of asymmetry depends on the volatility proxy employed in the estimation, with options' implied volatility generally exhibiting much more pronounced asymmetry (see e.g. Bates (2000), Wu and Xiao (2002), Eraker (2004)).

Several different parametric models and volatility-return regressions can be employed for empirically assessing the leverage effect. We use a regression model that is able to capture the asymmetric effect, by separating positive and negative returns. The regression is specified for IV as both levels and log returns:

$$IV_t = \alpha + \beta IV_{t-1} + \gamma_1 R_t^+ + \gamma_2 R_t^- + \epsilon_t \text{ (Levels)}$$

$$R_t^{IV} = \alpha + \gamma_1 R_t^+ + \gamma_2 R_t^- + \epsilon_t, \text{ (Returns)}$$
(2)

Table 2

Stationarity tests using Augmented Dickey Fuller (ADF) test and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test for the implied volatility (IV) indices at both levels and log returns. The ADF and KPSS test statistics are indicated with star(s) if the nullhypothesis is rejected. The null-hypothesis for the ADF test is non-stationarity, whereas the null-hypothesis for the KPSS test is stationarity.

IV index	Sample	Levels		Returns	
		ADF	KPSS	ADF	KPSS
NOVIX	2006–Feb. 2016	-2.79	2.79**	-23.42**	0.06
	2011-Feb. 2016	-1.73	1.33**	-17.32**	0.12
	2006-2010	-2.15	1.14**	-23.87**	0.11
VDAX-NEW	2006-Feb. 2016	-3.45**	0.68*	-13.96**	0.03
	2011-Feb. 2016	-2.47	0.79**	-10.37**	0.05
	2006-2010	-2.70	1.21**	-10.88**	0.07
VIX	2006-Feb. 2016	-3.33*	1.27**	-23.97**	0.04
	2011-Feb. 2016	-2.80	1.29**	-10.66**	0.03
	2006-2010	-2.19	1.55**	-11.76**	0.11

The number of lags is determined according to the formula $\sqrt[4]{12 \times (\frac{n}{100})}$ of Schwert (2002).

* Significant at the 5% level.

** Significant at the 1% level.

where R^{IV} denotes the log difference for the IV index and R_t the log return of the underlying index. We separate the log returns into positive and negative returns as

$$R_t^+ = \max(R_t, 0)$$
 and $R_t^- = \min(R_t, 0)$

 IV_t is the close value of IV at the end of day t, and we include its lagged value in order to account for the strong temporary dependencies in volatility in the levels regression. R_t is the return from the end of day t - 1 to the end of day t. Negative γ values indicate an opposite movement to returns in the IV index. We expect both γ_1 and γ_2 to be negative, and a larger magnitude of γ_2 will indicate asymmetry.

Table 3 displays the results of the regressions. From the levels regression we see that the NOVIX has negative γ coefficients for the entire sample, with the largest magnitude for negative returns. More interestingly, we see a change in the leverage effect over the two sub-samples. Both γ coefficients increase in magnitude and γ_1 become significant in the second sample, which reflect that NOVIX has become more sensitive to changes in OBX. The γ coefficients of the reference indices have a larger magnitude and the asymmetric relationships are more pronounced.

We see similar effects when modeling IV as log returns. The magnitude of the γ coefficients have increased from the first to the second sub-sample, and a clear asymmetry is evident in the second sub-sample. Again, the asymmetry is more pronounced for the reference indices and the explanation power from the returns regression is lower for NOVIX.

Our results show that NOVIX reacts less to market changes than VDAX-NEW and VIX. This smaller response may imply that the Norwegian option market is less efficient than the German and U.S. markets. However, the response has increased from the first to the second sub-sample, which means that the relationship between the NOVIX and its underlying index is now more similar to VIX and VDAX-NEW and their underlying indices.

5. NOVIX in volatility forecasting

We use realized volatility as a proxy for the true volatility as suggested by Andersen and Bollerslev (1998), and assess the incremental value of the information in the NOVIX for realized volatility forecasts. The realized volatility measure exploits information in high-frequency data, and one can in effect treat volatility as observable.

In the literature, the value of implied volatility indices for realized volatility forecasts has been studied extensively. Jiang and Tian (2005) find that the model-free implied volatility of S&P500 options subsumes all information contained in past realized volatility, and is a more efficient forecast for future realized volatility. We see clear signs of a relationship between the NOVIX and realized volatility in Fig. 3, and they clearly capture much of the same information.

5.1. Calculation of realized volatility

We let one trading day be split into *m* equidistant intervals, the intraday return r_i over the time period $[i - \frac{1}{m}, i]$ is given by

$$r_i = \ln\left(\frac{P_i}{P_{i-\frac{1}{m}}}\right), \quad \text{for} \quad i = \frac{1}{m}, \frac{2}{m}, \dots, 1,$$
(3)

where P_i is the price at time *i*. By taking the square root of the sum of the *m* squared intraday returns over a given trading day, we obtain daily realized volatility

$$RV^D = \sqrt{\sum_{i=1}^m r_i^2}.$$
(4)

We annualize RV^D by the conventional square-root-of-time rule with 250 trading days a year, i.e. $RV^D \times \sqrt{250}$.

Under perfect conditions, RV should be based on intra-day returns sampled at the highest possible frequency. However this runs into the challenge of market micro-structure in real world applications (Zhang, & Aït-Sahalia, 2005). This stems from the fact that efficient prices cannot be observed directly. Empirical work suggests that the estimate seems to diverge if RV is calculated using too frequent observations (Andreou & Ghysels, 2002; Bai, & Tiao, 2001; Bandi & Russell, 2008). We follow the common practice of using a sampling frequency of 5 min that allows us to ignore much of the microstructure noise.

The RV calculations were implemented by resampling the dataset to 5-minute intervals using a calendar time sampling scheme with equidistant samples. Our procedure for cleaning the high-frequency OBX data is based on the steps proposed by Barndorff-Nielsen, and Shephard (2009). Oslo Børs is open between 09:00 and 16:20, giving us 88 five-minute intra-day returns over a 440-minute typical trading day.

Our data set spans the period from April 13th 2010 to February 8th 2016. On average, there were 15,000 trades per day. For the DAX and S&P500, we obtain RV from the Oxford-Man Institute of Quantitative Finance (2016). Their values have been calculated with a 5-minute sampling interval, an equivalent procedure to ours, and cleaned as described in Barndorff-Nielsen et al. (2009).

