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A public firm in a vertically linked price discriminating spatial duopoly



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ARTICLE INFO

Article history:

Received 27 July 2015

Received in revised form 13 August 2015

Accepted 1 September 2015

Available online 23 January 2016

JEL classification:

L1

L4

D4

R3

Keywords:

Product differentiation

Spatial competition

Public firm

Price discrimination

Firm location

ABSTRACT

We show that, in a vertically linked duopoly where neither firm can produce all varieties demanded, spatial competition between a public and a private firm induces them to deviate from the socially optimal location. We identify specific conditions under which a change in the degree of privatization induces one firm to move toward, while the other moves away from the socially optimal location. There exists a critical level of privatization above (below) which the public and private firms will come close (drift apart) with a rise in the degree of privatization.

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1. Introduction

We capture the responsiveness of equilibrium locations of public and private firms selling different varieties of a product in a vertically related industry to a change in the degree of privatization. Examples of such vertically structured industries, where the co-existence of public and private firms is a “fundamental feature”, are abundant (e.g., auto, steel, banking, insurance, housing, health, education, energy, transportation, telecommunications, etc.).² Empirical evidence by [Hollas and Stansell \(1988\)](#) and [Hollas \(1990\)](#) indicate that property rights, as evidenced by public and private ownership, affect pricing patterns. The latter study provides evidence that, holding demand and cost characteristics constant, public and private pricing patterns vary across customer groups. The evident “movement towards (at least partial) privatization of public firms”³ operating in such industries motivates us to enquire whether the location decisions of firms operating in mixed oligopolies, when vertically structured, vary with the degree of privatization. In this paper, we build on [Braid \(2008\)](#) and [Beladi et al. \(2008, 2014\)](#) to find an answer to this question.

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² See [De Donder and Roemer \(2006\)](#) and [De Fraja \(2009\)](#).

³ See [De Donder and Roemer \(2006\)](#).

Braid (2008) established that the equilibrium locations of any two firms are partially centralized to a social optimum in case of spatial discrimination in price when neither of them can supply all varieties demanded. Beladi et al. (2008) demonstrated that vertical integration with an upstream manufacturer will tempt each downstream retailer (whether part of a merger or not) engaged in spatial competition for a market where neither of the downstream firms can produce all varieties demanded to deviate from Braid's (2008) socially optimal location. With the interest in the role of public firms in location decisions continuing to mount since the dramatic financial events of this millennium have given rise to new sectors where public and private companies compete to serve the same market, Beladi et al. (2014, 2015) showed that the equilibrium locations of two spatially price discriminating firms (none of which can produce all varieties demanded) are invariant with the degree of privatization when firms move simultaneously, but sensitive to the degree of privatization when the public and private firms move sequentially.⁴

We demonstrate that the Nash equilibrium locations of a public and a private firm competing spatially in a vertically structured mixed duopoly are not socially optimal and can vary with the degree of privatization when no firm can produce all varieties demanded and the demands for all product varieties are not identical. When the degree of privatization rises, the private firm will move toward, while the public firm moves away from, the socially optimal location if the fraction of consumers wanting to buy the commonly produced good falls short of the fraction of those wanting to buy one of the goods produced exclusively by either firm. The public firm moves toward, while the private firm moves away from, the socially optimal location if the degree of privatization rises when the fraction of consumers wanting to buy the commonly produced good exceeds the fraction of those wanting to buy one of the goods produced exclusively by either firm. There exists a critical level of privatization below which the public and private firms will drift apart and above which the firms will come closer with a rise in the degree of privatization.

The practical relevance of our results follows directly from the apparent “movement towards (at least partial) privatization of public firms” operating in vertically structured mixed oligopolies ranging from “network industries (energy, transportation, telecommunications) to the service sectors (banking, insurance), and from health care provision to education”.⁵ The policy implications are particularly important for transition economies experiencing unprecedented scales of privatization as they are moving away from economic systems based on central planning. For instance, while the former Soviet Union and Central and Eastern Europe led the waves of privatization in transition economies, there is significant cross-country variation in the degrees of privatization, with some governments opting for low degrees relative to others. Our analysis suggests that different degrees of privatization can induce varying deviations of the location of private and public firms from the social optimum. As such, the choice of the degree of privatization must take into cognizance the consequent impact on firm locations.

2. Model and results

Following Beladi et al. (2008), visualize a stylized representation of a vertically related industry where an upstream manufacturer (M) produces an intermediate good and sells this good to two downstream retailers ($R_j; j = 1, 2$). The downstream retailers transform each unit of the intermediate good into one unit of a differentiated final good. The final good is sold to consumers that are uniformly distributed with unit density on a linear (uni-dimensional) market interval. The location of R_1 and R_2 are denoted by x and y , respectively, on this market interval with support $[0, 1]$. R_1 sells products A and C and R_2 sells products B and C. A fraction c of buyers demand good A; a fraction c of buyers demand good B; and a fraction b of buyers demand good C.⁶ Suppose, as in Beladi et al. (2014, 2015), that one of the downstream retailers (say R_2 , without loss of any generality) is publicly owned with $\alpha \in (0, 1)$ parameterizing the proportion of privately held shares in R_2 .

