Economic Modelling 61 (2017) 113–125

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/econmod

Dynamic asset allocation and consumption under inflation inequality: The impacts of inflation experiences and expectations☆

Shaoyu Li[a], Lijia Wei[b,*], Zhiwei Xu[a,c]

[a] School of Securities and Futures, Southwestern University of Finance and Economics, China
[b] School of Economics and Management, Wuhan University, Wuhan 430072, China
[c] Rutgers Business School-Newark and New Brunswick, Rutgers University, United States

ARTICLE INFO

JEL classification:
G11
E31
O53

Keywords:
Inflation inequality
Inflation expectations
Asset allocation
Consumption

ABSTRACT

Differences in spending patterns and in price increases across goods and services lead to the unequal inflation experiences of households (called inflation inequality). These differences then cause disagreements in inflation expectations and eventually have a significant effect on households' asset allocation and consumption decisions. The asset allocation model in this paper explains how inflation experiences affect household investment and consumption through corresponding inflation expectations, which are characterized by long-term expected inflation, the impact coefficient of the expected inflation and the correlation between expected inflation and the risky return. Using China's economic data, the empirical results show that significant differences in inflation expectation arise from income gap, regional inequality, different inflation measures and economic sector spending differences. Using the estimated coefficients, the calibration results have policy implications that households need more financing channels to resist inflation, especially in rural areas and in the raw material sector.

1. Introduction

Households have different inflation experiences based on their overall spending patterns (called inflation inequality),[1] and interpret its tendency differently. This paper investigates how inflation inequality affects the asset allocation and consumption choices of households. First, we develop an intertemporal asset allocation model considering inflation risk, which suggests that households' investment and consumption should hedge the inflation risk according to the expected inflation dynamic. Second, using China's economic data, we find evidence of inflation inequality among different household groups. Moreover, the China Household Finance Survey (CHFS) data also shows large differences in households' asset allocation ratios and consumption ratios. Third, the estimation coefficients of the constraint vector auto-regression (VAR) model are applied to calculate the asset allocation model's parameters for calibration. Finally, the calibration results for optimal asset allocation and consumption ratios are consistent with the CHFS data.

This paper is closely related to studies of asset allocation problem by Campbell et al. (2004), Liu (2010), Maenhout (2006), and others. The asset allocation model in this paper adopts a time-varying expected return of production, which extends the asset allocation model in Anderson et al. (2000). Expected return, which is related to the inflation rate, is set as a state variable with an affine structure. It follows a stochastic process that is mean reverting in a price level model setup, which is consistent with the model of Brennan and Xia (2002) and Munk et al. (2004). However, our model extends the asset allocation model by additionally considering inflation risk.

This paper is also related to a growing literature document the inflation effect on the assets allocation problem. For example, Brennan and Xia (2002) developed a framework for the asset allocation problem of a long horizon investor within which a zero-coupon bond bears the inflation risk. Similarly, Munk et al. (2004) proposed a relevant model to resolve the Samuelson puzzle and Canner, Mankiw and Weil puzzle[2].

☆ This research is supported by research grants NSFC71301131 and NSFC71403192 from National Natural Science Foundation of China.
* Corresponding author.
E-mail address: ljwei.whu@gmail.com (L. Wei).

[1] Hobijn and Lagakos (2005) used the term inflation inequality to describe households faced different inflation levels.

[2] Investment advisors tend to recommend that younger investors – who have a long investment horizon – invest a higher fraction of their wealth in stocks than older investors should. This piece of investment advice is not consistent with rational portfolio allocation in basic portfolio choice models; and is often referred to as the Samuelson puzzle. The CMW puzzle is when the asset allocations recommended by professional investment advisors systematically display increasingly higher bonds to stocks ratios for increasingly higher risk aversion. This recommendation is at odds with the standard mean-variance two-fund separation results.

http://dx.doi.org/10.1016/j.econmod.2016.11.013
Received 9 March 2016; Received in revised form 18 November 2016; Accepted 20 November 2016

Chou et al. (2011) investigated the intertemporal portfolio choice problem by considering interim consumption under stochastic inflation and also compared the optimal allocation strategies of an aggressive investor and a conservative investor. Other literature has discussed the asset allocation problem with inflation risk in pension funds or pension plans (e.g., Han and Hung, 2012; Yao et al., 2013; Zhang and Ewald, 2010). Furthermore, heterogeneous beliefs on asset allocation with inflation risk have recently been discussed in the theoretical literature (e.g., Barberis et al., 2015; Ehling et al., 2013; He and Li, 2012; Piazzesi and Schneider, 2012). Our work is different in that we consider the effect of inflation experiences rather than inflation beliefs on asset allocation and consumption choices. Obviously, inflation experiences of households impact their inflation expectations. Therefore, the introduction of inflation inequality allows to classify and study household investment and consumption. However, there is no significant asset allocation literature in the context of inflation inequality. This paper intends to fill that gap.

Our work is also related to previous studies by Bruin et al. (2010), Bryan and Venkatu (2001a, 2001b), Diamond et al. (2016), Fratzscher et al. (2014), Johannsen (2014), Meyer and Venkatu (2011), Reid (2015), and Xu et al. (2016), among others, which used survey data that showed differences in households' inflation experiences and expectations. As far as we know, only a few studies have examined households' asset allocation and consumption in the context of these differences. This paper emphasizes the effects of inflation inequality on the allocation of financial and physical assets and consumption of household by considering income levels, regions, and economic activities as well as different inflation measures. This paper compares different asset allocation and consumption choices by introducing inflation inequality and correspondingly different inflation expectations to calibrate the model. The numerical results are investigated to determine whether they explain the allocation strategies and consumption choices in the CHFS data. So this paper also contributes to the literature that studied the differences in inflation expectations and inflation experiences through investigating their effects on asset allocation and the consumption ratio.

In this paper, different sets of inflation data are used to denote inflation inequality. Several groups of coefficients are estimated by the constraint VAR model, which represent divergent inflation expectations. By applying these estimated coefficient groups to the strategic allocation and consumption formulae, the calibration results reflect the asset allocation strategies and consumption choices of the households with different inflation experiences and expectations. Our results suggest that diverse inflation expectations caused by inflation inequality result in different allocation and consumption decisions, which is consistent with the CHFS data. Our findings also demonstrate how inflation experiences across different income groups, economic sectors, regions, and measures affect household choices.

The macroeconomic effects of inflation on investment and consumption are also frequently documented in macroeconomic and asset pricing studies. For instance, Bansal and Shaliastovich (2012), Hasseltoft (2012), Mallick and Mohsin (2010, 2016), Piazzesi and Campbell (2006), and others, empirically examined the effect of inflation on investment and consumption. In particular, Mallick and Mohsin (2010, 2016) found that inflation negatively affects investment and consumption, and proposed an open economy model to explain their empirical evidences. This paper contributes to the literature through study of the inflation effect. Our model is not only related to financial asset allocation but is also close to the resource allocation problem in the stochastic growth models.[3] On the one hand, it has an advantage over previous literature to theoretically explain the differences of the investment in either financial assets or physical assets (called allocation in financial assets or physical assets). Our findings generally support the evidences of financial asset allocation seen in the survey data. On the other hand, we find that the effects of inflation on investment and consumption among economic units are related to inflation inequality, which further enrich the homogeneous findings of Mallick and Mohsin (2010, 2016).

The rest of this paper is organized as follows. Section 2 introduces an asset allocation model by considering inflation risk and derives the analytic formulae of asset allocation and the consumption ratio. Section 3 describes the survey data and analyzes several types of inflation experiences by inflation data. Section 4 uses the inflation data to estimate different coefficient groups and then analyzes the different inflation expectations. Section 5 applies the estimation results to the analytic formulae and discusses the calibration results. Section 6 presents the conclusions.

2. An asset allocation model with a mean-reverting inflation dynamic

We consider a modification of the models in Anderson et al. (2000), Brock (1978), Brock and Mirman (1972), and Cox et al. (1985) by adding an inflation process from the traditional price level model.

In this economy, the production sector adopts a linear production technique (a special case of the stochastic growth model) and produces only a numeraire. Multiple technologies can be used to transfer goods from one instant to the next. Capital is freely transferable across the different technologies. Newly produced outputs are split between consumption and new capital that invested in economic sectors with various technologies. We suppose that households determine the consumption ratio and invest in a particular economic sector, which maximize their intertemporal expected utility.

Households attempt to balance the present and future consumption. Furthermore, households are assumed to have recursive preferences over consumption. As such, we use the continuous-time parameterization of Duffie and Epstein (1992a, b):

$$V_t = \int_t^{\infty} f(\overline{C}_S, V_S)dS, \tag{1}$$

where $\overline{C}_t$ and V_t denote the real level of consumption and utility, respectively. $f(\overline{C}, V)$ is a normalized aggregator of current consumption and continuation utility.

The continuation utility takes the form

$$f(\overline{C}, V) = \beta(1 - 1/\phi)^{-1}(1 - \gamma)V[((\overline{C}(1-\gamma)V)^{-1/(1-\gamma)})^{(1-1/\phi)} - 1], \tag{2}$$

where $\beta > 0$ is the rate of the time preference, $\gamma > 0$ is the parameter of relative risk aversion, $\phi > 0$ is the elasticity of intertemporal substitution (EIS) that signifies the time preference value of the utility, $\phi > 1$ indicates that households prefer to save for future consumption rather than current consumption, and ϕ=1 means that household are indifferent to consumption allocation in either period.

There are two special cases of the normalized aggregator Eq. (2). If $\phi = 1/\gamma$, then Eq. (2) will be the aggregation of the CRRA utility function. If ϕ approaches 1, then Eq. (2) will be $f(\overline{C}, V) = \beta(1 - \gamma)V[log(\overline{C}) - log((1 - \gamma)V)/(1 - \gamma)]$. Both of these cases allow us to derive the analytic solutions of this optimization problem.

In the production sector, suppose there are two technologies that can convert one good to a different good. Again, we suppose that K_t represents the nominal accumulated wealth. Household could allocate a proportion of new assets to either the riskless technical sector

[3] Anderson et al. (2000) provided a simple form of the resource allocation problem, based on the stochastic growth model literature, such as Brock (1978), Brock and Mirman (1972), and Cox et al. (1985). Different from Bansal and Shaliastovich (2012), Hasseltoft (2012), Mallick and Mohsin (2010, 2016), Piazzesi and Campbell (2006), the investment and consumption in our model are driven by expected inflation. So, the relationship we study between inflation and investment and consumption is not empirical but theoretical.

(riskless sector) or the risky technical sector (risky sector). The riskless sector has a risk-free return. Let y_t and dy_t be the nominal levels of cumulative excess returns and instantaneous change of excess returns, respectively; $r + \mu_t$ denotes the expected return where $\mu_t>0$ and is time-varying; $\sigma_y^2 dt$ is an instantaneous variance and φ_t represents the percentage of assets allocated to the risky sector.

