



# Strategic carbon taxation and energy pricing under the threat of climate tipping events



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## ABSTRACT

An appropriate design of climate mitigation policies such as carbon taxes may face a lot of challenges in reality, e.g., the strategic behavior of fossil fuel producers, and huge uncertainty surrounding the climate system. This paper investigated the effect of possible climate tipping events on optimal carbon taxation and energy pricing, taking into account the strategic behavior of energy consumers/producers and the uncertainty of tipping points through a stochastic dynamic game. The game was solved numerically to get some insights into the equilibrium carbon taxation and energy pricing strategies under the threat of possible tipping events. The results suggest that the sudden occurrence of tipping events will shift the carbon tax upwards but shift the (wellhead) fuel price downwards, which generates an overall effect of a sudden increase in the consumer price. The irreversibility of the damage caused by a tipping point implies more aggressive carbon taxation. Moreover, the design of climate policy should adjust to tipping probabilities, damage uncertainty, and types of tipping events.

## 1. Introduction

Climate change has been considered as one of the most important environmental issues at our time and the accumulated greenhouse gases (GHGs) in the atmosphere are believed to be the main cause of this. Mitigating climate change needs to address the externalities caused by GHGs emissions from fossil fuel consumption, through policy instruments such as carbon taxes or emission trading systems (see, e.g., Wu et al., 2014; Ouchida and Goto, 2016; Fan et al., 2016). However, the optimal design of climate policy is subject to many issues in reality, e.g., oligopoly in fossil energy markets, strategic behavior of agents, and uncertainties in the climate system.

It is known that there are huge uncertainties surrounding climate system, which implies that it is almost impossible to be sure about the exact effect of greenhouse gas (GHG) emissions on global climate change, and thus the ecological and socio-economic damage in the future. Therefore, the decisions on regulating emissions have to be made before we realize the exact consequences of climate change and the design of climate policy needs to take such uncertainties into consideration. However, there is a great variation in the approaches for modeling climate uncertainty across different studies. Many studies assume that the damage from climate change is a continuous function of atmospheric temperature which is stochastic due to the uncertainty in the effect of CO<sub>2</sub> concentration on temperature increase (e.g., Wirl,

2007). However, the scientific evidences show that the mechanism of climate change in reality can be much more complex. For instance, global warming could push the climate and ecosystem toward passing a threshold, where sudden, irreversible events would occur and lead to remarkable and persistent damage. These tipping events can be the melting of the Antarctic ice sheet, the die-back of the Amazon rainforest, and so on. What could be even worse is that we are not sure when these tipping events would occur. The sudden, irreversible damage caused by the tipping events implies a shift in the damage function at some certain levels of temperature, which are so-called tipping points. After the occurrence of tipping events, the damage will still be persistent even if there is no further warming or even there is cooling. That is, the regime after the occurrence of a tipping event will be different from the one before, as shown in Fig. 1.

In addition to the tipping events, another prevailing issue in global warming studies is the strategic behavior of stakeholders occurring around the taxation and pricing of fossil energy resources. Given the unbalanced resource endowment in the world, there are always conflicts of interest between different stakeholders, e.g., fossil energy consumers and producers, during the mitigation of climate change. From a more general perspective of energy production and consumption worldwide, a phenomenon that is high-likely to be observed in climate change mitigation can be described as follows: A possible coalition of energy importing countries coordinating their carbon

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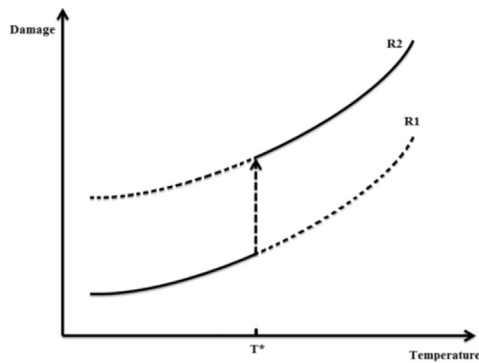


Fig. 1. The tipping point and irreversible damage.

taxation can affect the pricing strategy of energy produce/export countries, and meanwhile, the energy producer can react strategically and preempt carbon taxes by raising the producer price (Wirl, 1995).

As two of the most prevailing issues, climate uncertainty and strategic interaction have been shown their importance in the climate policy design. Following Wirl (2007), this paper tries to integrate the strategic interaction issue between carbon taxation and energy pricing with climate uncertainty in a dynamic game framework, which allows us to examine the optimal carbon taxes and energy prices under the threat of climate tipping events. More precisely, within a stochastic dynamic game framework, this study models the strategic interactions between energy seller side and buyer side on rent contest with integrating the uncertainty of climate tipping points. Two representative players have been modeled here: the collective consumer's benevolent government (or a consumer coalition/cartel, such as an empowered Organisation for Economic Co-operation and Development (OECD), or Annex I Countries); and the (cartelized) energy producers (such as Organization of the Petroleum Exporting Countries, OPEC). While the strategic interaction issues have been extensively examined in the literature, none of the previous studies (to the best of our knowledge) has incorporated the climate tipping points and its uncertainty issues which are well supported by the climate-economy integrated assessment models, into the investigation of strategic interaction issues. This paper tries to fill this gap in the literature. Based on a similar framework that was employed in Wirl (1994), Tahvonen (1996, 1997), Rubio and Esriche (2001) and Liski and Tahvonen (2004), this paper makes the extension by incorporating the uncertain climate tipping points through introducing a first order Markov chain for the climate state indicating whether a climate tipping event occurs or not. The probability of climate tipping events occurring increases as the temperature rises and the occurrence of tipping event is irreversible. Through theoretical analysis and numerical simulation, we find some interesting results such as how the carbon taxes and energy prices should look like in different cases and how they would differ before and after the occurrence of climate tipping events, and so on.

This paper is organized as follows. Section 1 gives an introduction on these issues and Section 2 describes the model and some optimality analysis. Section 3 describes the initialization and parameterization for numerical solution of the game. Section 4 presents the numerical results and discussions. Concluding remarks and their policy implications are summarized in the final section.

## 2. The model

### 2.1. Two players

As in Wirl (1994, 2007), Tahvonen (1996, 1997), Rubio and Esriche (2001) and Liski and Tahvonen (2004), there are two players in the dynamic game of strategic interaction: the collective consumers'

benevolent government (e.g., an empowered OECD) who maximizes the net present value of consumers' welfare by choosing a carbon tax  $\tau_t$ ; and the (cartelized) energy producers (e.g., OPEC) who maximize the net present value of profits by setting the (wellhead) fossil energy price  $p_t$ . Consequently the consumer price in period  $t$  would be  $\pi_t = p_t + \tau_t$ , which will determine the demand for fossil energy (measured in emissions). As in Wirl (1994, 2007), and Rubio and Esriche (2001), we use a linear function form  $D(\pi) = a + b\pi$ , where  $D(\pi)$  stands for the demand for fossil energy, and  $a$  and  $b$  are constants.

The net present value of consumers' welfare, which the collective consumers' government wants to maximize, consists of consumers' surplus plus carbon tax revenues minus the damage cost of climate change. In a discrete time framework, the maximization problem for the consumers can be expressed as:

$$W = \max_{\{\tau_t\}} E \left\{ \sum_{t=0}^{\infty} \frac{1}{(1 + \rho)^t} [u(p_t + \tau_t) + \tau_t D(p_t + \tau_t) - \Omega(T(S_t), J_t)] \right\} \quad (1)$$

where  $u(p_t + \tau_t) = u(\pi_t) = \int_{\pi_t}^{\pi^c} D(x) dx = \frac{1}{2} a \pi^c - \frac{1}{2} b \pi_t^2 - a \pi_t$  is the consumers' surplus and  $\pi^c$  is the choke price which makes  $D(\pi^c) = 0$ . Given the linear demand function, we have  $\pi^c = -a/b$ . The term  $\tau_t D(p_t + \tau_t)$  represents the tax revenues and they are reimbursed to the consumers. Since these tax revenues are not taken into account by the consumers' surplus  $u(\pi_t)$ , they have to be added explicitly in (1). The external cost of climate change is represented by  $\Omega(T(S_t), J_t)$ , where  $T$  is the atmospheric temperature depending on the carbon concentration (cumulative emissions) in the atmosphere, which is represented by  $S$ .  $J$  is a discrete variable indicating the state of the climate, i.e., whether the climate is in the normal/pre-tipping regime (before the occurrence of a tipping event) or in the post-tipping regime (after the occurrence of a tipping event).

With the producers' surplus being neglected by the consumers' government, the external cost of climate change is ignored by the energy producer and thus the producers' cartel concentrates only on maximizing the present value of its net profits through the pricing strategy  $p_t$ :

$$V = \max_{\{p_t\}} E \left\{ \sum_{t=0}^{\infty} \frac{1}{(1 + \rho)^t} [p_t \cdot D(p_t + \tau_t)] \right\} \quad (2)$$

Consistent with Wirl (2007), the discount rate is the same for both the consumers' government and the producers' cartel, and the extraction costs of producer are ignored.

