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How does issuing contingent convertible bonds improve bank's solvency? A Value-at-Risk and Expected Shortfall approach[☆]

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ABSTRACT

This paper examines how issuing an innovative financial instrument called contingent convertible bond (CoCo) may enhance bank's solvency in comparison to issuing a conventional bond. CoCos convert automatically into common equity or have a principal write-down when bank's regulatory capital fails to meet a predetermined level. They have been invented and put into legislation with an objective to absorb losses thus preventing institutions from bankruptcy. From the standpoint of an issuer CoCos bring about two counter effects regarding his solvency: on one hand they recapitalize a bank approaching insolvency on the other hand CoCos pay much higher coupon comparing to conventional bonds. In our model a bank has two funding alternatives: either to issue CoCos or conventional bonds. We measure issuer's default risk using the concept of Value-at-Risk (VaR) and Expected Shortfall (ES). We conclude that CoCos have the potential to strengthen the resilience of the issuer on the condition that the probability of conversion triggering is higher than the VaR's significance level. Our findings can be helpful to the policymakers and banks to better understand the impact of CoCos on issuer's solvency.

1. Introduction

Recent financial crisis has shown that capital buffer is highly needed to absorb losses in manner that will not force regulators to bail out bank with public money. Before the financial crisis, banks relied on debt financing too much. Debt financing is more effective source of capital in times of prosperity but not in times of crisis. For, unlike equity, debt does not absorb losses: coupon must be paid regardless the financial condition of a bank and the same goes with repayment of debt face value, when maturity of debt comes. That is why the Basel III Capital Accord sets new prudential capital requirements for banks.[1] Requiring banks to have more equity and less debt directly addresses the questions of the banks' solvency. When the financial crisis began in 2007, the equity of some of the major financial institutions worldwide was 2% or 3% of their total assets (Admati and Hellwig, 2013). The fact that these margins of safety were so thin played a major role in the crisis (Blundell-Wignall and Roulet, 2013; Demirguc-Kunt et al., 2013).

The pre-crisis regulatory regime also failed in the sense that it did not provide a sufficient level of loss absorption capacity on a going-concern basis. A mechanism of loss absorption sets in where occurrence of a predetermined event (reflecting the weakening of the institution's financial condition) automatically triggers either a conversion to a more subordinated instrument or a write-down of a bond value. The burden of an institution's failure is imposed on the bond holders, but the institution may continue to operate, allowing to avoid a disruption to the financial system (De Spiegeleer et al., 2014). This observation is a cornerstone for the contingent conversion into common equity that forms the footprint of new Basel III.

Prudent capital requirements for the EU banks are set out in the CRD IV package – the fourth Capital Requirements Directive and Capital Requirements Regulation. These requirements are based on internationally accepted principles of Basel III. Under the CRD IV regulatory package, EU banks have to fulfil certain capital require-

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[1] Basel regulation of banks on a global basis started with the publication of Basel I (or Basel Capital Accord) in 1988 to be implemented from January 1992. Basel I dealt with minimum capital requirements against credit risk. It introduced the concept of risk-weighted assets (RWAs), against which banks were required to hold at least 8% capital. Basel II Accord followed Basel I and was released by the Basel Committee in June 2004. Financial crisis of 2008–2009 has exposed the insufficiency of the Basel II capital structure, which was not able to preserve banks from bankruptcy.

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ments, which are grouped *inter alia* as Tier 1 and Tier 2 financial instruments. Tier 1 instruments constitute the 'going-concern capital' of the financial institution, while Tier 2 instruments form the 'gone-concern capital'. This means that Tier 1 instruments may absorb losses of the institution on an ongoing basis, while Tier 2 instruments absorb losses when a financial institution becomes insolvent or faces liquidation. The core of this system is an instrument known as the Common Equity Tier 1 (CET1). A CET1 must be composed of the highest quality of capital and possess maximum loss absorption capacity. CET1 instruments are mainly common shares and retained earnings. CRD IV capital requirements call for a minimum of 4.5% of risk weighted assets (RWAs)[2] as CET1. However, new regime still allows for other instruments than only genuine equity to be included as bank capital. So called hybrid bonds may be assigned to either the category of Additional Tier 1 (AT1) or that of Tier 2, provided that certain criteria are met. Upon the occurrence of the trigger event, the AT1 instrument converts into a CET1 instrument, or its principal amount is written down. Trigger event occurs when the CET1 capital ratio[3] falls below 5,125% or below some higher ratio, specified in the terms and conditions of the instruments. These hybrid instruments categorized AT1 capital are commonly referred to as contingent convertibles (CoCos). CoCos are specific type of convertible bonds where conversion is not an option at the discretion of a bondholder but is forced when regulatory capital fails to meet a predetermined level. Therefore CoCos contain built-in mechanisms for absorbing losses when trigger points are reached. CRD IV allows CoCos to account for 1.5% of RWAs in the additional Tier 1 layer, and 2% in the Tier 2.