With the linearization of the equation, instantaneous outputs in the risky sector and riskless sector are $K_t(dy_t+rdt)$ and $rK_t dt$, respectively. Consequently, the budget constraint of household is the change in its assets, which is equal to $\varphi_t K_t(dy_t+rdt)+(1-\varphi_t)rK_t dt$ minus instantaneous nominal consumption $C_t dt$. The nominal budget constraint of a household can be written as

$$dK_t=\varphi_t K_t(dy_t+rdt)+(1-\varphi_t)rK_t dt-C_t dt, \tag{3}$$

where $dy_t=\mu_t dt+\sigma_y dB_y$ is a state-variable process given by the economic environment and μ_t is defined as an affine structure where $\mu_t=\lambda+\alpha\pi_t$ and λ represents the real part of the expected excess return in the risky sector, excluding the inflation effect.B_y is an independent Brownian motion. In fact, asset's return is often affected by the inflation dynamics; hence, the expected inflation should be considered when modeling the asset return dynamics. The expected excess return μ_t is governed by the dynamic of expected inflation π_t. Suppose the impact coefficient $\alpha>0$, which suggests that a higher expected inflation leads to a larger excess return. This affine structure is verified in the later part of this paper.

As Brennan and Xia (2002) and Munk et al. (2004) suggested, we introduce the price level model to describe the dynamics of expected inflation. The state variable π_t follows:

$$\begin{aligned} d\Pi_t &= \Pi_t(\pi_t dt+\sigma_\Pi dB_I)\\ d\pi_t &= \omega(\bar{\pi}-\pi_t)dt+\sigma_\pi dB_I\\ dB_I &= \sigma_\pi\rho dB_y+\sigma_\pi\sqrt{1-\rho^2}\,dB_\pi \end{aligned} \tag{4}$$

where B_y and B_π are independent Brownian motions, respectively. $\sigma_\Pi dt$ is the instantaneous volatility of price level, and ρdt is the instantaneous correlation coefficient between dB_y and dB_I. The real levels of cumulative wealth $\bar{K}_t$ and consumption $\bar{C}_t$ are therefore $\bar{K}_t=K_t/\Pi_t$ and $\bar{C}_t=C_t/\Pi_t$, respectively. By applying Ito lemma, we obtain the real level of budget constraint:

$$d\bar{K}_t=\varphi_t\bar{K}_t(dy_t-\sigma_y\sigma_\Pi\rho dt)+(r-\pi_t+\sigma_\Pi^2)\bar{K}_t dt-\bar{C}_t dt. \tag{5}$$

Given the budget constraint $\bar{K}_t$ and state variable π_t, households allocate assets and determine the optimal input ratio and consumption level to maximize the intertemporal expected utilities. This programming problem could be written as

$$\max_{\varphi_t,\bar{C}_t} E_t\left[\int_t^\infty f(\bar{C}_S, V_S)ds\right], \tag{6}$$

where φ_t and $\bar{C}_t$ are the control variables. By using the Hamilton-Jacobi-Bellman equation (HJB), the optimal ratio of assets allocated to the risky sectors can be obtained:

$$\varphi_t=-\frac{\lambda+\alpha\pi_t-\sigma_y\sigma_\Pi\rho}{\sigma_y^2}\frac{V_{\bar{K}}}{\bar{K}_t V_{\bar{K}\bar{K}}}-\frac{\rho\sigma_\pi}{\sigma_y}\frac{V_{\bar{K}\pi}}{\bar{K}_t V_{\bar{K}\bar{K}}}+\frac{\rho\sigma_\Pi}{\sigma_y}. \tag{7}$$

where $V_{\bar{K}}$ denotes the partial derivative of V with respect to $\bar{K}$, and V_t represents V at time t.

The right-hand side of Eq. (7) is the demand for the current input, indicating the percentage of assets allocated to the risky sector during the current period. Therefore, $-\frac{V_{\bar{K}}}{\bar{K}_t V_{\bar{K}\bar{K}}}$ indicates relative risk aversion, which indicates how the households trade off the risk and returns. The second term and third item on the left-hand side are the intertemporal hedge allocation, which hedges the risk of an abnormal change in output caused by the state variables and smooth the consumption. The Eq. (7) differs from Campbell et al. (2004) in the items $\rho\sigma_\Pi/\sigma_y$ and $\sigma_y\sigma_\Pi\rho$, which suggest additional demands of myopic allocation and hedging allocation. As such, $\bar{C}_t$ is expressed as follows:

$$\text{If } \phi=1,\ \bar{C}_t=\beta(1-\gamma)V/V_{\bar{K}},\quad \text{If } \phi\neq 1,\ \bar{C}_t=V_K^{-\phi}\beta^{\phi}[(1-\gamma)V]^{(1-\phi\gamma)/(1-\gamma)}. \tag{8}$$

We solve for φ_t and $\bar{C}_t$ (refer to Appendix A for details). If $\phi\neq1$, then we could obtain asymptotically analytic formulae for the two control variables by creating a log-linearized transformation of the original model:

$$\begin{aligned}\varphi_t&=\frac{\lambda+\alpha\pi_t-\sigma_y\sigma_\Pi\rho}{\sigma_y^2\gamma}+\frac{1}{\phi-1}\frac{1-\gamma}{\gamma}\frac{\rho\sigma_\pi}{\sigma_y}(\hat{B}+\hat{C}\pi_t)+\frac{\rho\sigma_\Pi}{\sigma_y},\\ \bar{C}_t&=K_t\beta^{\phi}e^{-\hat{A}-\hat{B}\pi_t-0.5\hat{C}\pi_t^2}\end{aligned} \tag{9}$$

where $\hat{B}$ and $\hat{C}$ are functions of the primitive parameters of the model describing investment opportunities and preferences. The two expressions signify the asset allocation strategies and consumption choices of households with different degrees of risk aversion under a fixed EIS. The analytical formulae of the two control variables under $\phi=1$ are the simplified form of Eq. (9). If $\bar{K}_t$ is standardized to 1, then $\bar{C}_t$ is the ratio of assets allocated to consumption. Moreover, the parameters α, $\bar{\pi}$, and ρ reflect the inflation expectations.

According to the theoretical results, inflation expectations can affect asset allocation and consumption in the following three ways. First, a larger impact coefficient α incurs a higher future risky return; thus, households allocate a higher proportion of assets to the risky sector. Second, a higher long-term expected inflation level $\bar{\pi}$ implies larger inflation expectation, causing assets to be allocated to the risky sector and more favorable current consumption of the households. Third, if the return of the risky sector has a stronger positive correlation ρ with the expected inflation, then a higher expected inflation drives households to allocate more assets to the risky sector and reduce consumption.

3. Data and preliminary analysis

In this section, we first show the differences in inflation experiences by using China's economic datasets. Then, we summarize the differences in the asset allocation and consumption data of CHFS. Sections 4 and 5 explain the relation between these two differences.

3.1. Inflation inequality of China's households

We use economic data, including inflation data, provided by the NBSC from the third quarter of 2000 to the fourth quarter of 2011.[4] We consider four types of inflation inequality: high-income versus low-income (urban versus rural households), year-on-year versus quarter-on-quarter, different economic sectors (household consumption, raw materials, corporate and retail), and for different regions (Guangzhou in eastern China, Zhengzhou in central China, and Yinchuan in western China).

Fig. 1(a) shows eight indices of commodity and service prices: food, clothing, housing, household equipment, medical care, transportation and communication, entertainment and education, and cigarettes and wine. Using these indices, we develop weighted average CPIs for different income groups and for rural and urban areas (see Figs. 1(b) and 1(c), respectively). High income and low income households as well as urban and rural households have different baskets of consumer goods, so they experience inflation differently. For example, low income urban households usually spend a higher proportion of their income on food than high income urban households. In this way, low income urban households usually suffer more when food prices rise;urban households are more sensitive than rural households to food price inflation because rural households are able to consume home-grown foods.

[4] The inflation data of Guangzhou, Zhengzhou, and Yinchuan are only available from the third quarter of 2001.

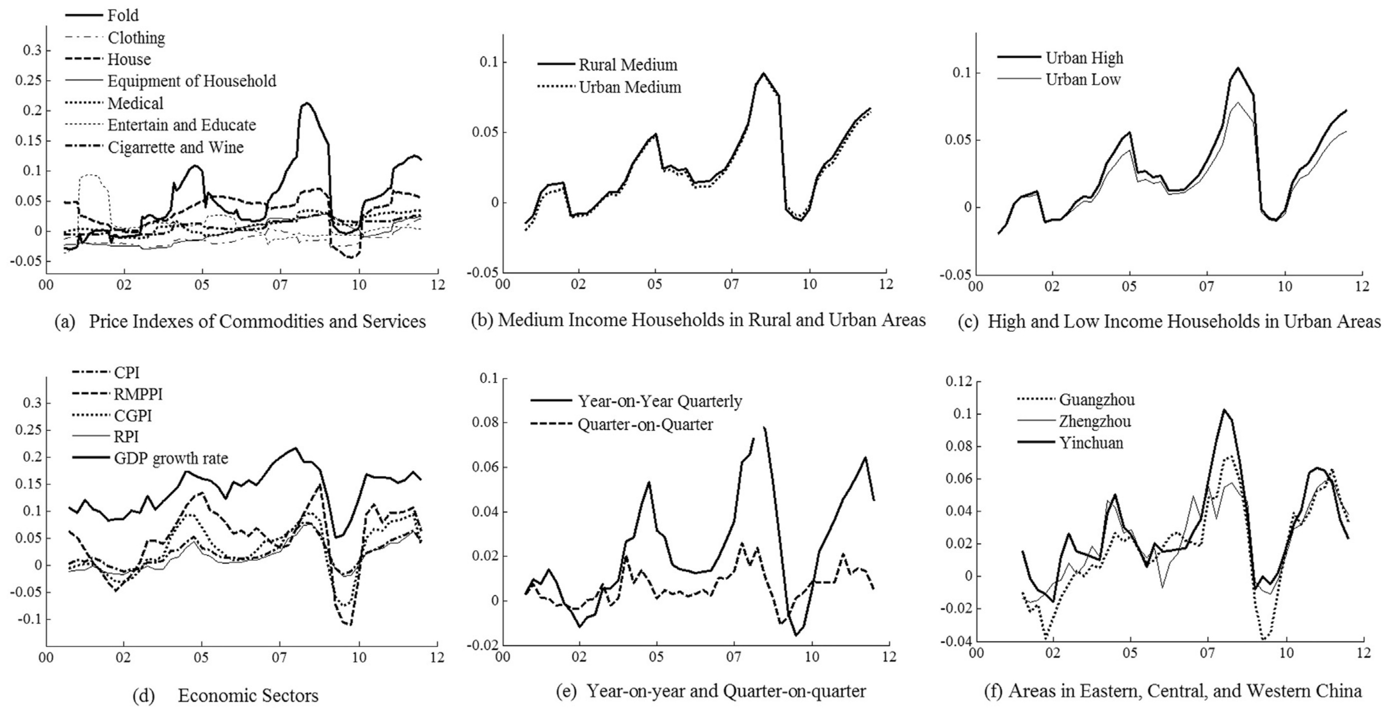

Fig. 1. Inflation inequality among households.

Fig. 1(d) displays the price indices of the following economic sectors: consumption (CPI), raw materials (RMPPI), corporate (CGPI), and retail (RPI). The raw materials sector is more likely to experience material price rises and price volatility. Households from the retail sector see price rises from both the manufacture and the transportation of goods. Fig. 1(e) shows the year-on-year quarterly inflation data series derived by subtracting one from the year-on-year quarterly price index series; while the quarter-on-quarter inflation data are derived from the year-on-year quarterly price index series.