5.2. Model

We use the Heterogeneous Autoregressive model of Realized Volatility (HAR-RV) by Corsi (2009) to forecast volatility on OBX,

Table 3

Leverage effect regression for Eq. (2) with levels and returns	. Results for the entire time period, as well as two sub-	samples.
--	---	----------

\dot{a} V_{t-1} R_{i}^{+} R_{i}^{-} R_{kd}^{2} NOVIX 2006-Feb. 2016 0.00" 0.01" 0.060" 0.08" 2011-Feb. 2016 0.00" 0.001" 0.006" 0.08" 2006-2010 0.01" 0.98" -0.03" 0.93" VDAX-NEW 2006-Feb. 2016 0.00" 0.0011 0.008" 0.09" 2006-2010 0.00" 0.0011 0.007" 0.101 0.08" 2006-2010 0.00" 0.0011 0.011 0.08" -0.35" 0.97 VDAX-NEW 2006-Feb. 2016 0.00" 0.001 0.071 0.08" -0.35" 0.97 2006-2010 0.01" 0.96" -0.31" -1.31" 0.98 2006-2010 0.00" 0.97" -0.93" -1.50" 0.99 2006-2010 0.00" 0.97" -0.93" -1.71" 0.98 2006-2010 0.00" 0.97" -0.94" -1.42" 0.99 2006-Feb.2016	LHS	Sample	Regression coefficients				
NOVIX 2006-Feb.2016 0.00 ⁺⁺ 0.98 ⁺⁺ -0.13 ⁺ -0.37 ⁺⁺ 0.98 2011-Feb.2016 0.00 ⁺⁺ 0.98 ⁺⁺ -0.25 ⁺⁺ -0.43 ⁺⁺ 0.96 2006-2010 0.01 ⁺ 0.98 ⁺⁺ -0.08 -0.35 ⁺⁺ 0.97 VDAX-NEW 2006-Feb.2016 0.00 ⁺⁺ 0.96 ⁺⁺ -0.31 ⁺⁺ -1.21 ⁺⁺ 0.98 VDAX-NEW 2006-Feb.2016 0.00 ⁺⁺ 0.96 ⁺⁺ -0.31 ⁺⁺ -1.21 ⁺⁺ 0.98 2006-2010 0.00 ⁺⁺ 0.96 ⁺⁺ -0.58 ⁺⁺ -1.17 ⁺⁺ 0.98 2006-2010 0.01 ⁺⁺ 0.96 ⁺⁺ -0.13 -1.31 ⁺⁺ 0.98 2006-2010 0.01 ⁺⁺ 0.96 ⁺⁺ -0.13 -1.31 ⁺⁺ 0.98 VIX 2006-Feb.2016 0.00 ⁺⁺ 0.97 ⁺⁺ -0.93 ⁺⁺ -1.1 ⁺⁺ 0.98 2011-Feb.2016 0.00 ⁺⁺ 0.97 ⁺⁺ -0.93 ⁺⁺ -1.1 ⁺⁺ 0.98 2006-Feb.2016 0.00 ⁺⁺ 0.97 ⁺⁺ -0.94 ⁺⁺ -1.1 ⁺⁺ 0.98 20			â	IV_{t-1}	R_t^+	R_t^-	$R^2_{Adj.}$
RMOVE(0.00)(0.00)(0.06)(0.08)2011-Feb.2016(0.00)(0.01)(0.08)(0.09)2006-2010(0.01)(0.01)(0.07)(0.10)VDAX-NEW2006-Feb.2016(0.00)(0.01)(0.01)(0.10)(0.00)(0.01)(0.01)(0.10)(0.07)(0.08)2011-Feb.2016(0.00)(0.01)(0.11)(0.08)(0.08)2006-Feb.2016(0.00)(0.08)-0.58**-1.07**0.98(0.00)(0.01)(0.01)(0.07)(0.08)(0.08)2006-2010(0.01)(0.01)(0.15)(0.12)(0.11)(0.00)(0.01)(0.01)(0.08)(0.08)(0.08)VIX2006-Feb.2016(0.00)(0.01)(0.08)(0.08)(0.08)(0.00)(0.01)(0.08)(0.08)(0.08)(0.08)(0.08)(0.00)(0.01)(0.08)(0.08)(0.08)(0.08)(0.08)(0.08)(0.00)(0.01)(0.08)(0.08)(0.08)(0.08)(0.08)(0.08)(0.08)(0.00)(0.01)(0.08	NOVIX	2006-Feb. 2016	0.00**	0.98**	-0.13*	-0.37**	0.98
2011-Feb.2016 0.00' 0.88*' -0.26*' -0.43*' 0.96 2006-2010 0.01' 0.98*' -0.08 (0.09)' (0.97)' VDAX-NEW 2006-Feb.2016 0.00'' 0.96'' -0.31*' -1.21*' 0.98 VDAX-NEW 2006-Feb.2016 0.00'' 0.96'' -0.31*' -1.21*' 0.98 2006-2010 0.00'' 0.96'' -0.13 -1.31*'' 0.98 2006-2010 0.00'' 0.96'' -0.13 -1.31*'' 0.98 2006-2010 0.00'' 0.96'' -0.13'' -1.50*'' 0.98 2006-2010 0.00'' 0.97''' -0.93*'' -1.50*'' 0.98 2006-2010 0.00''' 0.97''' -0.93*'' -1.50*'' 0.98 2006-2010 0.00''' 0.97''' -0.94*'' -1.42*'' 0.99 2006-2010 0.00''' 0.97'' -0.94*'' -1.42*'' 0.98 2006-2010 0.00''' 0.97''' -0.94*'' -1.42*''''''''''''			(0.00)	(0.01)	(0.06)	(0.08)	
NAME (000) (001) (008) (009) 2006-2010 0.01* 0.98* -0.08 -0.35** 0.9 VDAX-NEW 2006-Feb.2016 0.00** 0.96** -0.31** -1.21** 0.98 2011-Feb.2016 0.00* 0.98** -0.58** -1.07** 0.98 2011-Feb.2016 0.00* 0.98** -0.58** -1.07** 0.98 2006-2010 0.00* 0.98** -0.58** -1.07** 0.98 2006-2010 0.00* 0.99** -0.13 -1.31** 0.98 2006-2010 0.00* 0.99** -0.13 -1.31** 0.98 2006-2010 0.00* 0.99** -0.13 -1.31** 0.98 2006-Feb.2016 0.00* 0.97** -0.93** -1.50** 0.99 2006-Feb.2016 0.00** 0.97** -0.94** -1.42** 0.99 2006-Feb.2016 -0.00 0.01* 0.01* 0.01* 0.01* 2006-Feb.2016 -0.00*		2011-Feb. 2016	0.00**	0.98**	-0.26**	-0.43**	0.96
2006-2010 0.01' 0.98*' -0.08 -0.35*' 0.97 VDAX-NEW 2006-Feb.2016 0.00'' 0.96'' -0.31'' -1.21'' 0.98 VDAX-NEW 2016-Feb.2016 0.00'' 0.96'' -0.31'' -1.21'' 0.98 2011-Feb.2016 0.00'' 0.98'' -0.58'' -1.07'' 0.98 2006-2010 0.01'' 0.00'' 0.99''' -0.33''' -1.51''' 0.98 VIX 2006-Feb.2016 0.00'' 0.01''' 0.01''' 0.02'''' 0.02'''''''''''''''''''''''''''''''''''			(0.00)	(0.01)	(0.08)	(0.09)	