These instruments were first introduced in 2009 by Lloyd's even before Basel III came into effect. The market has experienced dramatic growth over the last few years. The supply of such hybrids appears closely related to a need of banks to increase their capital ratios in line with the new Basel III standards. On the demand side, the higher coupons paid to CoCo holders in comparison to other bonds of the same issuer have proven to be very attractive in the current low-yield environment.

Apparently a CoCo bond is an instrument designed to improve the issuer's solvency in comparison to strategy of issuing only 'conventional' bonds (those without the contingent conversion or write-down characteristics, often called 'straight' bonds). Thus contingent convertibles are widely supported by the legislators and supervisory authorities in such a way, that these primarily debt instruments may be qualified as a part of financial institutions' regulatory capital not being treated much inferior to common stocks. However the risk embedded in CoCos results in their relatively higher coupon (or yield) comparing to straight bonds which in turn may undermine issuer's solvency.

As far as institution's solvency is concerned, contingent convertibles deliver two counter effects. When things go wrong, CoCos are automatically converted into issuer's common equity or just wiped out: in both cases debt is reduced. Much higher coupons of CoCos relatively to straight bonds though make CoCos issuers less resilient.

There is a quite numerous literature on CoCos impact on issuing entities that is going to be presented in the Section 2.2 of this paper. Existing papers examine impact of CoCos issuance on bank's senior debt prices, management and shareholders incentives for risk taking. They study the wealth and risk transfer mechanism between CoCo investors, senior bondholders and shareholders. To authors' best knowledge, no one has examined so far in what circumstances does the strategy of issuing CoCo bonds instead of straight ones reduce issuer's default risk, taking into account a higher coupon CoCos must pay. The aim of the article is to investigate when the CoCos' loss absorption capacity offsets higher cost of coupon in terms of reducing risk of bank's insolvency.

We formulate and prove hypothesis that AT1 CoCos may enhance the issuer's solvency in comparison to situation, when a financial institution issues conventional senior bonds. On certain conditions the solvency improvement is achieved irrespective of straight bonds cost advantage over CoCos. The rule also holds irrespective of CoCo issue amount. We propose Value-at-Risk and Expected Shortfall as a proper default risk measure.

Our findings could be useful for regulatory authorities and legislators determining shape of eligible CoCos on the market, so that these securities might globally contribute to individual banks bankruptcy risk reduction avoiding therefore so unwelcomed bail-outs at the expense of taxpayers. From this perspective we do not need to distinguish between conversion into equity or principal write-down CoCos as both result in the same level of debt reduction after being triggered.

In the Section 2 we discuss incentives for issuing CoCos. In the same section we explain and show some empirical evidence for significant cost disadvantage of CoCos relatively to other types of bonds from issuer's perspective. The rest of the paper is planned as follows: Section 3 is devoted to solvency modelling which is treated as a surplus of issuers' wealth process over his debt process. The Section 3.3 presents VaR application for quantifying default risk referring to financing strategy with- and without CoCos. Section 3.4 describes the Expected Shortfall analysis for different financing strategies. Section 4 concludes.

2. Regulatory and economic incentives for issuing CoCos

2.1. CoCos origins and main characteristics

The very idea of automatic conversion of distressed debt into equity to avoid the expense of bankruptcy and the cost of exchanges is not new. In debt exchange situations, bondholders surrender outstanding bonds in exchange for new bonds with significantly lower interest rates or principal amounts or extended maturity. As one possible contractual solution of that problem, a new security called distress-contingent convertibles (DCCs) was proposed in 1991 to address the excessive costs of over-leveraged US corporate issuers' junk bond defaults at that time (The Harvard Law Review Association, 1991). In the case of standard convertibles, bondholders will generally exercise their option only in good times, not in times of distress when the company desperately wishes to reduce its debt. DCCs would operate in reverse. Conversion would not occur at the holder's option as the stock price rises. Instead, DCC conversion would be automatic if the equity value falls below a certain threshold. Flannery (2002) proposed reverse convertible debentures (RCDs) that automatically convert into common equity of the bank's capital ratio falls below a prespecified value. In his view, loss absorption on a going-concern basis addresses resolution problem in the banking sector: for the sake of preserving the stability of the financial system it is better to bail-in systemically important banks at the cost of bondholders than to let them go bankrupt. Marquardt and Wiedman (2005) examine convertible bonds than cannot be converted into shares of common stock until a prespecified stock price is reached; that is, conversion is contingent on reaching the price threshold[4].

[2] Risk – weighting is a complex system in which some assets count less against capital requirements than others. Under the Basel system total RWAs are defined as:$RWA = 12,5(OR+MR)+\sum_{i=1}^{N} W_i A_i$where:

OR – operational risk

MR – market risk

W – an asset risk weight

A – an assetN - the number of assets categories

[3] Common Equity Tier 1/Risk Weighted Assets ratio, that pursuant to Capital Requirements Regulation Article 92(1)(a) should be at least 4,5%.