Spillover of inflation across economic sectors is another reason for divergent household inflation experiences. Inflation usually emerges in one or another sector of the production chain and then spreads to all sectors. Inventory and menu costs also help spread inflation. For instance, the fast growth of the China economy ensures soaring demand for raw materials and energy. This leads to price rises of bulk commodities like oil, which results in agricultural production prices also rising. Subsequently, food and industrial production prices increase and finally, prices in the service sector go up. Therefore, different sectors experience inflation at different points in time.

Asset endowment, production, and consumption habits in China are quite diverse, so the inflation levels that households in different regions experience are also different. Fig. 1(f) illustrates that an underdeveloped area is more likely to tolerate a higher inflation rate than a developed area. It is also more sensitive to an upward trend in inflation (e.g., Yinchuan, which has endured ongoing inflation with large fluctuations). However, a developed area, such as Guangzhou, is sensitive to a descending trend in inflation levels and adjusts to inflation in advance, unlike underdeveloped areas. Therefore, they experience less volatile inflation levels.

3.2. Investment and consumption data from the CHFS

In China, the household is the basic economic unit. The Survey and Research Center for China Household Finance carried out a country-wide survey (the CHFS) of Chinese households' economic activities in 2011. The survey data includes numerous variables related to household asset allocation and consumption (e.g., gross assets and debts for activities, such as agriculture, industry, commerce, deposits, trading amounts of derivatives/cash/stocks/ bonds, household consumption expenditure and credit card consumption rates), all of which allow us to calculate the ratios of assets allocated to the risky sector and to consumption.

For the allocation to the risky sector, we choose the ratio of physical or financial assets relative to the gross income of the individual household as the proxy. For physical assets, the main source of risk faced by households is from agriculture, industry, and commerce activities. The gross assets of these activities are categorized as the assets of the risky sector. Since no information about the assets of the riskless sector as available, we use household aggregate income to represent gross assets for both the risky and the riskless sectors. The first, second, and fifth columns of Table 1 provide the proportions of the different assets in aggregate income. We use the proportions to denote the optimal allocation to risky assets and the optimal consumption ratio.

For financial assets, we consider stocks, bonds, funds, financial derivatives, non-RMB assets, gold, and other financial products as the value of risky assets. We treat deposits and cash as riskless assets. In this way, the value of gross assets is the sum of the value of the risky

Table 1
Household asset allocations and consumption.

	Physical asset allocation		Financial asset allocation		Consumption ratio
Households (8438)	Risky assets / income	Risky debts / income	Risky assets /total financial assets	Derivative assets / total financial assets	Consumption / income
Mean	0.153	0.398	0.044	–	0.112
Std. Dev.	9.770	0.767	0.166	–	0.861
Aggregate ratio	0.396	0.189	0.188	0.013	0.611

Note: The data is from the Survey and Research Center for China Household Finance in the Southwestern University of Finance and Economics. Detailed information about the database can be found in Gan et al. (2013). Since a limited sample is available for derivative trading, this table includes calculations at the aggregate level only.

assets and of the riskless assets. Therefore, the optimal allocation ratio of assets to the risky sector is the value of the risky assets that the household holds over the gross value of the riskless assets and the risky assets. Furthermore, we calculate the ratio of the financial derivative assets relative to the gross financial assets and considered it as the hedging ratio of the household. The third columns of Table 1 provide the ratios of the financial assets allocated to the risky sector.

Table 1 provides the means and the standard deviations of the ratios for each household. The mean ratio is the average level (homogenous case) of households' asset allocation strategies and consumption choices. The large standard deviations (the heterogeneous case) of these ratios imply that the allocation and consumption of these households are significantly different. Some of them either enter into a short position or turn to financing; thus, the ratios either become negative (a higher debt ratio) or are larger than one.

The aggregate ratio row in Table 1 (the homogeneous case) denotes the aggregate level of risky assets, risky debt, and consumption over the aggregate income of all households. Notably, derivative trading occurs only in the futures market. Households rarely participate in this market, so we only calculate the aggregate hedging ratio.

Table 1 illustrates that China's households are inclined to allocate a high proportion of their wealth to risky assets but rarely participate in derivatives markets. The results revealed large standard deviations in household asset allocations and consumption. However two things remain unclear: how to explain these results and what exactly induced these large standard deviations.

4. The empirical results on divergent inflation expectations

We consider the nominal GDP growth rate calculated by year-on-year quarterly GDP data as the proxy of the return of the risky sector. We combine it with the inflation data to constitute the time series data necessary for a bivariate VAR model. On this basis, we estimate the groups of characteristic coefficients in the VAR model through altering inflation data and then considered the coefficients as the differing expectations of inflation due to inflation inequality.

4.1. Estimation of the coefficients

To obtain the coefficients of the continuous-time model, we transform the estimated coefficients of the discrete-time model with a one-to-one econometric transformation method. More specifically, based on the state-variable model mentioned previously, we obtain the first-order VAR via discretization where $\Delta y_{t_n+\Delta t}$ denotes the return of the risky sector, namely the GDP growth rate, and $\pi_{t_n+\triangle t}$ is the inflation rate:

$$\begin{bmatrix}\Delta y_{t_n+\Delta t}\\ \pi_{t_n+\Delta t}\end{bmatrix}=\begin{bmatrix}\frac{-\alpha}{\omega}(1-e^{-\omega\Delta t})\overline{\pi}+(\lambda+\alpha\overline{\pi})\Delta t\\ (1-e^{-\omega\Delta t})\overline{\pi}\end{bmatrix}+\begin{bmatrix}0 & \frac{\alpha}{\omega}(1-e^{-\omega\Delta t})\\ 0 & e^{-\omega\Delta t}\end{bmatrix}\begin{bmatrix}\Delta y_{t_n}\\ \pi_{t_n}\end{bmatrix}+\begin{bmatrix}u_{y,t_n+\Delta t}\\ u_{\pi,t_n+\Delta t}\end{bmatrix} \tag{10}$$

$$\begin{bmatrix}\Delta y_{t_n+\Delta t}\\ \pi_{t_n+\Delta t}\end{bmatrix}=\begin{bmatrix}a_1\\ a_2\end{bmatrix}+\begin{bmatrix}0 & a_3\\ 0 & a_4\end{bmatrix}\begin{bmatrix}\Delta y_{t_n}\\ \pi_{t_n}\end{bmatrix}+\begin{bmatrix}e_{y,t_n+\Delta t}\\ e_{\pi,t_n+\Delta t}\end{bmatrix} \tag{11}$$

Because of the complex structure of Eq. (10), we first adopt the maximum likelihood method to estimate the coefficients of Eq. (11) using the data of the GDP growth rate and the inflation rates, namely the coefficients of the first-order constraint VAR.

It is worth noting that the setup of the VAR model resembles that of Campbell and Viceira (1999).[5] The instantaneous volatility σ_Π of the price level equation was derived using the fitted value of inflation from VAR model and calculating the standard deviation of the error term series $\Delta\Pi_t/\Pi_t - \pi_t\Delta t$. We then convert the coefficients of the state-variable process (Appendix B) using the one-to-one relationship between Eqs. (10) and (11). Finally, we use the coefficient sets that are consistent with the continuous-time model to calibrate it (see Table 2). The significances of α and λ in Table 2 support the affine setup of μ_t. Furthermore, the delta method was used to convert the standard deviations of the coefficients. Please see Table C1 in the Appendix C for details.

4.2. Analysis of estimation results

Table 2 reports the estimation results after the one-to-one transformation. The estimation results specify that four types of inflation inequality could lead to disagreements about inflation expectations manifested among the groups of coefficients.

The estimation results show disagreements in inflation expectations.[6] First, the disagreements among different income groups are quite obvious. The long-term expected inflation was highest for the low-income group in the urban area. The return of the risky sector is more susceptible to inflation to the medium-income group in the rural rather than in the urban area. Furthermore, by observing the instantaneous correlation between the GDP growth rate and inflation, we note that the risky returns of the low-income households are more susceptible to the expected inflation.

Second, the difference in the estimated coefficients between year-on-year quarterly and quarter-on-quarter inflations was significant. The differences in αimply that the expected inflation has a larger effect on risky return if households measure the inflation using year-on-year quarterly data rather than quarter-on-quarter data. The long-term expected inflation $\overline{\pi}$ was 3%, as estimated using the year-on-year quarterly inflation, and it approached zero when using the quarter-on-quarter inflation.

Third, along the industrial chain, the closer to the final consumption stage the economic sector is, the larger the effects of the expected inflation on the risky return of the sector are. Meanwhile, the long-term expected inflation is larger in the raw materials sector and the corporate sector compared to any other sector. Furthermore, the positive feedback of the inflation rate in the raw materials sector to the risky return is approximately equal to one, which suggests that a change of the inflation has greater effect on the risky return of the asset allocation in the raw materials sector compared to other sectors.

Fourth, among the three areas (Guangzhou, Zhengzhou, and Yinchuan), the expected inflation of Zhengzhou in central China had the most significant effect on its risky return. Guangzhou, a more developed area in eastern China, had the lowest long-term expected inflation while Zhengzhou tended to have the highest long-term expected inflation.

5. Asset allocation strategies and consumption choices dominated by inflation experiences

In this section, the coefficient groups that were previously estimated were applied to analyze the mechanism through which inflation expectations generated by inflation experiences affect asset allocation strategies, consumption choices, and other phenomena in the economy. Thus, this section first presents the asset allocation strategies and

[5] Higher-order unconstrained VAR models may have better time series property. However, the first-order lag has been empirically found to play the most significant role in VAR models. Moreover, the first-order constraint VAR is consistent with the state-variable formulas in this paper. Therefore, we include only the first-order lag in the estimations. The setting also simplifies the calibration analysis.

[6] Impulse responses between two variables can interpret the effects of inflation on the risky return. However, there is a singular matrix (uninvertiable) in the constraint VAR model of formula (11). Our constraint VAR method is related to Campbell and Viceira (1999) and Campbell et al. (2004). Instead of impulse responses, the coefficients are estimated to calibrate the optimal allocation and consumption ratios.

Table 2
Parameters estimation results after the transformation.