VDAX-NEW (0.00) (0.01) (0.07) (0.10) VDAX-NEW (0.00) (0.01) (0.11) (0.08) 2011-Feb. 2016 0.00* 0.98** -0.58** -1.07** 0.98 2006-2010 (0.01) (0.01) (0.07) (0.08) -0.13 -1.31** 0.98 2006-2010 (0.00) (0.01) (0.15) (0.12) -0.13 -1.50** 0.98 2006-2010 0.00* 0.97** -0.93** -1.50** 0.98 2011-Feb. 2016 0.00** 0.97** -0.93** -1.71** 0.98 2011-Feb. 2016 0.00** 0.97** -0.94** -1.42** 0.99 2011-Feb. 2016 0.00** 0.97** -0.94** -1.42** 0.99 2010-Feb. 2016 -0.00 (0.11) (0.10) (0.09) -1.37** 0.12 2006-Feb. 2016 -0.00 -1.39** -1.73** 0.12 -1.39** -1.73** 0.12 2000-Feb. 2016 -0.00 -0.		2006-2010	0.01*	0.98**	-0.08	-0.35**	0.97
VDAX-NEW 2006-Feb. 2016 0.00 ⁺⁺ 0.96 ⁺⁺ -0.31 ⁺⁺ -1.21 ⁺⁺ 0.98 2011-Feb. 2016 0.00 ⁺⁺ 0.98 ⁺⁺ -0.58 ⁺⁺⁺ -1.07 ⁺⁺ 0.98 2006-2010 0.00 ⁺ 0.96 ⁺⁺ -0.13 ⁺⁺ 0.98 2006-2010 0.01 ⁺ 0.96 ⁺⁺ -0.13 ⁺⁺ 0.98 0.00 ⁺ 0.96 ⁺⁺ -0.13 ⁺⁺ 0.98 0.00 ⁺ 0.96 ⁺⁺ -0.93 ⁺⁺ -1.3 ⁺⁺ 0.99 0.00 ⁺ 0.96 ⁺⁺ -0.93 ⁺⁺ -1.5 ⁺⁺ 0.99 0.00 ⁺ 0.96 ⁺⁺ -0.93 ⁺⁺ -1.5 ⁺⁺ 0.99 2011-Feb. 2016 0.00 ⁺⁺ 0.96 ⁺⁺ -0.89 ⁺⁺ -1.71 ⁺⁺ 0.98 2016-2010 0.00 ⁺⁺ 0.96 ⁺⁺ -0.89 ⁺⁺ -1.42 ⁺⁺ 0.99 2006-2010 0.00 ⁺⁺ 0.97 ⁺⁺ -0.94 ⁺⁺ -1.42 ⁺⁺ 0.99 2010-Feb. 2016 -0.00 ⁺ 0.77 ⁺⁺ -0.88 ⁺⁺ 0.12 ⁺⁺ 0.99 2006-2010 -0.00 ⁺ -0.01 ⁺⁺ -0.01 ⁺			(0.00)	(0.01)	(0.07)	(0.10)	
NOVE (0.00) (0.01) (0.11) (0.08) 2011-Feb. 2016 (0.00) (0.01) (0.07) (0.08) 2006-2010 (0.01) (0.01) (0.07) (0.02) VIX 2006-Feb. 2016 (0.00) (0.01) (0.08) (0.09) VIX 2006-Feb. 2016 (0.00) (0.01) (0.08) (0.09) 2011-Feb. 2016 (0.00) (0.01) (0.08) (0.09) 2006-2010 (0.00) (0.01) (0.08) (0.09) 2006-2010 (0.00) (0.01) (0.08) (0.09) 2006-Feb. 2016 (0.00) (0.01) (0.09) (0.09) (0.00) (0.01) (0.14) (0.09) (0.01) (0.09) (0.00) (0.01) (0.01) (0.01) (0.01) (0.09) (0.01) (0.00) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) <	VDAX-NEW	2006–Feb. 2016	0.00**	0.96**	-0.31**	-1.21**	0.98
2011-Feb.2016 0.00° 0.98° -0.58° -1.07° 0.98 2006-2010 0.001° (0.00) (0.07) (0.08) VIX 2006-2016 0.00° (0.01) (0.13) -1.31° 0.98 VIX 2006-Eb.2016 0.00° 0.97° -0.93° -1.50° 0.99 2011-Feb.2016 0.00° 0.96° -0.89° -1.71° 0.98 2011-Feb.2016 0.00° 0.96° -0.89° -1.71° 0.99 2011-Feb.2016 0.00° 0.011 (0.08) 0.99 -0.93° -1.42° 0.99 2006-2010 0.00° 0.97° -0.94° -1.42° 0.99 (0.00) (0.01) (0.10) (0.09) -1.13° 0.13 -1.13° 0.13 (0.00) -0.01° -0.01° -0.01° -0.01° -0.01° 0.01° 0.01° (0.00) -0.00 -0.01° -0.02° -0.01° -0.02° -0.01° -0.01° -0.01° -0.01° </td <td></td> <td></td> <td>(0.00)</td> <td>(0.01)</td> <td>(0.11)</td> <td>(0.08)</td> <td></td>			(0.00)	(0.01)	(0.11)	(0.08)	
RNOVIX (0.00) (0.01) (0.07) (0.08) VIX 2006-Feb. 2016 0.00** 0.96** -0.13 -1.31** 0.98 VIX 2006-Feb. 2016 0.00** 0.97** -0.93** -1.50** 0.99 2011-Feb. 2016 0.00** 0.96** -0.88** -1.71** 0.98 2006-C010 0.00** 0.96** -0.88** -1.42** 0.98 2006-C010 0.00** 0.97** -0.94** -1.42** 0.98 2006-Feb. 2016 -0.00 0.01* 0.10 0.09 0.00* 2006-Feb. 2016 -0.00 -0.70** -0.88** 0.06 0.000 (0.01) (0.14) (0.15) 0.12 2010-Feb. 2016 -0.00 -0.70** -0.88** 0.06 (0.00) (0.14) (0.15) (0.16) 0.12 RVDAX-NEW 2006-Feb. 2016 -0.01** -1.38** -3.51** 0.45 (0.00) (0.028) (0.15) (0.16) <t< td=""><td></td><td>2011–Feb. 2016</td><td>0.00*</td><td>0.98**</td><td>-0.58**</td><td>-1.07**</td><td>0.98</td></t<>		2011–Feb. 2016	0.00*	0.98**	-0.58**	-1.07**	0.98
2006-2010 0.01* 0.96* -0.13 -1.31** 0.98 VIX 2006-Feb. 2016 (0.00*) (0.01) (0.15) (0.12) VIX 2006-Feb. 2016 (0.00*) 0.97** -0.93** -1.50** 0.98 0.00 (0.01) (0.08) (0.08) (0.08) (0.08) 2011-Feb. 2016 0.00** 0.96** -0.89** -1.71** 0.98 (0.00) (0.01) (0.14) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.00) (0.01) (0.09) (0.09) (0.00) (0.01) (0.09) (0.00) (0.01) (0.01) (0.02) (0.02) (0.01)			(0.00)	(0.01)	(0.07)	(0.08)	
VIX (0.00) (0.01) (0.15) (0.12) VIX 0.00*- 0.00*+ 0.97** -0.93** -1.50* 0.98 (0.00) (0.01) (0.08) (0.08) 0.98 2011-Feb.2016 0.00*+ 0.96** -0.89** -1.71** 0.98 (0.00) (0.01) (0.14) (0.09) 0.97* -0.94** -1.42** 0.99 2006-2010 0.00** 0.07** -0.94** -1.42** 0.99 (0.00) (0.01) (0.10) (0.09) 0.97* -0.88** 0.99 2006-2010 (0.00) (0.01) (0.10) (0.09) 0.12 0.12 2010-Feb.2016 -0.00 -1.39** -1.73** 0.12 0.12 RVDAX-NEW 2006-2010 -0.01** -0.42** -0.64** 0.40 (0.00) (0.15) (0.16) 0.14* 0.15* 0.12* RVDAX-NEW 2006-2010 -0.01** -2.61** -4.03** 0.55		2006-2010	0.01*	0.96**	-0.13	-1.31**	0.98