[4] They used 'COCOs' abbreviation for contingent convertibles.

The origins of the trigger event mechanism, which is a core feature of CoCo bonds, may be found elsewhere. Catastrophe Bonds (or 'CAT' bonds) as well as CAT Mortality Bonds (CATM Bonds) are financial instruments that may be regarded as predecessors to CoCos in terms of the conversion and mandatory write-downs of principle value upon the occurrence of the trigger event. CAT bonds were developed in the mid-1990s as a tool to transfer catastrophic risk. The provisions of such instruments specify in a very detailed manner the catastrophic 'trigger event' that will cause a write-down of the bond's nominal value (Lee and Yu, 2007), (Cummins, 2006). Usually trigger event is linked with industry loss indices such as the Property Loss Services (PCS) loss index (Bauer and Kramer, 2015). With CATM Bonds, the write-down trigger event does not depend on any underlying loss index, but on 'less artificial' events: the catastrophic evolution of death rates in a given population (Bauer and Kramer, 2015). When it comes to CoCos, the trigger event of a natural disaster is replaced by an accounting trigger, but the underlying idea remains similar.

The new generation of AT1 CoCos embeds a unique feature of contingent conversion into common equity or mandatory write-down of principal value. Thus, the concept of pre-bankruptcy loss absorption establishes the fundament both for instruments converting into equity (CE CoCos) and for instruments that may suffer a principal write-down (PWD CoCos).

In case of a CE CoCo instrument, terms and conditions will have to specify the rate of conversion, permitted amount of conversion and a range within the instruments will convert into common shares. As a result of conversion, common shares amount will increase by the amount of converted AT1 instruments.

Another manner of capital ratio restoration is AT1 face value write-down (full or partial). The purpose of write-down requirement is loss absorption, so write-down of a 'principal amount' of AT1 hybrid bonds results in its partial cancellation. Such action will reduce debt and capitalization of the institution will boost. That in turn will lead to restoration of the CET1 capital ratio amount.

2.2. Rationale for issuing CoCos

To discuss rationale for issuing CoCos, one must in the first place answer the basic questions: why does a financial institution need a capital buffer and why cannot this buffer be composed of regular forms of equity, such as shares, reserves and retained earnings? In other words, why is it necessary to engage financial engineering and create highly complex and sophisticated financial instruments? The financial crisis has shown that a capital buffer is highly needed to absorb losses in a manner that will not force regulators to bail out banks with public money. An alternative to current regulation would be to raise capital to 20% or more. Recent empirical research supports this view (Blundell-Wignall and Roulet, 2013). Bankers, in turn, argue against more capital. They explain that equity capital is expensive because shareholders require higher returns on capital than debt holders. Besides, banks have to generate a minimum ROE that will not be achievable if they have to increase capital (Admati and Hellwig, 2013). More equity reduces the ability of banks to lend money: banks will miss opportunities that would be attractive if they could fund themselves with more debt (Crouhy and Galai, 2015). Besides, issuing CoCos but does not dilute the rights of shareholders, as does the issuance of equity.

There is not much literature on how the CoCo actually affects banking sector stability. The three main aspects of this problem are worth considering.

1. Banks that issue contingent capital instead of subordinated debt are significantly less likely to default. The financial crisis of 2008–2009 has exposed the insufficiency of the Basel II capital structure, which was not able to preserve banks from bankruptcy. Basel II Tier 2 instruments became to bond-like and were proven by the credit crunch to not be truly loss absorbing. Coupons were discretionary but had to be paid if the bank made a profit or paid any ordinary dividend (Liberadzki and Liberadzki, 2016). Most coupons were paid throughout the credit crisis unless there was some form of state involvement in the bank (Lally, 2013). In fact, lack of banks' willingness to defer coupons on Upper Tier 2 (UT2) instruments (that would ruin their credit history) and lack of ability to defer coupons on Lower Tier 2 (LT2) subordinated debt led to necessity of bail-outs. Financial institutions which strongly relied on LT2 instruments had no tool to convert these instruments to more loss absorbing ones. On the ground of these experiences the Basel III introduced a rule that all the instruments eligible for Additional Tier 1 purposes (that has to a large extent replaced the Basel II Upper Tier 2) must be subject to such contingent conversion.
2. Contingent capital provides a form of buffer capital, like common stock but at a lesser cost. CoCo bonds are the most cost-effective recapitalization vehicle to provide buffer capital to a bank at a time of limited or no access to the capital markets due to its weakened capital position. Therefore it makes possible to recapitalize the bank from debt to equity without infusing new cash into the bank (Liberadzki and Liberadzki, 2016). CoCos are even more than a capital buffer – PWD CoCos and their coupon deferral clauses with no dividend stopper shaped a mechanism of wealth transfer from bondholders to shareholders (Roggi et al., 2013). From this perspective such CoCos may be in fact evaluated as being junior to equity.
3. Contingent capital design (in particular the conversion ratio, the fraction of post-conversion common equity that contingent capital holders receive) has an important impact on risk-taking motivation. For relatively low conversion ratios stockholders have an incentive to increase asset risk, while a high conversion ratio leads to a desire to reduce risk (Hilscher and Raviv, 2014). It creates an incentive for an excessive risk taking by bank shareholders and management and needs regulator's involvement (Berg and Kaserer, 2015).

