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Indeterminacy, capital maintenance expenditures and the business cycle

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ABSTRACT

This paper examines how maintenance expenditures affect the occurrence of indeterminacy in a two-sector model economy, motivated by the empirical fact that equipment and structures are maintained and repaired. McGrattan and Schmitz's (1999) survey on 'Capital and Repair Expenditures' in Canada indicates that maintenance expenditures account for a substantial fraction of output and new investment. It is shown that the endogenous maintenance expenditures reduce the requirement of the degree of increasing returns to scale to generate sunspot equilibria. In fact, the minimum level of the returns to scale required could be as low as 1.0179. This aspect is important since empirical works such as Basu and Fernald (1997) suggest that returns to scale is close to constant.

1. Introduction

Recent years have witnessed the formulation of business cycle models with multiple equilibria. In particular, many researchers explore the mechanisms that give rise to indeterminacy.[1] It has been recognized that the indeterminacy could arise if the assumption of a perfect market is relaxed. In earlier research such as Benhabib and Farmer (1994), the existence of a continuum of equilibria relies on a high degree of increasing returns to scale in production. However, empirical work by Basu and Fernald (1997) depicts that the presence of production externalities is rather modest, if any, which led researchers to pursue model structures with lower scale economies to induce indeterminacy. The increasing returns to scale are often exhibited via external effects.

This paper works on such a model. It examines how maintenance expenditures affect the occurrence of indeterminacy in a two-sector model economy. This model provides an extension of the two-sector, endogenous capital utilization model of Guo and Harrison (2001). The main feature of this model is that the capital depreciation rate varies with capital utilization rate and maintenance expenditures, whereas in many other two-sector model papers the evolution of the depreciation rate is solely determined by variable capital utilization. McGrattan and Schmitz (1999) define maintenance expenditures as "the expenditures made for the purpose of keeping the stock of fixed assets or productive capacity in good working order during the life originally intended". Licandro and Puch (2000) point out that such expenditures are important factors affecting depreciation, as machines are better preserved if maintenance activity is engaged during the production process. In the model, the amount of maintenance expenditures affects the capital accumulation law and is upon the representative agent's optimal decisions.

Empirical studies affirm the importance of maintenance expenditures. McGrattan and Schmitz (1999) conduct a survey on 'Capital and Repair Expenditures' in Canada and show that expenditures on maintenance activity are large relative to that on other activities. In this survey, total maintenance and repair expenditures accounted for 5.7 percent of gross domestic product (GDP) over 1981–1993.[2] Over the same period, these expenditures averaged about 28 percent of spending on new investment. Expenditures on R & D were 1.4 percent of GDP which was much lower than maintenance-to-GDP ratio. Moreover, the proportion of public spending on education was 6.8 percent which was only slightly higher than that of maintenance expenditures, indicating that maintenance expenditures are 'too big to ignore'.

This model relates to Guo and Lansing (2007). They investigate the indeterminacy properties of a one-sector model with maintenance expenditures. As there is a lack of data on maintenance expenditures in the U.S., they calibrate maintenance-to-GDP ratio using Canadian data as the proxy for U.S. data. In this paper I consider a two-sector case as subsequent research has indicated that models with two-sector or multi-sectors of production require much lower increasing returns to obtain indeterminacy.[3] Furthermore, I allow households to make decisions on capital maintenance expenditure as households own the capital, whereas in Guo and Lansing's (2007) economy the sequence of

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[1] Examples include Benhabib and Farmer (1996), Wen (1998), Guo and Harrison (2001).

[2] It accounted for 6.1 percent of GDP if the period covers from 1961 to 1993.

[3] For example, Benhabib and Farmer (1996), and Weder (2000).

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maintenance expenditures is the firms' choice. The study has quantitatively shown that maintenance expenditures could reduce the minimum required level of increasing returns to scale. The minimum level of returns to scale is 1.0179 which is close to constant.

It has been criticized that a model combining both two production sectors and variable capital utilization tends to generate an extremely narrow range of increasing returns that give rise to indeterminacy (Guo and Lansing, 2007). Under this circumstance it is not possible to generate pro-cyclical consumption with such low degree of externalities. Therefore, this paper also considers a model variant in which capital utilization is assumed to be constant over time. This model is in fact an extension of Benhabib and Farmer's (1996) model by incorporating maintenance activities into their model specification. The results that indeterminacy requires lower returns to scale in models with maintenance activities than non-maintenance economic variants are robust. Under this formation, the countercyclical consumption puzzle is solved and most features of the model moments are comparable to the U.S. data.