Panel A	Grouping by measures		Grouping by income levels			
	Year-on-year	Quarter-on-quarter	Low-income in urban	Med-income in urban	High-income in urban	Med-income in rural
λ	-0.076	0.561	-0.035	-0.034	-0.034	-0.038
	[0.007]	[0.004]	[0.006]	[0.007]	[0.007]	[0.007]
α	3.210	20.768	1.292	1.457	1.732	1.525
	[0.002]	[0.001]	[0.003]	[0.003]	[0.002]	[0.003]
ω	0.509	2.506	0.647	0.648	0.678	0.685
	[0.193]	[0.588]	[0.168]	[0.190]	[0.215]	[0.187]
$\overline{\pi}$	0.031	-0.0002	0.040	0.035	0.030	0.036
	[0.015]	[0.0106]	[0.074]	[0.077]	[0.077]	[0.080]
σ_y	0.628	0.792	0.199	0.224	0.255	0.222
	[0.003]	[0.003]	[0.004]	[0.004]	[0.004]	[0.004]
ρ	0.282	0.116	0.525	0.469	0.413	0.482
	[0.002]	[0.001]	[0.002]	[0.001]	[0.001]	[0.001]
σ_π	0.024	0.016	0.034	0.031	0.027	0.032
	[0.009]	[0.116]	[0.084]	[0.087]	[0.084]	[0.083]
σ_Π	0.011	0.006	0.016	0.014	0.012	0.015
Log likelihood	240.1	264.5	221.6	226.8	232.2	225.4

Panel B	Grouping by areas			Grouping by economic sectors			
	Guangzhou	Zhengzhou	Yinchuan	CPI	RMPPI	CGPI	RPI
λ	-0.065	-0.134	-0.102	-0.076	-0.064	-0.064	-0.040
	[0.005]	[0.006]	[0.006]	[0.007]	[0.006]	[0.005]	[0.005]
α	3.710	4.477	3.599	3.210	1.303	2.156	2.538
	[0.003]	[0.003]	[0.003]	[0.002]	[0.007]	[0.004]	[0.002]
ω	0.616	0.771	0.695	0.509	0.866	0.664	0.502
	[0.145]	[0.148]	[0.149]	[0.193]	[0.077]	[0.096]	[0.123]
$\overline{\pi}$	0.024	0.035	0.030	0.031	0.049	0.036	0.026
	[0.081]	[0.083]	[0.078]	[0.015]	[0.086]	[0.081]	[0.000]
σ_y	0.600	0.577	0.516	0.628	0.149	0.323	0.504
	[0.003]	[0.003]	[0.003]	[0.003]	[0.003]	[0.003]	[0.003]
ρ	0.310	0.290	0.323	0.282	0.988	0.549	0.336
	[0.002]	[0.002]	[0.002]	[0.002]	[0.001]	[0.002]	[0.000]
σ_π	0.031	0.032	0.033	0.024	0.083	0.052	0.027
	[0.127]	[0.145]	[0.140]	[0.009]	[0.129]	[0.100]	[0.121]
σ_Π	0.014	0.015	0.016	0.011	0.036	0.024	0.012
Log likelihood	212.8	213.3	207.5	240.1	240.1	232.6	236.5

Note: The first line in each panel displays the inflation rate series with different measures, income levels, regions, and economic sectors. The standard deviation before the transformation is in the squared brackets; the log-likelihood denotes the log likelihood value after the maximum likelihood estimation.

consumption choices with homogeneous inflation experience. It then extends the analysis to the condition with various inflation experiences.

5.1. Homogeneous inflation experiences

Assuming that households have the same expectation of inflation, to conduct a numerical analysis, we first use the groups of the coefficients estimated by the year-on-year quarterly inflation to calibrate and alter the inflation rate in the range of two standard deviations from the long-term expected inflation rate. Additionally, the EIS is set to be $\phi = 4/3$ and $\phi = 3/4$, denoting the preference that current consumption dominates saving for future consumption and vice versa, respectively. The risk aversion coefficient is set as $\gamma = 0.5$, and $\gamma = 4$, which represent risk lover and risk averter, respectively. In this way, we can analyze asset allocation strategies and consumption choices based on different EIS and risk attitudes.

Theoretically, when the inflation is high, the real return of the risky sector will shrink; therefore, the revenue effect will decrease the consumption ratio. Meanwhile, the high inflation signifies that the real return of the risky sector decreases and that saving or investing for the future is less attractive; hence, the substitution effect will increase the consumption ratio.

The numerical analysis of the EIS yielded the following two cases. On the one hand, if the EIS is larger than one, as Fig. 2(a-c) shows, the consumption ratio changes little as inflation increases, and the ratio is close to 61.1% (aggregate level of household). When facing high inflation, households who prefer risk will allocate more assets to the risky sector by using short positions while risk-averse households will use the long positions. In particular, those who are risk averse will reduce the hedge allocation and those who prefer risk will increase the hedge allocation.

On the other hand, if the EIS is less than one, as Fig. 2(d-e) displays, then the substitution effect dominates the revenue effect; therefore, the consumption ratio will increase as the inflation rises. Accordingly, households that are risk averse allocate more to the risky sector by long position, and those that prefer risk follow the opposite rule, both of which are reflected in the hedge allocation.

It is necessary to note that households are usually involved in the trading of physical assets and financial assets when allocating assets to the risky sector. On the one hand, if the weight of the asset allocation exceeds its present revenue level, then households need indirect financing from banks, private financing (usury), or direct financing from the inter-bank borrowing market.

Fig. 2(e) indicates that if the inflation exceeds 8%, with $\gamma = 4$, then the ratio of assets allocated to the risky sector will be larger than one. This means that a household's budget is in deficit and that external financing is required, which is consistent with the financing phenomena that emerge during high inflation periods.

On the other hand, households will engage in derivative markets, such as futures, when doing hedge allocations. Figs. 2(b) and (c) demonstrate that when the inflation is close to the long-term expected inflation rate, with $\gamma = 4$, the ratio of assets allocated to the risky sector

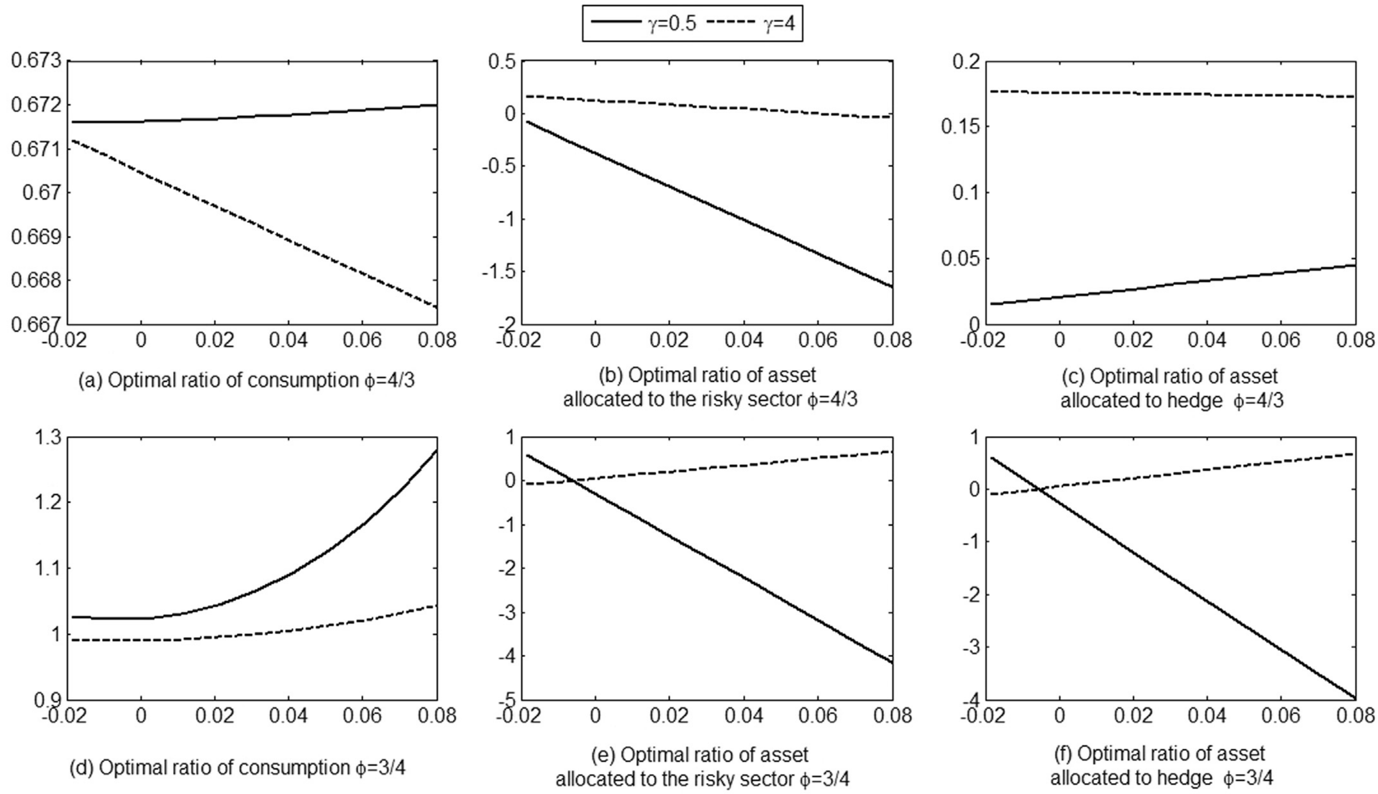

Fig. 2. Asset allocation and consumption with different EIS and risk attitudes.

approaches 15.3% (mean of the households). When the inflation is around 4%, with $\gamma = 4$ in Fig. 2(e), the ratio of assets allocated to the risky sector rises to 39.6% (aggregate level). The percentage of assets allocated to a hedge demand is only the absolute value of 0.013% (aggregate level) at around 1% level of inflation in Fig. 2(f). The limited hedge activity indicates that the ratio of assets allocated to the derivative is too low.

The calibration values are consistent with those collected by the investigation of the CHSF, which is apparent in Table 1. However, we also observe a large deviation of households' allocation and consumption, as seen in Table 1, which indicates the necessity to investigate the case with inflation inequality.

5.2. Heterogeneous inflation experiences (inflation inequality)

With various inflation experiences, we conduct calibration by using the coefficient groups estimated with different inflation rate data. Here, we set the EIS as $\phi>1$ and $\phi<1$, respectively, and fixed the risk aversion coefficient as $\gamma = 4$. In this setup, inflation experiences affect asset allocation strategies and consumption choices in following four cases.

Under the experiences of the different income groups, as illustrated in Fig. 3(a), the ratio of assets allocated to the risky sector is greater for the urban household with a low income compared to the urban households with other income levels. Two reasons can explain this phenomenon; specifically, the risky return has the largest positive correlation with the exogenous shock of the inflation, and the long-term expected inflation of the urban household was the highest. However, these two factors may have contradictory effects on the consumption ratios, so there is not much difference in households' consumption ratios.

Fig. 3(b) shows that hedging motivates urban low-income households to allocate more assets to the risky sector. To fulfill the risky investment, households with low incomes face enormous financing demands. However, the medium-income and high-income groups, or the dominant enterprises in China, are more likely to obtain financing from commercial banks or security markets while the low-income group or inferior firms can only obtain financing from the private market with a higher cost. This, to some extent, indicates the problem of access to financing from the regular financial market that the small-medium firms or individuals face. As such, this cultivates the boom of usury activity in the private financial market. That's why governments must create an enabling environment that low-income households have access to formal financial services.

Most income in medium-income households in the rural area is related to agricultural products, which are susceptible to price volatility. As such, their ratios of hedging allocation approximate the hedge ratios of the medium-income households in the urban area in the case of $\phi>1$. This confirms the fact that an increasing number of farmers from the main agricultural areas are involved in the derivatives market (commodities futures). Although the ratio of assets allocated to the risky sector and the consumption ratio of the medium-income households in the urban area are close to those of the medium-income households in the rural area, the gross wealth of the urban household is several times greater than that of the rural household, on average. As such, rural households allocate fewer assets to the risky sector than urban households. Therefore, providing abundant investment channels to rural households could overcome the problem.