Contingent capital bears more advantages both from an issuer's and regulator's perspective. It allows banks and insurers to avoid paying the extra cost of an equity issue. Instead, bank purchases a 'capital line commitment' – a sort of guarantee that is drawn only when necessary (Bolton and Samama, 2012). Contingent capital replaces the bankruptcy process and thus does not depend on regulators properly exercising their resolution authority. During the great financial crisis governments injected capital to rescue from failure a select group of large banks commonly referred to as 'too big to fail' or systemically important financial institutions (SIFIs). The others were left to survive or fall on their own. CoCos can be designed to make the government intervention remote and leave the fate of all banks in the capital markets' domain. Contingent capital builds loss absorption into a firm's capital structure instead of drawing down funds form the bail-out fund, making whole industry less exposed to contagion risk.

From investors' point of view, CoCo bonds provide the safety of bond during prosperous times capping the maximum possible payoff to the face value of the bond. In bad times, however, the payoff will decrease as the share price drops. As a result CoCos predominantly expose investors to downside risk, while their maximum value is restricted to the face value and the coupons.

The CoCos limited gain potential together with investor's exposure for high losses result in relatively high coupons for investors (Fig. 1), which may be confirmed empirically (Fig. 2 and Fig. 3).The higher interest rate paid by the issuer on contingent capital may be deemed as an equivalent to "bank tax" to offset bail-out costs (Coffee, 2011).

Pure arithmetic proves that in case of high-leveraged issuers conversion or write-down radically improves leverage ratio.

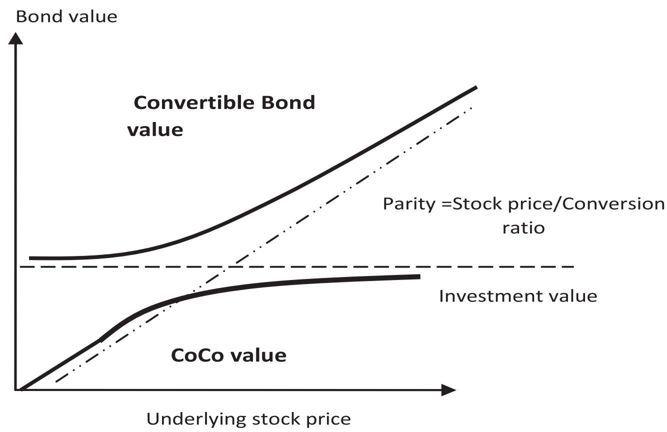

Fig. 1. A CoCo bond yield should remain relatively higher (the bond price is cheaper assuming the same coupon rate) than of comparable straight bond (with no conversion option) or traditional convertible bond (conversion into shares effects at discretion of bondholders). This is because CoCos price is capped by the straight bond value (denoted here as Investment value). CoCos have no upside potential whereas convertible bond value floor equals to straight bond value.

3. CoCos and solvency

3.1. Solvency

We will discuss the problem of the financial institution solvency. Therefore we will deal with the following characteristics of the company:

- the welfare process A_t understood as a market value of all the assets owned by the company in case of its liquidation at a moment t;
- the debt process D_t understood as an aggregated amount which would be chargeable in case of liquidation of the company at t due to all its debts.

The surplus of the welfare process A_t over the debt process D_t indicates the solvency of the company.

We denote by E_t the surplus (net welfare) process

$$E_t = A_t - D_t. \tag{1}$$

Note that when the company is in a good shape then E_t coincides with the market value of the company. Hence in case of a joint-stock company we may approximate E_t by the capitalization of the company, i.e.

$$E_t \approx N\ S_t, \tag{2}$$

where S_t is a stock price at time t of one share issued by the company and N is the total number of shares.

3.2. CoCos and the surplus process

We will consider two schemes of rising the money. The first one is classic, the company issues the senior straight corporate bonds. The second one is an innovative one, the company issues CoCos. Note that rising the money by issuing CoCos is more expensive but is less risky from the supervision authority's point of view. Hence there is a tradeoff, when company is doing well the issuance of CoCos is decreasing the surplus process but when company is doing not so well it is increasing this process. To explain this in more details we will apply the following simplified model:

3.2.1. Model notation and assumptions

1. T_0 is the day of issuance and T is the first subsequent day of the payment of the interest. Furthermore we assume that the CoCo is periodically triggered and T is the day of the first check.
2. We denote by C the amount of money raised by the company (the same for both strategies) and by i_1 and i_2 the interest rates respectively for bonds and CoCos, $0 < i_1 < i_2 < 1$, i.e. the interest in the first case equals $i_1 C$ and $i_2 C$ in the second. For simplicity we assume that the nominal value of the debt in both case equals C.
3. We denote by A_T^i, D_T^i and E_T^i the wealth, debt and surplus processes respectively for the first ($i = 1$) and the second ($i = 2$) strategy.
4. Since most often the CoCos are triggered when the market price of underlying shares falls below some strike, say k, and the surplus process coincides with the capitalization of the company, we assume that the CoCo is triggered when the surplus process E_T^2, calculated after the cash flows, falls below some threshold (strike) K, where $K = Nk$. Putting it other way, we assume that in case of CoCos conversion at time T only the nominal is affected, the interest remains chargeable.
5. We assume that after the payment of interests the market value of the debt in both strategies is equal to the nominal value.

