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Speculative behavior in a housing market: Boom and bust

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ABSTRACT

We study a housing market with household buyers, speculative investors and property developers in a Walrasian scenario. We show that in addition to the factors that affect the real demand of household buyers and the development cost of property developers, investors' speculative behavior is an important factor explaining housing price evolution and dynamics. In particular, investors' extrapolative expectations may drive the housing price to persistently deviate from its benchmark value and even to explode. In contrast, investors' mean-reverting strategy can balance out the position of trend extrapolators, which may stabilize an otherwise explosive housing market. Moreover, the evolutionary process of housing prices driven by investors' speculative behavior is path-dependent in the sense that different initial market conditions may result in different price paths, which corresponds to the localization property empirically documented in the real housing market. In addition, within the stylized model, we provide some policy implications through analyzing the limitation and effectiveness of policy adjustments via down payment and development cost, and find that the decrease of development cost is a better measure to adjust the housing market when it booms or busts.

1. Introduction

In the traditional economic paradigm that underpins the influential rational expectation real estate models of Alonso (1964), Rosen (1979) and Roback (1982), housing prices are determined by fundamental economic factors including national income, monetary policy, population growth, rents and interest rates. Many empirical studies of housing prices, however, have shown that there are often large movements in housing prices that apparently cannot be explained by these fundamental factors (Shiller, 2005, 2008). In a review of the Fed's forecasting record leading up to the financial crisis, Potter (2011) acknowledges a "misunderstanding of the housing boom … [which] downplayed the risk of a substantial fall in house prices.".

A weak explanatory power of economic factors on housing prices is known as the housing price puzzle or anomaly. One of the most important puzzles for housing economists is the strong persistence of price changes from one year to the next. Over the last three decades, an increasing number of anomalies and puzzles have been uncovered in empirical research. They include, for example, (i) price changes are predictable (Case and Shiller, 1989; Clayton, 1998; Schindler, 2013); (ii) price changes and construction levels are quite volatile in many markets; (iii) over longer time periods, the price changes mean revert while quantity changes persist; (iv) most variations in housing prices are local, not national. We refer to Glaeser et al. (2014) for additional discussion of these empirical anomalies. Therefore, the explanations of housing price movements based solely on fundamental economic factors are unsatisfactory. As a result, researchers have shifted their attention from economic factors towards incorporating the microstructure of housing prices, in particular, by considering agents' demand and supply based on their bounded rationality and heterogeneity.

In fact, the idea of heterogeneity and bounded rationality among agents has long been successfully applied to asset pricing in the stock market. Empirical studies (such as Lux (1998), Chiarella et al. (2014), Kabundi et al. (2015)) and theoretical analyses (such as Day and Huang (1990), Brock and Hommes (1997, 1998), Föllmer et al. (2005), He et al. (2009), Wang et al. (2013)) have shown that heterogeneity and bounded rationality among investors are important factors that affect the volatility of asset pricing. They can be used to explain many puzzles and stylized facts in the stock market, including the equity-premium puzzle, interest rate puzzle, volatility clustering, excess volatility and fat tails (see Basak (2005), Boswijk et al. (2007), De Grauwe (2012), He and Li (2015), and therein). We refer to Hommes (2006) and Chiarella et al. (2009) for surveys of the developments in this literature.

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In the past decades, an increasing number of studies provide ample evidences about heterogeneity and bounded rationality of investors in housing markets. For example, Piazzesi and Schneider (2009) use data on expectations from the Michigan Survey of Consumers to study household beliefs during the US housing boom in the early 2000s and show that the data are characterized by heterogeneity and mutual feedback between housing price expectations and actual housing prices. Applying different methodologies, two studies on a broad set of international securitized real estate markets, conducted by Schindler et al. (2010) and Serrano and Hoesli (2010), provide the evidence that persistence and predictability in real estate returns can be used to earn excess returns compared to a passive strategy. Schindler (2013) finds that investors can obtain excess returns from both autocorrelation-based and moving average-based trading strategies compared to a buy-and-hold strategy. Gelain and Lansing (2014) show that under fully-rational expectations, the model significantly underpredicts the volatility of the US price-rent ratio for reasonable levels of risk aversion but the moving-average model predicts a positive correlation such that agents tend to expect high future returns when prices are high relative to fundamentals - a feature that is consistent with a wide variety of survey evidences from real estate markets. Bolt et al. (2014), by analyzing the housing markets of eight different countries, find that the data support heterogeneity in expectations, with temporary endogenous switching between fundamental mean-reverting and trend-following beliefs based on their relative performance. Shiller (2005, 2008) concludes that speculative thinking among investors, the use of heuristics such as extrapolative expectations, market psychology in the form of optimism and pessimism, herd behavior and social contagion of new ideas (new era thinking), and positive feedback dynamics are elements that play an important role in determining housing prices.

There is still a lack of theoretical studies about the impact of heterogeneity and bounded rationality on housing prices. Granziera and Kozicki (2015) show only the extrapolative expectation model with time-varying extrapolation coefficient is consistent with the run up in housing prices observed over the 2000–2006 period and subsequent sharp downturn. Gelain and Lansing (2014) demonstrate that the model can approximately match the volatility of the price-rent ratio in the data if agents employ a simple moving-average forecast rule for the price-rent ratio. These studies focus on bounded rationality of agents but ignore potential heterogeneity among them. Malpezzi and Wachter (2005) and Dieci and Westerhoff (2012, 2013) consider both heterogeneity and bounded rationality and show that the models can generate real estate cycles. However, in the models, the demand and supply of agents are given exogenously rather than coming from their own objectives. Bolt et al. (2014) consider the endogenous demand by optimizing agents' mean-variance utility but the supply is fixed.

This paper revisits the traditional supply-demand mechanism in housing markets in light of the understanding that fundamental economic factors alone are not the sole drivers of housing price movements. We present a discrete-time model of housing prices considering both economic factors and heterogeneous agents. An important difference with the models of Malpezzi and Wachter (2005) and Dieci and Westerhoff (2012) is that they use a price adjustment rule based on excess demand which is exogenously given, while our approach uses a temporary equilibrium price model where all demands and supplies are endogenously determined based on agents' own objectives and financial constraints. The equilibrium model is similar to Bolt et al. (2014) but with two important differences. Firstly, they fix the housing supply, whereas the supply amount in our model is endogenously decided by property developers through maximizing their profit. Secondly, their fundamental housing price is determined by rents while in our approach, the fundamental value of houses depends on real demand and supply which rely on the fundamental economic factors like national income, development costs and down payment. Within the stylized model, we analytically study the impacts of real demand, development cost and speculative behavior on housing prices using stability and bifurcation theories. An interesting theoretical contribution of our analysis is that from the viewpoints of speculative behavior and heterogeneity among agents, it can explain various stylized facts in housing markets, like persistence and predictability in housing price movements which are connected with the stability of different steady states and the local characteristics of housing price changes which corresponds to the co-existence of two attractors.

The paper proceeds as follows. In Section 2, we set up an equilibrium framework with three types of agents, house buyers, investors and property developers, and study a benchmark market without speculative investors. Compared with the benchmark market, Section 3 studies a one-period case and analyzes the impact of investors' speculative behavior on housing prices. In Section 4, we examine investors with dynamic beliefs who follow two specific strategies - extrapolation and mean-reversion, by stability and bifurcation theories. In Section 5, within our framework, the paper gives some policy implications on a housing market with speculative investors by analyzing the limitation and effectiveness of policy adjustments through down payment and development cost. In addition, we use real data on the Beijing housing market to demonstrate the applicability of the model and the economic meaning of it. Section 6 concludes the paper. All the proofs of the technical results are given in the Appendix.

2. The model

We consider a housing market with two types of property consumers, including house buyers and investors, and one type of property developers who supply houses. At any given time, house buyers and investors decide their property demands while property developers decide how many units of houses to be constructed, based on their own objectives and budget constraints. In particular, the three types of agents have the following specification of preferences.