The role of maintenance expenditures on the occurrence of indeterminacy is obvious. Starting from an equilibrium path where the rate of discount equals the overall (net) rate of return on capital. Suppose an optimistic agent believes that there will be an increase in the rate of return on capital, the agent will reallocate resources from consumption to investment. In order to validate the agent's expectations as a new equilibrium, the return on capital has to be actually increased at higher level of economic activity and the associated first order conditions still hold. The model has two major features that could achieve this. Firstly, a mild degree of returns to scale exhibits in the present model economy. The marginal product of capital increases when labour flows from the consumption sector into the investment sector. Secondly, engaging in maintenance activities makes capital better preserved and thus increases its productivity. This results in a higher rate of return on capital as well. Combining these two features, the return on capital can easily increase with higher level of capital stock even if the degree of externalities is small.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 analyzes the local dynamics and the indeterminacy properties. In Section 4 I show the business cycle properties generated from the models and address the cyclicality of consumption issue. Section 5 concludes.

2. The model

The model incorporates maintenance expenditures into Guo and Harrison's (2001) two-sector model. The economy consists of a continuum of identical households who make decisions about consumption, labour hours worked, utilization rate of capital and maintenance expenditures. Households own the capital and lend capital and labour services to firms, taking rent and real wage rate as given. Firms produce consumption and investment goods which are sold to households. Households own the firms and therefore the profits are remitted to households.

2.1. Preferences and household's choices

A representative household chooses the sequences of consumption C_t and hours worked L_t to maximize his lifetime utility

$$\int_0^\infty \left(\ln C_t - \frac{L_t^{1+\chi}}{1+\chi}\right)e^{-\rho t}dt, \tag{1}$$

where χ captures the inverse elasticity of labour supply and ρ is the discount rate. The budget constraint faced by the household is

$$C_t + P_t I_t = r_t u_t K_t + w_t L_t, \tag{2}$$

where I_t is the household's investment in new capital, P_t is the relative price of investment goods in units of consumption goods. r_t and w_t are the rental rate of capital and the real wage rate, respectively. u_t is the rate of capital utilization. Let K_t denotes economy-wide capital stock. The law of motion for capital accumulation is given by

$$\dot{K}_t = I_t - \delta_t K_t - M_t, \tag{3}$$

where M_t is goods expenditure on maintenance. $\delta_t \in (0, 1)$ is the rate of capital depreciation which is variable over time. Following Guo and Lansing (2007), δ_t has the form of

$$\delta_t = \tau \frac{u_t^\theta}{(M_t/K_t)^\phi}, \tag{4}$$

where $\tau > 0$, $\theta > 1$, and $\phi \geq 0$. θ is the elasticity of depreciation with respect to capital utilization. ϕ captures the elasticity of depreciation rate with respect to maintenance cost rate:

$$\phi = -\frac{\partial \delta_t}{\partial (M_t/K_t)} \times \frac{(M_t/K_t)}{\delta_t}. \tag{5}$$

Licandro and Puch (2000) define M_t/K_t as 'the maintenance cost rate' that captures the intensity of maintenance activities. Above form of the depreciation rate implies that the depreciation rate depends on both capital utilization and maintenance activities. Higher capital utilization rate accelerates the depreciation whereas higher maintenance expenditures has the opposite effect (Guo and Lansing, 2007).

Let Λ_t be the co-state variable associated with the Hamiltonian set-up of the household's optimization problem. It is often explained as the shadow price of capital, meaning the marginal utility gain if agent's capital constraint is relaxed. The Hamiltonian set-up is

$$H_t = \left(\ln C_t - \frac{L_t^{1+\chi}}{1+x}\right) + \Lambda_t [(r_t u_t K_t + w_t L_t - C_t)P_t^{-1} - \tau u_t^\theta k_t^{\phi+1} M_t^{-\phi} - M_t] \tag{6}$$

Then the first-order conditions are given by

$$\frac{1}{C_t} = \Lambda_t P_t^{-1}, \tag{7}$$

$$\Lambda_t w_t P_t^{-1} = L_t^\chi, \tag{8}$$

$$r_t P_t^{-1} u_t = \theta \delta_t, \tag{9}$$

$$\phi \delta_t \frac{K_t}{M_t} = 1, \tag{10}$$

$$\frac{\dot{\Lambda}_t}{\Lambda_t} = \rho - r_t u_t P_t^{-1} + (\phi + 1)\delta_t. \tag{11}$$

The transversality condition is $\lim_{t\to\infty} e^{-\rho t}\Lambda_t K_t = 0$. Eqs. (7) and (8) show the intratemporal trade-off between consumption and leisure. Eq. (9) shows that the household utilizes capital by equating the marginal gain and marginal loss of a change in utilization rate. Eq. (10) indicates that the household equates one unit of good expenditure on maintenance to marginal maintenance cost rate with respect to the depreciation rate. Eq. (11) is the intertemporal Euler equation.

2.2. Production technology

The production functions for the consumption sector and investment sector are given by

$$Y_{ct} = A_t (u_t K_{ct})^\alpha L_{ct}^{1-\alpha}, \tag{12}$$

where $A_t = [(\overline{u}_t \overline{K}_{ct})^\alpha \overline{L}_{ct}^{1-\alpha}]^\eta$ and

$$Y_{xt} = B_t (u_t K_{xt})^\alpha L_{xt}^{1-\alpha}, \tag{13}$$

where $B_t = [(\overline{u}_t \overline{K}_{xt})^\alpha \overline{L}_{xt}^{1-\alpha}]^\eta$.