### 2.2. Temperature and CO<sub>2</sub> concentration

The temperature (relative to the pre-industrial level) in period  $t$  is a function of the atmospheric CO<sub>2</sub> concentration:

$$T_t = \lambda \left( \ln \frac{S_t}{\bar{S}} / \ln 2 \right) \quad (3)$$

where  $\lambda$  is the climate sensitivity, which is often expressed as the temperature change associated with a doubling of CO<sub>2</sub> concentration in the atmosphere with respect to the preindustrial level  $\bar{S}$ . Eq. (3) states that the temperature (relative to the pre-industrial level) will be  $\lambda$  when the current CO<sub>2</sub> concentration is the double of the pre-industrial level.<sup>1</sup>

The accumulation of CO<sub>2</sub> in the atmosphere depends on the consumption of fossil fuels and the natural decay of existing CO<sub>2</sub> in the atmosphere:

$$S_{t+1} = (1 - \delta)S_t + D_t \quad (4)$$

where  $S$  is the carbon dioxide concentration in the atmosphere and  $\delta$  is

<sup>1</sup> As can be seen in Eq. (3), the effect of other greenhouse gases is ignored for the sake of simplicity.

the natural depreciation (decay) rate.

As can be seen, the external cost of climate change  $\Omega(T(S_t), J_t)$  is actually a function of the two state variables in the dynamic optimization problem: the atmosphere CO<sub>2</sub> concentration  $S$  (which determines atmosphere temperature  $T$  through Eq. (3)) and the state of climate  $J$ . While there can exist several different types of tipping points, this study focuses on the most typical ones to illustrate the effect of uncertain tipping points on carbon taxation and energy pricing under the strategic interactions between the energy buyer side and the seller side.

### 2.3. Climate tipping events

One typical tipping event that has been widely studied in the literature is a sudden, irreversible change (such as the collapse of ice sheets), which increases the damage cost for a certain temperature substantially. However, as argued by some recent studies on climate-economy integrated assessments (e.g., Cai et al., 2012), this approach of modeling climate tipping events may not be appealing because it implies that we can immediately know that we are safe provided that we stay at or below a certain level, no matter how long we stay there. That is, modeling the sudden, irreversible climate change with a known threshold location is not that plausible. A more realistic case would be that even if a tipping event has not happened yet, it may still occur later, even if there is no further warming. Therefore, it will be more appropriate to model the occurrence of tipping events as a stochastic process in which the temperature threshold (tipping point) is unknown but the probability for the occurrence of these events increases as the temperature rises. This approach to modeling the climate change with uncertain climate tipping points is expected to have an effect on the optimal climate policy. Within this modeling framework, the case of deterministic threshold can be considered as a special case where the probability of tipping is zero if the temperature does not pass the threshold and one otherwise.

We model the consequence of such a tipping event through increasing the coefficient of the damage function in the post-tipping regime. In this way, the external cost of climate change  $\Omega(T(S_t), J_t)$  can be written as:

$$\Omega(T(S_t), J_t) = \begin{cases} \frac{d_0}{2} T_t^2 & \text{if } J_t = 0 \\ \frac{d_1}{2} T_t^2 & \text{if } J_t = 1 \end{cases} \quad (5)$$

where  $J_t = 0$  represents the pre-tipping regime (i.e., normal state of climate), and  $J_t = 1$  represents the post-tipping regime (after the occurrence of a tipping event). And higher damage in the post-tipping regime implies  $d_1 > d_0$ . However, since the temperature threshold for tipping is uncertain, the occurrence of tipping events becomes stochastic. That is, the occurrence of a tipping event can happen in any period. After the occurrence of a tipping event, the climate will stay in the post-tipping regime since the effect of tipping events is irreversible. Such a specification implies that the evolution of the state of the climate will follow a first order Markov process with a transition probability matrix  $\Gamma = \begin{bmatrix} 1 - \Psi_t & \Psi_t \\ 0 & 1 \end{bmatrix}$  where  $\Psi$  represents the probability that the state of the climate will transit to  $J = 1$  in the next period, given that the current state of the climate is  $J = 0$ . The (2,2) entry of the matrix being ‘1’ implies that the state of the climate will stay in  $J = 1$  after the occurrence of a tipping event. A number of previous studies on pollution control and regulation under environmental catastrophes, such as Tsur and Zemel (1998, 2006, 2008), employed a similar approach to describe the regime switching, but in a continuous-time framework. It can be seen that with our model specification, the external cost of climate change is stochastic due to the uncertainty of occurrence of tipping events, which is different from the previous studies on uncertainty and strategic interactions (e.g., Wirl, 2007).

### 2.4. Solving the model

The strategic interaction between a consumers’ government and a producers’ cartel is thus modeled by a stochastic dynamic game where the state of climate is stochastic due to the uncertainty of climate tipping points. This game can be solved for the Nash equilibrium in Markov strategies where the players’ strategies in period  $t$  are contingent on the CO<sub>2</sub> concentration  $S_t$  (temperature level  $T_t$ ) and the state of climate  $J_t$ . As in many previous studies such as Hoel (1993), Wirl (1994), Tahvonen (1994, 1996, 1997), Rubio and Esriche (2001), Liski and Tahvonen (2004), and Wirl (2007), the natural resource constraints are ignored since with finite fossil resources the global warming will not be a serious problem in the long run.

Recursively, the optimization problems of consumers’ government and producers’ cartel can be rewritten as:

$$W(S_t, J_t) = \max_{\{\tau_t\}} \left\{ [u(p_t + \tau_t) + \tau_t D(p_t + \tau_t) - \Omega(T(S_t), J_t)] + \frac{1}{1 + \rho} E_{J_{t+1}|J_t} [W(S_{t+1}, J_{t+1})] \right\} \quad (6)$$

$$V(S_t, J_t) = \max_{\{p_t\}} \left\{ [p_t \cdot D(p_t + \tau_t)] + \frac{1}{1 + \rho} E_{J_{t+1}|J_t} [V(S_{t+1}, J_{t+1})] \right\} \quad (7)$$

In such a way, the Nash equilibrium for the stochastic dynamic game can be found through the solution of the two Bellman Eqs. (6) and (7) simultaneously. Here it needs to be pointed out that, as can be seen above, there are two state variables in the game model presented here: one is the CO<sub>2</sub> concentration in the atmosphere, i.e.,  $S_t$ , which is continuous; the other one is a discrete variable indicating the state of the climate,  $J$ , which follows a Markov process where the probability of transition depends on the temperature. Unfortunately, it would be difficult to find an explicit analytical solution for such a stochastic dynamic game.

However, our model can be solved numerically using infinite-horizon dynamic programming techniques combined with Gauss-Seidel algorithm. More specifically, value function iteration was employed to find the optimal solution for the stochastic DP Eqs. (6) and (7) simultaneously. And the value functions of players are approximated by the Chebychev polynomial using Chebychev nodes for the continuous state variable  $S$  and by linear approximation for the discrete state variable  $J$ .<sup>2</sup> Given a pricing strategy  $p(\cdot)$  of the producer’s cartel, one can obtain the optimal carbon tax  $\tau(\cdot)$  of the consumers’ government through the solution of the stochastic DP problem (6), and this would be the best response of the consumers’ government to the producer’s pricing strategy  $p(\cdot)$ . Using this carbon taxation strategy  $\tau(\cdot)$  of the consumers’ government, the best response of the producer’s cartel  $p(\cdot)$  can be attained by the solution of the DP problem (7). The procedure continues by updating the strategy of the consumers’ government given the new pricing strategy of the producers, and then using the new strategy of consumers’ government to solve for the new optimal pricing strategy of the producers. The procedure repeats until the variation in the coefficients of the value function approximants passed a convergence test on successive iterations passes a convergence test. If  $\tau^*(\cdot)$  is the best response to  $p^*(\cdot)$  and vice versa, it can be concluded that the strategy pair  $(\tau^*(\cdot), p^*(\cdot))$  is a Nash equilibrium for the two players in the game.

### 2.5. Optimality conditions

The resulting complexity with the incorporation of uncertain tipping points suggests a large role for numerical methods to obtain the solutions to the game and to examine the effects of changes in

<sup>2</sup> See Judd (1998) for discussions on value function iteration and approximation.

variables of interest on the solutions, as one will see in Sections 3 and 4. However, by looking at the optimality conditions for the Bellman equations in the game model, it is possible to find some interesting observations which can complement the numerical solutions obtained later soon.

**Proposition 1.** The optimal carbon tax in period  $t$  for the consumers' government in the Nash equilibrium equals to the discounted expected shadow price of cumulative emissions in the next period, given the current cumulative emissions, the current state of the climate, and the optimal energy pricing of energy producers' cartel.

**Proof.** Given the optimal energy pricing of producers' cartel  $p_t^*$ , the first order condition for the Bellman equation of the consumers' government with respect to carbon tax  $\tau$  (Eq. (6)) can be rewritten as:

$$0 = \frac{\partial u(p_t^* + \tau_t^*)}{\partial \tau} + D(p_t^* + \tau_t^*) + \tau_t^* \frac{\partial D(p_t^* + \tau_t^*)}{\partial \tau} + \frac{1}{1 + \rho} E_{J_{t+1}|J_t} \left[ \frac{\partial W(S_{t+1}^*, J_{t+1})}{\partial S} \cdot 1 \cdot \frac{\partial D(p_t^* + \tau_t^*)}{\partial \tau} \right]$$

Since  $\frac{\partial u(p_t^* + \tau_t^*)}{\partial \tau} = -b \cdot (p_t^* + \tau_t^*) - a$ ,  $D(p_t^* + \tau_t^*) = a + b \cdot (p_t^* + \tau_t^*)$ , and  $\frac{\partial D(p_t^* + \tau_t^*)}{\partial \tau} = b$ , we have:  $0 = b\tau_t^* + \frac{1}{1 + \rho} E_{J_{t+1}|J_t} \left[ \frac{\partial W(S_{t+1}^*, J_{t+1})}{\partial S} \cdot b \right]$

That is,

$$\tau_t^*(S_t, J_t) = -\frac{1}{1 + \rho} E_{J_{t+1}|J_t} \left[ \frac{\partial W(S_{t+1}^*, J_{t+1})}{\partial S} \right] \tag{8}$$

where  $S_{t+1}^* = (1 - \delta)S_t + D(p_t^* + \tau_t^*)$ .