2.1. Preference of house buyers

A house buyer (denoted by agent B) has real demand for houses and thinks a house as a necessity. At time n, there are $\varphi_n^B(>0)$ house buyers who enter into the housing market. Each house buyer makes a decision about how many units of houses to buy by maximizing his/her consumption utility subject to his/her ability to make down payment at the time. Without bankrupt and solvency constraint,[2] agent B's optimization problem can be written as

$$(OP^B):\ \begin{array}{ll}\max\limits_{C_n^B,h_n^B} & U^c(C_n^B)+k^BU^h(h_n^B)\\ \text{s. t.} & C_n^B+\theta^BP_nh_n^B=Y_n^B,\end{array}\tag{2.1}$$

where P_n is the housing price (without the rent) per unit of houses at time n, C_n^B and h_n^B denote the consumption amounts, respectively, of goods and houses by agent B at time n, $U^c(\cdot)$ and $U^h(\cdot)$ are agent B's utility functions respectively of goods and houses, k^B is the significance factor of properties to agent B compared with his/her goods consumption, $\theta^B\in(0,1]$ is the down payment ratio on agent B's mortgage and Y_n^B is agent B's income at time n. In (2.1), it shows that as long as the house buyer has the financial capacity to pay the down payment, he/she will buy houses to maximize his/her consumption utility at that time and then exit from the housing market. At time $n+1$, a total of φ_{n+1}^B new house buyers will enter into the housing market and make their buying decisions. Without loss of generality, we assume that agent B's income is constant, that is $Y_n^B\equiv Y^B$ for every time n. In addition, to simplify the analysis, we assume that agent B adopts the logarithmic utilities[1] for both goods and houses, that is $U^c(x)=U^h(x)=\ln(x)$.

[2] In a general case, the solvency of a house buyer should be considered, which can be done within the framework given by Adam et al. (2011). In this paper, we emphasize the impact of speculative behavior from investors and leave the general case for future work.

From the first order condition of (2.1), the optimal volume traded by agent B at time n is

$$h_n^B = \frac{Y^B k^B}{P_n \theta^B (k^B + 1)} > 0, \tag{2.2}$$

which means that the demand of agent B depends on his/her income (Y^B), his/her down payment per unit of houses ($\theta^B P_n$) and the significance of houses to him/her (k^B). Thus, to agent B, the higher income, the lower down payment and/or the higher significance, the higher demand of houses.

2.2. Preference of investors

At time n, there are $\varphi_n^I(>0)$ investors in the housing market. An investor (denoted by agent I) optimizes his/her portfolio by putting his/her money into his/her bank account to obtain gross return[4] $R_f(>1)$ or buying houses to make money on capital gain. We denote at time n, his/her portfolio wealth by Π_n^I and the unit amount of houses he/she invests by h_n^I. Thus, the wealth process of agent I can be formulated as

$$\Pi_{n+1}^I = (\Pi_n^I - P_n h_n^I) R_f + (P_{n+1} + R_f Q_n) h_n^I,$$

where Q_n is the price for renting one unit of houses in the period between times n and $n+1$. Since rents are typically paid up-front (just after buying houses, that is at time n^+), to express the rent at time n^+ in terms of currency at time $n+1$, it should be inflated by the opportunity cost[3] R_f. Assume that agent I is a myopic mean-variance maximizer with very deep pockets and no budget or short selling constraints. That is, he/she maximizes his/her wealth expectation and minimizes its variance based on his/her beliefs. Therefore, we can write the objective of agent I as follows

$$(OP^I): \max_{h_n^I} \; E_n^I(\Pi_{n+1}^I) - \frac{\alpha^I}{2} V_n^I(\Pi_{n+1}^I), \tag{2.3}$$

where E_n^I and V_n^I denote agent I's 'beliefs' about the expectation and variance based on a publically available information set $\mathcal{F}_n$ consisting of past prices and rents, that is $\mathcal{F}_n = \{P_{n-1}, P_{n-2}, \ldots; Q_{n-1}, Q_{n-2}, \ldots\}$, and α^I is his/her risk aversion coefficient.

By $E_n^I(\Pi_{n+1}^I) = R_f \Pi_n^I + E_n^I(P_{n+1} + R_f Q_n - R_f P_n) h_n^I$ and $V_n^I(\Pi_{n+1}^I) = (h_n^I)^2 V_n^I(P_{n+1} + R_f Q_n - R_f P_n)$, we can obtain the optimal trading volume of agent I at time n from (2.3) as follows

$$h_n^I = \frac{E_n^I(P_{n+1} + R_f Q_n - R_f P_n)}{\alpha^I (\sigma^I)^2}, \tag{2.4}$$

where $(\sigma^I)^2 = V_n^I(P_{n+1} + R_f Q_n - R_f P_n)$ is assumed constant for all times and $E_n^I(P_{n+1} + R_f Q_n - R_f P_n)$ is time-varying. When $E_n^I(P_{n+1} + R_f Q_n - R_f P_n) > 0$, that is when investors expect the future gross return of houses would be higher than the opportunity cost R_f, then they want to buy houses rather than saving money in their bank accounts to increase their wealth. Otherwise, investors want to sell houses.

2.3. Preference of developers

There are $\varphi_n^D(>0)$ property developers in the housing market at time n. Each developer (denoted by agent D) decides how many units of houses to be constructed and sold based on his/her profit maximization and financing cost. Without the consideration of delay effect of construction, agent D's objective can be described as

$$(OP^D): \begin{array}{ll} \max\limits_{h_n^D} & P_n h_n^D - F^D(h_n^D) - \frac{R_f - 1}{R_f} L_n^D \\ \text{s. t.} & F^D(h_n^D) = B_n^D + L_n^D, \end{array} \tag{2.5}$$

where h_n^D is agent D's construction/sale amount at time n, B_n^D and L_n^D are, respectively, the amount of self-financing and bank loan, and $F^D(x) = cx^2/2$ is the developer's cost function[5] with $c > 0$.

From (2.5), the developer's optimal supply is given by

$$h_n^D = \frac{P_n}{C}, \tag{2.6}$$

where $C = c\left(2 - \frac{1}{R_f}\right) > 0$ is regarded as the development cost per unit of houses and the higher development cost, the lower supply.

2.4. Equilibrium

Equilibrium housing prices are derived by the market clearing condition, that is, the demand of property consumers is equal to the supply of proper developers. This implies,

$$\varphi_n^B h_n^B + \varphi_n^I h_n^I = \varphi_n^D h_n^D. \tag{2.7}$$

For simplicity, we assume that φ_n^i $(i = B, I, D)$ is constant, denoted by φ^i. In addition, without loss of generality, we assume $\varphi^D = 1$. Thus, φ^B and φ^I can be regarded as the normalized market populations of agents B and I by the number of property developers or market forces of a property developer.

2.4.1. Without speculative investors

To highlight the impact of speculative behavior among investors on housing prices, we first consider a benchmark notion without speculative investors. No investors enter into the housing market and then only house buyers and developers can affect the housing price. Therefore, the equilibrium housing price only relies on the real demand of house buyers and the supply of developers, which serves as a benchmark case for the rest of the paper. Combining (2.2) and (2.6) with (2.7), we can easily obtain that at any time n, the equilibrium housing price is a constant, that is $P_n \equiv \overline{P}$ where

$$\overline{P} = \left(C \frac{\varphi^B Y^B k^B}{\theta^B (k^B + 1)}\right)^{\frac{1}{2}}, \tag{2.8}$$

which is called a benchmark price and is determined by some fundamental economic factors like the income (Y^B), the development cost (C) and the down payment (θ^B). The higher income, the higher cost or the lower down payment directly contributes to the higher housing price.

2.4.2. With speculative investors

For a general case, we consider that there are speculative investors in the housing market. They base their investment decision on their own forecast about the future housing price, which can be described as

$$E_n^I(P_{n+1} + R_f Q_n) = R_f P_{n-1} + R_{n+1}^I, \tag{2.9}$$

where, considering the existence of the opportunity cost (R_f), $R_f P_{n-1}$ is a basic level of the future housing price (including the rent) based on the historical information $\mathcal{F}_n$, and R_{n+1}^I represents a speculative component, which is adapted with $\mathcal{F}_n$ and based on investors' own beliefs.

(footnote continued)

[1] Logarithmic utility functions are used to simplify the analysis. If other forms of utility functions were used, some but not all results could hold and some more complex phenomena were expected. Thank one referee for pointing it out.

[4] In fact, this gross return (R_f) also can be regarded as the opportunity cost of agent I, including not only a riskfree rate but also the returns generated by other investment choices such as hedge funds and trusts.

[3] Here the opportunity cost is only one way to inflate the rent and we also can use another factor like mortgage rate as an inflation factor, which would not change the main results in this paper.