Our goal is to compare the surplus processes after the checking of the trigger. We denote them by E_{T+}^i.

We have the following equalities before "triggering".

$$A_T^2 = A_T^1 - (i_2 - i_1)C \text{ and } D_T^2 = D_T^1. \tag{3}$$

Hence

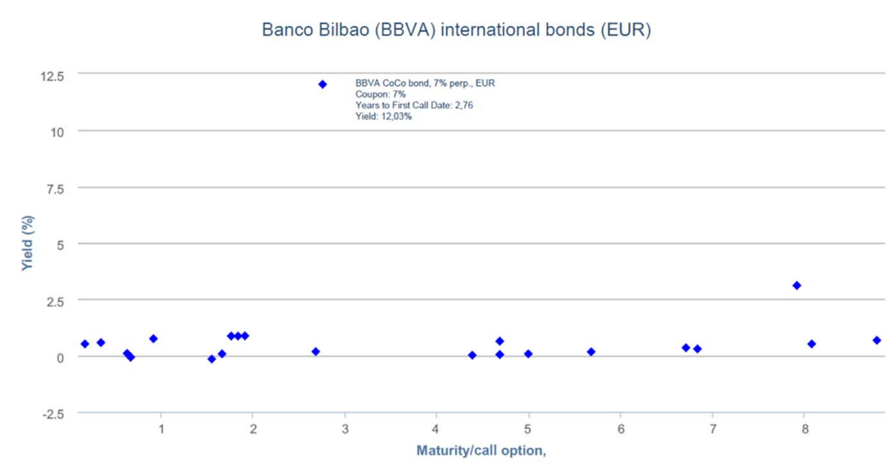

Fig. 2. Yields of Banco Bilbao (BBVA) euro-denominated international bonds as on 05/17/2016. Yields quoted according Yield to Maturity (YTM) convention. In case of bonds embedding call option, Yield to First Call Date was used.
Data source: cbonds.com.

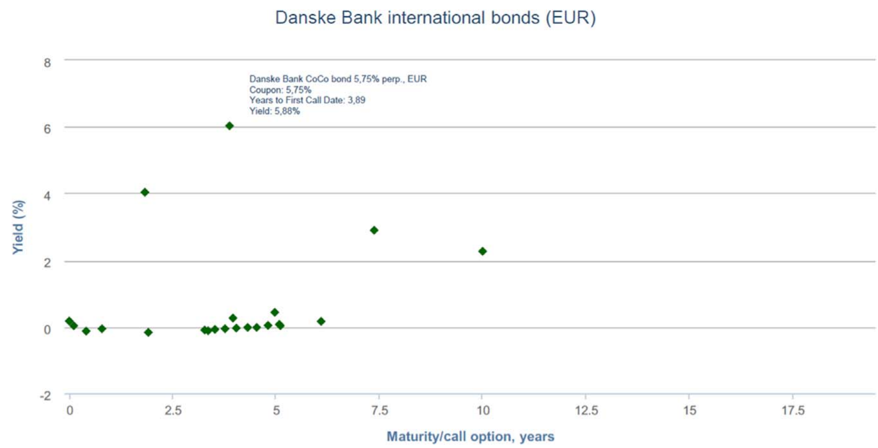

Fig. 3. Yields of Danske Bank euro-denominated international bonds as on 05/17/2016. Yields quoted according to Yield to Maturity (YTM) convention. In case of bonds embedding call option, Yield to First Call Date was used.
Data source: cbonds.com.

$$E_T^2=A_T^2-D_T^2=A_T^1-(i_2-i_1)C-D_T^1=E_T^1-(i_2-i_1)C. \quad (4)$$

We obtain that the triggering condition $E_T^2 < K$ is equivalent to

$$E_T^1 < K+(i_2-i_1)C. \quad (5)$$

After checking the trigger we get

$$E_{T+}^1=E_T^1=A_T^1-D_T^1. \quad (6)$$

While

$$E_{T+}^2=\begin{cases} E_T^1-(i_2-i_1)C & \text{when } E_T^1\geq K+(i_2-i_1)C,\\ E_T^1+(1-i_2+i_1)C & \text{when } E_T^1<K+(i_2-i_1)C. \end{cases} \quad (7)$$

The function describing the dependence of E_{T+}^2 on E_T^1 is discontinuous and not monotnic (see Fig. 4). Since E_{T_0} is known in advance, before making decision on issuance of CoCos, it is a "deterministic" variable in our stochastic model.