**Proposition 2.** The optimal energy pricing strategy for the producers' cartel in the Nash equilibrium follows the rule  $p_t^*(S_t, J_t) = \frac{1}{2} \left( \pi^c - \tau_t^* - \frac{1}{1 + \rho} E_{J_{t+1}|J_t} \left[ \frac{\partial V(S_{t+1}^*, J_{t+1})}{\partial S} \right] \right)$ .

**Proof.** Given the optimal carbon tax from the consumers' government  $\tau_t^*$ , The first order condition for the Bellman equation of the producers' cartel (Eq. (7)) can be rewritten as:

$$0 = p_t^* \frac{\partial D(p_t^* + \tau_t^*)}{\partial p} + D(p_t^* + \tau_t^*) + \frac{1}{1 + \rho} E_{J_{t+1}|J_t} \left[ \frac{\partial V(S_{t+1}^*, J_{t+1})}{\partial S} \cdot 1 \cdot \frac{\partial D(p_t^* + \tau_t^*)}{\partial p} \right]$$

Since  $\frac{\partial D(p_t^* + \tau_t^*)}{\partial p} = b$  and  $D(p_t^* + \tau_t^*) = a + b(p_t^* + \tau_t^*)$ , we can have:

$$0 = bp_t^* + a + b(p_t^* + \tau_t^*) + \frac{1}{1 + \rho} E_{J_{t+1}|J_t} \left[ \frac{\partial V(S_{t+1}^*, J_{t+1})}{\partial S} \cdot b \right]$$

And this leads to

$$p_t^*(S_t, J_t) = \frac{1}{2} \left[ \pi^c - \tau_t^*(S_t, J_t) - \frac{1}{1 + \rho} E_{J_{t+1}|J_t} \left[ \frac{\partial V(S_{t+1}^*, J_{t+1})}{\partial S} \right] \right] \tag{9}$$

where  $S_{t+1}^* = (1 - \delta)S_t + D(p_t^* + \tau_t^*)$ .

Thus, the Nash equilibrium for the stochastic dynamic game defined by Bellman Eqs. (6) and (7) can be characterized by the simultaneous Eqs. (8) and (9).

As can be seen from Eqs. (8) and (9), the optimal pair of carbon taxes and producer prices in each period would be state-dependent, i.e., they depend upon the pollutant concentration (cumulative emissions) and the state of the climate system in that period. It can also be expected from these optimality conditions that the changes in some factors (such as the transition probability matrix for states of the climate, the parameters of the (post-tipping) damage function, and so on) which may affect the discounted shadow price of cumulative emissions can have an impact on the optimal carbon taxation of the consumers' government, and consequently affect the optimal energy

pricing of the producers' cartel, since the (wellhead) price and the tax are strategic substitutes for the producers (see Eq. (8)). These impacts will be better evidenced by the numerical results in the following sections.

### 3. The implementation of the stochastic dynamic game

To complement the analytic illustration above, the stochastic dynamic game model needs to be solved numerically. The discrete time in this study is on an annual (yearly) basis. In the simulation part of Wirl (2007), the choke price was set at \$100 per barrel of oil equivalent (thus around \$780 per ton of carbon). Considering the facts of high oil prices in recent years (e.g., in late 2000s), we set the choke price in this study as \$200 per barrel of oil equivalent, i.e., around \$1560 per ton of carbon.<sup>3</sup> With this choke price plus the facts that the emissions by the world from fossil fuels are 9.14GtC (33.5 GtCO<sub>2</sub>) in 2010 at a final price of \$624 per ton of carbon (\$80 per barrel of oil equivalent), we can figure out the fossil fuel demand (in terms of emissions) function  $D = 15.2 - 0.0097\pi$ , which is in billion tons of carbon (GtC) and  $\pi$  is given in US\$ per ton of carbon.

The atmospheric CO<sub>2</sub> depreciation rate  $\delta$  is set at 0.005 per year (i.e., CO<sub>2</sub> emissions approximately stay in the atmosphere for 200 years), which is based on International Panel on Climate Change (IPCC) (2002). Pre-industrial atmospheric CO<sub>2</sub> stock is 595.6GtC and initial atmospheric CO<sub>2</sub> stock (for the year 2010) is 833.2 GtC (3055GtCO<sub>2</sub>). Climate sensitivity  $\lambda$  is set at +2.5 °C per doubling of atmospheric CO<sub>2</sub> concentration with respect to preindustrial concentrations.

The damage function is assumed to be in a quadratic form, as in Eq. (5). The coefficient in the damage function in the pre-tipping regime is set at  $d_0 = 265$ , which implies a loss of around 2% of the GDP if the temperature increase (w.r.t. pre-industrial level) was 2.5 °C (International Panel on Climate Change (IPCC), 2002). In the post-tipping regime, a 2.5 °C increase in the temperature would result in a 10% loss of GDP, which implies  $d_1 = 1325$ .

Regarding to the probability of the occurrence of tipping events, it is usually assumed to be an increasing function of the temperature increase. Following the integrated assessment study of Cai et al. (2012), the probability of the occurrence of tipping events in period  $t$  is in a form:

$$\Psi_t = 1 - \exp\{-\gamma \max\{0, (T_t - 1)\}\}$$

where  $\gamma$  is a parameter indicating the hazard rate. As you can see, the probability of triggering a tipping point for temperature lower than a certain level is set at zero. The parameter  $\gamma$  is set at 0.08, which is chosen in such a way to try to fit the expert subjective probability elicited in Krieglner et al. (2009) where they found conservative lower bounds for the probability of triggering at least one of the tipping events studied of 0.16 for medium global mean temperature change (2–4 °C) and 0.56 for high climate change (4–8 °C) relative to year 2000 level.

As mentioned above, the value functions of players are approximated by the Chebychev polynomial using Chebychev nodes for the continuous state variable  $S$  and by linear approximation for the discrete state variable  $J$ . For the initial runs, we use 10 nodes for the state variable  $S$  (CO<sub>2</sub> concentration) and 2 nodes for the state variable  $J$  (the two states of climate system). It is assumed that the carbon taxes, (wellhead) producer prices and the fossil fuel demand are non-negative. Given the sequence of the states of the climate, and the initial carbon concentration, it is possible to obtain the optimal paths for the carbon taxes and energy prices over a certain time horizon, based on the equilibrium of the game that has been solved. Through extensive numerical simulations, it is also possible to investigate the

<sup>3</sup> Actually \$1567 per ton of carbon if calculation is based on rounded parameters.



effect of different factors on the equilibrium carbon taxation and energy pricing under the uncertainty of climate tipping points. Of course, the occurrence of the tipping points can be sequential. In other words, the tipping of climate can be a multi-stage process, with several tipping points differing in levels of damage. The case when there is only one tipping event will be examined first in the next section. The effect of the probabilities of tipping, damage parameters, and so on will be comprehensively investigated based on this single-tipping point case. And then the case of a multi-stage tipping process will be investigated as well at the end of the next section.

#### 4. Numerical results and discussions

The numerical results of the stochastic dynamic game model would be the equilibrium strategies of carbon taxation and energy pricing as functions of the two state variables: one indicating the state of the climate, which is stochastic and exogenous (though the probability of tipping is endogenous and depends on both players' decisions); and the other one representing the CO<sub>2</sub> concentration in the atmosphere, which is endogenous and rely on the players' decisions. Therefore, to illustrate the results, the time period when the tipping event will happen needs to be assumed, which consequently specifies a time path for the states of the climate. Along with the initial carbon concentration in the atmosphere, the evolution over time of carbon taxes and energy prices in the equilibrium can then be calculated. For purposes of illustration, only a few representative cases are examined here and the time paths of carbon taxes, wellhead energy prices, consumer prices, and the CO<sub>2</sub> concentration, are presented for a time horizon of 500 years.<sup>4</sup>

##### 4.1. Baseline results

Given the specifications and initializations in Section 3, the results from the first run of the stochastic dynamic game model can be obtained and illustrated in Fig. 2. The solid lines in the figure represent the results if we are lucky and no tipping event occurs over the time horizon. As can be seen, the carbon tax starts at \$33/tC in the initial period and increases at a diminishing rate in the following periods. Increasing rapidly in the early periods, the carbon tax reaches \$138/tC after one century. In the later periods, it increases slower and slower, reaching a level of around \$166/tC in the 500th period. While the consumers' government is gradually increasing its carbon tax, the producers' energy price shows a decreasing path over time (Fig. 2B). Starting at about \$816/tC, the optimal energy price for the producers' cartel in the equilibrium would decrease gradually to a level of around \$734/tC in 100 years and decrease further to \$704/tC in period 500. It should be noted that the producer price decreases at a diminishing rate as well. However, the consumer price shows an increasing pattern, implying that the increase in the carbon tax outweighs the decrease in the producer price. Based on the optimal decisions of the two players, it is possible to calculate the time path of CO<sub>2</sub> concentration in the equilibrium, which is increasing but at a decreasing rate, as can be seen in Fig. 2D. It should be noted that the probability for the occurrence of a tipping point is endogenous here. That is, the players' decisions will affect the probability of tipping.