VIX 2006-Feb. 2016 0.00** 0.97** -0.93** -1.50** 0.99 (0.00) (0.01) (0.08) (0.08) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.09) (0.00) (0.01) (0.09) (0.09) (0.00) (0.01) (0.09) (0.00) (0.01) (0.09) (0.00) (0.01) (0.09) (0.00) (0.01) (0.09) (0.00) (0.01) (0.01) (0.09) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.02) (0.01) (0.02) (0.01) (0.02) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01) (0.01)<			(0.00)	(0.01)	(0.15)	(0.12)	
NOVIX (0.00) (0.01) (0.08) (0.08) 2006-2010 (0.00) (0.01) (0.14) (0.09) (0.00) (0.01) (0.14) (0.09) (0.00) (0.01) (0.14) (0.09) (0.00) (0.01) (0.10) (0.09) (0.00) (0.01) (0.10) (0.09) (0.00) (0.01) (0.10) (0.09) (0.00) (0.01) (0.10) (0.09) (0.00) (0.11) (0.10) (0.09) (0.00) (0.14) (0.15) (0.12) (0.00) (0.14) (0.12) (0.02) (0.00) (0.030) (0.32) (0.16) (0.00) (0.15) (0.16) (0.16) (0.00) (0.15) (0.16) (0.16) (0.00) (0.28) (0.18) (0.18) (0.00) (0.28) (0.21) (0.21) (0.00) (0.31) (0.21) (0.21) (0.00)	VIX	2006–Feb. 2016	0.00**	0.97**	-0.93**	-1.50**	0.99
2011-Feb. 2016 0.00* 0.96** -0.89** -1.71** 0.98 (0.00) (0.01) (0.14) (0.09) (0.09) (0.00) 0.97** -0.94** -1.42** 0.99 (0.00) 0.97** -0.94** -1.42** 0.99 (0.00) 0.00* 0.97** -0.94** -1.73** 0.95 (0.00) 0.010 (0.09) -0.00 (0.14) (0.09) 0.97** -0.84** 0.06 (0.00) (0.01) (0.10) (0.09) -0.70** -0.88** 0.06 (0.00) (0.14) (0.15) -0.1** -0.00 (0.30) (0.32) RVDAX-NEW 2006-Feb. 2016 -0.01** -0.42** -0.64** 0.04 (0.00) (0.01) (0.15) (0.18) -0.1** -2.61** -4.03** 0.55 (0.00) (0.01) (0.026) (0.25) -0.1** -0.1** -0.1** -3.67** -4.65** 0.51 (0.00) <td< td=""><td></td><td></td><td>(0.00)</td><td>(0.01)</td><td>(0.08)</td><td>(0.08)</td><td></td></td<>			(0.00)	(0.01)	(0.08)	(0.08)	
RNOVX 0.00 0.01 0.14 0.09 RNOVX 0.967** -0.94** -1.42** 0.99 RNOVX 0.00-Feb.2016 -0.00 (0.01) (0.10) (0.09) RNOVX 2006-Feb.2016 -0.00 (0.01) (0.14) (0.09) 2010-Feb.2016 -0.00 (0.01) (0.14) (0.15) (0.12) 2006-2010 -0.00 (0.30) (0.32) (0.32) (0.30) (0.32) RVDAX-NEW 2006-2010 -0.01** -1.88** -3.51** 0.45 (0.00) (0.01) (0.28) (0.18) (0.18) RVDAX-NEW 2006-Feb.2016 -0.01** -1.88** -3.51** 0.45 (0.00) (0.01) (0.28) (0.25) (0.28) (0.25) RVTAX-NEW 2006-2010 -0.01** -1.34** -3.11** 0.38 (0.00) (0.00) (0.35) (0.21) (0.21) (0.21) RVTAX-NEW 2006-2010 -0.01** -3		2011–Feb. 2016	0.00**	0.96**	-0.89**	-1.71**	0.98
2006-2010 0.00* 0.97* -0.94** -1.42** 0.99 RNOVIX (0.00) (0.01) (0.10) (0.09) (0.09) RNOVIX 2006-Feb.2016 -0.00 -0.00 (0.14) (0.15) 0.12 (0.00) (0.01) (0.13) (0.32)			(0.00)	(0.01)	(0.14)	(0.09)	
RNOVEX (0.00) (0.01) (0.10) (0.09) RNOVEX 2006-Feb. 2016 -0.00 -0.70** -0.88** 0.06 (0.00) (0.14) (0.15) -0.73** 0.12 (0.00) (0.30) (0.32) -0.64** 0.04 (0.00) (0.030) (0.32) -0.64** 0.04 (0.00) (0.15) (0.16) -0.16** 0.04 (0.00) (0.15) (0.16) -0.16** 0.04 (0.00) (0.028) (0.18) -0.18** -3.51** 0.55 (0.00) (0.26) (0.25) -0.11** 0.38 -0.11** 0.38 (0.00) (0.028) (0.15) (0.15) -0.11** 0.38 (0.00) (0.026) (0.25) -0.11** 0.36 (0.00) (0.35) (0.21) -0.261** -4.03** 0.55 (0.00) (0.035) (0.21) -0.11** -0.11** 0.54 (0.00) (0.31)		2006-2010	0.00**	0.97**	-0.94**	-1.42**	0.99
RNOVIX 2006-Feb. 2016 -0.00 -0.70** -0.88** 0.06 (0.00) (0.14) (0.15) 2010-Feb. 2016 -0.00 -1.39** -1.73** 0.12 (0.00) (0.30) (0.32) 0.032) 2006-2010 -0.00 -0.42** -0.64** 0.04 (0.00) (0.15) (0.16) 0.15 0.15 RVDAX-NEW 2006-Feb. 2016 -0.01** -1.88** -3.51** 0.45 (0.00) (0.15) (0.18) 0.18 0.15 0.18 0.15 RVDAX-NEW 2006-2010 -0.01** -1.34** -3.51** 0.45 (0.00) (0.028) (0.18) 0.18 0.14 0.14 0.14 (0.00) (0.26) (0.25) 0.21			(0.00)	(0.01)	(0.10)	(0.09)	