3.3. CoCos and Value-at-Risk calculation

Now we show how the issuance of CoCos instead of straight corporate bonds affects the Value-at-Risk (VaR) calculations. We keep the notation from the previous section. Let X^i denotes the change of the surplus (net wealth) in the time period $[T_0,T]$, due to the i-th strategy.

$$X^1=E_{T+}^1-E_{T_0}, \quad X^2=E_{T+}^2-E_{T_0}, \quad (8)$$

where E_{T_0} is the surplus prior to the issuance.

We recall that Value-at-Risk of random variable Y modelling a position and a given significance level $\alpha\in(0,1)$ is defined as follows

$$VaR_\alpha(Y)=\inf\{v\in\mathbb{R}:\mathbb{P}(Y+v\leq 0)\leq\alpha\}. \quad (9)$$

So if we interpret Y as the profit/loss random variable, $VaR_\alpha(Y)$ is the smallest amount of capital v that ensures that $Y+v$ is solvent with probability at least equal to $1-\alpha$.

The above definition can be expressed in terms of the upper (right-continuous) quantile function associated with the Y via the formula

$$VaR_\alpha(Y)=-Q_\alpha^+(Y). \quad (10)$$

For more details concerning Value-at-Risk the reader is referred to Föllmer and Schied (2004) or Jorion (2007). Although the dependence between E_{T+}^2 and E_{T+}^1 is not monotonic in general, it is monotonic in intervals, therefore we get the following estimates.

Theorem. *Let $\alpha\in(0,1)$ be a fixed significance level. Then the following estimates hold:*

- $VaR_\alpha(X^2)=VaR_\alpha(X^1)-(1-i_2+i_1)C$ *when* $VaR_\alpha(X^1)\geq E_*+C$;
- $E_{T_0}-K\geq VaR_\alpha(X^2)\geq VaR_\alpha(X^1)-(1-i_2+i_1)C$ *when* $E_*+C>VaR_\alpha(X^1)\geq E_*$;
- $VaR_\alpha(X^1)+(i_2-i_1)C\geq VaR_\alpha(X^2)\geq E_{T_0}-K-C$ *when* $E_*>VaR_\alpha(X^1)\geq E_*-C$;
- $VaR_\alpha(X^2)=VaR_\alpha(X^1)+(i_2-i_1)C$ *when* $E_*-C>VaR_\alpha(X^1)$,

where $E_*=E_{T_0}-K-(i_2-i_1)C$.

Proof.
X_2 depends on X_1 in a similar way as E_{T+}^2 on E_{T+}^1. We have

$$X^2=f(X_1), \quad (11)$$

Where for $x\in\mathbb{R}$

$$f(x)=\begin{cases} x-(i_2-i_1)C & \text{for } x\geq -E_*,\\ x+(1-i_2+i_1)C & \text{for } x<-E_*. \end{cases} \quad (12)$$

We derive from f two continuous nondecreasing piece-wise linear

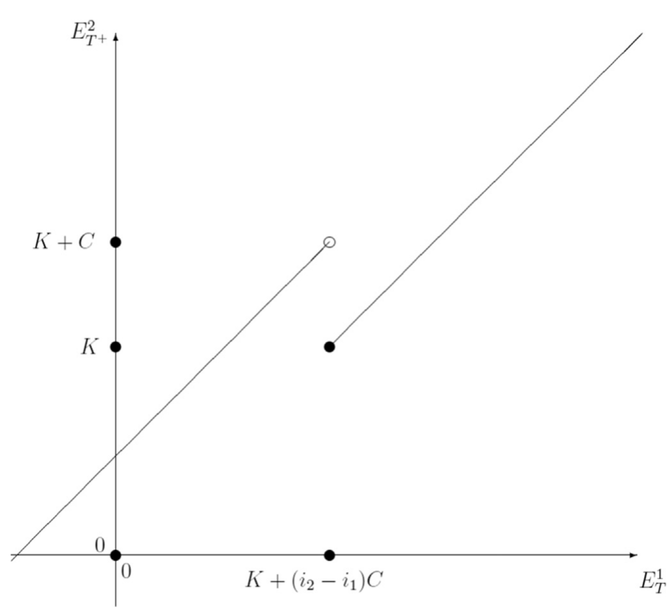

Fig. 4. Graph of E_{T+}^2 as a function of E_T^1.

functions f_1 and f_2 defined for $x \in \mathbb{R}$ by:

$$f_1(x) = \begin{cases} x-(i_2-i_1)C & \text{for } x \geq -E_*, \\ K-E_{T_0} & \text{for } -E_* > x \geq -E_*+C, \\ x+(1-i_2+i_1)C & \text{for } x < -E_*-C, \end{cases} \tag{13}$$

$$f_2(x) = \begin{cases} x-(i_2-i_1)C & \text{for } x \geq -E_*+C, \\ K+C-E_{T_0} & \text{for } -E_*+C > x \geq -E_*, \\ x+(1-i_2+i_1)C & \text{for } x < -E_*. \end{cases} \tag{14}$$