[5] Here the corresponding marginal cost of development equals cx, which reflects the possibility that there are scarce inputs into housing production, so that the marginal cost of development rises linearly with the amount of development, see Glaeser et al. (2008).

When $R^I_{n+1} > (<)0$, investors optimistically (pessimistically) believe the housing price should be higher (lower) than its basic level. Thus, the trading volume (2.4) of houses by agent I is[6]

$$h^I_n = \frac{R_f P_{n-1} + R^I_{n+1} - R_f P_n}{\alpha^I(\sigma^I)^2}. \tag{2.10}$$

In this case, equilibrium housing prices not only depend on the real demand of house buyers and the supply of property developers, but also rely on the behavior of investors. To better clarify the role of investors in the housing market, we respectively consider one-period and multi-period cases as follows. In the one-period case, we study the impact of different factors on housing prices like income, down payment ratio, development cost and speculative behavior, while in the multi-period case, we emphasize the dynamic evolution of housing prices based on investors' updating expectations.

3. One-period model

In this section, we consider a one-period case when investors just invest for one period. Compared with the benchmark case, different roles of three types of agents in the housing market are analyzed. Denote investors' expectation about the future housing price (including the rent) as P^I_Q, that is $P^I_Q = E^I_1(P_2 + R_f Q_1)$, and let

$$m_0 = \frac{1}{C} + \mathcal{A}^I R_f, \quad m_1 = \mathcal{A}^I P^I_Q \quad \text{and} \quad m_2 = \frac{\varphi^B Y^B k^B}{\theta^B(k^B+1)}, \tag{3.1}$$

where $\mathcal{A}^I = \frac{\varphi^I}{\alpha^I(\sigma^I)^2}$ is called the risk characteristics of investors. From (3.1), we can see that m_0 represents[7] developers' cost factor, m_1 represents investors' speculative behavior factor and m_2 represents buyers' real demand factor. Combining (2.2), (2.4) and (2.6) with (2.7), we denote the equilibrium housing price as $\mathcal{P}$ and obtain the following result.

Proposition 3.1. *The equilibrium housing price $\mathcal{P}$ satisfies*

$$m_0\mathcal{P}^2 - m_1\mathcal{P} - m_2 = 0, \tag{3.2}$$

that is,

$$\mathcal{P} = \frac{m_1 + \sqrt{m_1^2 + 4m_0 m_2}}{2m_0} > 0, \tag{3.3}$$

which negatively depends on the cost factor but positively relies on the factors of speculative behavior and real demand, that is

$$\frac{\partial\mathcal{P}}{\partial m_0} < 0, \quad \frac{\partial\mathcal{P}}{\partial m_1} > 0 \quad \text{and} \quad \frac{\partial\mathcal{P}}{\partial m_2} > 0.$$

Proposition 3.1 shows that the equilibrium price is the result of combined action from m_0, m_1 and m_2. Note that $\frac{\partial\mathcal{P}}{\partial C} = -\frac{1}{C^2}\frac{\partial\mathcal{P}}{\partial m_0}$. Thus, similar to the benchmark case, the equilibrium price $\mathcal{P}$ positively depends on development cost and real demand, as shown in Fig. 3.1. But those two factors are not unique impact factors any more and speculative behavior of investors also has the positive effect on the equilibrium price. Especially, investors' risk characteristics ($\mathcal{A}^I$) and expectation (P^I_Q) have the following impacts on the equilibrium housing price ($\mathcal{P}$).

Proposition 3.2 (*Impact of Investors*). *Let $\mathcal{H}^I = \varphi^I h^I_1$ denoted as the total optimal volume traded by investors. If $P^I_Q \gtreqless R_f\overline{P}$, then $\mathcal{H}^I \gtreqless 0$ and furthermore, $\mathcal{P} \gtreqless \overline{P}$ and $\frac{\partial\mathcal{P}}{\partial\mathcal{A}^I} \gtreqless 0$.*

Proposition 3.2 shows how the equilibrium housing price with speculative behavior compares to the benchmark price. If $P^I_Q = R_f\overline{P}$, we are back to the benchmark case with $\mathcal{P} = \overline{P}$. For $P^I_Q \neq R_f\overline{P}$, investors' speculative behavior becomes an important factor for housing prices.

The liquidity supplied to and taken away from the market by investors changes with their expectations about future prices. If investors' expectation P^I_Q is higher than the future value $R_f\overline{P}$ of the benchmark price, investors are in the position to buy, that is $\mathcal{H}^I > 0$, which pushes the price beyond the benchmark level, as illustrated in the first quadrants of the top panel of Fig. 3.2. Meanwhile, the equilibrium housing price increases with their risk characteristics. This is because the higher risk characteristics ($\mathcal{A}^I$) of investors, the lower risk aversion coefficient (α^I) or expectation about the volatility (σ^I) of the housing market and/or the more investors in the market, which increases their demand to buy houses. Furthermore, the housing price is pushed up, see the bottom panel of Fig. 3.2.

Similarly, when investors are in the position to sell, that is $P^I_Q - R_f\overline{P} < 0$, their role in the housing market is similar to a developer and they can be regarded as suppliers of houses. Therefore, it can drive the equilibrium price below the benchmark value and decreasing with the risk characteristics of investors, as shown in Fig. 3.2.

Therefore, changes in investors' expectations will result in changes in the market clearing price. If investors updated their expectations based on the past market information, then investors' expectations were time-varying, which would give us more interesting price evolution and dynamics, like the persistence and predictability of housing prices and even the localization property, shown in the empirical literature. We will explore this in the following section.

4. Multi-period models

To analyze the impact of investors' dynamic expectations, we follow the idea of Dieci and Westerhoff (2012) and consider that investors can update their beliefs about the trend of the housing price, which are based on the historical information about housing prices but not the contemporaneous realizations. In addition, considering heterogeneity in beliefs, in the paper, we assume that investors can adopt two ways to update their beliefs - extrapolation and mean-reversion. To examine the role of different types of investors, we first assume that there is only one type of investors who use an extrapolative method to update their expectations about future housing prices. Second, we consider investors update their expectations about future housing prices based not only on an extrapolative component but also on a mean-reverting one. We contrast the two cases to explain different impacts of the extrapolative and mean-reverting beliefs on the dynamic evolution of housing prices.

4.1. The model with extrapolation

We consider investors who use an extrapolative method to update their beliefs, meaning they are confident in the continuation of the price trend in the next period. Investors' belief under the extrapolative method can be formalized as

$$R^I_{n+1} = R^E_{n+1} = f(P_{n-1} - \overline{P}), \tag{4.1}$$

where $f(>0)$ is the extrapolative intensity to the housing price trend. Then (4.1) implies that when the housing price is above (below) its benchmark value, trend followers optimistically (pessimistically) believe in a further price increase (decrease).

Note that each investor is a mean-variance maximizer. The demand of each investor is therefore given by

$$h^I_n = h^E_n = \frac{R_f P_{n-1} + R^E_{n+1} - R_f P_n}{\alpha^I(\sigma^I)^2}, \tag{4.2}$$

Then combined with (2.2) and (2.6), the equilibrium price is deter-

[6] P_n is determined by the market clearing condition (2.7) and then is $\mathcal{F}_n$-adapted.

[7] In fact, m_0 also includes the risk characteristics $\mathcal{A}^I$ of investors. However, in this paper, we focus on the impact of investors' belief P^I_Q which is in the coefficient m_1 while corresponding to the benchmark case, m_0 includes the cost of developers. Therefore, we call m_0 the cost factor of developers. The impact of $\mathcal{A}^I$ will be analyzed in Proposition 3.2 separately.