Y_{ct} and Y_{xt} denote the production of consumption goods and investment goods, respectively. K_{it} and L_{it} are, respectively, the capital and labour inputs used in the production of sector i for $i = C, I$. α is the capital share in each sector. A_t and B_t are scaling factors that capture the external effects. A bar over variables means the economy-wide average which firms taken as given. η captures the degree of sector-specific externalities[4] and is assumed to be non-decreasing, $\eta > 0$.

Under the assumption that the factor markets are perfectly competitive, the first-order conditions for the representative firm are

$$u_t r_t = \frac{\alpha Y_{ct}}{K_{ct}} = P_t \frac{\alpha Y_{xt}}{K_{xt}}, \tag{14}$$

$$w_t = \frac{(1-\alpha) Y_{ct}}{L_{ct}} = P_t \frac{(1-\alpha) Y_{xt}}{L_{xt}}. \tag{15}$$

3. Equilibrium and local dynamics

This paper focuses on perfect foresight equilibrium which is defined as a path $\{K_t, L_t, M_t, u_t, \Lambda_t, s_t, C_t\}_{t=0}^{\infty}$ and a set of prices $\{P_t, r_t, w_t\}_{t=0}^{\infty}$ satisfying household and firm's first-order conditions and their resource constraints. Let s_t be the ratio of the aggregate capital and labour used in the consumption sector,

$$s_t = \frac{K_{ct}}{K_t} = \frac{L_{ct}}{L_t}. \tag{16}$$

In equilibrium, the consistency requires that

$$u_t = \bar{u}_t, \quad K_{ct} = \overline{K}_{ct}, \quad L_{ct} = \overline{L}_{ct}, \quad K_{xt} = \overline{K}_{xt}, \quad L_{xt} = \overline{L}_{xt}. \tag{17}$$

As capital and labour are only used in the production of consumption and investment goods, the following conditions must be satisfied:

$$K_{ct} + K_{xt} = K_t, \quad L_{ct} + L_{xt} = L_t. \tag{18}$$

And as total production consists of consumption and investment goods, one must have

$$Y_t = Y_{ct} + P_t Y_{xt}, \tag{19}$$

where Y_t denotes the aggregate output produced in the economy.

Then one can derive the following production functions:

$$Y_{ct} = s_t^{1+\eta} (u_t K_t)^{\alpha(1+\eta)} L_t^{(1-\alpha)(1+\eta)}, \tag{20}$$

$$Y_{xt} = (1-s_t)^{1+\eta} (u_t K_t)^{\alpha(1+\eta)} L_t^{(1-\alpha)(1+\eta)}, \tag{21}$$

$$Y_t = s_t^{\eta} (u_t K_t)^{\alpha(1+\eta)} L_t^{(1-\alpha)(1+\eta)}, \tag{22}$$

It is assumed that $\alpha(1+\eta) < 1$, implying moderate size of increasing returns so that it is not able to generate endogenous growth.

The consumption and investment goods demanded by household and supplied by firms are equal, implying

$$C_t = Y_{ct}, \; I_t = Y_{xt}. \tag{23}$$

From Eqs. (14) and (15) the factor prices and the relative price of investment good in terms of consumption goods are given by

$$u_t r_t = \alpha s_t^{\eta} \frac{(u_t K_t)^{\alpha(1+\eta)} L_t^{(1-\alpha)(1+\eta)}}{K_t}, \tag{24}$$

$$w_t = (1-\alpha) s_t^{\eta} \frac{(u_t K_t)^{\alpha(1+\eta)} L_t^{(1-\alpha)(1+\eta)}}{L_t}, \tag{25}$$

$$P_t = \left(\frac{s_t}{1-s_t}\right)^{\eta}. \tag{26}$$

[4] The sizes of the externalities in the consumption and investment sectors are assumed to be the same. Harrison (2001) points out that if the utility function is logarithmic in consumption then the indeterminacy properties are independent of the degree of the externalities in the consumption sector.

I use Eqs. (9), (10) and (24) to derive the expression for optimal capital utilization in terms of aggregate capital and labour and then substitute it into Eq. (22).[5] The reduced form of aggregate production function is then given by

$$Y_t = D_t K_t^{\frac{\alpha(1+\eta)(\theta-\phi-1)}{\theta-\alpha(1+\eta)(\phi+1)}} L_t^{\frac{(1-\alpha)(1+\eta)\theta}{\theta-\alpha(1+\eta)(\phi+1)}}, \tag{27}$$

where D_t is an expression in terms of parameters and s_t the fraction that the aggregate capital and labour used in the consumption sector. The variable utilization rate changes the production function and induces Wen's (1998) so-called 'return-to-scale effect' as

$$\frac{\alpha(1+\eta)(\theta-\phi-1)}{\theta-\alpha(1+\eta)(\phi+1)} + \frac{(1-\alpha)(1+\eta)\theta}{\theta-\alpha(1+\eta)(\phi+1)} > 1+\eta. \tag{28}$$

The output elasticity with respect to capital and labour coincide with Guo and Lansing (2007). I restrict the analysis by $0 < \frac{\alpha(1+\eta)(\theta-\phi-1)}{\theta-\alpha(1+\eta)(\phi+1)} < 1$ to ensure that the positive externalities exhibit and that there is no endogenous growth, implying that $\theta - \phi - 1 > 0$.