Since the occurrence of the tipping point is stochastic, one should expect that the tipping point could happen at any time, even though the probability of occurrence is higher at a higher temperature. With no loss of generality, one can assume that a tipping point suddenly occurs in one period, say, the 100th period, and then the time paths for the carbon tax, prices and CO<sub>2</sub> concentration in this case can be illustrated, as shown with the dashed lines in Fig. 2. As can be seen, the occurrence of a tipping point triggers an upward shift in the carbon tax path of the

consumers' government. More specifically, the carbon tax will be raised from \$138/tC to \$161/tC as the tipping point occurs and will be maintained at a higher level above that for the case where we are lucky to have no occurrence of the tipping point. That is, the carbon tax in the post-tipping regime would be kept at a higher level. At the same time, the occurrence of the tipping point entails a downward adjustment in the producer price, but the magnitude of the adjustment is smaller than that in the carbon tax, which gives a total effect of an upward adjustment in the consumer price. Also, higher consumer prices in the post-tipping regime result in lower CO<sub>2</sub> concentrations in the atmosphere, as shown in Fig. 2D.

##### 4.2. The effect of irreversibility in tipping events

One of the most important characteristics for a tipping event is the irreversibility of the damage that is caused. It is of significance to investigate how the irreversibility of a tipping event would affect the carbon taxation and energy pricing in the equilibrium. To do this, the transition matrix of the climate states is specified as follows in this sub-section:

$$\Gamma_1 = \begin{bmatrix} 1 - \Psi_1 & \Psi_1 \\ 0.6 & 0.4 \end{bmatrix}$$

It can be seen that the second row of the matrix is different from that of the transition matrix  $\Gamma$  (in the baseline case). In such way, one can see that the tipping event is reversible now (with a 0.6 probability in the setting here). That is, after the occurrence of a tipping point (from climate state 0 to state 1), it is still likely that the state of the climate can be reversed to the pre-tipping regime. The results by employing such a transition matrix in the solution of the game model are shown in Fig. 3.

If one compares the results for this case with those for the baseline case, some interesting observations can be found. As can be seen from Fig. 3, the carbon tax in this case is generally much lower and less aggressive compared with the baseline results (see Fig. 2). Even though the initial carbon tax is about the same as that in the baseline case, the increase in carbon tax over time is less substantial. For instance, in the first 100 years, the carbon tax increases less than 30\$/tC only while in the baseline case it increases around 100\$/tC during the same time. And by the 500th period, the carbon tax will be increased to a level of 81 \$/tC in this case, which is much lower than that in the baseline case. Regarding the producers' energy pricing, one can see that the initial producer price for this case is \$758/tC (lower than that in the baseline case, \$816/tC). Even though the producer price is decreasing over time in this case as well (see Fig. 3B), one can see that the decreasing trend is less dramatic than that in the baseline case. More specifically, the (wellhead) fuel price in this case decreases by 17\$/tC only during the first 100 years and by 29\$/tC only within the 500 years. In contrast, the fuel price in the baseline case decreases by 92\$/tC within the first 100 years and decreases further by 20\$/tC in the following 400 years. Therefore, it can be seen that compared with the baseline case, the decrease in the producer price is less dramatic in this case where the effect of a tipping event is reversible, especially for the early periods. Also, the consumer price increases from \$808/tC to \$829/tC within the 500 years in this case, suggesting that the increase in the consumer price is also less aggressive than that in the baseline case. With the less aggressive consumer prices, this case would end up with a much higher CO<sub>2</sub> concentration, compared with the baseline case (1390GtC in Fig. 3D VS 1313 GtC in Fig. 2D). It can also be seen from Fig. 3 that the magnitudes of shifts in the equilibrium carbon tax and energy price due to the occurrence of a tipping point is also smaller, compared with those in the baseline case.

Therefore, one can see that the irreversibility of tipping events plays an important role in the climate policy design of the consumers' government and the energy pricing of the producers' cartel in the

<sup>4</sup> Since the model is solved with infinite time horizon, the results can be illustrated for an arbitrary time horizon.

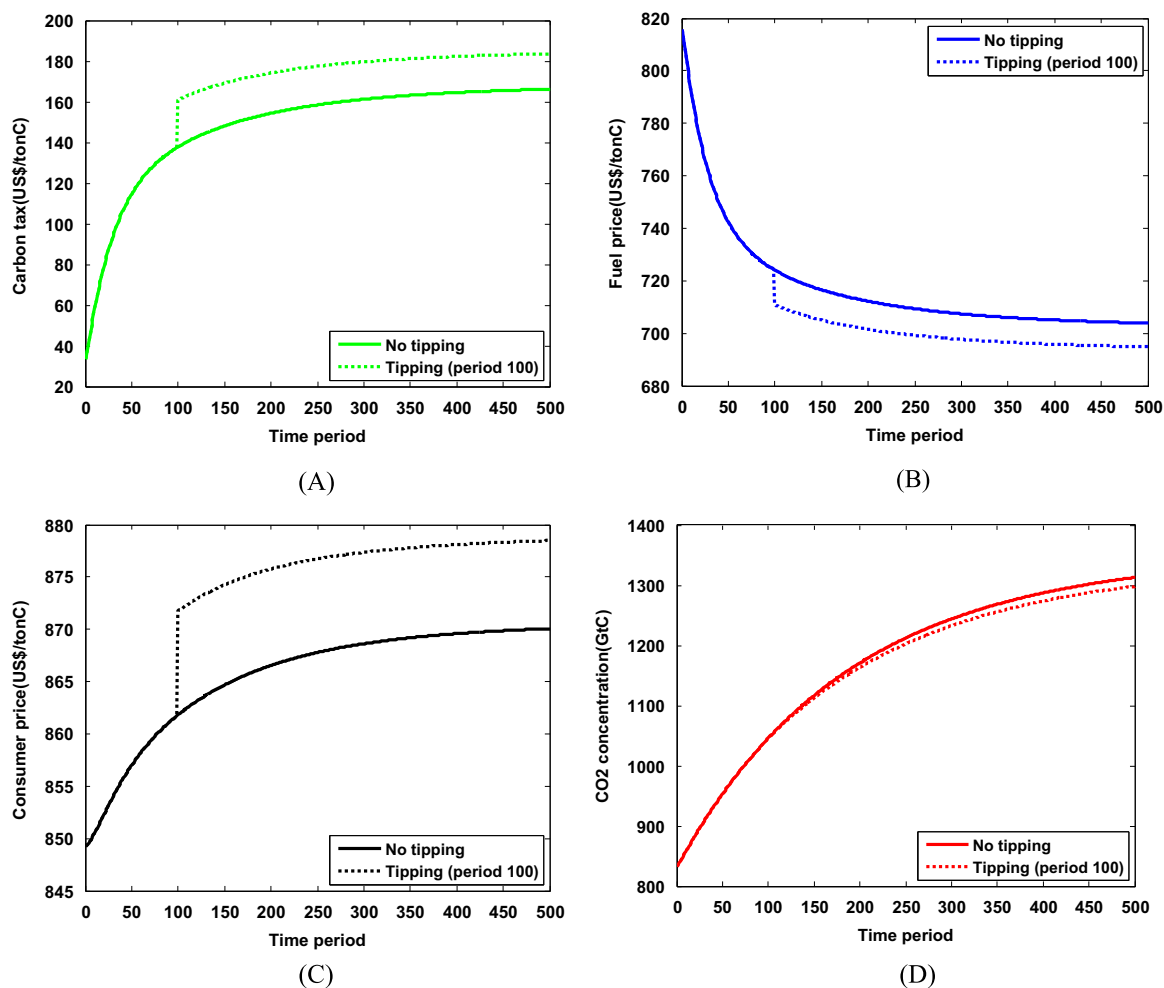


Fig. 2. Baseline results for the stochastic dynamic game.

equilibrium. More specifically, the irreversibility of a tipping event requires the government to impose much more aggressive carbon taxes, especially in the early periods. Meanwhile, the irreversibility of a tipping event may also lead up to more rapid decrease in the producer (fuel) price in the early periods. With a higher incentive to reduce emissions due to the higher consumer price in the case of an irreversible tipping event, the CO<sub>2</sub> concentration will be lower than that in the case where the tipping event is reversible.

### 4.3. The effect of tipping probabilities

Since the occurrence of a tipping event is stochastic, the tipping probability would be important for the players' decisions. As mentioned earlier, the tipping probability is somewhat endogenous in this study. That is, the probability that a tipping point occurs depends directly on the atmospheric temperature (and thus the CO<sub>2</sub> concentration) which will be affected by both players' decisions. Besides, the probability will be affected by some other parameters as well, e.g., the parameter  $\gamma$ . It can be seen from the expression of  $\Psi_t$  in Section 3 that the tipping probability at a certain temperature is an increasing function of the parameter  $\gamma$ . Therefore, in this section, we decrease the parameter  $\gamma$  from 0.08 to 0.05 to examine the effect of lower tipping probabilities on the players' optimal decisions.