RVDAX-NEW (0.00) (0.14) (0.15) RVDAX-NEW 2006-2010 (0.00) (0.30) (0.32) RVDAX-NEW 2006-Feb. 2016 -0.00 (0.00) (0.15) (0.64) RVDAX-NEW 2006-Feb. 2016 -0.01** -1.88** -3.51** 0.45 (0.00) (0.28) (0.18) (0.25) (0.25) (0.25) RVDAX-NEW 2006-2010 -0.01** -3.67** -4.65** 0.55 (0.00) (0.26) (0.21) (0.21) (0.21) (0.21) RVDX 2006-Feb. 2016 -0.00* -3.67** -4.65** 0.54 (0.00) (0.31) (0.41) (0.41) (0.41) (0.41) Q006-Feb. 2016 -0.00* -5.12** -7.37** 0.64 (0.00) (0.65) (0.49) (0.49) (0.41) (0.41) (0.41) (0.41) (0.41) (0.41) (0.41) (0.41) (0.41) (0.41) (0.41) (0.41) (0.41) (0.41) (0.41)	R ^{NOVIX}	2006–Feb. 2016	-0.00		-0.70**	-0.88**	0.06
2010-Feb. 2016 -0.00 -1.39* -1.73* 0.12 (0.00) (0.30) (0.32) 2006-2010 -0.00 -0.42* -0.64* 0.04 (0.00) (0.15) (0.16) 0.17 RVDAX-NEW 2006-Feb. 2016 -0.01** -1.88** -3.51** 0.45 (0.00) (0.28) (0.18) 0.15 0.18 2010-Feb. 2016 -0.01** -2.61** -4.03** 0.55 (0.00) (0.26) (0.25) 0.15 0.21 RVTX 2006-2010 -0.01** -1.34** -3.11** 0.38 (0.00) (0.35) (0.21) 0.21 0.			(0.00)		(0.14)	(0.15)	
RVDAX-NEW (0.00) (0.30) (0.32) RVDAX-NEW 2006-2010 -0.00 (0.15) (0.16) RVDAX-NEW 2006-Feb.2016 -0.01** -1.88** -3.51** 0.45 (0.00) (0.028) (0.18) (0.18) (0.18) (0.18) (0.25) (0.25) (0.25) (0.25) (0.25) (0.21) (0		2010–Feb. 2016	-0.00		-1.39**	-1.73**	0.12
2006-2010 -0.00 -0.42** -0.64** 0.04 (0.00) (0.15) (0.16) (0.17) (0.17) (0.17) (0.17) (0.17) (0.17) (0.14) (0.16) (0.16) (0.16) (0.16) (0.16) (0.16) (0.16) (0.16) <td></td> <td></td> <td>(0.00)</td> <td></td> <td>(0.30)</td> <td>(0.32)</td> <td></td>			(0.00)		(0.30)	(0.32)	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		2006-2010	-0.00		-0.42**	-0.64**	0.04
RVDAX-NEW 2006-Feb. 2016 -0.01** -1.88** -3.51** 0.45 (0.00) (0.28) (0.18) (0.28) (0.18) 2010-Feb. 2016 -0.01** -2.61** -4.03** 0.55 (0.00) (0.26) (0.25) (0.27) 2006-2010 -0.01** -1.34** -3.11** 0.38 (0.00) (0.35) (0.21) (0.21) RVIX 2006-Feb. 2016 -0.00* -3.67** -4.65** 0.54 (0.00) (0.31) (0.41)			(0.00)		(0.15)	(0.16)	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	R ^{VDAX-NEW}	2006–Feb. 2016	-0.01**		-1.88**	-3.51**	0.45
2010-Feb. 2016 -0.01** -2.61** -4.03** 0.55 (0.00) (0.26) (0.25) 2006-2010 -0.01** -1.34** -3.11** 0.38 (0.00) (0.35) (0.21) R ^{VIX} 2006-Feb. 2016 -0.00* -3.67** -4.65** 0.54 (0.00) (0.31) (0.41) 0.40* 0.64 (0.00) (0.65) (0.49) 0.64 (0.00) (0.65) (0.49) 0.54 (0.00) (0.65) (0.49) 0.54 (0.00) (0.65) (0.49) 0.54 (0.00) (0.28) (0.37) 0.54			(0.00)		(0.28)	(0.18)	
(0.00) (0.26) (0.25) 2006-2010 -0.01** -1.34** -3.11** 0.38 (0.00) (0.35) (0.21) R ^{VIX} 2006-Feb. 2016 -0.00* -3.67** -4.65** 0.54 (0.00) (0.31) (0.41) (0.41) 2010-Feb. 2016 -0.00 -5.12** -7.37** 0.64 (0.00) (0.65) (0.49) -4.65** 0.54 2006-2010 -0.00* -2.97** -3.78** 0.54 (0.00) (0.28) (0.37) -54		2010-Feb. 2016	-0.01**		-2.61**	-4.03**	0.55
2006-2010 -0.01** -1.34** -3.11** 0.38 (0.00) (0.35) (0.21) R ^{VIX} 2006-Feb. 2016 -0.00* -3.67** -4.65** 0.54 (0.00) (0.31) (0.41) (0.41) 0.64 2010-Feb. 2016 -0.00* -5.12** -7.37** 0.64 (0.00) (0.65) (0.49) -2.97** -3.78** 0.54 2006-2010 -0.00* -2.97** -3.78** 0.54			(0.00)		(0.26)	(0.25)	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		2006-2010	-0.01**		-1.34**	-3.11**	0.38
RVIX 2006-Feb. 2016 -0.00* -3.67** -4.65** 0.54 (0.00) (0.31) (0.41)			(0.00)		(0.35)	(0.21)	
(0.00) (0.31) (0.41) 2010-Feb. 2016 -0.00 -5.12** -7.37** 0.64 (0.00) (0.65) (0.49) 2006-2010 -0.00* -2.97** -3.78** 0.54 (0.00) (0.28) (0.37) -3.78** 0.54	R ^{VIX}	2006–Feb. 2016	-0.00*		-3.67**	-4.65**	0.54
2010-Feb. 2016 -0.00 -5.12** -7.37** 0.64 (0.00) (0.65) (0.49) 2006-2010 -0.00* -2.97** -3.78** 0.54 (0.00) (0.28) (0.37)			(0.00)		(0.31)	(0.41)	
(0.00) (0.65) (0.49) 2006-2010 -0.00* -2.97** -3.78** 0.54 (0.00) (0.28) (0.37)		2010-Feb. 2016	-0.00		-5.12**	-7.37**	0.64
$\begin{array}{cccc} 2006-2010 & -0.00^{*} & -2.97^{**} & -3.78^{**} & 0.54 \\ (0.00) & (0.28) & (0.37) \end{array}$			(0.00)		(0.65)	(0.49)	
(0.00) (0.28) (0.37)		2006-2010	-0.00*		-2.97**	-3.78**	0.54
			(0.00)		(0.28)	(0.37)	