With different measures of the inflation, as shown in Fig. 4, households that are long-term observers harness the year-on-year quarterly inflation data to establish their inflation expectations. Hence, if they exert greater effect of expected inflation on the risky return, long-term expected inflation, and feedback effect to the inflation shock, they usually allocate more assets to the risky sector. On the other hand, households who use the quarter-on-quarter inflation data put fewer assets in the risky sector. However, due to the dominant role of feedback effect to the inflation shock, households using short-term inflation measure tend to consume more at present. To smooth households' consumption and investment, the inflation rates are reported to the public using the year-on-year price indices.

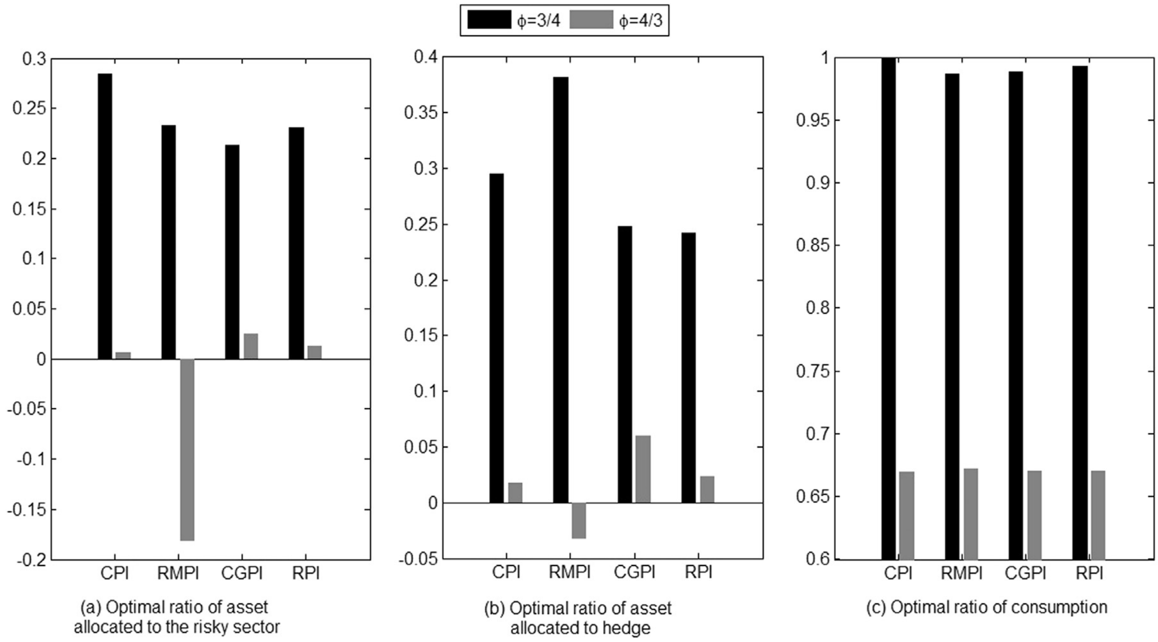

Fig. 3. Asset Allocation and Consumption with Inflation Experiences from Different Income Levels.

Due to inflation inequality in different economic sectors, asset allocation strategies and consumption choices are significantly different, as illustrated in Fig. 5. Since the expected inflation has the most significant effect on the consumption sector, the largest allocation is made to the risky asset and consumption in this sector. Since the risky return has an enormously positive feedback on the exogenous inflation shock, more assets are allocated to hedge the risk in the raw materials sector. Fig. 5(b) shows that the percentage of assets allocated to hedge the risky sector exceeds 35%. If people prefer to substitute future consumption for current consumption, then they need to employ the negative hedging strategies; thus, the hedging values of the raw materials sector become relatively large.

Fig. 5(b) implies that in a high inflation environment, these sectors are more involved in the derivative market. In this way, the hedging position is the main composition of the assets allocated to the risky sector. This calibration result highlights that when facing inflation risk, the raw materials sector engages more in the derivatives market. For instance, an agricultural company called China National Cereals, Oils and Foodstuffs Corporation has been frequently involved in the derivatives trading. Since the derivative markets including raw materials are still underdeveloped in China, more derivatives need to be created for the raw material sector.

Under dispersion of inflation experiences among different regions, the asset allocation strategies and consumption choices of the three areas in eastern China, central China and western China are also quite different. In general, Fig. 6 shows that when current consumption is emphasized more compared to future one, the ratio of assets allocated to the risky sector decreases from Zhengzhou to Yinchuan and finally to

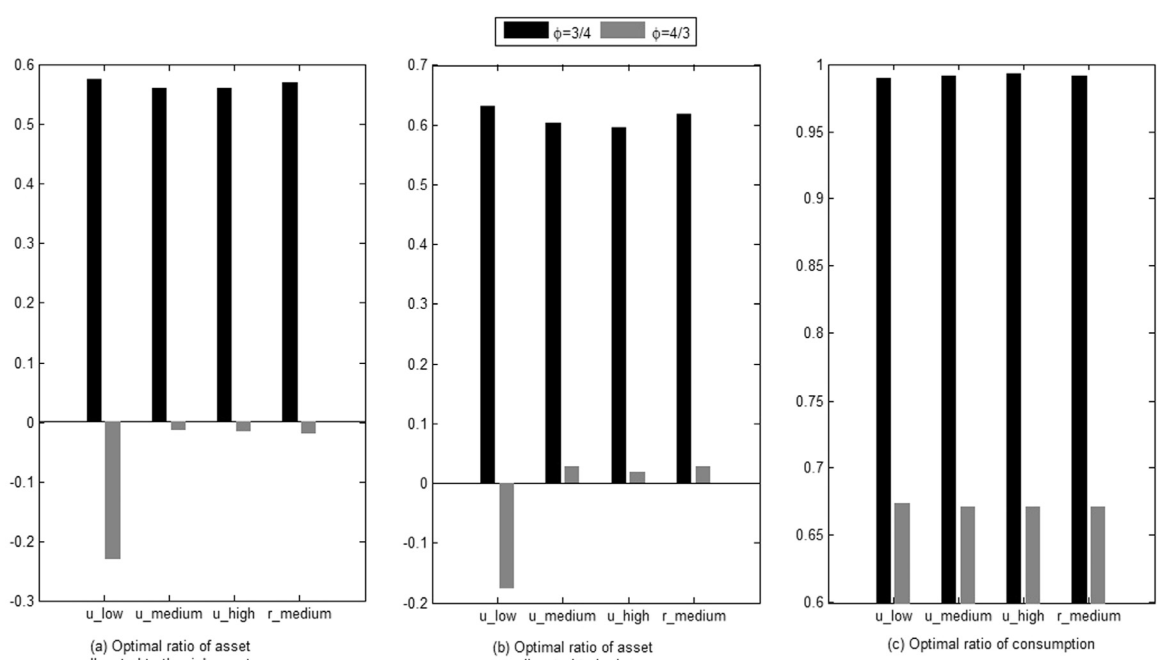

Fig. 4. Asset allocation and consumption with inflation experiences from different measures.

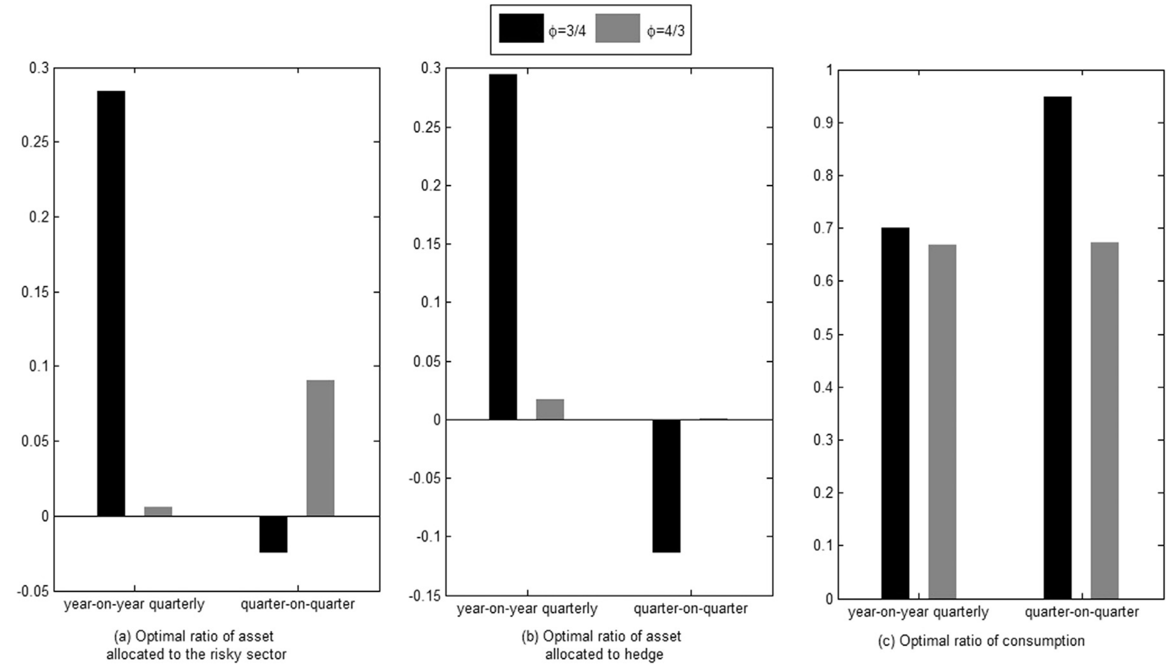

Fig. 5. Asset allocation and consumption with inflation experiences from different economic sectors.

Guangzhou and that Zhengzhou also has the largest ratio of the hedging allocation and consumption.

There are three reasons for this. First, Guangzhou's economy is the largest of the three areas, and it has considerable industry foundation and high marketization. As households in Guangzhou forecast a low long-term expected inflation rate and a weak influential magnitude of the inflation, they allocate fewer assets to the risky sector.

Second, Zhengzhou is the capital of Henan, a traditional agricultural province. The households in this area have a higher long-term expected inflation than any other areas, so the ratio of assets allocated to the risky sector and consumption ratio in Zhengzhou are the largest. Nonetheless, Zhengzhou is the principal commodity derivatives market in China, where households can hedge inflation risk; hence, the ratio of the hedging allocation in the risky sector is higher compared to the ratios of any other areas. It is worth noting that the consumption ratio of Zhengzhou exceeds that of Guangzhou, suggesting that agrarian areas have a larger consumption ratio.

Third, Yinchuan, an important area in western China, engages mainly in exploiting and processing minerals and energy assets. As it faces a high long-term expected inflation, the ratio of assets allocated to the risky sector is higher compared to that of Guangzhou.

To summarize, households in central China (like Zhengzhou) invest and consume more than households in other regions, which also implies that hedge instruments and agricultural price stability are indispensible to households in central China.