The results in this case are illustrated in Fig. 4, from which some changes in the players' optimal strategies can be found. It can be seen that the initial carbon tax is \$26/tC, lower than that in the baseline case. The carbon tax reaches \$123/tC in the 100<sup>th</sup> period and \$156/tC in the 500<sup>th</sup> period, respectively, if we are lucky and the tipping event

does not occur. That is, the carbon taxes in this case would be generally lower than those in the baseline case. However, the producer (fuel) prices in this case are generally higher than those in the baseline case. For instance, the producer price in the 100<sup>th</sup> period will be \$733/tC, higher than that in the baseline case (\$724/tC). Besides, the consumer prices in this case are lower than those in the baseline case, as can be seen in Figs. 4C and 2C. This consequently leads to higher CO<sub>2</sub> concentrations over time. As for the case with the sudden occurrence of a tipping event in, say, the 100<sup>th</sup> period, one can find that the carbon taxes and fuel prices after the occurrence of the tipping event are the same with those in the baseline case. This implies that the changes in the tipping probabilities will change the equilibrium strategies of both players in the normal state of the climate (pre-tipping regime) but will not change the carbon taxes and fuel prices in the post-tipping regime. The intuition behind this is as follows: the changes in the expectation of how likely the tipping event would happen will lead to changes in players' expected payoffs when the state of the climate is normal but will not affect their expectations on the states of the climate in the future (and thus the expected payoffs) if the climate is already in post-tipping regime, since the probability of staying in the post-tipping regime is one after the occurrence of a tipping event.

It is important to note that under the lowered tipping probabilities, the equilibrium paths without the occurrence of a tipping event are shifted (lower carbon taxa and higher fuel prices) but those with the occurrence of a tipping event are unchanged. Therefore, it is not surprising to see the larger gaps between the solid lines and dashed lines in Fig. 4, compared with Fig. 2, i.e., the baseline case.

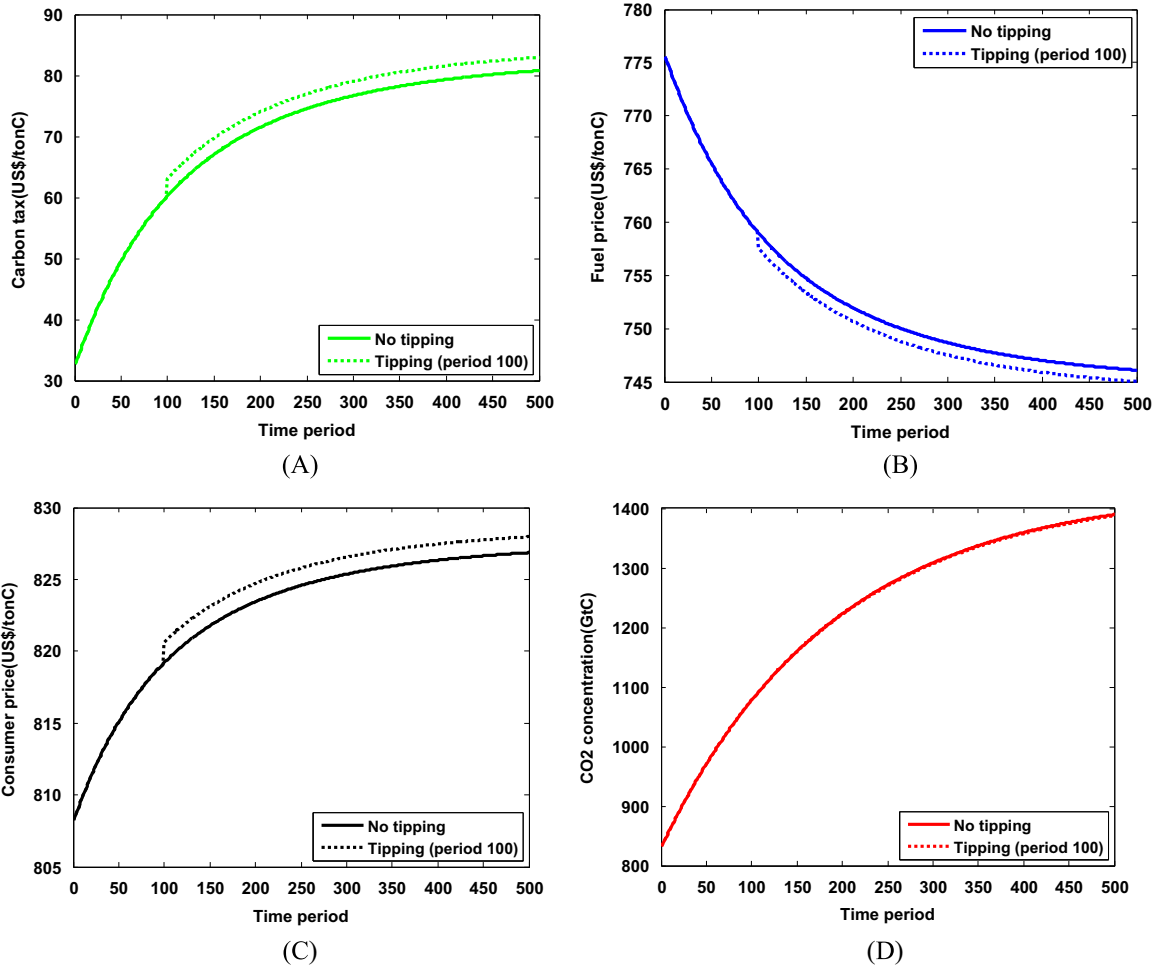


Fig. 3. The results had it been the tipping event is reversible.

4.4. The effect of uncertain damage

In addition to the uncertainty of the climate tipping point, the damage caused by a tipping event can also be uncertain. First of all, the uncertainty of damage associated with the tipping events is well supported by the natural science research (see, e.g., International Panel on Climate Change (IPCC), 2002). Also, the aggregation of the multitudes of sectorial and regional impacts is not an easy task. As pointed out by Cai et al. (2012), another reason is that we don't know when a tipping point will occur (whether in 2050 or 2100, or in other years), but the impact of a tipping event will depend on socio-economic factors at the time when the tipping point occurs. Therefore, it makes sense to investigate the effect on the players' optimal carbon taxation and energy pricing when the impact of a future tipping event is uncertain. To do this, we suppose that if the tipping event occurs, the damage function in the post-regime will be:

$$\Omega(T(S_t), 1) = \begin{cases} \frac{d'_1}{2} T_t^2, & \text{with probability } 0.5 \\ \frac{d''_1}{2} T_t^2, & \text{with probability } 0.5 \end{cases}$$

where we set  $d'_1 = 0.5d_1$  and  $d''_1 = 0.5d_1$ . By this way, the expected post-tipping damage would coincide with that specified in the baseline case since  $0.5d'_1 + 0.5d''_1 = d_1$ . The uncertain damage actually implies that now the climate system has three states: pre-tipping regime, post-tipping regime with low damage, and post-tipping regime with high damage. The corresponding transition probability matrix in this case for the evolution of climate states would be:

$$\begin{bmatrix} 1 - \Psi_t & 0.5\Psi_t & 0.5\Psi_t \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

That is, the tipping probability from a normal state is still  $\Psi_t$  and the occurrence of the tipping event will result in either high damage or low damage in the post-tipping regime with equal probabilities. And then the system will stay in the high-damage or low-damage state and cannot reverse.

The same as before, the results without and with the occurrence of a tipping point in period 100 are illustrated in Fig. 5 with solid and dashed lines, respectively. It can be seen that the optimal strategies of players without the occurrence of a tipping point in this case are the same as those in the baseline case, which is reasonable since the expected damage of the tipping event conditional on the current normal climate state is the same as that in the baseline case by construction. Regarding the results with the occurrence of a tipping event, it can be seen that the optimal strategies of both players will depend on the magnitude of the damage in the post-tipping regime. For instance, if a tipping event happens in period 100 and results in high damage, the carbon tax will be raised immediately from the pre-tipping level to \$241/tC, which is much higher than the carbon tax in the case without the occurrence of a tipping event (\$138/tC). Meanwhile, the producer (fuel) price will be reduced immediately. The overall effect of the changes in the carbon tax and producer price is an increase in the consumer price (as can be seen from Fig. 5C), which results in lower CO<sub>2</sub> concentrations in the high-damage case (see Fig. 5D). In contrast, if the tipping event leads to low damage, the carbon tax will be shifted downwards instead from the pre-tipping level to \$81/tC and the