I analyse the properties of local dynamics of the model by taking log-linear approximations around the steady state. Then dynamic system becomes

$$\begin{bmatrix} \log \dot{\Lambda}_t \\ \log \dot{K}_t \end{bmatrix} = J \begin{bmatrix} \log \Lambda_t - \log \Lambda \\ \log K_t - \log K \end{bmatrix}. \tag{29}$$

The variables without time subscript refer to their steady state level and J is a 2×2 Jacobian matrix. Λ_t is a non-predetermined variable and K_t is a pre-determined variable. Indeterminacy requires that both eigenvalues of the Jacobian matrix J are negative. Since the trace of J measures the sum of the roots and the determinant measures the product of them, indeterminacy requires that $\mathrm{Tr}\, J < 0 < \mathrm{Det}\, J$. When indeterminacy arises, equilibria may be driven by sunspots.

3.1. The one-sector model

In this subsection I first consider the case where there are no sector-specific externalities, instead we allow for the aggregate externalities, denoted by γ. Hence, $\alpha(1+\gamma) < 1$. The model is reduced to a one-sector model that corresponds to the continuous time version of Guo and Lansing's (2007) model, implying $P_t = 1$. Firm maximizes its profit subject to the following production function:

$$Y_t = A_t (u_t K_t)^{\alpha} L_t^{1-\alpha}, \tag{30}$$

where $A_t = [(\bar{u}_t \overline{K}_t)^{\alpha} \overline{L}_t^{1-\alpha}]^{\gamma}$.

The following analysis focuses on the case of χ=0, a standard assumption in real business cycle models, implying infinite labour supply elasticity or indivisible labour. The household's utility function becomes

$$\int_0^{\infty} (\ln C_t - L_t) e^{-\rho t} dt, \tag{31}$$

which is essentially the Hansen–Rogerson preference.

Then the determinant of J is given by

$$\mathrm{Det}\, J = \frac{[\alpha(1+\gamma)-1]\theta\rho^2(\theta-\alpha\phi-\alpha)}{\alpha[-\gamma\theta+\alpha(1+\gamma)(\theta-\phi-1)](\theta-\phi-1)}, \tag{32}$$

and the trace of the matrix is

$$\mathrm{Tr}\, J = \frac{\alpha(1+\gamma)\rho(\theta-\phi-1)}{-\gamma\theta+\alpha(1+\gamma)(\theta-\phi-1)}. \tag{33}$$

The necessary and sufficient conditions for indeterminacy require negative trace and positive determinant. As $\alpha(1+\gamma) - 1 < 0$ and $\theta - \phi - 1 > 0$, the determinant is always positive when

[5] For simplicity, τ is set to equal $\frac{1}{\theta}$ as it does not play any role in the model's steady state and indeterminacy properties.

$-\gamma\theta + \alpha(1+\gamma)(\theta - \phi - 1) < 0$. Trace is always negative when this condition is satisfied. Both necessary and sufficient conditions together imply

$$\frac{\alpha(\theta - \phi - 1)}{\theta - \alpha(\theta - \phi - 1)} < \gamma < \frac{1}{\alpha} - 1. \tag{34}$$

The necessary condition can be rewritten as

$$\frac{(1-\alpha)(1+\gamma)\theta}{\theta - \alpha(1+\gamma)(\phi + 1)} - 1 > 0. \tag{35}$$

The left-hand side refers to the labour demand elasticity. Hence this condition implies that the equilibrium wage-hours locus in the labour market is positively sloped and steeper than the slope of labour supply curve. The larger the labour demand elasticity is, indeterminacy occurs more easily (Harrison, 2001).