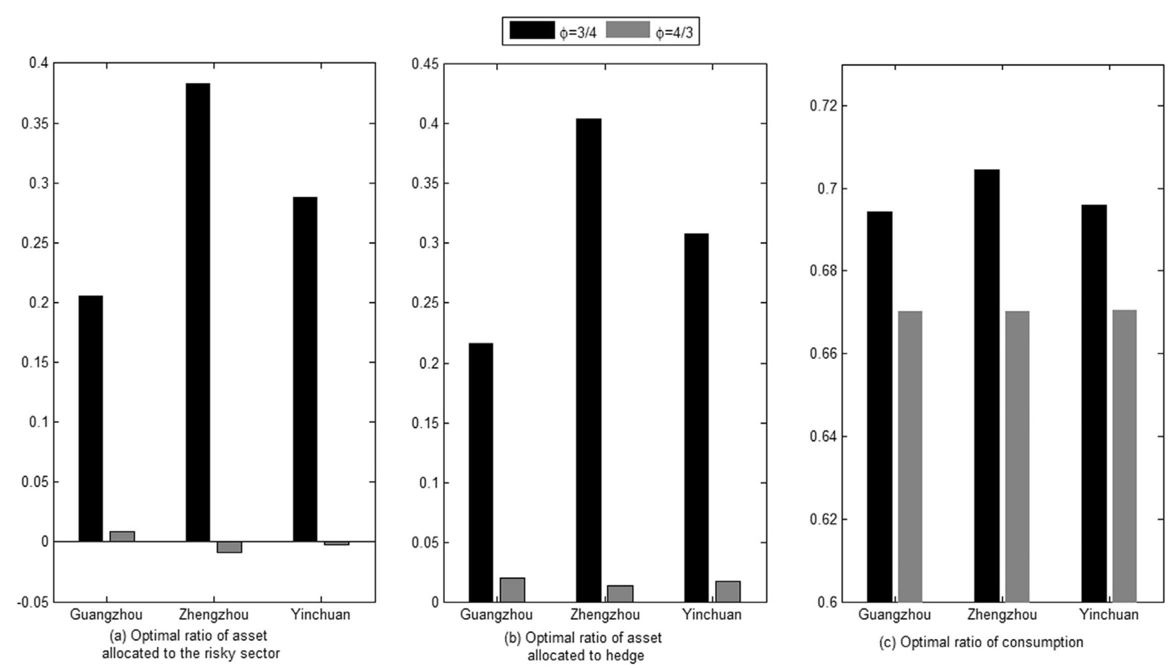

Fig. 6. Asset allocation and consumption with inflation experiences from different regions.

6. Conclusions

Inflation inequality has a significant effect on households' asset allocation strategies and consumption choices. This paper elicits disagreements in inflation expectations based on various dispersions of inflation experiences. An asset allocation model is employed to determine how diverse inflation expectations affect the decisions of households. This provides some evidence of the asset allocation strategies and consumption choices of households in China.

In this paper, we use CHFS survey data to find that the average ratio of assets allocated to the risky sector matches the average asset allocation situation of China's households. Households do not participate significantly in the derivatives market. Based on the analysis of the CHFS data, under the dispersions of inflation experiences, the diverse asset allocation strategies and consumption choices of households accounted for the high standard deviations of the ratios of risky allocation and consumption.

More specifically, we discover that the inflation experiences from the revenue gap between the low-income group and the high-income group forced the low-income groups to take a stronger position in the risky sector and demand more financing. The inflation experiences from the income gap between the urban and rural households encourage the rural households to be more involved in the derivatives market to hedge against inflation risk. Second, we find that if households measured inflation quarter-on-quarter, then they would allocate fewer assets to the risky sector but consume more. Third, we find that households with inflation experience in the consumption sector allocates more to the risky assets and consumes more. Meanwhile, households with inflation experience in the raw materials sector have a far higher hedge ratio compared to other sectors. Hence, they are more involved in the derivatives market when they are hedging against the risk of inflation. Finally, households in central China allocate more assets to the risky sector, rely more on the future markets to hedge inflation risk, and consume more.

Our findings shed lights on policy implications, showing that the government should take measures to develop the private financing market and eliminate the barriers of financing from state-owned commercial banks. To hedge the risks, various types of derivatives should be created to help households in rural area or households that mainly work for raw material sectors. It is also very important to reduce the income gaps among regions or economic sectors. If the government wants to reduce the wide disagreements in inflation expectations, the trustworthiness and authority of NBSC should be enhanced.

Appendices

As noted in Section 2, we provide the details of the deduction as follows.

Appendix A

Given that:

$$\Upsilon = \{\varphi_t : \overline{K}_t \geq 0 \quad \text{and} \quad E[f(\overline{C}_t, V_t) < \infty], t \geq 0\},$$

the Hamilton-Jacobi-Bellman (HJB) equation is:

$$=\max_{\varphi_t, C_t} \left\{ \begin{array}{c} f(\overline{C}_t, V_t) + V_{\overline{K}}(\overline{K}_t(\varphi_t(\lambda + a\pi_t - \sigma_y\sigma_\Pi\rho) + r + \sigma_\Pi^2 - \pi_t) - \overline{C}_t) \\ + V_\pi\omega(\overline{\pi} - \pi_t) \quad + 0.5 V_{\overline{K}\overline{K}} K_t^2 (\varphi_t^2\sigma_y^2 - 2\varphi_t\sigma_y\sigma_\Pi\rho + \sigma_\Pi^2) \\ + V_{\overline{K}\pi}\overline{K}_t(\varphi_t\rho\sigma_y\sigma_\pi - \sigma_\Pi\sigma_\pi) + 0.5 V_{\pi\pi}{\sigma_\pi}^2 \end{array} \right\},$$

where $V_{\overline{K}}$ and V_π denote the first-order derivatives with respect to $\overline{K}_t$ and π_t. The second-order derivatives with respect to $\overline{K}_t$ and π_t are $V_{\overline{K}\overline{K}}$, $V_{\overline{K}\pi}$, and $V_{\pi\pi}$. Taking the derivative of the HJB equation with respect to φ_t and $\overline{C}_t$, we obtain:

$$\varphi_t = -\frac{\lambda + \alpha\pi_t - \sigma_y\sigma_\Pi\rho}{\sigma_y^2}\frac{V_{\overline{K}}}{\overline{K}_t V_{\overline{K}\overline{K}}} - \frac{\rho\sigma_\pi}{\sigma_y}\frac{V_{\overline{K}\pi}}{\overline{K}_t V_{\overline{K}\overline{K}}} + \frac{\rho\sigma_\Pi}{\sigma_y}.$$

(1) When $\phi = 1$

Given that $f(\overline{C}, V) = \beta(1-\gamma)V[log(\overline{C}) - log((1-\gamma)V)/(1-\gamma)]$, we have:

$$\overline{C}_t = \beta(1-\gamma)V/V_{\overline{K}}.$$

We assume $V(\overline{K}_t, \pi_t) = \frac{H(\pi_t)^{1-\gamma}\overline{K}_t^{1-\gamma}}{(1-\gamma)}$ with $H(\pi_t) = exp\{\dot{A} + \dot{B}\pi_t + 0.5\dot{C}\pi_t^2\}$, then we obtain:

$$\varphi_t = \frac{\lambda + \alpha\pi_t - \sigma_y\sigma_\Pi\rho}{\sigma_y^2\gamma} + \frac{1-\gamma}{\gamma}\frac{\rho\sigma_\pi}{\sigma_y}(\dot{B} + \dot{C}\pi_t) + \frac{\rho\sigma_\Pi}{\sigma_y}, \quad \overline{C}_t = \beta\overline{K}_t$$

We substitute φ_t and $\overline{C}_t$ into the HJB equation and collected the coefficients in the constant item, π_t, and π_t^2, which constitute an equation system:

$$0 = \beta log(\beta) - \beta\dot{A} + r - \beta + \frac{\sigma_\pi^2}{2}\dot{C} + \left(1 - \frac{\gamma}{2}\right)\sigma_\Pi^2 + \left(\omega\overline{\pi} + \frac{1-\gamma}{2}\sigma_\pi^2 - (1-\gamma)\sigma_\Pi\sigma_\pi + \frac{1-\gamma}{\gamma}\frac{\rho\sigma_\pi(\lambda - \sigma_y\sigma_\Pi\rho)}{\sigma_y}\right)\dot{B} + \frac{(\lambda - (1-\gamma)\sigma_y\sigma_\Pi\rho)^2}{2\sigma_y^2\gamma} + \frac{(1-\gamma)^2}{2\gamma}\rho^2\sigma_\pi^2\dot{B}^2,$$

$$0 = -1 + \left(\frac{1-\gamma}{\gamma}\frac{\rho\sigma_\pi\alpha}{\sigma_y} - \beta - \omega\right)\dot{B} + \frac{(1-\gamma)^2}{\gamma}\rho^2\sigma_\pi^2\dot{B}\dot{C} + \left(\omega\overline{\pi} + \frac{1-\gamma}{2}\sigma_\pi^2 - (1-\gamma)\sigma_\Pi\sigma_\pi - \frac{(1-\gamma)^2}{\gamma}\rho^2\sigma_\Pi^2\right)\dot{C},$$

$$0 = \frac{\alpha^2}{2\sigma_y^2\gamma} + \left(\frac{1-\gamma}{\gamma}\frac{\rho\sigma_\pi\alpha}{\sigma_y} - \frac{\beta}{2} - \omega\right)\dot{C} + \frac{(1-\gamma)^2}{2\gamma}\rho^2\sigma_\pi^2\dot{C}^2.$$

Solving the equation system, we first get the analytic formula for $\dot{C}$, then we arrive at the formula $\dot{B}$. Finally, we combine $\dot{C}$ and $\dot{B}$ to derive the formula for $\dot{A}$.

(2) When $\phi \neq 1$.

Given $f(\overline{C}, V)=\beta(1-1/\phi)^{-1}(1-\gamma)V[(\overline{C}((1-\gamma)V)^{-1/(1-\gamma)})^{(1-1/\phi)}-1]$, we have:

$\overline{C}_t=V_{\overline{K}}{}^{-\phi}\beta^{\phi}[(1-\gamma)V]^{(1-\gamma\phi)/(1-\gamma)}$.

We guess $V(\overline{K}_t, \pi_t)=\frac{I(\pi_t)^{-\frac{(1-\gamma)}{(1-\phi)}}\overline{K}_t^{1-\gamma}}{(1-\gamma)}$, where $I(\pi_t)=exp\{\widehat{A}+\widehat{B}\pi_t+0.5\widehat{C}\pi_t^2\}$, we get the analytic formulae:

$\varphi_t=\frac{\lambda+\alpha\pi_t-\sigma_y\sigma_\Pi\rho}{\sigma_y^2\gamma}+\frac{1}{\phi-1}\frac{1-\gamma}{\gamma}\frac{\rho\sigma_\pi}{\sigma_y}(\widehat{B}+\widehat{C}\pi_t)+\frac{\rho\sigma_\Pi}{\sigma_y}$, and $\overline{C}_t=I(\pi_t)^{-1}\beta^{\phi}\overline{K}_t$.

Applying log-linearization to $I^{-1}\beta^{\phi}$, we have:

$\frac{\overline{C}_t}{\overline{K}_t}=exp(c_t-k_t)\approx exp[E(c_t-k_t)]+exp[E(c_t-k_t)][c_t-k_t-E(c_t-k_t)]=k_0+k_1[\phi log(\beta)-(\widehat{A}+\widehat{B}\pi_t+0.5\widehat{C}\pi_t^2)]$,

where $c_t=log(\overline{C}_t)$, $k_t=log(\overline{K}_t)$, $k_1=exp[E(c_t-k_t)]$, and $k_0=k_1(1-log(k_1))$.