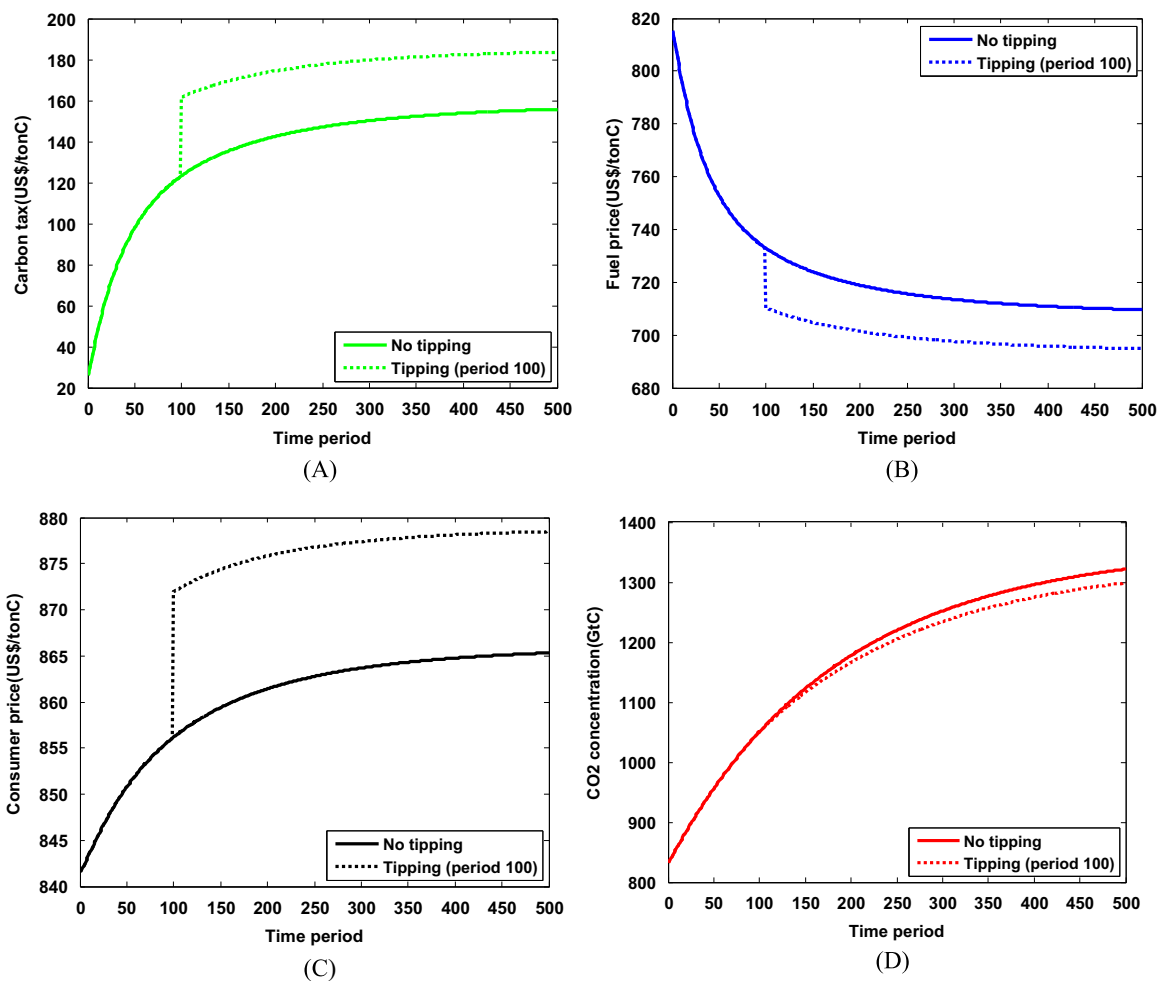


Fig. 4. The results for lower tipping probabilities.

producer price will be shifted upwards to \$747/tC, as illustrated in Fig. 5A and B. The overall effect is that the consumer prices will be lower and CO<sub>2</sub> concentrations will be higher after the occurrence of the tipping event (as in Fig. 5C and D).

Therefore, the uncertainty of the damage in the post-tipping regime will result in damage-dependent equilibrium strategies. If the damage is high, the carbon tax would be adjusted upwards and the producer price would be adjusted downwards when the tipping event occurs. In contrast, if the damage is lower than the expected damage, the carbon tax would be shifted downwards and the producer price would be shifted upwards. It can be seen that the changes in the carbon tax and those in the producer price are in opposite directions and the magnitude of the changes is larger for the carbon tax, which makes the changes in the consumer price would follow the direction of changes in the carbon tax.

#### 4.5. A different tipping point

So far we have been investigating the effect of a typical tipping point that can be represented by a direct change in the parameter of the damage function. However, there are some other forms of tipping points, which may lead to the change in damage through an indirect way. For instance, because of the complexity of the climate system, it is likely that there exists a certain tipping point so that the climate sensitivity will be significantly different after the occurrence of the tipping event (i.e., in the post-tipping regime). Therefore, in this section, we would like to investigate the effect of a tipping point that results in different climate sensitivity in the post-tipping regime. To

model such a tipping point, we increase the climate sensitivity in the post-tipping regime:  $\hat{\lambda} = 2\lambda$ . That is, the climate sensitivity in the post-tipping regime would be twice of that in the pre-tipping regime.

Fig. 6 illustrates the results in this case. It can be seen that, similar to the patterns of the results above, the carbon tax in this case also increases over time and the producer price decreases over time. Once the tipping event occurs, the carbon tax would be adjusted upwards while the producer price would be adjusted downwards, resulting in a total effect of an increase in the consumer price. However, comparing the results in this case with those in the baseline case, we can see that the carbon tax in this case is less aggressive. More specifically, the carbon tax without the occurrence of a tipping event increases from the initial level \$27/tC to \$113/tC in the 100th period. These figures are lower than those in the baseline case. The decrease in the producer price also becomes less dramatic. Therefore, it can be seen that a different type of tipping event can probably affect the optimal carbon taxation and energy pricing of the players.

#### 4.6. Three stage tipping process

In the previous sections, the effect of only one tipping point is studied. However, in reality, it is likely that some tipping events may happen in sequence. For instance, a further tipping event can happen after the occurrence of a tipping event. How will this affect the carbon taxes and producer prices in the equilibrium? To examine this effect, we add another state of the climate into the baseline case. More specifically, it is assumed that in this case:



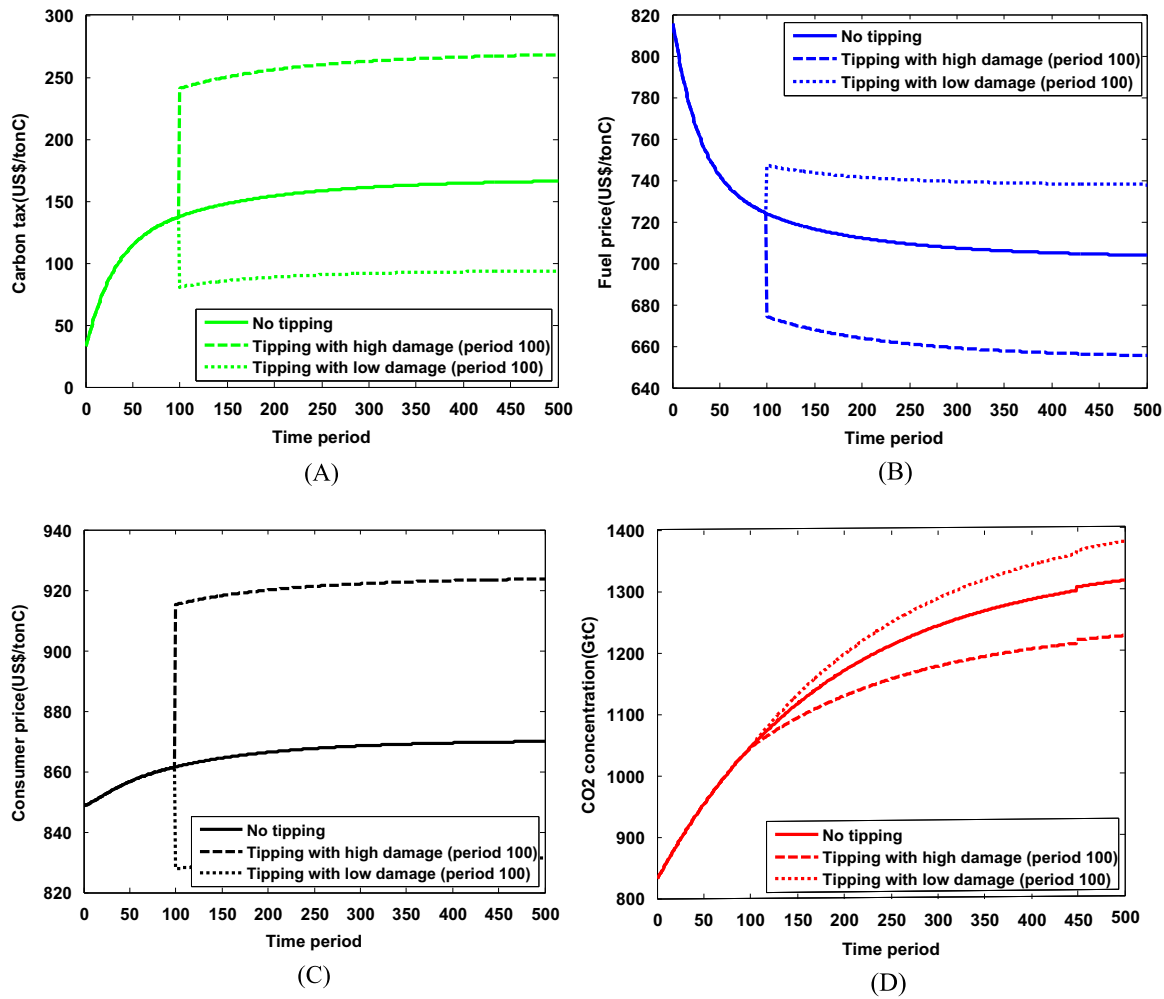


Fig. 5. The results for uncertain post-tipping damage.