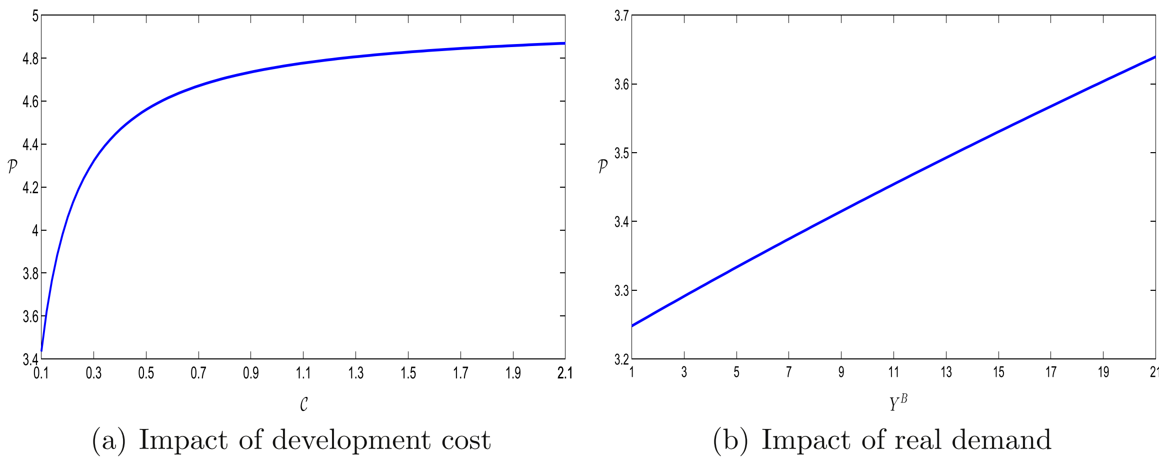


Fig. 3.1. The relationships between the equilibrium housing price and different factors. Here R_f=1.05, k^B=2, $\theta^B = 0.3$, $\alpha^I = 5$, $\sigma^I = 0.1$, $\varphi^I = \varphi^B = 1$ with (a) $P_Q^I = 5$, Y^B=10 and (b) $P_Q^I = 5$, $C = 0.1$.

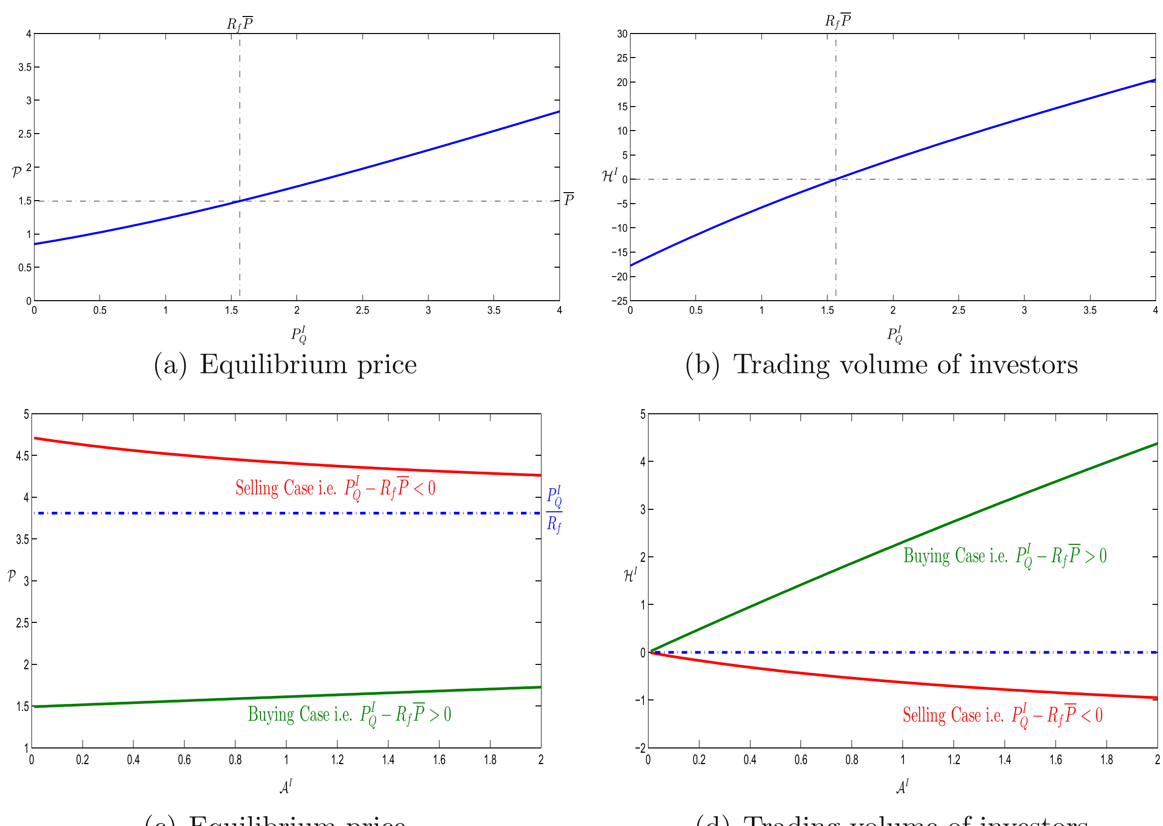


Fig. 3.2. The relationships between the equilibrium housing price and investors' expectation P_Q^I P_Q^I or risk characteristics $\mathcal{A}^I$. Here Y^B=10, R_f=1.05, k^B=2, $\theta^B = 0.3$ and $\varphi^B = 1$. (a–b) For the impact of investors' expectation, take $\alpha^I = 5$, $\sigma^I = 0.1$, $\varphi^I = 1$ and $C = 0.1$. (c–d) For the impact of investors' risk characteristics, take $P_Q^I = 4$ and let $C = 1$ for the selling case and $C = 0.1$ for the buying case.

mined by the market clearing condition given by (2.7), satisfying

$$m_0 P_n^2 - \widetilde{m}_{1,n} P_n - m_2 = 0, \tag{4.3}$$

where

$$\widetilde{m}_{1,n} = \widetilde{m}_{1,n}(P_{n-1}) = \mathcal{A}^I (R_f P_{n-1} + R_{n+1}^E). \tag{4.4}$$

This gives us a one-dimensional discrete dynamic system which can be described as

$$P_n = \widetilde{G}(P_{n-1}) = \frac{\widetilde{m}_{1,n}(P_{n-1}) + \sqrt{\widetilde{m}_{1,n}^2(P_{n-1}) + 4m_0 m_2}}{2m_0}. \tag{4.5}$$

The following proposition characterizes the price dynamics of the above dynamic system.

Proposition 4.1 (*Stability and Bifurcation*). *Denote* $\widetilde{f}^* = (\mathcal{A}^I C)^{-1}$, $f^{**} = 2\widetilde{f}^*$ *and* $\widetilde{P} = \frac{\overline{P}}{\mathcal{A}^I C f - 1}$.

(1) *When* $0 < f < \widetilde{f}^*$, *the benchmark price* $\overline{P}$ *is a unique steady state of* (4.5), *which is stable.*
(2) *When* $\widetilde{f}^* < f < f^{**}$, *there are two steady states in* (4.5), *the benchmark price* $\overline{P}$ *and a non-benchmark steady state* $\widetilde{P}$, *where* $\overline{P}$ *is stable and* $\widetilde{P}$ *is unstable.*
(3) *When* $f = f^{**}$, (4.5) *undergoes a transcritical bifurcation.*