RIV denotes the IV returns, computed as log differences. Newey-West standard errors in parenthesis below the regression coefficients, using five lags.

* Significant at the 5% level.

** Significant at the 1% level.

and augment the model with an additional independent variable for the NOVIX. The HAR-RV model is a long-memory model utilizing RV calculated from high-frequency data. Empirically, the volatility forecasts calculated from the HAR-RV model have performed much better than traditional GARCH models (Andersen, & Labys, 2003), and perform well compared to other more complicated long memory models (Andersen, & Diebold, 2007). Following Corsi (2009), we include three different volatility components based on different

Fig. 3. The NOVIX and OBX realized volatility in the period from April 13th 2010 to February 8th 2016, quoted in percentage values.

Table 4

In-sample regression results for HAR-RV and HAR-RV-IV models where the left-hand-side (LHS) variable in Eqs. (5) and (6) are solved using OLS regression, using daily (Panel A), weekly (Panel B) and monthly (Panel C) horizons.

LHS	Model	Regression coefficients					
		Intercept	RV_{t-1}^d	RV_{t-1}^w	RV_{t-1}^m	IV _{t-1}	R^2_{Adj}
Panel A: daily horizon							
OBX RV	HAR-RV	0.01** (0.00)	0.43** (0.07)	0.36** (0.08)	0.15 [*] (0.07)		0.672
	HAR-RV-IV	-0.00	0.36** (0.07)	0.33** (0.07)	-0.03 (0.08)	0.23** (0.05)	0.683
DAX RV	HAR-RV	0.01** (0.00)	0.37** (0.08)	0.39** (0.10)	0.17 (0.09)		0.613
	HAR-RV-IV	-0.02** (0.00)	0.25** (0.08)	0.24** (0.09)	-0.14 (0.09)	0.57** (0.06)	0.641
SP500 RV	HAR-RV	0.01** (0.00)	0.34** (0.07)	0.34** (0.07)	0.21** (0.07)		0.517
	HAR-RV-IV	-0.03** (0.01)	0.19** (0.07)	0.19** (0.06)	-0.15 (0.08)	0.66** (0.07)	0.600
Panel B: weekly horizon							
OBX RV	HAR-RV	0.01**	0.31**	0.33**	0.25**		0.704
	HAR-RV-IV	0.00	0.23**	0.31**	0.07	0.23** (0.05)	0.715
DAX RV	HAR-RV	0.02**	0.26**	0.35**	0.26**		0.673
	HAR-RV-IV	-0.00 (0.01)	0.15** (0.06)	0.25** (0.05)	0.05 (0.08)	0.41** (0.06)	0.693
SP500 RV	HAR-RV	0.02** (0.00)	0.23** (0.05)	0.31** (0.06)	0.29** (0.07)		0.580
	HAR-RV-IV	-0.01 (0.01)	0.11*** (0.04)	0.19** (0.06)	0.01 (0.08)	0.51*** (0.07)	0.625
Panel C: monthly horizon							
OBX RV	HAR-RV	0.03**	0.16**	0.18**	0.42**		0.618
	HAR-RV-IV	0.02**	0.11*	0.16**	0.28**	0.17**	0.626
DAX RV	HAR-RV	0.04**	0.15**	0.23**	0.36**	ו•• ,	0.586
	HAR-RV-IV	0.03** (0.01)	0.10* (0.05)	0.18** (0.06)	0.26** (0.08)	0.20* (0.08)	0.591

Newey-West standard errors in parenthesis below the regression coefficients, using five lags.

Significant at the 5% level.
Significant at the 1% level.