$$\Omega(T(S_t), J_t) = \begin{cases} \frac{d_0}{2} T_t^2 & \text{if } J_t = 0 \\ \frac{d_1}{2} T_t^2 & \text{if } J_t = 1 \\ \frac{d_2}{2} T_t^2 & \text{if } J_t = 2 \end{cases}$$

where  $d_0 < d_1 < d_2$  and  $d_2 = 1.5d_1$ . It can be noted we have an additional state  $J = 2$  in this case.  $J = 1$  is a stage that has to be passed to reach stage  $J = 2$ . This implies that once a tipping event that will lead to the transition of states from  $J = 0$  to  $J = 1$ , it is possible that the state of the climate can become even worse ( $J = 2$ ) in the future. To investigate the effect of such a multi-stage tipping process, the transition probability matrix in this case is specified as:

$$\begin{bmatrix} 1 - \Psi_t & \Psi_t & 0 \\ 0 & 0.8 & 0.2 \\ 0 & 0 & 1 \end{bmatrix}$$

That is, once a tipping event changes the state of the climate from  $J = 0$  to  $J = 1$ , the climate will not always stay in the state of  $J = 1$  but can become even worse and transit to the state  $J = 2$  in the future. Using the new specifications, the results for this case can be obtained and illustrated in Fig. 7. It should be noted that for illustration purpose only, we assume that the first tipping point happens in the 100th period and the second tipping point happens in the 200th period.

Compared with the results in the baseline case, it can be seen that the carbon tax in this case is much higher. Starting at \$44/tC, the carbon tax increases with a diminishing rate over time and reaches \$183/tC in the 100th period. These figures are substantially higher than those in the baseline case, which implies that the carbon taxation

would become more aggressive when it is expected that the occurrence of a tipping event can lead to a worse tipping event in the future. In terms of producers' (wellhead) energy pricing, the producer sets an initial producer price at \$830/tC in this case, higher than that in the baseline case. However, it decreases to \$707/tC in period 100 and declines further to \$678/tC in period 500. Therefore, we can see that the producer prices are higher than those in the baseline case for early periods but lower than those in the baseline case for later periods, which suggests that the decline in the producer price over time is more dramatic in this case, compared with that in the baseline case. Even so, the consumer price will be increasing over time, which implies that the increase in the carbon tax outweighs the decrease in the producer price.

Besides, it can be seen that once a tipping event occurs, the carbon price will be shifted upwards. Since it is expected that the climate situation can become even worse in the future, the carbon tax after the occurrence of the first tipping event is higher than that in the baseline case (\$183/tC VS \$138/tC). Meanwhile, the producer (fuel) price will be decreased to \$707/tC when the first tipping point occurs in this case, lower than that in the baseline case. If another tipping event happens (assumed to be in period 200 in the illustrations), there will be an further upward adjustment in the carbon tax and an further downward adjustment in the producer price. The overall effect on the consumer price will be a shift upward upon the occurrence of the second tipping event. Due to the more aggressive consumer prices in this case, the CO<sub>2</sub> concentrations will be lower than those in the baseline case, no matter the tipping events actually occur or not.

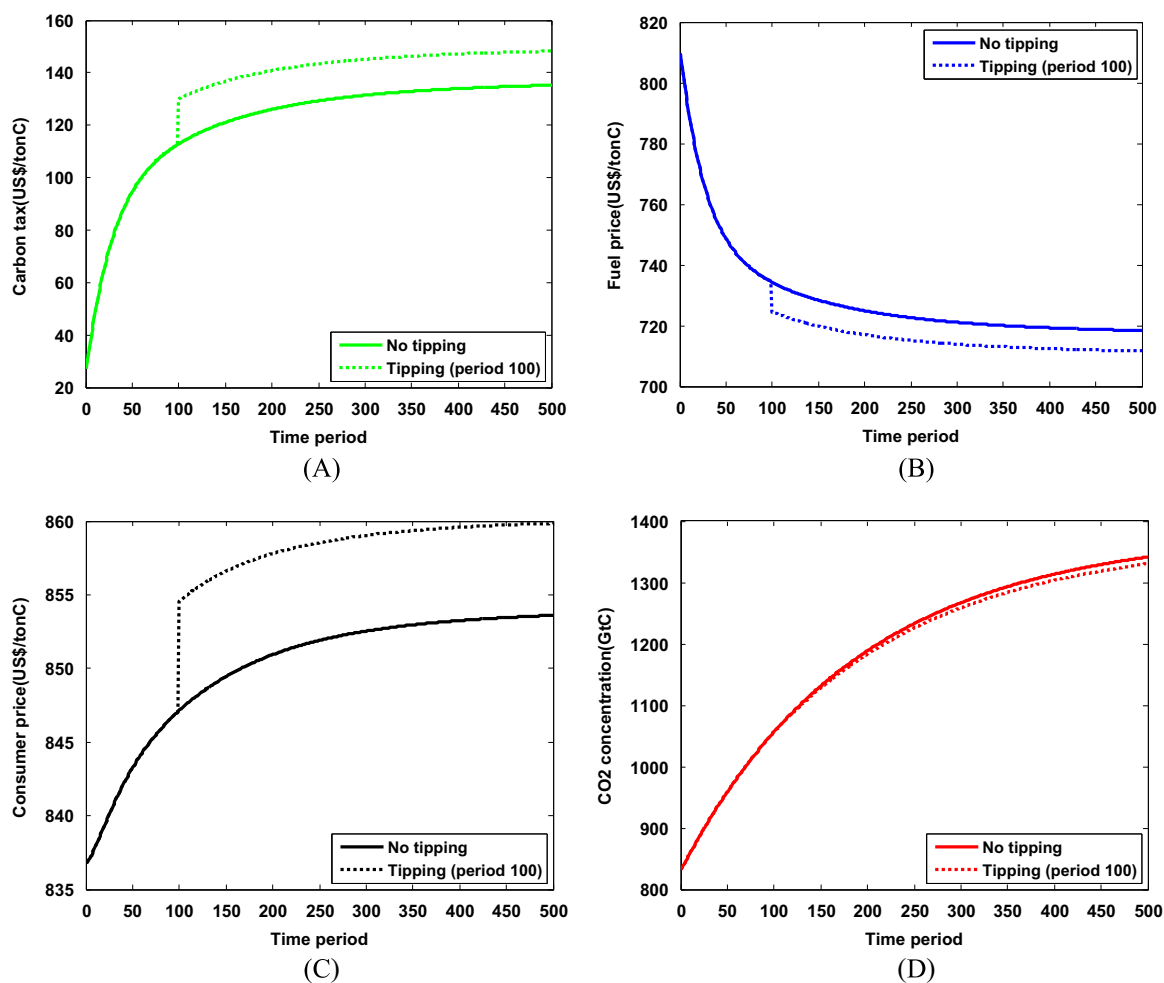


Fig. 6. The results for a different tipping point.

### 5. Concluding remarks and policy implication

This paper investigates the effect of possible climate tipping events on optimal carbon taxation and energy pricing, taking into account the strategic behavior of energy consumers/producers and the uncertainty of climate tipping points through a stochastic dynamic game. The tipping probability is increasing with the rise in temperature, which can be affected by both players' decisions (indirectly through the CO<sub>2</sub> concentration). The consumers' government chooses the optimal carbon taxation to maximize the social welfare with taking into account the externality of the climate change and the producers' cartel chooses its optimal energy pricing strategy to maximize its profits. Due to the difficulties of finding the equilibrium analytically, this paper uses numerical methods to solve the model and thus to examine the effect of different factors on the outcomes of the game, from which some interesting findings and policy implications can be obtained as follows.

Since the occurrence of the tipping events is stochastic, the optimal strategies of players need to take into account the potential effect of a tipping event through the players' expected payoffs. Under the threat of possible tipping events, the carbon tax from the consumers' government will be increasing over time while the producer (energy) price set by the producer's cartel will be decreasing over time, both at a diminishing rate. The increases in the carbon tax outweigh the decreases in the producer price, which results in the increasing consumer price over time. Once a tipping event occurs, the carbon tax will be shifted upwards significantly and stay in the new trajectory. Meanwhile, the occurrence of a tipping event will lead to a downward shift in the cartel's producer price. The overall effect of the shift in the

carbon tax and that in the producer price would be a shift upward in the consumer price.

The irreversibility of a tipping event implies much more aggressive carbon taxation and more rapid declines in the producer price, especially in the early periods. That is, the irreversibility of a tipping event gives the consumers' government a larger incentive to reduce emissions, which somehow restrains the producer's energy pricing in the strategic interaction game. And the overall effect of the irreversibility of a tipping event is higher consumer prices, which consequently leads to lower CO<sub>2</sub> concentrations. From a policy perspective of view, more complete information on the irreversibility of a tipping would help the decision-makers to design sound climate policies.