I quantitatively investigate the local dynamic properties using Benhabib and Farmer's (1996) model calibration. The capital share, α, equals 0.3, implying that the labour share $1-\alpha$ is 0.7, the depreciation rate δ is 0.1, the discount value ρ equals 0.05. The values of θ and ϕ depend on the maintenance cost rate M/K and the maintenance-to-GDP ratio M/Y. Given the household and firm's first-order conditions together with the equilibrium conditions, the maintenance cost rate and maintenance-to-GDP ratio are

$$\frac{M}{K} = \frac{\phi\rho}{\theta - \phi - 1}, \tag{36}$$

$$\frac{M}{Y} = \frac{\alpha\phi\delta(\theta - \phi - 1)}{\rho\theta}. \tag{37}$$

In equilibrium both the maintenance cost rate and the maintenance-to-GDP ratio are positive constants and depend on parameter values only, implying that maintenance expenditures are procyclical to capital and output. Following Guo and Lansing (2007), I also use Canada's data estimated by McGrattan and Schmitz (1999) as the proxy of the steady state maintenance-to-GDP ratio (ranged between 5.7 percent and 6.1 percent). Here we set $M/Y = 0.061$, implying that the value of θ and ϕ is 1.8828 and 0.3828, respectively. Given these parameter values, the model requires $\gamma_{min} = 0.0866$ for the steady state to be indeterminate.

When the economy is in the absence of maintenance activities, meaning that ϕ=0 and M=0, the minimum degree of externalities required for indeterminacy is 0.1111 which is higher than the case involving maintenance activities. The results quantitatively imply that a positive equilibrium ratio of maintenance costs leads to lower minimum required degree of increasing returns, which is consistent with Guo and Lansing's (2007) discrete time version of the one-sector model.

Above analysis gives the following proposition

Proposition 1. *In the one-sector model with variable capital utilization, lower returns to scale are needed in order for indeterminacy to result if maintenance expenditures are considered.*

Suppose agents become more optimistic about the future. The anticipated increase in output induces a rise in consumption and draws labour out of leisure. With a sufficiently upward sloping labour demand curve, a shift in the supply curve to the left implies that in the new labour market equilibrium output increases and allows agents' expectations to be self-fulfilling. If maintenance on capital increases with economic activity, the net returns to labour may be decreasing with the need of lower returns to scale to validate the agent's expectation.

3.2. The two-sector models

In this subsection, I examine two different versions of the model with maintenance expenditures. The first version considers the endogenous capital utilization. It is an extension of Guo and Harrison's (2001) model. In fact, their specification is a special case of the present model – there are no maintenance activities. In the second version, the capital utilization rate is assumed exogenous. This model is essentially Benhabib and Farmer's (1996) model incorporating maintenance expenditures. In both versions, the required minimum level of returns to scale is reduced compared to Guo and Harrison (2001) and Benhabib and Farmer (1996).

3.2.1. Endogenous capital utilization

First I consider a two-sector model with sector-specific externalities. Then the trace and determinant of J are given by

$$\text{Tr } J = \frac{\rho[-\eta\theta^2 + \alpha\eta\theta(1+\phi) + \alpha^2(1+\eta)(\theta - \phi - 1)(1+\phi)]}{-\eta\theta^2 + \alpha^2(1+\eta)(\theta - \phi - 1)(1+\phi)}, \tag{38}$$

$$\text{Det } J = \frac{-[\alpha(1+\eta) - 1]\theta\rho^2(1+\phi)(\theta - \alpha\phi - \alpha)}{(\theta - \phi - 1)[\eta\theta^2 - \alpha^2(1+\eta)(\theta - \phi - 1)(1+\phi)]}. \tag{39}$$

Under the assumption of $\alpha(1+\eta) < 1$ and $\theta - \phi - 1 > 0$, the determinant is always positive when $\eta\theta^2 - \alpha^2(1+\eta)(\theta - \phi - 1)(1+\phi) > 0$. The trace is negative if $-\eta\theta^2 + \alpha\eta\theta(1+\phi) + \alpha^2(1+\eta)(\theta - \phi - 1)(1+\phi) > 0$. These inequalities translate into the following condition:

$$\frac{\alpha^2(1+\phi)(\theta - \phi - 1)}{\theta^2 - \alpha^2(1+\phi)(\theta - \phi - 1)} < \eta < \frac{\alpha^2(1+\phi)(\theta - \phi - 1)}{\theta^2 - \alpha^2(1+\phi)(\theta - \phi - 1) - \alpha\theta(1+\phi)}. \tag{40}$$

Given the above calibration, indeterminacy emerges when $0.0179 < \eta < 0.0231$ which is within the empirically plausible range estimated by Basu and Fernald (1997). When there are no maintenance activities, multiple equilibria require $0.0204 < \eta < 0.0256$.[6] Therefore, as in the one-sector model, in the presence of maintenance activities indeterminacy can occur with lower degree of externalities or increasing returns to scale. An important feature of this indeterminacy condition is that indeterminacy rests on a downward sloping demand curve. In the two-sector model with endogenous capital utilization, Proposition 1 still holds.