Putting them into the HJB equation and collecting the coefficients of the constants, π_t and π_t^2, we arrive at the following equation system:

$$0=\frac{-k_1\widehat{A}+k_1(\phi\log(\beta)-log(k_1)+1)-\phi\beta+0.5\widehat{C}\sigma_\pi^2}{\phi-1}+r+(1-\frac{\gamma}{2})\sigma_\Pi^2+\frac{\sigma_\pi^2}{2(\phi-1)}\widehat{C}+\frac{(\lambda-(1-\gamma)\sigma_y\sigma_\Pi\rho)^2}{2\sigma_y^2\gamma}+\frac{(1-\gamma)^2}{2\gamma(\phi-1)^2}\rho^2\sigma_\pi^2\widehat{B}^2\left(\frac{\omega\pi}{\phi-1}+\frac{1-\gamma}{(\phi-1)}\left(0.5\sigma_\pi^2+\sigma_\Pi\sigma_\pi+\frac{\rho\sigma_\pi(\lambda-(1-\gamma)\sigma_y\sigma_\Pi\rho)}{\gamma\sigma_y}\right)\right)\widehat{B}$$

$$0=\frac{\alpha\sigma_\pi(\lambda-(1-\gamma)\sigma_y\sigma_\Pi\rho)}{\sigma_y^2\gamma}-1+\left(\frac{1-\gamma}{\gamma(\phi-1)}\frac{\rho\sigma_\pi\alpha}{\sigma_y}+k_1-\frac{\omega}{\phi-1}\right)\widehat{B}+\left(\frac{1-\gamma}{\gamma(\phi-1)}\frac{\rho\sigma_\pi(\lambda-(1-\gamma)\sigma_y\sigma_\Pi\rho)}{\sigma_y}+\frac{\omega\pi}{\phi-1}+\frac{1-\gamma}{(\phi-1)}\left(\frac{\sigma_\pi^2}{2(\phi-1)}-\sigma_\Pi\sigma_\pi\right)\right)\widehat{C}+\frac{2(1-\gamma)^2}{\gamma(\phi-1)^2}\rho^2\sigma_\pi^2\widehat{B}\widehat{C}$$

$$0=\frac{\alpha^2}{2\sigma_y^2\gamma}+\frac{1-\gamma}{\gamma(\phi-1)}\frac{\rho\sigma_\pi\alpha}{\sigma_y}+\left(\frac{(1-\gamma)^2}{\gamma(\phi-1)^2}\rho^2\sigma_\pi^2-\frac{k_1}{2(\phi-1)}-\frac{\omega}{\phi-1}\right)\widehat{C}-\frac{(1-\gamma)^2}{2\gamma(\phi-1)^2}\rho^2\sigma_\pi^2\widehat{C}^2$$

Likewise, if we solve this equation system, we obtain the analytic formulae for $\widehat{C}$, $\widehat{B}$, and $\widehat{A}$.

Appendix B

Given that the return of the risky sector follows a stochastic process, we have

$dy_t=\mu_t dt+\sigma_y dB_y$,

where the expected return is defined as:

$\mu_t=\lambda+\alpha\pi_t$ and $d\pi_t=\omega(\overline{\pi}-\pi_t)dt+\sigma_\pi\rho dB_y+\sigma_\pi\sqrt{1-\rho^2}dB_\pi$.

We then have:

$$d\begin{bmatrix} y_t-(\lambda+\alpha\overline{\pi})t \\ \pi_t-\overline{\pi} \end{bmatrix}=\begin{bmatrix} 0 & \alpha \\ 0 & -\omega \end{bmatrix}\begin{bmatrix} y_t-(\lambda+\alpha\overline{\pi}) \\ \pi_t-\overline{\pi} \end{bmatrix}dt+\begin{bmatrix} \sigma_y & 0 \\ \sigma_\pi\rho & \sigma_\pi\sqrt{1-\rho^2} \end{bmatrix}d\begin{bmatrix} B_y \\ B_\pi \end{bmatrix}.$$

Let $Y_t=\begin{bmatrix} y_t-(\lambda+\alpha\overline{\pi})t \\ \pi_t-\overline{\pi} \end{bmatrix}$, $A=\begin{bmatrix} 0 & \alpha \\ 0 & -\omega \end{bmatrix}$, $B=\begin{bmatrix} B_y \\ B_\pi \end{bmatrix}$, and $C=\begin{bmatrix} \sigma_y & 0 \\ \sigma_\pi\rho & \sigma_\pi\sqrt{1-\rho^2} \end{bmatrix}$, then the equation will be reduce to:

$dY_t=AY_t dt+CdB_t$,

where

$$Var(dY)=CC'=\begin{bmatrix} \sigma_y^2 & \rho\sigma_y\sigma_\pi \\ \rho\sigma_y\sigma_\pi & \sigma_\pi^2 \end{bmatrix}dt.$$

Dividing the entire time period into N equal periods $\{t_0, t_1, \ldots, t_n, \ldots, t_N\}$, where the length of every period is $\Delta t=t_n-t_{n-1}$, we have $Y_{t_n+\Delta t}=exp(\Delta tA)Y_{t_n}+u_{t_n+\Delta t}$, where $u_{t_n+\Delta t}=\int_{\tau=0}^{\Delta t}exp\{(\Delta t-\tau)A\}CdB_{t_n+\tau}$.

We know that $\exp\{A\}=I+\sum_{r=1}^{\infty}\frac{A^r}{r!}$. After some calculation, we arrive at:

$$\begin{bmatrix} y_{t_n+\Delta t}-(\lambda+\alpha\overline{\pi})(t_n+\Delta t) \\ \pi_{t_n+\Delta t}-\overline{\pi} \end{bmatrix}=\begin{bmatrix} 1 & \frac{\alpha}{\omega}(1-exp(-\omega\Delta t)) \\ 0 & exp(-\omega\Delta t) \end{bmatrix}\begin{bmatrix} y_{t_n}-(\lambda+\alpha\overline{\pi})t_n \\ \pi_{t_n}-\overline{\pi} \end{bmatrix}\begin{bmatrix} u_{y,t_n+\Delta t} \\ u_{\pi,t_n+\Delta t} \end{bmatrix}.$$

Meanwhile, $Var\begin{bmatrix} u_{y,t_n+\Delta t} \\ u_{\pi,t_n+\Delta t} \end{bmatrix}=\int_0^{\Delta t}\begin{bmatrix} B_{11} & B_{12} \\ B_{12} & B_{22} \end{bmatrix}d\tau$ and:

$B_{11}=\sigma_y^2+\frac{2\alpha}{\omega}\rho\sigma_y\sigma_\pi(1-exp(-\omega(\Delta t-\tau)))+\left(\frac{\alpha}{\omega}\right)^2(1-exp(-\omega(\Delta t-\tau)))\sigma_\pi^2$, $B_{12}=\rho\sigma_y\sigma_\pi exp(-\omega(\Delta t-\tau))+\frac{\alpha}{\omega}[exp(-\omega(\Delta t-\tau))-exp(-2\omega(\Delta t-\tau))]\sigma_\pi^2$

, $B_{22}=\sigma_\pi^2 exp(-2\omega(\Delta t-\tau))$,

with

$$\int_0^{\Delta t}B_{11}d\tau=\sigma_y^2\Delta t-2\rho\sigma_y\sigma_\pi\frac{\alpha}{\omega}\left(\frac{1-exp(-\omega\Delta t)}{\omega}-\Delta t\right)+\left(\frac{\alpha}{\omega}\sigma_\pi\right)^2\Delta t-2\frac{\alpha^2}{\omega^3}(1-exp(-\omega\Delta t))+\frac{\alpha^2}{2\omega^3}(1-exp(-2\omega\Delta t)),\ \int_0^{\Delta t}B_{12}d\tau=\frac{\rho\sigma_y\sigma_\pi}{\omega}(1-exp(-\omega\Delta t))+\frac{\alpha}{\omega^2}$$

$$\sigma_\pi^2(1-exp(-\omega\Delta t))-\frac{\alpha}{2\omega^2}\sigma_\pi^2(1-exp(-2\omega\Delta t)),\ \int_0^{\Delta t}B_{22}d\tau=\frac{\sigma_\pi^2}{2\omega}(1-exp(-2\omega\Delta t)).$$

Finally, this can be expressed as:

$$\begin{bmatrix}\Delta y_{t_n+\Delta t}\\ \pi_{t_n+\Delta t}\end{bmatrix}=\begin{bmatrix}\frac{-\alpha}{\omega}(1-exp(-\omega\Delta t))\overline{\pi}+(\lambda+\alpha\overline{\pi})\Delta t\\ (1-exp(-\omega\Delta t))\overline{\pi}\end{bmatrix}+\begin{bmatrix}0 & \frac{\alpha}{\omega}(1-exp(-\omega\Delta t))\\ 0 & exp(-\omega\Delta t)\end{bmatrix}\begin{bmatrix}\Delta y_{t_n}\\ \pi_{t_n}\end{bmatrix}+\begin{bmatrix}u_{y,t_n+\Delta t}\\ u_{\pi,t_n+\Delta t}\end{bmatrix},$$

where $\Delta y_{t_n+\Delta t}$ represents the return of the risky sector and Δt is dependent on the frequency of the data.

This model corresponds to the original continuous-time model. To obtain the coefficient, we used the one-to-one matching method. We first used a discrete first-order VAR model to estimate a system of coefficients:

$$\begin{bmatrix}\Delta y_{t_n+\Delta t}\\ \pi_{t_n+\Delta t}\end{bmatrix}=\begin{bmatrix}a_1\\ a_2\end{bmatrix}+\begin{bmatrix}0 & a_3\\ 0 & a_4\end{bmatrix}\begin{bmatrix}\Delta y_{t_n}\\ \pi_{t_n}\end{bmatrix}+\begin{bmatrix}e_{y,t_n+\Delta t}\\ e_{\pi,t_n+\Delta t}\end{bmatrix}.$$

We then used the matching method as follows. After estimating a_4, we have ω. Using ω and a_2, we obtain $\overline{\pi}$ and further combine ω, a_3, and $\overline{\pi}$. We then get α, put the coefficients that we know into a_1, and arrive at λ. Based on B_{22} and $Var(e_{\pi,t_n+\Delta t})$, we can derive σ_π^2. Based on B_{12} and $Cov(e_{y,t_n+\Delta t}, e_{\pi,t_n+\Delta t})$, we calculate $\rho\sigma_y\sigma_\pi$. Finally, by using σ_π^2, $Var(e_{\pi,t_n+\Delta t})$, B_{11}, and $\rho\sigma_y\sigma_\pi$, we can calculate σ_y^2 and ρ.

This paper used the Delta method (Kim and Nelson, 2000) to transform the standard deviation of these coefficients. Due to the complicated formula of the standard deviation, and because this paper used only the coefficients to calibrate it, Appendix C provides Table C1, the transformed standard deviations, for the reference.

Appendix C

See Table C1 here.

Table C1
Parameters estimation results and standard deviations after the transformation.