When the players expect lower tipping probabilities, the producer price in the equilibrium will be higher and the carbon tax will be less aggressive, which has an overall effect of a downward adjustment in the consumer price. Though the equilibrium strategies of both players after the occurrence of the tipping event will remain unchanged, those before the occurrence of the tipping would adjust to the changes in tipping probabilities. Therefore, the decision-makers should effectively assess the risk of occurrence of climate tipping events, and once the risk increases, it would be wise to have more aggressive carbon taxes.

The uncertainty of the damage in the post-tipping regime will result in damage-dependent equilibrium strategies. If the damage is high, the carbon tax would be shifted upwards and the producer prices would be shifted downwards when the tipping event occurs. In contrast, if the damage is low, the carbon tax would be reduced and the producer price would be raised immediately upon the occurrence of the tipping event. With the magnitude of the change in the carbon tax outweighing that in

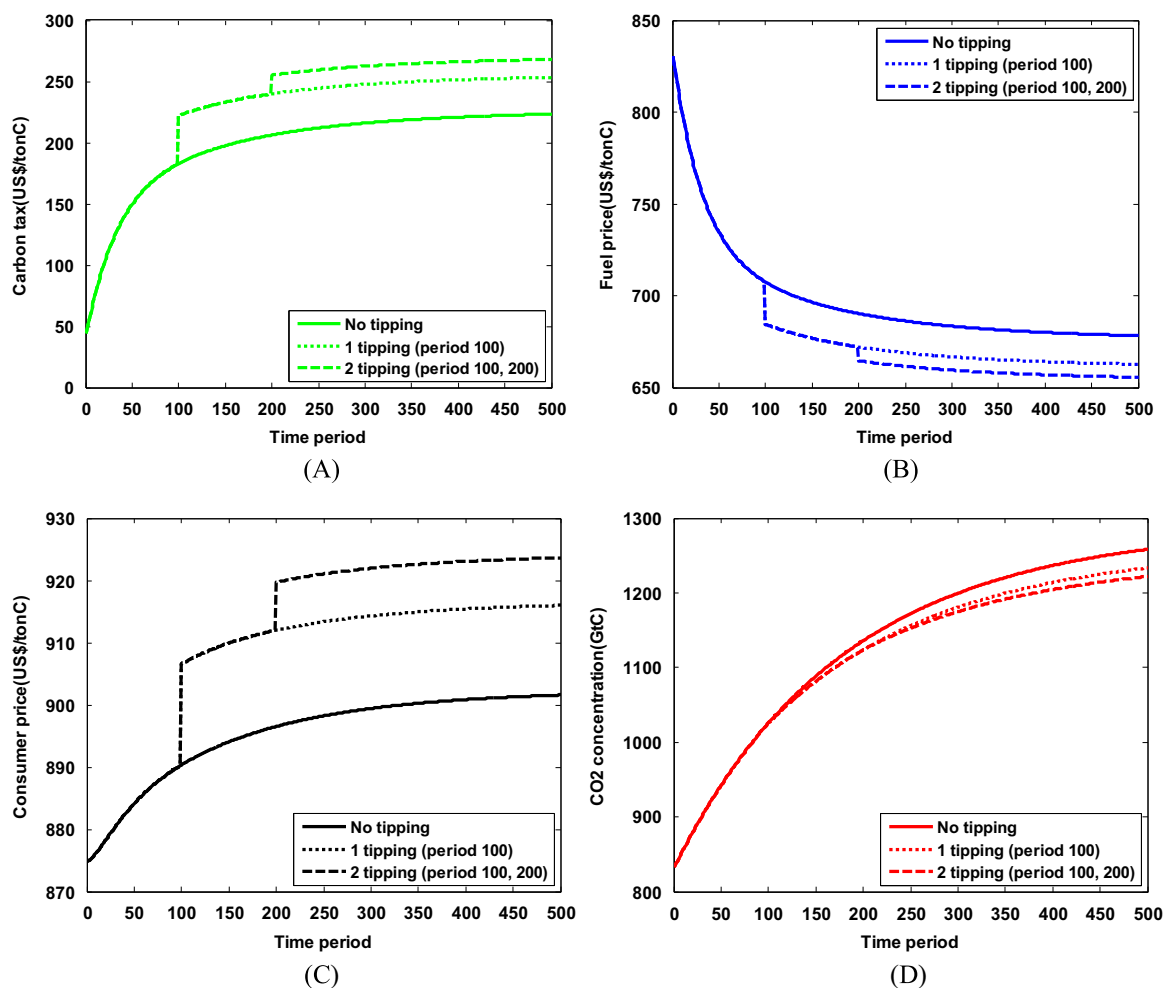


Fig. 7. The results for a three-stage tipping process.

the producer price, the overall effect follows the direction of the change in the carbon tax.

The worries that the climate situation can become even worse after the occurrence of a tipping event will stimulate the carbon taxation of the consumers' government. That is, the carbon taxation under the strategic interactions will become more aggressive when the players expect that the occurrence of a tipping event may lead to a worse tipping event in the future. Even though the declines in the producer price also become more dramatic, the overall effect of the changes in the carbon tax and those in the producer price will be increases in the consumer price over time. Once the tipping event occurs, the carbon tax would be shifted upwards while the producer price would be shifted downwards. With the worries that the current tipping event may lead to a worse one, the shifts in carbon tax and energy are more dramatic than the case when there are no such worries. This implies that the decision-makers should treat the different types of tipping events differently and impose more aggressive carbon taxes for those may lead to further catastrophic events.

Modeling the optimal carbon taxation and energy pricing under the strategic interaction between energy consumers and producers when the occurrence of climate tipping points is stochastic is a difficult task. To simplify this issue, we have made many assumptions in this paper. It should be acknowledged that the numerical results derived here might be subject to these restrictions and have several limitations. For instance, to lower the difficulty of computing the equilibrium, the climate module in the stochastic dynamic game here is rather simple and in reality the climate system and the damage function can be very complex. Also, a fixed form of energy demand function is applied here

for the sake of simplicity, but in reality the demand is changing over time and there is a lot of uncertainty in future demand. It would be of great interest to investigate the optimal carbon taxation and energy pricing issues with taking into account the roles of deep oceans in the carbon concentration and the change of energy demand function over time. Moreover, it would be interesting as well to simplify the model appropriately to make it analytically solvable (e.g., through piecewise deterministic differential games), which could give us more analytical insights on this issue.

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**References**

Cai, Y., Judd, K.L., Lontzek, T.S., 2012. The Social Cost of Abrupt Climate Change. Working paper. Available at: ([www-2.ies.su.se/Nobel2012/Papers/Lontzek\\_et\\_al.pdf](http://www-2.ies.su.se/Nobel2012/Papers/Lontzek_et_al.pdf)).

Fan, J., He, H., Wu, Y., 2016. Personal carbon trading and subsidies for hybrid electric vehicles. *Econ. Model.* 59, 164–173.

Hoel, M., 1993. Intertemporal properties of an international carbon tax. *Resour. Energy*

- Econ. 15, 51–70.
- International Panel on Climate Change (IPCC), 2002. IPCC Third Assessment Report Climate Change 2001. < www.ipcc.ch >
- Judd, K.L., 1998. Numerical Methods in Economics. MIT Press, Cambridge (Mass).
- Kriegler, E., Hall, J., Helda, H., Dawson, R., Schellnhuber, H.J., 2009. Imprecise probability assessment of tipping points in the climate system. Proceedings of the National Academy of Sciences of the United States of America, 106, 5041–5046.
- Liski, M., Tahvonen, O., 2004. Can carbon tax eat OPEC's rents? J. Environ. Econ. Manag. 45, 416–432.
- Ouchida, Y., Goto, D., 2016. Environmental research joint ventures and time-consistent emission tax: endogenous choice of R & D formation. Econ. Model. 55, 179–188.
- Rubio, S., Eliche, L., 2001. Strategic pigouvian taxation, stock externalities, and polluting nonrenewable resources. J. Public Econ. 79, 297–313.
- Tahvonen, O., 1994. Carbon dioxide abatement as a differential game. Eur. J. Polit. Econ. 10, 685–705.
- Tahvonen, O., 1996. Trade with polluting nonrenewable resources. J. Environ. Econ. Manag. 30, 1–17.
- Tahvonen, O., 1997. Tariffs and Pigouvian Taxation in Trade With Polluting Fossil Fuels. University of Helsinki, Mimeo.
- Tsur, Y., Zemel, A., 1998. Pollution control in an uncertain environment. J. Econ. Dyn. Control 22, 967–975.
- Tsur, Y., Zemel, A., 2006. Welfare measurement under threats of environmental catastrophes. J. Environ. Econ. Manag. 52, 421–429.
- Tsur, Y., Zemel, A., 2008. Regulating environmental threats. Environ. Resour. Econ. 39 (3), 297–310.
- Wirl, F., 1994. Pigouvian taxation of energy for stock and flow externalities and strategic, non-competitive pricing. J. Environ. Econ. Manag. 26, 1–18.
- Wirl, F., 1995. The exploitation of fossil fuels under the threat of global warming and carbon taxes: a dynamic game approach. Environ. Resour. Econ. 5, 333–352.
- Wirl, F., 2007. Energy prices and carbon taxes under uncertainty about global warming. Environ. Resour. Econ. 36, 313–340.
- Wu, P.I., Chen, C.T., Cheng, P.C., Liou, J.L., 2014. Climate game analyses for CO<sub>2</sub> emission trading among various world organizations. Econ. Model. 36, 441–446.