Now I focus the analysis on the lower bound of the degree of externalities to further explore the indeterminacy properties,

$$\eta > \eta_{min} = \frac{\alpha^2(1+\phi)(\theta - \phi - 1)}{\theta^2 - \alpha^2(1+\phi)(\theta - \phi - 1)}. \tag{41}$$

It is easy to show that the following first derivatives of η_{min} with respect to θ and ϕ hold:

$$\frac{\partial\eta_{min}}{\partial\theta} > \text{or} < 0, \quad \frac{\partial\eta_{min}}{\partial\phi} < 0, \tag{42}$$

implying that the elasticity of depreciation with respect to capital utilization has ambiguous effect on the occurrence of multiple equilibria as there are two opposite effects. On the one hand, higher elasticity implies that more intensive capital utilization leads to a faster depreciation rate, which lowers the net rate of return on capital. On the other hand, from Eq. (9) we can see that as the elasticity parameter θ increases, the marginal benefit of more output increases, which boost the net return on capital.

And the main result of this analysis is

Proposition 2. *Indeterminacy occurs more easily the larger the elasticity of depreciation rate with respect to maintenance cost rate.*

When depreciation rate is very sensitive to a percentage change of the maintenance cost rate, it is easier for extra expenditures spending on maintenance to have positive impact on the net return on capital.

[6] This case corresponds to a continuous time version of Guo and Harrison's (2001) model.

Table 1
Regions of indeterminacy.

Models	Maintenance	No maintenance
One-sector variable utilization	$0.0866 < \gamma < 2.3333$	$0.1111 < \gamma < 2.3333$
Two-sector variable utilization	$0.0179 < \eta < 0.0231$	$0.0204 < \eta < 0.0256$
Two-sector constant utilization	$0.0638 < \eta < 0.1764$	$0.0708 < \eta < 0.6342$

Suppose that agents expect a higher rate of return on capital. In response to this expectation, they increase investment goods expenditure. Labour flows from the consumption goods sector into the investment goods sector, increasing the production of investment goods and therefore increasing future capital stock. With sufficient increasing returns to scale, the increase in capital stocks is associated with a higher rate of return, and the agents' expectation can be self-fulfilling. Involving maintenance activities reduces the required level of increasing returns to scale to justify multiple equilibria as it makes the existing capital such as machines and equipment more productive, which tends to increase the marginal product of capital and therefore boost the net return on capital.

3.2.2. Constant capital utilization

Now I discuss the indeterminacy region when $u_t = u$. Under this formulation, the trace and determinant of J are given by

$$\text{Tr } J = \frac{\alpha^2\delta(1+\eta)\rho(1+\phi) - \eta\rho(\delta+\rho+\delta\phi) + \alpha\delta\eta(1+\phi)(\delta+\rho+\delta\phi)}{\alpha^2\delta(1+\eta)(1+\phi) - \eta(\delta+\rho+\delta\phi)}, \tag{43}$$

$$\text{Det } J = \frac{\delta[\alpha(1+\eta)-1](1+\phi)(\delta+\rho+\delta\phi)[\rho+(1-\alpha)\delta(1+\phi)]}{\alpha^2\delta(1+\eta)(1+\phi) - \eta(\delta+\rho+\delta\phi)}. \tag{44}$$

The necessary condition for a positive determinant is $\alpha^2\delta(1+\eta)(1+\phi) - \eta(\delta+\rho+\delta\phi) < 0$,
indeterminacy requires that the trace to be negative, implying $\alpha^2\delta(1+\eta)\rho(1+\phi) - \eta\rho(\delta+\rho+\phi\delta) + \alpha\delta\eta(1+\phi)(\delta+\rho+\delta\phi) > 0$. This
implies:

$$\frac{\alpha^2\delta(1+\phi)}{\rho+\delta(1+\phi)-\alpha^2\delta(1+\phi)} < \eta < \frac{\alpha^2\delta\rho(1+\phi)}{[\rho-\alpha\delta(1+\phi)][\rho+\delta(1+\phi)]-\alpha^2\delta\rho(1+\phi)}, \tag{45}$$

I use the same parameterization as above. Then $\eta_{min} = 0.0638$ and $\eta_{max} = 0.1764$, which allows the degree of externalities to have a much wider parameterization range than that in endogenous capital utilization case. The minimum value remains empirically plausible and smaller than the case where there are no maintenance activities. Therefore the importance of maintenance expenditures on generating multiple equilibria still presents in this framework. With sufficient increasing returns and maintenance activities, well-maintained capital equipment may increase the productivity in the investment good sector. Moreover, as the elasticity of depreciation rate with respect to maintenance cost rate ϕ increases, the minimum required η decreases.

Table 1 compares the regions of indeterminacy for three different economies discussed in this section. It is shown that indeterminacy is easier to obtain in models with maintenance activities than in models without that.

4. Simulation

In this section, I simulate the discrete time economies involving maintenance activities by introducing both technology and sunspot shocks into the models discussed in Section 3. I use Benhabib and Farmer's (1996) discrete time parameterization.[7] The capital share α is set to 0.35, the quarterly depreciation rate δ is 0.025, the quarterly discount value ρ equals 0.01, and the inverse elasticity of labour supply χ is 0.