We assume that there is spatial price discrimination for good C of the sort originally examined by Lerner and Singer (1937), where a Nash equilibrium exists in delivered price schedules. Consumers are willing to pay a maximum reservation price (k) that is sufficiently high so that it becomes relevant only in the absence of any inter-firm competition. td measures the costs of transportation, with t being a constant and d the distance covered. Monopoly goods A and B are priced at a uniform delivered price that is infinitesimally below k .

As in Beladi et al. (2008), the downstream retailers choose their locations simultaneously, while the upstream manufacturer's offer takes the form of a two-part tariff. The role of the two-part tariff, *ceteris paribus*, is instrumental since this allows no perturbation in retail prices to raise the profits of any of the retailers, leaving no incentive for any deviation from the Nash equilibria identified in Beladi et al. (2008). Decisions are taken in stages with perfect monitoring, with each past action becoming common knowledge at the completion of every stage. At the initial stage, a two-part tariff contract is offered to each of the downstream retailers by the upstream manufacturer on a take-it-or-leave-it basis: M 's offer takes the form (F_j, w_j) , extracting all the profits from R_j , where w_j is a uniform wholesale price and F_j is a fixed fee. It is at this same

⁴ More specifically, Beladi et al. (2015) have shown that a rise (fall) in the degree of privatization will induce the public and private firms to move closer to (farther from) the socially optimal Nash equilibrium when the public firm leads.

⁵ See De Donder and Roemer (2006).

⁶ It is possible to contemplate an equivalent scenario where one of the downstream firms sells one variety while the other sells a different variety, and some consumers want to buy only one of the two varieties and some are indifferent between the two. Following Braid (2008), if neither firm can price discriminate, it is possible to assume mixed price strategies. Unlike Dasgupta and Maskin (1986), who had a single mixed-strategy Nash equilibrium in mill prices for any given set of firm locations, there would be a different mixed-strategy Nash equilibrium in delivered prices for any given locations of firms.

stage of the game that the downstream retailers choose their locations simultaneously. In the following stage, the retailers must simultaneously commit to accepting or declining M 's offer. The fixed fee (F_j) is collected by M at this stage only if R_j decides to accept the contract offered. Spatial discrimination in prices by each retailer occurs in the next stage. In the final stage, consumers reveal their demand for goods. The downstream retailers pay the wholesale price (w_j) for each unit that is ordered from the upstream manufacturer and then sell the final goods to the consumers. A solution is reached by backward induction. R_1 's (located at x) profits from a) selling A , at a uniform delivered price (k), are $c(k - w_1) - \frac{ct}{2}[x^2 + (1 - x)^2]$; b) selling C , to consumers located in the market interval from 0 to x , are $b[t(y - x) - w_1]$; and c) selling C , to consumers located in the market interval from x to $\frac{x+y}{2}$, are $b[t(x + y - 2z) - w_1]$. R_1 chooses x to maximize its own profits

$$\left(c(k - w_1) - \frac{ct}{2}[x^2 + (1 - x)^2] \right) + \left(\int_0^x b[t(y - x) - w_1]dz + \int_x^{\frac{x+y}{2}} b[t(x + y - 2z) - w_1]dz \right).$$

The objective of the publicly-owned firm (R_2 , located at y), as in [Beladi et al. \(2014\)](#), is to maximize a weighted average of its own producer's surplus and social welfare, where the weight is the degree of privatization. Social welfare comprises the profits of both firms as well as the consumer surplus. An underlying model of bargaining between the public and the private shareholders, where the board of the firm consists of government's representatives who advocate welfare (consumer and producer surplus) and representatives of the private shareholders who advocate profit, can be used to rationalize such a welfare function: bargaining will involve α percent of representatives who have an objective of maximizing profits and $(1 - \alpha)$ percent of representatives who have an objective of maximizing welfare, since $(1 - \alpha)$ is the proportion of publicly held shares in the R_2 and the rest is privately owned.⁷ The monopoly goods A and B are priced to leave zero consumer surplus, while the spatial duopoly good C generates consumer surplus that consists of a) $b(k - t(y - z))$ for consumers located in the market interval from 0 to $\frac{x+y}{2}$, where $t(y - z)$ is the delivered price, and b) $b(k - t(z - x))$ for consumers located in the market interval from $\frac{x+y}{2}$ to 1, where $t(z - x)$ is the delivered price. Thus, R_2 chooses to maximize

$$\begin{aligned} & \left(c(k - w_2) - \frac{ct}{2}[y^2 + (1 - y)^2] \right) - \left(\int_{\frac{x+y}{2}}^y b[t(x + y - 2z) - w_2]dz + \int_y^1 b[t(x - y) - w_2]dz \right) \\ & + (1 - \alpha) \left[\left(c(k - w_1) - \frac{ct}{2}[x^2 + (1 - x)^2] \right) + \left(\int_0^x b[t(y - x) - w_1]dz + \int_x^{\frac{x+y}{2}} b[t(x + y - 2z) - w_1]dz \right) \right. \\ & \left. + \int_0^{\frac{x+y}{2}} b(k - t(y - z))dz + \int_{\frac{x+y}{2}}^1 b(k - t(z - x))dz \right] \end{aligned}$$