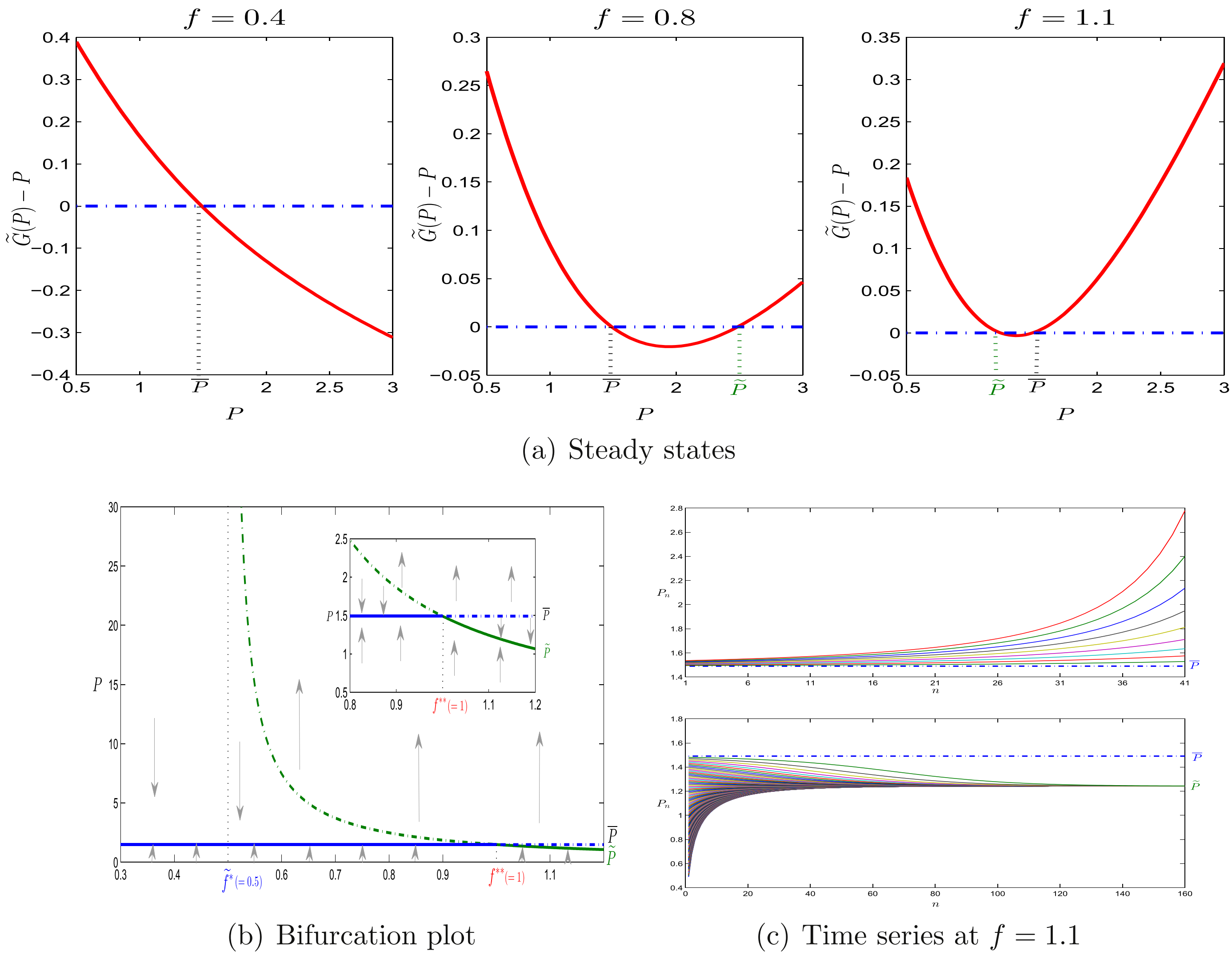


Fig. 4.1. $\widetilde{f}^{*} = 0.5$ and $f^{**} = 1$ at Y^B=10, k^B=2, $\theta^B = 0.3$, $C = 0.1$, R_f=1.05, $\alpha^I = 5$, $\sigma^I = 0.1$ and $\varphi^I = \varphi^B = 1$. Here in (b), the dash-dot lines correspond to unstable steady states while stable steady states are given by solid lines.

(4) *When $f > f^{**}$, the benchmark price $\overline{P}$ loses its stability and becomes unstable while $\widetilde{P}$ becomes stable.*

From Proposition 4.1, we see that when investors' extrapolative intensity increases, system (4.5) has different steady states, as illustrated in Fig. 4.1(a) and their stabilities can change, as shown in Fig. 4.1(b). In detail, when investors' extrapolative intensity is low, that is $0 < f < \widetilde{f}^{*}$, then (4.5) maintains the characteristics of the benchmark case. The benchmark price is a unique steady state and is stable in the sense that price deviations from the benchmark one will eventually vanish in the long run. However, if the extrapolative intensity is fairly strong so that $\widetilde{f}^{*} < f < f^{**}$, then the small upward deviations from the benchmark level (for $\overline{P} < P_0 < \widetilde{P}$) do not persist while the big upward deviations (for $P_0 > \widetilde{P}$) may evolve into permanent deviations which will eventually blow up the housing price. The fairly strong extrapolative behavior causes large upward deviations from the benchmark level to be further reinforced by the increasing housing prices and demand, which is consistent with the survey result of Piazzesi and Schneider (2009). Nevertheless, deviations below the benchmark level will converge to the benchmark price. This is because there exists a real demand from house buyers in the market and the extrapolative behavior of investors is only fairly strong, which results the house price cannot deviate downward from the benchmark price very far and it eventually converges back to the benchmark price $\overline{P}$ (see Fig. 4.1(b)).

Finally, when the extrapolative intensity is very strong, that is $f > f^{**}$, then the benchmark price loses its stability and deviations of housing prices from the benchmark price are inevitable. Whether the deviation is upward or downward depends on the market situation. If the initial price is higher than the benchmark level, then the upward price trend will persist and lead to an eventual blow up, as illustrated in the top panel of Fig. 4.1(c). On the other hand, if the initial price is lower than the benchmark value, then the downward price trend will continue until it hits the level $\widetilde{P}$ which is strictly positive because of the existence of the real demand from house buyers at any time, as illustrated in the bottom panel of Fig. 4.1(c). In this case, investors act as suppliers of properties, which increase the supply of houses and drive the housing price down.

Note that except the unique steady state case of the benchmark equilibrium as the extrapolative intensity is small, that is $0 < f < \widetilde{f}^{*}$, the evolution of housing prices is path-dependent. That is to say, different initial prices will result in different price dynamics, either explosion or convergence to some bounded level. If the determinants of initial prices were some local factors, like local income, cultures, commodity prices and policy environment, then the price changes would display the localization rather than national property, which phenomenon has been shown in the empirical literature. In addition, from Fig. 4.1, we can see that the price trends are continuous, which corresponds to the persistence and predictability of price changes existing in the real housing market. Therefore, our model has the good ability to explain some stylized facts in housing markets.

4.2. The model with extrapolation and mean reversion

In Section 4.1, we see that extrapolative investors who believe in the persistence of price trend may cause the housing market to explode because of their increasing demand and reinforced price expectations. In this section, we show that the interaction among different types of investors increases the complexity of the system, nevertheless, stabilizing the system.

Similar to Dieci and Westerhoff (2012), we consider two types of speculative behavior in the housing market. In addition to the extrapolative component shown in Section 4.1, there is a mean-reverting strategy used in the market. The mean-reverting component can be written as

$$R_{n+1}^{MR} = g(\overline{P} - P_{n-1}), \tag{4.6}$$

where $g \in (0, 1)$ measures the mean-reverting speed adopted by investors. Eq. (4.6) implies that if the housing price is above (below) its benchmark price, mean-reverting investors believe that the price is overestimated (underestimated) and the future price will decrease (increase). In other words, they believe that the housing price cannot deviate far away from its benchmark price in the long run. Thus, the mean-reverting strategy should play a part in stabilizing the housing price.[8]

Thus, the total speculative demand from investors is

$$\mathcal{H}_n^I = \omega_n \varphi^I h_n^E + (1 - \omega_n)\varphi^I h_n^{MR}, \tag{4.7}$$

where

$$h_n^u = \frac{R_f P_{n-1} + R_{n+1}^u - R_f P_n}{\alpha^I (\sigma^I)^2}, \quad \text{where} \quad u = E \quad \text{or} \quad MR, \tag{4.8}$$

and ω_n and $1 - \omega_n$ stand for the market fractions respectively of extrapolative and mean-reverting investors. Furthermore, the speculative demand[9] of agent I is

$$h_n^I = \frac{\mathcal{H}_n^I}{\varphi^I} = \omega_n h_n^E + (1 - \omega_n) h_n^{MR} = \frac{R_f P_{n-1} + R_{n+1}^I - R_f P_n}{\alpha^I (\sigma^I)^2}, \tag{4.9}$$

where R_{n+1}^I is a forecast variable with a nonlinear law of motion given by

$$R_{n+1}^I = \omega_n R_{n+1}^E + (1 - \omega_n) R_{n+1}^{MR}, \tag{4.10}$$

which is the average of different investors' beliefs weighted by their market fractions.

We consider social interactions between different investors and assume investors can switch their forecasting rules with respect to market circumstances such that in each step the impacts of those two components are time-varying. The switching mechanism is following a formulation by He and Westerhoff (2005) and Bauer et al. (2009). Investors seek to exploit price trends (that is, bull and bear markets). However, the more the price deviates from its benchmark value, the more agents come to the conclusion that a benchmark market correction is about to set in. As a result, an increasing number of investors opt for the mean-reverting strategy. Furthermore, similar to Dieci and Westerhoff (2012), the relative impact of extrapolators is formalized as

$$\omega_n = \frac{1}{1 + \mu (P_{n-1} - \overline{P})^2}, \tag{4.11}$$

where $\mu \geq 0$ measures the intensity of switching speed. When $\mu = 0$, then it is reduced to the case in Section 4.1. Hereafter, if not specified, we assume that $\mu > 0$.