	Grouping by measures		Grouping by income levels			
	Year-on-year	Quarter-on-quarter	Low-income in urban	Med-income in urban	High-income in urban	Med-income in rural
λ	-0.076	0.561	-0.035	-0.034	-0.034	-0.038
	[0.007]	[0.004]	[0.006]	[0.007]	[0.007]	[0.007]
	(0.019)	(0.016)	(0.0212)	(0.023)	(0.023)	(0.020)
A	3.210	20.768	1.292	1.457	1.732	1.525
	[0.002]	[0.001]	[0.003]	[0.003]	[0.002]	[0.003]
	(1.446)	(4.756)	(0.592)	(0.652)	(0.764)	(0.675)
ω	0.509	2.506	0.647	0.648	0.678	0.685
	[0.193]	[0.588]	[0.168]	[0.190]	[0.215]	[0.187]
	(0.296)	(0.797)	(0.349)	(0.361)	(0.366)	(0.379)
$\overline{\pi}$	0.031	-0.0002	0.040	0.035	0.030	0.036
	[0.015]	[0.0106]	[0.074]	[0.077]	[0.077]	[0.080]
	(0.014)	(0.001)	(0.014)	(0.015)	(0.013)	(0.013)
σ_y	0.628	0.792	0.199	0.224	0.255	0.222
	[0.003]	[0.003]	[0.004]	[0.004]	[0.004]	[0.004]
	(0.123)	(0.005)	(0.032)	(0.038)	(0.040)	(0.036)
ρ	0.282	0.116	0.525	0.469	0.413	0.482
	[0.002]	[0.001]	[0.002]	[0.001]	[0.001]	[0.001]
	(0.007)	(0.002)	(0.008)	(0.008)	(0.007)	(0.008)
σ_π	0.024	0.016	0.034	0.031	0.027	0.032
	[0.009]	[0.116]	[0.084]	[0.087]	[0.084]	[0.083]
	(0.058)	(0.084)	(0.027)	(0.029)	(0.029)	(0.031)
σ_Π	0.011	0.006	0.016	0.014	0.012	0.015
Log likelihood	240.1	264.5	221.6	226.8	232.2	225.4

	Grouping by areas			Grouping by economic sectors			
	Guangzhou	Zhengzhou	Yinchuan	CPI	RMPPI	CGPI	RPI
λ	-0.065	-0.134	-0.102	-0.076	-0.064	-0.064	-0.040
	[0.005]	[0.006]	[0.006]	[0.007]	[0.006]	[0.005]	[0.005]
	(0.017)	(0.0158)	(0.016)	(0.019)	(0.020)	(0.018)	(0.020)
α	3.710	4.477	3.599	3.210	1.303	2.156	2.538
	[0.003]	[0.003]	[0.003]	[0.002]	[0.007]	[0.004]	[0.002]
	(1.911)	(2.059)	(1.731)	(1.446)	(0.422)	(0.938)	(0.523)
ω	0.616	0.771	0.695	0.509	0.866	0.664	0.502
	[0.145]	[0.148]	[0.149]	[0.193]	[0.077]	[0.096]	[0.123]
	(0.380)	(0.402)	(0.371)	(0.296)	(0.0428)	(0.384)	(0.000)

(continued on next page)

Table C1 *(continued)*

	Grouping by measures		Grouping by income levels				
	Year-on-year	Quarter-on-quarter	Low-income in urban	Med-income in urban	High-income in urban	Med-income in rural	
$\overline{\pi}$	0.024	0.035	0.030	0.031	0.049	0.036	0.026
	[0.081]	[0.083]	[0.078]	[0.015]	[0.086]	[0.081]	[0.000]
	(0.0164)	(0.013)	(0.015)	(0.014)	(0.028)	(0.026)	(0.015)
σ_y	0.600	0.577	0.516	0.628	0.149	0.323	0.504
	[0.003]	[0.003]	[0.003]	[0.003]	[0.003]	[0.003]	[0.003]
	(0.113)	(0.082)	(0.080)	(0.123)	(0.018)	(0.056)	(0.000)
ρ	0.310	0.290	0.323	0.282	0.988	0.549	0.336
	[0.002]	[0.002]	[0.002]	[0.002]	[0.001]	[0.002]	[0.000]
	(0.009)	(0.008)	(0.008)	(0.007)	(0.003)	(0.014)	(0.000)
σ_π	0.031	0.032	0.033	0.024	0.083	0.052	0.027
	[0.127]	[0.145]	[0.140]	[0.009]	[0.129]	[0.100]	[0.121]
	(0.092)	(0.098)	(0.079)	(0.058)	(0.059)	(0.081)	(0.000)
σ_Π	0.014	0.015	0.016	0.011	0.036	0.024	0.012
Log likelihood	212.8	213.3	207.5	240.1	240.1	232.6	236.5

Note: The first line in each panel displays the inflation rate series for various measures, income levels, regions, and economic sectors. The standard deviation before the transformation is in the square brackets, and the standard deviation after the transformation is in parentheses.

References

Anderson, E., Hansen, L.P., Sargent, T., 2000. Robustness, detection and the price of risk. Manuscr. Stanf., 19.

Bansal, R., Shaliastovich, I., 2012. A long-run risks explanation of predictability puzzles in bond and currency markets. Rev. Financ. Stud. 26 (1), 1–33.

Barberis, N., Greenwood, R., Jin, L., Shleifer, A., 2015. X-CAPM: an extrapolative capital asset pricing model. J. Financ. Econ. 115 (1), 1–24.

Brennan, M.J., Xia, Y., 2002. Dynamic asset allocation under inflation. J. Financ. 57 (3), 1201–1238.

Brock, W.A., 1978. An Integration of Stochastic Growth Theory and the Theory of Finance; Part I: the Growth Model. Graduate School of Business and Department of Economics. *University of Chicago*.

Brock, W.A., Mirman, L.J., 1972. Optimal growth under uncertainty: the case of discounting. J. Econ. Theory 4 (3), 479–513.

Bryan, M.F., Venkatu, G., 2001a. The Curiously Different Inflation Perspectives of Men and Women. Federal Reserve Bank of Cleveland Economic Commentary Series.

Bryan, M.F., Venkatu, G., 2001b. The Demographics of Inflation Opinion Surveys. Federal Reserve Bank of Cleveland Economic Commentary Series.

Bruin, W.B.D., Vanderklaauw, W., Downs, J.S., Fischhoff, B., Topa, G., Armantier, O., 2010. Expectations of inflation: the role of demographic variables, expectation formation, and financial literacy. J. Consum. Aff. 44 (2), 381–402.

Campbell, J.Y., Chacko, G., Rodriguez, J., Viceira, L.M., 2004. Strategic asset allocation in a continuous-time VAR model. J. Econ. Dyn. Control 28 (11), 2195–2214.

Campbell, J.Y., Viceira, L.M., 1999. Consumption and portfolio decisions when expected returns are time varying. Q. J. Econ. 114, 433–495.

Chou, Y.Y., Han, N.W., Hung, M.W., 2011. Optimal portfolio-consumption choice under stochastic inflation with nominal and indexed bonds. Appl. Stoch. Model. Bus. Ind. 27 (6), 691–706.

Cox, J.C., Ingersoll, J.E., Ross, S.A., 1985. An intertemporal general equilibrium model of asset prices. Econometrica 53, 363–384.

Diamond, J., Watanabe, K., and Watanabe, T., 2016. . The Formation of Consumer Inflation Expectations: Evidence from Japan's Deflation Experience. Utokyo Price Project Working Paper.

Duffie, D., Epstein, L.G., 1992a. Stochastic differential utility. Econometrica 60 (3), 353–394.

Duffie, D., Epstein, L.G., 1992b. Asset pricing with stochastic differential utility. Rev. Financ. Stud. 5 (2), 411–436.

Ehling, P., Gallmeyer, M.F., Heyerdahl-Larsen, C., Illeditsch, P.K., 2013. Disagreement about Inflation and the Yield Curve (Working paper)Social Science Electronic Publishing.

Fratzscher, M., Gürkaynak, R.S., Ozcan, G., 2014. Inflat. Exp. Inflat. Expect.: Spat. Evid., (Working paper).

Gan, L., Yin, Z.C., Jia, N., Xu, S., Ma, S., Zheng, L., 2013. Data You Need to Know about China: research Report of China Household Finance Survey 2012. *Springer*.

Han, N.W., Hung, M.W., 2012. Optimal asset allocation for DC pension plans under inflation. Insur.: Math. Econ. 51 (1), 172–181.

Hasseltoft, H., 2012. Stocks, bonds, and long-run consumption risks. J. Financ. Quant. Anal. 47 (2), 309–332.

He, X.Z., Li, K., 2012. Heterogeneous beliefs and adaptive behavior in a continuous-time asset price model. J. Econ. Dyn. Control 36 (7), 973–987.

Hobijn, B., Lagakos, D., 2005. Inflation inequality in the United States. Rev. Income Wealth 51 (4), 581–606.

Johannsen, B.K., 2014. Inflation Experience and Inflation Expectations: dispersion and Disagreement Within Demographic Groups (Working Paper). Social Science Electronic Publishing.

Kim, C.J., Nelson, C.R., 2000. State Space Model with Regime Switching. The MIT Press, Cambridge, Massachusetts, London, England.

Liu, H.N., 2010. Robust consumption and portfolio choice for time varying investment opportunities. Annu. Financ. 6 (1), 435–454.

Maenhout, P., 2006. Robust portfolio rules and detection-error probability for a mean-reverting risk premium. J. Econ. Theory 128 (1), 136–163.

Mallick, S.K., Mohsin, M., 2010. On the real effects of inflation in open economies: theory and empirics. Empir. Econ. 39 (3), 643–673.

Mallick, S.K., Mohsin, M., 2016. Macroeconomic effects of inflationary shocks with durable and non-durable consumption. Open Econ. Rev. 27 (5), 895–921.

Meyer, B., Venkatu, G., 2011. Demographic Differences in Inflation Expectations: what Do They Really Mean. Federal Reserve Bank of Cleveland Economic Commentary Series.

Munk, C., Sørensen, C., Vinther, T.N., 2004. Dynamic asset allocation under mean-reverting returns, stochastic interest rates and inflation uncertainty. Int. Rev. Econ. Financ. 13 (2), 141–166.

Piazzesi, M., Campbell, J.Y., 2006. Equilibrium yield curves. Nber Macroecon. Annu. 21 (12609), 389–457, (459-472).

Piazzesi, M., Schneider, M., 2012. Inflation and the Price of Real Assets. Working paper, Chicago Graduate School of Business. Social Science Electronic Publishing.

Reid, M., 2015. Inflation expectations of the inattentive general public. Econ. Model. 46 (April), 157–166.

Xu, Y., Chang, H.L., Lobonţ, O.R., Su, C.W., 2016. Modeling heterogeneous inflation expectations: empirical evidence from demographic data? Econ. Model. 57 (May), 153–163.

Yao, H., Yang, Z., Chen, P., 2013. Markowitz's mean–variance defined contribution pension fund management under inflation: a continuous-time model. Insur. Math. Econ. 53 (3), 851–863.

Zhang, A., Ewald, C.O., 2010. Optimal investment for a pension fund under inflation risk. Math. Methods Oper. Res. 71 (2), 353–369.