The first order conditions for profit maximization yields

$$(x, y) = \left(\frac{2(3\alpha - 1)(b - c)}{48c - b(23 + 18\alpha + 9\alpha^2)}, \frac{12(b - c)}{48c - b(23 + 18\alpha + 9\alpha^2)} \right).$$

In comparison, [Braid \(2008\)](#) showed that the socially optimal locations are

$$\left(\frac{1}{2} - \frac{b}{4(b + c)}, \frac{1}{2} + \frac{b}{4(b + c)} \right)$$

Our main propositions follow.

Proposition I. The Nash equilibrium locations are not socially optimal, with or without privatization.

⁷ Following [Chao and Yu \(2006\)](#), such bargaining will yield a mixed objective between profits and welfare, in which each carries the respective weight of the representatives.

Proof:

$$\frac{2(3\alpha - 1)(b - c)}{48c - b(23 + 18\alpha + 9\alpha^2)} + \frac{b}{4(b + c)} \neq \frac{1}{2} \neq \frac{12(b - c)}{48c - b(23 + 18\alpha + 9\alpha^2)} - \frac{b}{4(b + c)} \text{ for any } \alpha \in (0, 1).$$

Proposition II. The private firm moves i) toward the socially optimal location, while the public firm moves away from it, when privatization rises if $b > c$, and ii) away from the socially optimal location, while the public firm moves toward it, when privatization rises if $b < c$.

Proof:

$$\frac{\partial x}{\partial \alpha} \underset{<}{\geq} 0 \text{ if } b \underset{<}{\geq} c \text{ and } \frac{\partial y}{\partial \alpha} \underset{<}{\geq} 0 \text{ if } b \underset{<}{\geq} c \forall \alpha \in (0, 1).$$

Proposition III. A rise in the degree of privatization i) above a critical level (α_c) induces the private and public firms to come close, and ii) below α_c induces the private and public firms to drift apart.

Proof:

$$x = \frac{1}{6}(3\alpha - 1)y \Rightarrow \frac{dx}{dy} = \frac{1}{6}(3\alpha - 1) \underset{<}{\geq} 0 \text{ if } \alpha \underset{<}{\geq} \frac{1}{3} = \alpha_c.$$

In sum, when a publicly owned firm and a private firm compete with neither firm producing all varieties demanded, firms do not locate at the socially optimal Nash equilibrium. Except when demand for all product varieties are identical (i.e., $b = c$), the Nash equilibrium locations of both firms are sensitive to the degree of privatization. A rise in the degree of privatization induces the private (public) firm to move toward (away from) the socially optimal location if the fraction of consumers wanting to buy the commonly produced good falls short of the fraction of those wanting to buy one of the goods produced exclusively by either firm. When the degree of privatization rises, the public (private) firm moves toward (away from) the socially optimal location if the fraction of consumers wanting to buy the commonly produced good exceeds the fraction of those wanting to buy one of the goods produced exclusively by either firm. There exists a critical level of privatization below which the public and private firms will drift apart and above which the firms will move closer with a rise in the degree of privatization.

3. Conclusion

Our results shed new light on the role of privatization in the location choice of vertically linked firms engaged in spatial competition. We show that, when a publicly owned firm competes with a private firm in a vertically related industry where neither firm can produce all varieties demanded, firm locations are not socially optimal as long as the demand for all product varieties are not identical. The private firm moves toward, while the public firm moves away from, the socially optimal location if the degree of privatization rises when the fraction of consumers wanting to buy the commonly produced good falls short of the fraction of those wanting to buy one of the goods produced exclusively by either firm. The public firm will move toward, while the private firm moves away from, the socially optimal location if the degree of privatization rises when the fraction of consumers wanting to buy the commonly produced good exceeds the fraction of those wanting to buy one of the goods produced exclusively by either firm. A rise in privatization above (below) a critical level will induce the public and private firms to come close (drift apart). We anticipate that our findings are likely to have important implications for firms' choice of entry mode as well.⁸ As such, some natural extensions of this paper may involve imposing trade barriers à la [Oladi \(2004, 2005\)](#) and/or allowing asymmetric costs à la [Mukherjee and Sinha \(2014\)](#).

Acknowledgments

The authors would like to thank two anonymous referees for valuable comments on an earlier draft of this manuscript. The usual disclaimer applies.

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⁸ See, for instance, [Haller \(2009\)](#), [Lahiri \(2009\)](#), [Kurata et al. \(2009\)](#), [Raff et al. \(2009\)](#), [Kikuchi and Long \(2010\)](#), and [Klomp \(2010\)](#).

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