Therefore, the equilibrium price is determined by the market clearing condition given by (2.7), satisfying

$$m_0 P_n^2 - m_{1,n} P_n - m_2 = 0, \tag{4.12}$$

where

$$m_{1,n} = m_{1,n}(P_{n-1}) = \mathcal{A}^I (R_f P_{n-1} + R_{n+1}^I), \tag{4.13}$$

which is also a one-dimensional discrete dynamic system as

$$P_n = G(P_{n-1}) = \frac{m_{1,n}(P_{n-1}) + \sqrt{m_{1,n}^2(P_{n-1}) + 4 m_0 m_2}}{2 m_0}. \tag{4.14}$$

To get the steady states of system (4.14), we let $P_{n-1} = P_n = P^*$ and obtain

$$(P^* - \overline{P}) T(P^*) = 0, \tag{4.15}$$

where

$$T(P^*) = \mathbb{a}(P^*)^3 + \mathbb{b}(P^*)^2 + \mathbb{c}P^* + \mathbb{d} = 0, \tag{4.16}$$

and

$$\mathbb{a} = \left[\mathcal{A}^I g + \frac{1}{C}\right]\mu,$$

$$\mathbb{b} = -\left[2\mathcal{A}^I g + \frac{1}{C}\right]\mu \overline{P}, \ \mathbb{c} = \frac{1}{C}(1 - \mu \overline{P}^2) - \mathcal{A}^I (f - \mu g \overline{P}^2),$$

$$\mathbb{d} = \frac{\overline{P}}{C}(1 + \mu \overline{P}^2).$$

Then the discriminant (Δ) of (4.16) is

$$\Delta = 27\mathbb{a}^2\mathbb{d}^2 - 18\mathbb{a}\mathbb{b}\mathbb{c}\mathbb{d} + 4\mathbb{a}\mathbb{c}^3 + 4\mathbb{b}^3\mathbb{d} - \mathbb{b}^2\mathbb{c}^2.$$

Furthermore, we can get the following proposition.

Proposition 4.2 (*Existence of Steady States*). *The benchmark price $\overline{P}$ is always a steady state of (4.14). When $\Delta \leq 0$, (4.14) has another two steady states (P_1^*, P_2^*) where $P_2^* \geq P_1^* > 0$.*

Proposition 4.2 demonstrates the structure change of (4.14) because of the existence of two new steady states when $\Delta \leq 0$ as shown in Fig. 4.2(a). At $\Delta = 0$, two new equal steady states ($P_1^* = P_2^*$) appear, which corresponds to a saddle-node bifurcation. So we call $\Delta = 0$ a saddle-node bifurcation boundary. In addition, similar to Proposition 4.1, we can show that the benchmark price $\overline{P}$ may lose its stability at $f = f^{**}$ which corresponds to a transcritical bifurcation and is therefore called a transcritical bifurcation boundary. If we use the extrapolative intensity (f) and mean-reverting speed (g) as a pair of parameters to plot the bifurcation boundaries, as illustrated in Fig. 4.2(b), we show that for any fixed g, with the increase of f, system (4.14) can undergo two types of bifurcations. However, when f is fixed and g increases, the benchmark price $\overline{P}$ may remain stable or unstable and at most, system (4.14) may just undergo a saddle-node bifurcation. Therefore, we choose f as a bifurcation parameter, on the one hand to illustrate those two types of bifurcations and on the other hand to compare with the case in Section 4.1.

Proposition 4.3 (*Stabilities and Bifurcations*). *Let f^* denote a unique positive solution of $\Delta = 0$ and $f^{**} = 2(\mathcal{A}^I C)^{-1}$. Assume $\partial^2 P_n / \partial P_{n-1}^2 |_{\{P_{n-1} = P_1^*, f = f^*\}} < 0$. Then*

(1) *When $0 < f < f^*$, the benchmark price $\overline{P}$ is always stable.*
(2) *At $f = f^*$, a saddle-node bifurcation occurs and two non-benchmark steady states (P_1^*, P_2^*) appear.*
(3) *When $f^* < f < f^{**}$, P_1^* is unstable and P_2^* is stable while the stability of the benchmark price $\overline{P}$ keeps invariant.*
(4) *At $f = f^{**}$, a transcritical bifurcation occurs.*
(5) *When $f > f^{**}$, the benchmark price $\overline{P}$ becomes unstable while both P_1^* and P_2^* are stable.*

[8] In fact, in Proposition 4.3, we show that the mean-reverting strategy can stabilize an otherwise unstable system to a certain price level.

[9] In the model, agent I can be interpreted as a speculative representative agent who uses a nonlinear mix of different forecasting rules. The representative agent then updates his/her mix in each time step.

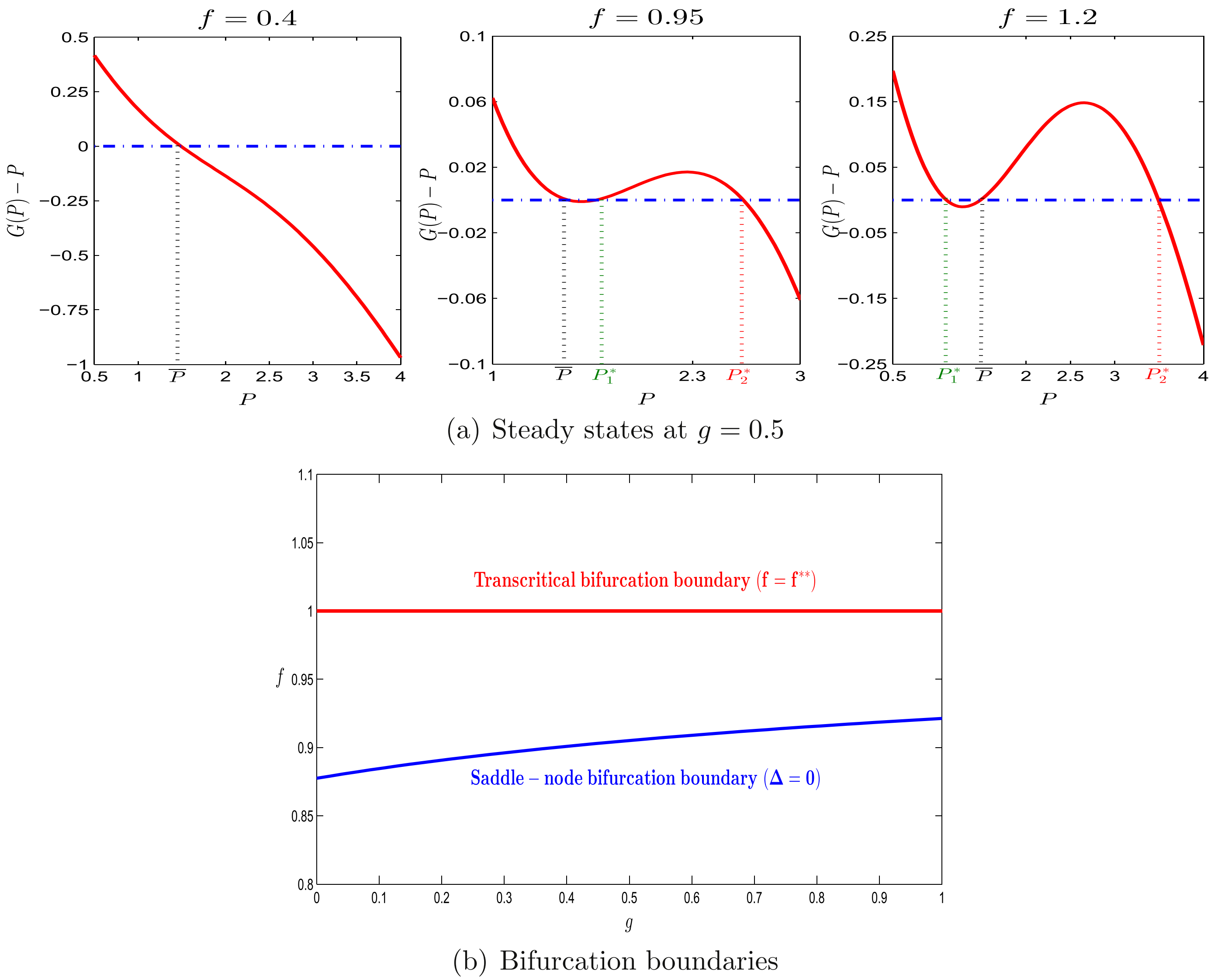
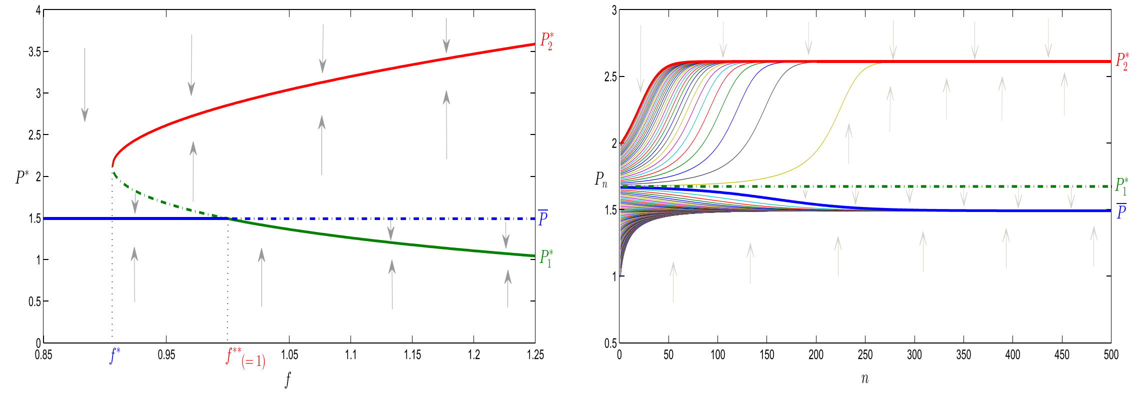