** Significant at the 1% level.

horizons: daily, weekly and monthly. The one-day ahead HAR-RV model is specified as

$$RV_{t+1}^{D} = \alpha + \beta_1 RV_t^{D} + \beta_2 RV_t^{W} + \beta_3 RV_t^{M} + \epsilon_{t+1},$$
(5)

where the superscripts *D*, *W* and *N* denote daily, weekly and monthly RV. These multi-period volatilities are defined as simple backward averages of the daily RV. Thus, weekly and monthly RV using 5 trading days a week and 22 per month is

$$RV_t^W = \frac{1}{5} \sum_{i=0}^4 RV_{t-i}^D$$
 and $RV_t^M = \frac{1}{22} \sum_{i=0}^{21} RV_{t-i}^d$

Table 5

Out-of-sample regression results for HAR-RV-IV models specified in Eq. (6). We use a window 500 observations, and evaluate the forecasts with mean squared errors (MSE) for daily, weekly and monthly horizons. The MSE values are normalized relative to HAR-RV.

	MSE					
LHS	Daily	Weekly	Monthly			
OBX RV DAX RV	0.955* 0.919**	0.921** 0.914**	0.944* 0.973*			
SP500 RV	0.820**	0.832**	0.921**			

The MSE values are marked with */** if the HAR-RV-IV model is significantly more accurate than the corresponding HAR-RV model according to the test of Diebold and Mariano (1995), using squared error loss function.

* Significant at the 5% level.

** Significant at the 1% level.

We augment the model similarly as Haugom, and Westgaard (2014) when we include the IV component, and denote this model HAR-RV-IV. The model is then specified as

$$RV_{t+1}^D = \alpha + \beta_1 RV_t^D + \beta_2 RV_t^W + \beta_3 RV_t^M + \beta_4 IV_t + \epsilon_{t+1}.$$
(6)

In addition to daily forecasts, we use the models in Eqs. (5) and (6) to produce weekly and monthly forecasts. For this, the left-hand-side variables in both models are changed to the forward averages of the monthly and weekly RVs, i.e.

$$RV_t^W = \frac{1}{5} \sum_{i=0}^4 RV_{t+i}^D$$
 and $RV_t^M = \frac{1}{22} \sum_{i=0}^{21} RV_{t+i}^D$

The models are estimated with OLS regression.

5.3. In-sample evaluation

In-sample results are displayed in Table 4. We first consider the Norwegian market. For the HAR-RV model, we see that all lagged values of daily, weekly and monthly RV coefficients are significant. The model puts the most weighting on the lagged variable matching the model's horizon. Moving to the HAR-RV-IV model, we see that the NOVIX coefficient is significant for all horizons and of relatively large magnitude. Consequently, the daily, weekly and monthly RV coefficients are reduced in both statistical significance and coefficient value. The largest decrease is observed for the monthly RV, which is no longer significant for the daily and weekly horizons. Also, the adjusted R² squared is increased across all three horizons when the NOVIX is included. Thus, our initial finding is that the NOVIX may be a useful predictor of future RV.

Turning to our reference indices, we observe similar dynamics when modeling future RV for DAX and S&P500. However, we note that the magnitude of the IV coefficients relative to the lagged RV coefficients in the HAR-RV-IV model is always greater for the reference indices compared to the NOVIX. This indicates that the VDAX-NEW and VIX are more useful than the NOVIX when predicting future RV in their respective markets.

5.4. Out-of-sample evaluation

Out-of-sample forecasts let us directly compare the performance of the models relative to the actual observed values. The forecasts use a rolling window of 500 observations for the time interval January 4th 2010 to February 22nd 2016.

We use mean squared errors (MSE) to evaluate the two models. MSE is a robust loss function when it comes to volatility forecast errors, as described in Patton (2011). The loss function is defined as

$$MSE = \frac{1}{T} \sum_{t=1}^{T} (\hat{RV}_t - RV_t)^2,$$
(7)

where \hat{RV}_t is the forecast value for time *t* and RV_t is the actual observed value at time *t*. Besides MSE, we employ the statistical test of Diebold and Mariano (1995) (DM) to determine if the model that includes the NOVIX provides significantly more accurate forecasts than the model without. We specify the loss function in the DM test as squared errors.

Table 5 shows the out-of-sample results for all forecast horizons. The MSE values improve by 4.5%, 7.9% and 5.6% for the daily, weekly and monthly horizons when we add the NOVIX as an additional variable. The DM test rejects the null hypothesis of equal predictive ability at the 5% level for daily and monthly forecasts, and at the 1% level for weekly forecasts. The alternative hypothesis is that the HAR-RV-IV model produces more accurate forecasts.

Turning to our reference indices, we see that all three considered implied volatility indices contain information not included in the historical volatility. However, both VDAX-NEW and VIX both improve volatility forecasts in their respective markets more than the NOVIX. In other words, the option prices contain forward looking information to largest degree in the US, then in Germany and to smallest extend in Norway.

6. Concluding remarks

We introduce a model-free implied volatility index for the Norwegian market, the NOVIX. We construct the NOVIX from options on the OBX index and analyze its properties in the period between January 3rd 2006 and February 22nd 2015. Throughout the paper we study the NOVIX in light of the popular VIX and VDAX-NEW implied volatility indices for the U.S. and German markets. We show that the index contains the same characteristics as VIX and VDAX-NEW when we study features such as stationarity and leverage effect. In order to facilitate for further research, we also calculate and provide continuously updated 5-minute intraday values for the NOVIX at https:// novix.xyz.

We find that the potential value of the created Norwegian implied volatility index has increased steadily over the last 15 years, as the NOVIX is more efficient at absorbing market information today than it was a decade ago. The correlation between NOVIX and OBX returns shows an increased negative relationship, and today the correlation between NOVIX and OBX returns is similar to that of VIX and S&P500 returns.

In the end, we study the value of NOVIX, VIX and VDAX-NEW in the volatility forecasting for their respective markets. We find that all three volatility indices are very useful in volatility forecasting and can improve even a precise HAR model of Corsi (2009) based on high-frequency data. This result holds across forecasting horizons we studied - daily, monthly and weekly. However, the largest improvement is achieved by the U.S. VIX index, followed by the German VDAX-NEW and Norwegian NOVIX. This indicates that option prices in larger and more liquid markets reflect more forward-looking information than option prices in smaller and less liquid markets.

Acknowledgments

We would like to thank Oslo Børs and Øyvind Skar for providing us with data, and for many useful insights.