The technology shocks follow the process

$$Z_t = Z_{t-1}^{\omega}\zeta_t, \tag{46}$$

where the persistence parameter ω is calibrated to 0.95. We also introduce i.i.d. sunspot shocks and then the law of motion of this economy becomes

$$\begin{pmatrix} \widehat{\Lambda_{t+1}} \\ \widehat{K_{t+1}} \\ \widehat{Z_{t+1}} \end{pmatrix} = J\begin{pmatrix} \widehat{\Lambda_t} \\ \widehat{K_t} \\ \widehat{Z_t} \end{pmatrix} + R\begin{pmatrix} \widehat{\zeta_{t+1}} \\ \widehat{e_{t+1}} \end{pmatrix}, \tag{47}$$

where e_{t+1} is i.i.d. expectation error which denotes the sunspot shocks.

Table 2
U.S. Moments and model moments.

Variable	U.S.		Model 1		Model 2		Model 3	
	σ_x/σ_Y	ρ_{xY}	σ_x/σ_Y	ρ_{xY}	σ_x/σ_Y	ρ_{xY}	σ_x/σ_Y	ρ_{xY}
C_t	0.38	0.71	0.09	0.76	0.11	−0.77	0.29	0.42
P_tI_t	3.62	0.97	3.57	0.99	9.58	0.99	3.48	0.98
L_t	0.84	0.83	0.94	0.99	1.08	0.99	0.42	0.96
w_t	1.14	0.76	0.09	0.76	1.11	−0.77	0.29	0.42
M_t	–	–	1.00	1.00	1.12	1.00	1.19	0.96

Notes: The U.S. statistical results of *C*, *PI* and *L* are taken from Pavlov and Weder (2012). Data is quarterly, seasonally adjusted and covers from 1948:I–2006:IV. The information about the real wage *w* is from King et al. (1988, cited in Weder 2000, p. 286). This data series shows the deviations from linear trend, quarterly, from 1948:I to 1986:IV. An HP-filter is introduced into the artificial time series data.

Table 2 shows the U.S. population moments. σ_Y denotes the standard deviation of output and σ_x refers to the standard deviation of variable x. ρ_{xY} is the correlation between variable x and output. The table reveals main stylized facts: all variables are procyclical to output. Consumption is less volatile and investment is more volatile than output.

The one-sector model corresponds to Guo and Lansing (2007). In this model, I let externality parameter equal 0.2. The moments derived from the one-sector model are shown in column 'Model 1' of Table 2. The table indicates that this model performs reasonably well except that consumption and real wage are too smooth relative to output.

I simulate the two-sector models using the same calibration, except the externality parameter. When I set the externality parameter to 0.2 for the model where the capital utilization is endogenous, it leads to a dynamics of a source instead of a sink as this externality parameter is much larger than the upper bound of our indeterminacy region.[8] Therefore it is set to 0.03. The 'Model 2' column in Table 2 shows this two-sector model moment.

In this two-sector model, investment, employment and maintenance expenditures are procyclical but consumption and real wage are countercyclical, although the model results in reasonable degree of externalities generating multiple equilibria. It is a well-known fact that the artificial data obtained from the models may have some time series properties that are not consistent with the U.S. data. In particular, the countercyclical behaviour of consumption in two-sector models has

[7] In this paper, the calibrations of both continuous and discrete time models are the same as Benhabib and Farmer (1996) for comparison purpose.
[8] In the discrete time version of this model, local indeterminacy arises when the degree of externalities is within 0.025 and 0.032.

been discussed in several business cycle literature, such as Benhabib and Farmer (1996), Weder (2000) and Harrison (2001). This is due to the fact that when the economy is driven by sunspot shocks, rates of return on capital increase with a higher level of capital stock, implying that the marginal product of labour decreases. To restore Eqs. (7) and (8), consumption must decline (Harrison, 2001). That is, at low levels of increasing returns to scale, consumption is countercyclical. Guo and Lansing (2007) indicate that a model combining both production sectors and endogenous capital utilization will generate an extremely narrow range of increasing returns that give rise to multiple equilibria. As the upper bound of the required degree of externalities in the present model is very small, it is not possible to get pro-cyclical consumption with such low degree of externalities when the model displays indeterminacy. Another major counterfactual property in this two-sector model is that investment is much too volatile.

Benhabib and Farmer (1996) point out that if externalities are sufficiently large then one may get procyclical consumption. If I consider a two-sector model with constant capital utilization, the model exhibit indeterminacy if η is as large as 0.3. With this sufficiently large degree of externalities, output may rise substantial enough to allow both consumption and investment to increase even if resources are shifted out from consumption goods sector to investment goods sector.

The artificial time series data gives pro-cyclical consumption.[9] The model moments results are shown in the column 'Model 3'. Most features of the model moments are comparable to the U.S. data. It is worth noting that the volatility of investment is quite close to the data in this version of model.