Fig. 4.2. Y^B=10, k^B=2, $\theta^B = 0.3$, $\mu = 0.1$, $C = 0.1$, $\alpha^I = 5$, $\sigma^I = 0.1$, $\varphi^I = \varphi^B = 1$ and R_f=1.05.

Proposition 4.3[10] shows that the benchmark price is not always stable and it may undergo two types of bifurcations. When investors' extrapolative intensity (f) is small, that is $0 < f < f^*$, the benchmark price is stable. It goes from stable to unstable as the extrapolative intensity becomes increasingly strong. In particular, when this intensity (f) increases to f^*, the system undergoes a saddle-node bifurcation. That is, there exist two non-benchmark steady states (P_1^*, P_2^*) at $f^* < f < f^{**}$, with one (P_2^*) being stable and the other (P_1^*) unstable. At the same time, the benchmark price ($\overline{P}$) remains stable. This implies that there are two attractors in the system - P_2^* and $\overline{P}$, as shown in Fig. 4.3(a). Thus, the two attractors split the price space into two parts separated by P_1^*, where prices exhibit different evolutionary processes. The coexistence of a stable benchmark price and a stable non-benchmark price in our simple evolutionary model can be explained by the following simple economic intuition. When investors' extrapolative intensity is relatively low (that is $f^* < f < f^{**}$), similar to Proposition 4.1, for prices near the benchmark level, investors' extrapolative behavior is not strong enough to cause permanent deviations from the benchmark price. In addition, because of the existence of real demand and relatively low extrapolation, the prices cannot deviate downward from the benchmark value very far and eventually converge back to it. For the upward price trend that has deviated far away from the benchmark level, the extrapolative trend-following behavior strengthens the upward deviation of prices with stronger demands that feed back into price increases, which, similar to the case in Section 4.1, lets the price trend persistent and predicable. Unlike that case, however, this up trend cannot be sustained endlessly because of mean-reverting investors in the market. As the prices deviate further from the benchmark level, more and more investors form strong beliefs that the prices will mean revert. As a result, an increasing number of investors opt for the mean-reverting strategy and begin to sell houses, which increases the total house supply in the market. This creates a balance between the buying and selling sides, which stabilizes the equilibrium price at P_2^*, as illustrated in Fig. 4.3(b). From this, we can see that the introduction of mean-reverting investors into the model, on one side, increases the complexity of the system; on the other side, has the role of stabilizing an otherwise explosive market because of the interaction between the opposite trading strategies of different investors. Therefore, the rich types of investors are useful to stabilize the system bounded in a certain range rather than exploding because of their counteracting behavior.

As the extrapolative intensity f increases beyond f^{**}, the bench-

[10] Note that in Proposition 4.3, if $\mu \to 0$ which means it is back to the case in Section 4.1, then $f^* \to \tilde{f}^*$. In addition, for the condition of $\partial^2 P_n/\partial P_{n-1}^2|_{\{P_{n-1}=P_1^*, f=f^*\}} < 0$, numerical results show that for all parameters, it is always true.



(a) Bifurcation plot with $f^* = 0.906$ and $f^{**} = 1$ (b) Two attractors at $f = 0.95$

Fig. 4.3. Y^B=10, k^B=2, $\theta^B = 0.3$, $\mu = 0.1$, $C = 0.1$, $\alpha^I = 5$, $\sigma^I = 0.1$, g=0.5, $\varphi^I = \varphi^B = 1$ and R_f=1.05. Here in (a), the dash-dot lines correspond to unstable steady states while stable steady states are given by solid lines.

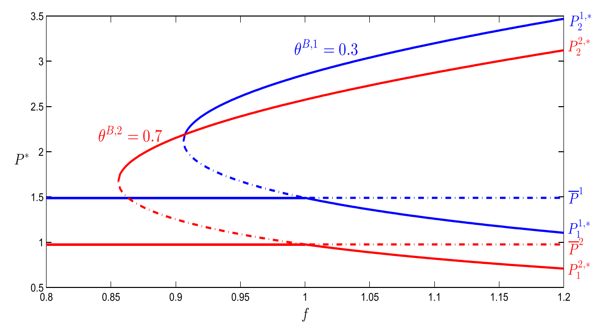


Fig. 5.1. Steady states at Y^B=10, k^B=2, $\mu = 0.1$, $C = 0.1$, $\alpha^I = 5$, $\sigma^I = 0.1$, g=0.5, $\varphi^I = \varphi^B = 1$ and R_f=1.05, where the dash-dot lines correspond to unstable steady states while stable steady states are given by solid lines.

mark price $\overline{P}$ loses its stability while the originally unstable steady state P_1^* becomes stable, because of a transcritical bifurcation at $f = f^{**}$. The system remains to be divided into two domains defined by the two attractors (P_1^*, P_2^*), separated by the benchmark price $\overline{P}$. When the initial price is higher than the benchmark value, the housing price will diverge from the benchmark level until it hits P_2^*. Otherwise, the housing price will converge to a lower level P_1^*. The following three reasons make the path-dependent phenomenon occur. First, the extrapolation from investors is strong enough for prices to move away from the benchmark price, which leads to the instability of the benchmark price. Second, investors with extrapolative strategy are trend chasers whose buying/selling decision depends on the initial market price relative to the benchmark level. Finally, mean-reverting investors act as contrarians in the market, whose strategy balances out the position of the trend extrapolators. Thus, in different initial situations, housing prices will converge to different non-benchmark levels, which corresponds to the localization property in housing markets.

5. Policy implications and application

Based on the analysis in Section 4, the structural knowledge of the system may yield important policy insights that can reduce the volatility of housing prices. For example, by (2.8) and Proposition 4.3, we know that the evolution of the housing market depends on the market environment because the benchmark price $\overline{P}$ and the bifurcation points f^* and f^{**} all depend on certain fundamental factors, for example, the down payment ratio and development cost. Thus, housing prices can be regulated through the adjustment of fundamental factors that change the market supply and/or demand. In the following, we take the down payment ratio and development cost as examples to illustrate the relationship between housing prices and the policy implementations within the framework given in Section 4.2. At last, with real data on Beijing housing prices, practical relevance of the paper is illustrated.