References

- Andersen, T. G., & Bollerslev, T. (1998). Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International economic review*, 885–905. http://dx.doi.org/10.2307/2527343.
- Andersen, T. G., Bollerslev, T., & Diebold, F. X. (2007). Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility. *Review of Economics and Statistics*, 89(4), 701–720. http://dx.doi.org/10.1162/rest. 89.4.701.
- Andersen, T. G., Bollerslev, T., Diebold, F. X., & Labys, P. (2003). Modeling and forecasting realized volatility. *Econometrica*, 71(2), 579–625. http://dx.doi.org/10.1111/ 1468-0262.00418.
- Andreou, E., & Ghysels, E. (2002). Detecting multiple breaks in financial market volatility dynamics. *Journal of Applied Econometrics*, 17(5), 579–600. http://dx.doi.org/ 10.1002/jae.684.
- Bai, X., Russell, J. R., & Tiao, G. C. (2001). Beyond merton's utopia (i): Effects of nonnorMality and dependence on the precision of variance estimates using highfrequency financial data,
- Bandi, F. M., & Russell, J. R. (2008). Microstructure noise, realized variance, and optimal sampling. *The Review of Economic Studies*, 75(2), 339–369. http://dx.doi.org/10. 1111/j.1467-937X.2008.00474.x.
- Barndorff-Nielsen, O. E., Hansen, P. R., Lunde, A., & Shephard, N. (2009). Realized kernels in practice: Trades and quotes. *Econometrics Journal*, 12(3), C1–C32. http:// dx.doi.org/10.1111/j.1368-423X.2008.00275.x.
- Bates, D. S. (2000). Post-'87 crash fears in the s&p 500 futures option market. *Journal of Econometrics*, 94(1), 181–238. http://dx.doi.org/10.1016/S0304-4076(99)00021-4.
- Black, F. (1976). The pricing of commodity contracts. Journal of financial economics, 3(1), 167–179. http://dx.doi.org/10.1016/0304-405X(76)90024-6.
- Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. The journal of political economy, 637–654. http://dx.doi.org/10.1086/260062.
- Britten-Jones, M., & Neuberger, A. (2000). Option prices, implied price processes, and stochastic volatility. *The Journal of Finance*, 55(2), 839–866. http://dx.doi.org/10. 1111/0022-1082.00228. http://dx.doi.org/10.1111/0022-1082.00228.
- Chicago Board Options Exchange(2015). VIX white paper. (visited on 06/09/2016) URL: https://www.cboe.com/micro/vix/vixwhite.pdf.
- Christie, A. A. (1982). The stochastic behavior of common stock variances: Value, leverage and interest rate effects. *Journal of financial Economics*, 10(4), 407–432. http:// dx.doi.org/10.1016/0304-405X(82)90018-6.
- Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics*, 7(2), 174–196. http://dx.doi.org/10.1093/jjfinec/ nbp001.
- Demeterfi, K., Derman, E., Kamal, M., & Zou, J. (1999). More than you ever wanted to know about volatility swaps. Goldman Sachs quantitative strategies research notes, 41,
- Derman, E., & Kani, I. (1994). Riding on a smile. Risk, 7(1), 32-39.
- Diebold, F. X., & Mariano, R. S. (1995). Comparing predictive accuracy. *Journal of Business &, Economic Statistics*, 13(3), 253–263. http://dx.doi.org/10.1080/07350015. 1995.10524599.
- Dupire, B. (1994). Pricing with a smile. *Risk*, 7(1), 18–20.
- Dupire, B. (1997). Pricing and hedging with smiles, Mathematics of derivative securities. Dempster and Pliska eds., Cambridge Uni. Press.
- Eraker, B. (2004). Do stock prices and volatility jump? Reconciling evidence from spot and option prices. *The Journal of Finance*, 59(3), 1367–1403. http://dx.doi.org/10. 1111/j.1540-6261.2004.00666.x.
- Figlewski, S., & Wang, X. (2000). Is the 'leverage effect' a leverage effect? Available at SSRN 256109, http://dx.doi.org/10.2139/ssrn.256109.
- González, M. T., & Novales, A. (2009). Are volatility indices in international stock markets forward looking? RACSAM-Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales. Serie A. Matematicas, 103(2), 339–352. http://dx.doi.org/10. 1007/BF03191911.
- Haugom, E., Langeland, H., Molnár, P., & Westgaard, S. (2014). Forecasting volatility of the U.S. oil market. *Journal of Banking &, Finance*, 47, 1–14. http://dx.doi.org/10. 1016/j.jbankfin.2014.05.026.
- Jiang, G. J., & Tian, Y. S. (2005). The model-free implied volatility and its information content. *Review of Financial Studies*, 18(4), 1305–1342. http://dx.doi.org/10.1093/ rfs/hhi027.

- Jiang, G. J., & Tian, Y. S. (2007). Extracting model-free volatility from option prices: An examination of the VIX index. *Journal of Derivatives*, 14(3), http://dx.doi.org/10. 2139/ssrn.880459.
- Oslo Børs(2015). Oslo Børs website, (visited on 06/09/2016) URL: http://www.oslobors. no.
- Oxford-Man Institute of Quantitative Finance(2016). *Realized library*. (visited on 06/09/2016) URL: http://realized.oxford-man.ox.ac.uk.
- Patton, A. J. (2011). Volatility forecast comparison using imperfect volatility proxies. Journal of Econometrics, 160(1), 246–256. http://dx.doi.org/10.1016/j.jeconom. 2010.03.034.
- Rubinstein, M., Britten-Jones, M., & Neuberger, A. (1994). Implied binomial trees. *The Journal of Finance*, 49(3), 771–818. http://dx.doi.org/10.1111/j.1540-6261.1994. tb00079.x.
- Schwert, G. W. (2002). Tests for unit roots: A Monte Carlo investigation. Journal of Business & Economic Statistics, 7(2), 147–159. http://dx.doi.org/10.1198/ 073500102753410354.

- Skiadopoulos, G. (2004). The greek implied volatility index: Construction and properties. Applied Financial Economics, 14(16), 1187–1196. http://dx.doi.org/10.1080/ 0960310042000280438.
- Ting, H. A. (2007). Fear in the Korea market. *Review of Futures Markets*, *16*(1), 106. Whaley, R. E. (1993). Derivatives on market volatility: Hedging tools long overdue. *The*
- *journal of Derivatives*, 1(1), 71–84. http://dx.doi.org/10.3905/jod.1993.407868. Wu, G., & Xiao, Z. (2002). A generalized partially linear model of asymmetric volatil-
- ity. Journal of Empirical Finance, 9(3), 287–319. http://dx.doi.org/10.1016/S0927-5398(01)00057-3.
- Zhang, L., Mykland, P. A., & Aït-Sahalia, Y. (2005). A tale of two time scales: Determining integrated volatility with noisy high-frequency data. *Journal of the American Statistical Association*, 100(472), 1394–1411. http://dx.doi.org/10.1198/ 016214505000000169.