Since $f(-E_*) = K - E_{T_0}$ and $f(-E_*^+) = K + C - E_{T_0}$, f is bounded by f_1 and f_2 ,

$$\forall\, x \in \mathbb{R}\, f_1(x) \leq f(x) \leq f_2(x). \tag{15}$$

Hence

$$f_1(X^1) \leq f(X^1) = X^2 \leq f_2(X^1) \tag{16}$$

and due to the monotonicity of VaR for profit/loss variables we get

$$VaR_\alpha(f_1(X^1)) \geq VaR_\alpha(X^2) \geq VaR_\alpha(f_2(X^1)). \tag{17}$$

Since f_1 and f_2 are nondecreasing and continuous, we observe that for $=1,2$:

$$VaR_\alpha(f_i(X^1)) = -Q_\alpha^+(f_i(X^1)) = -f_i(Q_\alpha^+(X^1)) = -f_i(-VaR_\alpha(X^1)). \tag{18}$$

All together this gives us the following estimates:

$$-f_1(-VaR_\alpha(X^1)) = VaR_\alpha(f_1(X^1)) \geq VaR_\alpha(X^2) \tag{19}$$

$$\geq VaR_\alpha(f_2(X^1)) = -f_2(-VaR_\alpha(X^1)). \tag{20}$$

Since

$$-f_1(-x) = \begin{cases} x+(i_2-i_1)C & \text{for } x \leq E_*, \\ -K+E_{T_0} & \text{for } E_* < x \leq E_*+C, \\ x-(1-i_2+i_1)C & \text{for } x > E_*+C, \end{cases} \tag{21}$$

and

$$-f_2(-x) = \begin{cases} x+(i_2-i_1)C & \text{for } x \leq E_*-C, \\ -K-C+E_{T_0} & \text{for } E_*-C < x \leq E_*, \\ x-(1-i_2+i_1)C & \text{for } x > E_*, \end{cases} \tag{22}$$

we get the equalities and inequalities stated in the theorem.

Basing on the above theorem one gets the following hints when the issuance of CoCos is improving VaR and when not. It shows that for $VaR_\alpha(X^1)$ greater than $E_{T_0}-K$, $VaR_\alpha(X^2)$ is smaller than $VaR_\alpha(X^1)$, while for $VaR_\alpha(X^1)$ smaller than $E_{T_0}-K-C$ it is bigger (see Fig. 5).

Putting it in other words, if for any spread $i_2 - i_1 < 1$ the significance level α is smaller than the probability of the conversion at time T then independently of the size of the issuance the value of VaR_α is decreasing.

Hence in the case of $\alpha = 0{,}01$, as suggested by the third Basel accord, any "reasonable" trigger will improve the performance. In more details:

Corollary. *For a given* α

$$VaR_\alpha(X^2) = VaR_\alpha(X^1)-(1-i_2+i_1)C \text{ if } \alpha \leq \mathbb{P}(E_T^2 \leq K-C), \tag{23}$$

$$VaR_\alpha(X^2) \leq VaR_\alpha(X^1) \text{ if } \alpha \leq \mathbb{P}(E_T^1 \leq K) \tag{24}$$

and

$$VaR_\alpha(X^2) \geq VaR_\alpha(X^1) \text{ if } \alpha \geq \mathbb{P}(E_T^1 \leq K+C). \tag{25}$$

Proof.

Since

$$X^1 = E_T^1 - E_{T_0} = E_T^2 + (i_2-i_1)C - E_{T_0}, \tag{26}$$

the conditions

$$\mathbb{P}(E_T^2 \leq K-C) \geq \alpha \tag{27}$$

and

$$\mathbb{P}(E_T^1 \leq K) \geq \alpha \tag{28}$$

are equivalent to

$$\mathbb{P}(X^1 + E_* + C \leq 0) \geq \alpha \tag{29}$$

and

$$\mathbb{P}(X^1 + E_{T_0} - K \leq 0) \geq \alpha. \tag{30}$$

Which implies (compare formula (9))

$$VaR_\alpha(X^1) \geq E_* + C \text{ and } VaR_\alpha(X^1) \geq E_{T_0} - K. \tag{31}$$

Hence due to Theorem, in the first case $VaR_\alpha(X^2)$ equals $VaR_\alpha(X^1)$ diminished by $(1-i_2+i_1)C$ and in the second one is just smaller than $VaR_\alpha(X^1)$.

In a similar way the condition

$$\mathbb{P}(E_{T+}^1 \leq K+C) \leq \alpha \tag{32}$$

is equivalent to

$$\mathbb{P}(X^1 + E_{T_0} - K - C \leq 0) \leq \alpha. \tag{33}$$

That implies

$$VaR_\alpha(X^1) \leq E_{T_0} - K - C. \tag{34}$$

Hence due to Theorem $VaR_\alpha(X^2)$ is bigger than $VaR_\alpha(X^1)$.