5. Conclusion

There is evidence that suggests that expenditures on the maintenance and repair of physical capital. However, with regard to indeterminacy properties, very few papers consider such expenditures in their artificial economies. Hence, I am interested in what changes it can make to dynamic properties of an artificial economy once maintenance expenditures are considered. In this model, capital depreciation is dependent on capital utilization and maintenance expenditure. I conclude that maintenance expenditures are instrumental in generating multiple equilibria: It can make indeterminacy occur under relatively milder degree of externalities compared with its model predecessors as maintenance activities offset the capital depreciation and boost the net rate of return on capital.

It has been an issue that a model incorporating both two-sector production and variable capital utilization rate will induce extremely narrow range of increasing returns. It is difficult to replicate the procyclical consumption behaviour. Therefore, I also present an alternative model where capital utilization rate is constant over time. The result that indeterminacy requires lower degree of externalities in a model with maintenance activities than a model without it is robust. Under this formation, the countercyclical consumption puzzle is solved and most features of the model moments are comparable to the U.S. data.

Understanding the importance of maintenance expenditures on multiple equilibria could have implications for policymakers to improve economic welfare. As an important part of economic activity, the maintenance expenditures should be taken into account in the formation of economic policy such as interest rates which are often reckoned as agents' confidence indicator.

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Appendix A. Elasticity of labour supply and the externalities

In this paper I only focus on χ=0, Hansen's indivisible labour case. In this appendix we illustrate how changes in the inverse elasticity of labour supply χ affect the indeterminacy results, leaving other parameters unchanged. I derive the χ–η relation to illustrate the indeterminacy region for the present two-sector variable capital utilization model.

The household's preference is

$$\int_0^\infty \left(\ln C_t - \frac{L_t^{1+\chi}}{1+\chi}\right)e^{-\rho t}dt, \tag{48}$$

where $\chi > 0$. The trace and the determinant of J are

$$\text{Tr}\,J = \frac{\theta\rho[\alpha^2(1+\eta)(1+\phi) - \eta\theta(1+\chi) + \alpha(1+\phi)(\eta\chi-1)]}{\alpha(1+\eta)\theta(1+\phi)\chi - \eta\theta^2(1+\chi) - \alpha^2(1+\eta)(1+\phi)(1-\theta+\phi+\chi+\phi\chi)} + \frac{\alpha(1+\eta)\rho(1+\phi)(\theta-\alpha\phi-\alpha)(1+\chi)}{\alpha(1+\eta)\theta(1+\phi)\chi - \eta\theta^2(1+\chi) - \alpha^2(1+\eta)(1+\phi)(1-\theta+\phi+\chi+\phi\chi)}, \tag{49}$$

$$\text{Det}\,J = -\frac{[\alpha(1+\eta)-1]\theta\rho^2(1+\phi)(\theta-\alpha\phi-\alpha)(1+\chi)}{(\theta-\phi-1)[\alpha(1+\eta)\theta(1+\phi)\chi - \eta\theta^2(1+\chi) - \alpha^2(1+\eta)(1+\phi)(1-\theta+\phi+\chi+\phi\chi)]}. \tag{50}$$

The necessary condition for a positive determinant is $\alpha(1+\eta)\theta(1+\phi)\chi - \eta\theta^2(1+\chi) - \alpha^2(1+\eta)(1+\phi)(1-\theta+\phi+\chi+\phi\chi) > 0$, indeterminacy requires that the trace to be negative, implying $\theta\rho[\alpha^2(1+\eta)(1+\phi) - \eta\theta(1+\chi) + \alpha(1+\phi)(\eta\chi-1)] + \alpha(1+\eta)\rho(1+\phi)(\theta-\alpha\phi-\alpha)(1+\chi) < 0$. I set α=0.3, θ=1.8828, and ϕ=0.3828. Then the relationship between the externality parameter η and the inverse elasticity of labour supply χ is shown in Fig. 1.

The area between the two curves represents the parameter combinations that lead to indeterminacy. The lowest η value can be achieved when χ=0, as indicated in Fig. 1, the corresponding externalities can be as low as 0.0179, as discussed in the paper. The results are in line with Benhabib and Farmer's (1996) finding that as the value of χ decreases, the lower level of increasing returns to scale is required to obtain indeterminacy.

[9] η=0.2 (Benhabib and Farmer's calibration) is not sufficient to generate strong procyclical consumption.

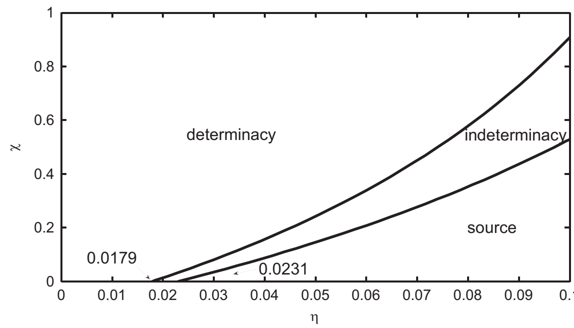

Fig. 1. Elasticity of labour supply and the externalities.

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