3.4. CoCos and Expected Shortfall Calculation

Expected Shortfall called also Conditional Value at Risk (CVaR) or Average Value at Risk (AVaR), is a risk measure given by the formula.

$$ES_\alpha(Y) = \frac{1}{\alpha}\int_0^\alpha VaR_\gamma(Y)\,d\gamma. \tag{35}$$

For more details on Expected Shortfall see, for example, Chen et al. (2014) and Moussa et al. (2014).

When the random variable Y modelling profit has a continuous distribution function then Expected Shortfall coincides with an expected loss given that the profit is occurring at or below the $\alpha -$ quantile.

$$ES_\alpha(Y) = \mathbb{E}(-Y)\,Y \leq -VaR_\alpha(Y)). \tag{36}$$

As a direct consequence of Corollary 1 and formula (35) we get

Lemma 1. *For any given* α *and* K

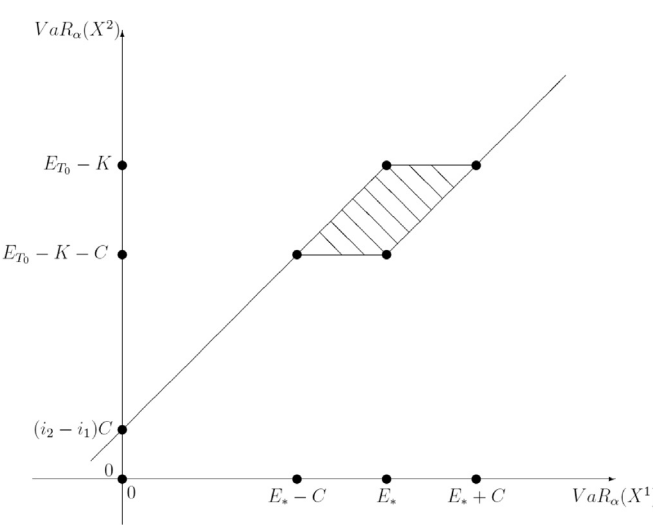

Fig. 5. Bounds for $VaR_\alpha(X^2)$ ($E_* = E_{T_0} - K - (i_2 - i_1)C$).

$$ES_{\alpha}(X^2) = ES_{\alpha}(X^1) - (1 - i_2 + i_1)C \ if \ \alpha \le \mathbb{P}(E_T^2 \le K - C) \tag{37}$$

and

$$ES_{\alpha}(X^2) \le ES_{\alpha}(X^1) \ if \ \alpha \le \mathbb{P}(E_T^1 \le K). \tag{38}$$

Whenever we measure default risk either using VaR or ES, if for any difference od coupon values (spread) $i_2 - i_1 < 1$ the significance level α is smaller than the probability of the conversion at time T then independently of the size of the issuance the issuer's default risk is decreasing.

4. Conclusion

Loss absorbing capacity of contingent convertibles made them a key element of Basel III regime and consequently CRD IV legal framework for bank capital requirements. Therefore these new regulatory packages deliver strong incentives for issuing contingent debt. The post-crisis legal regime is very intent on bailing-in banks' liabilities at the cost of creditors rather than accepting any future bail-out. Market uncertainty replaces the 'too big to fail' confidence of creditors. The previous approach was very safe for creditors, as losses were borne only by shareholders and taxpayers.

The main reason behind introducing CoCos into the legal system has been to enhance pre-bankruptcy loss absorbing capacity of an institution (that is on a going-concern basis). It is the CoCo-holders who became exposed to the risk of issuer's distress on an ongoing basis which protects other creditors and sometimes – the shareholders. This is the main reason why CoCo bonds must offer coupon (yield) for the investors significantly higher than of other debt instruments with no contingent conversion option embedded. For these reasons a strategy of financing with CoCos instead of using straight bonds results in two opposite effects: on one hand it reduces institution's default risk. On the other hand high coupon of CoCo bonds undermines issuer's solvency.

Since our analysis was conducted from the standpoint of issuing bank or supervisory authority rather than creditors and shareholders we didn't differentiate between contingent conversion and write-down mechanisms of loss absorption. Both effects have the same impact on bank solvency.

We proposed to measure the issuer's default risk with a VaR method given alpha significance level. Issuing CoCos makes sense (that is: improves issuer's solvency) only if they are structured so that the probability of the triggering is more than the alpha significance level. As it was shown this rule also holds when Expected Shortfall is used as default risk measure. The theorem is valid for any spread (the difference between coupon of CoCo and of straight bond) less than 100%. Apparently the past and present spread values observed on the market do not exceed the value of 100%. The conclusion is, that whenever the probability of contingent conversion is high enough, the issuer's default risk will be reduced irrespective of cost disadvantage of CoCo bonds in comparison to straight bonds.

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