5.1. Down payment ratio

From (2.2) and (2.8), we know that the down payment ratio can affect the real demand of house buyers and the benchmark level of housing prices, which is the reference price adopted by investors. Therefore, policy makers can regulate housing markets by changing the down payment ratio. However, whether the adjustment of the down payment ratio has the desired effect depends on the timing of the policy adjustment. We take $\theta^B = 0.3$ and $\theta^B = 0.7$ to illustrate our point.

In the long run, we know that because of a higher real demand corresponding to a lower down payment, the benchmark price at $\theta^B = 0.3$ is higher than that at $\theta^B = 0.7$ and so are the corresponding non-benchmark prices, as shown in Fig. 5.1. Therefore, in the long run, an increase(decrease) in the down payment ratio may cool(warm) the housing market to some extent. However, this may not be true in the short run.

For instance, if investors have very strong extrapolative intensity like f=1.1 which makes the benchmark price unstable, then when the market is booming, the extrapolative belief of investors will push the housing price up and up. At that time, the policy maker might want to increase the down payment ratio in order to avoid the housing market overheating. However, the effectiveness of this adjustment is state-dependent. If the price is still in its up-trend (for example, at around n=70), then the market will continue its trend after the increase in the down payment ratio from $\theta^B = 0.3$ to $\theta^B = 0.7$ as illustrated in Fig. 5.2(a). Therefore, in this case, the adjustment of the down payment ratio does not have a significant impact on the housing market. Only when the market has been booming for a while (for example, at n=200), can the increase in the down payment ratio push the housing price down, as shown in Fig. 5.2(c). The amount of the drop in prices, however, is limited. The intuition is as follows. Although the down payment ratio plays a role in setting the benchmark value, it has the limited impact on investors' beliefs about the housing price trend. Increasing the down payment ratio, therefore, mainly drives some real demand out of the market but not the speculative demand, and sometimes even increases investors' demand. Similarly, when the market goes down, decreasing the down payment ratio provides the limited price support no matter when this policy adjustment is put into effect, as illustrated in the right panel of Fig. 5.2.

In summary, the effectiveness of adjusting the down payment ratio depends on the timing of the policy implementation and policy regulations based on the down payment ratio have limited power.

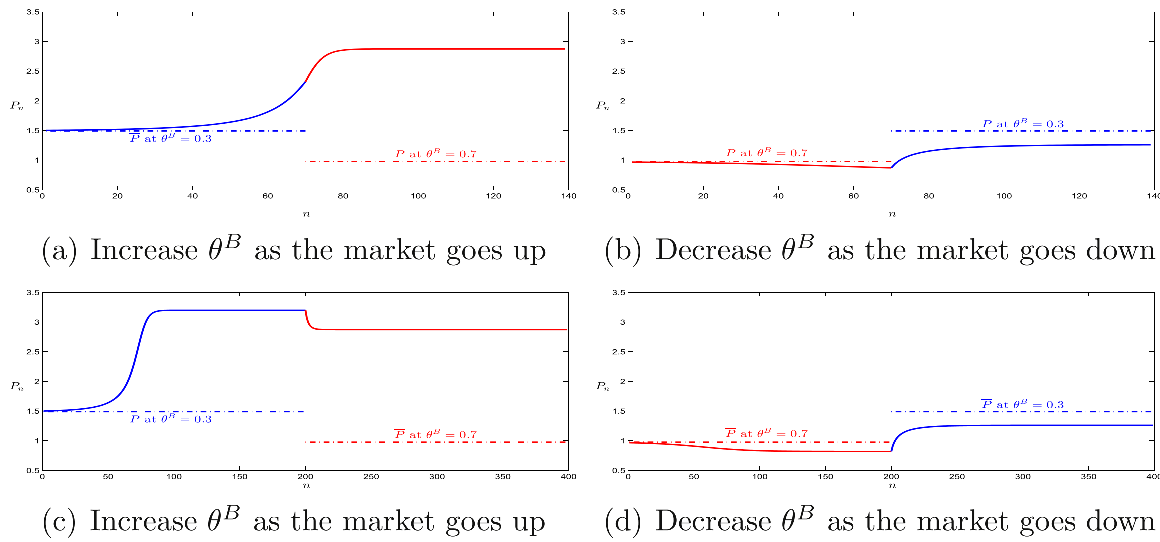


Fig. 5.2. Time series at Y^B=10, k^B=2, μ = 0.1, C = 0.1, α^I = 5, σ^I = 0.1, g=0.5, f=1.1, $\varphi^I = \varphi^B$ = 1 and R_f=1.05.

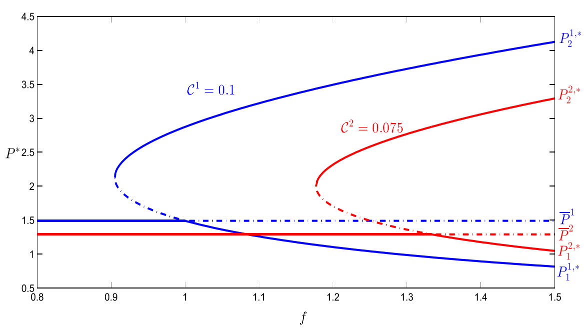


Fig. 5.3. Steady states at Y^B=10, k^B=2, θ^B = 0.3, μ = 0.1, α^I = 5, σ^I = 0.1, g=0.5, $\varphi^I = \varphi^B$ = 1 and R_f=1.05, where the dash-dot lines correspond to unstable steady states while stable steady states are given by solid lines.

5.2. Development cost

Different from the down payment ratio, by Propositions 4.2 and 4.3, we know that the development cost (C) not only affects the supply of developers and the benchmark price, but also determines the impact of investors on housing prices because f^* and f^{**} both depend on C. The development cost in our model could include land cost, financial cost, and construction cost. Among them, land cost should be the biggest cost for developers. Lands are owned by governments in some countries, who can adjust the land supply to change the development cost of developers and furthermore, to regulate the housing market. We take $C = 0.075$ and $C = 0.1$ as an example to illustrate the effectiveness of adjusting the development cost. From Fig. 5.3, we can see that when the development cost decreases, the corresponding benchmark price decreases because of the increase of developers' supply. However, different from the case of the down payment ratio, the corresponding bigger non-benchmark price (P_2^*) becomes smaller while the corresponding smaller non-benchmark price (P_1^*) becomes bigger. Therefore, the policy adjustment based on the development cost has different impact on housing prices, compared to the down payment ratio.

When the market goes up, if the policy maker can decrease the development cost of developers (for example, by supplying cheaper lands to developers), then the housing market can be effectively cooled down. We take $C = 0.1$ and f=1.1 to illustrated it. Because of the high extrapolative intensity of investors, an up-trend of the housing market is reinforced as the benchmark price is unstable. At that time, a drop of the development cost from $C = 0.1$ to $C = 0.075$, on one side, similar to the case of the down payment ratio, decreases the benchmark level of the housing price and increases the rationality of the market because of the increasing market fraction of mean-reverting investors. On the other side, it weakens the sensitivity of extrapolative investors corresponding to the increase of f^{**}, which leads the new benchmark price stable at $C = 0.075$. Therefore, the housing market can be cooled down to converge to the new benchmark level, see Fig. 5.4(a). Similarly, when the market goes down, decreasing the development cost is also a better measure to adjust the housing market because in this case, the new benchmark price is stable, which attracts the market back to its fundamental level, as illustrated in Fig. 5.4(b).

On the contrary, no matter whether the market is overheated with bubbles or stuck in depression, an increase of the development cost is not a good way to adjust it. In some cases, this method even can deteriorate the market. This is because the increase of the development cost can raise the benchmark price and strengthen the sensitivity of extrapolative investors (that is the decrease of f^{**}), which makes the whole housing market more unstable and catalyzes the further boom or bust of the market, as shown in the bottom panel of Fig. 5.4.

Summing up, decreasing the development cost can increase the rationality of the market and decrease the sensitivity of extrapolative investors, which helps the stability of the benchmark price, reviving the market and stabilizing housing prices.

5.3. Application

To illustrate the effectiveness of the model, we use monthly Beijing housing prices from the WIND Financial Terminal[11] to test the explanation power of our model on the real housing market. For the data from February, 2002 to February, 2016, shown in Fig. 5.5, although during the Financial crisis in 2008, there are some adjustments in the housing prices, we can see that the real time series of the Beijing housing market has a significantly increasing pattern. In particular, after the financial crisis, the housing prices of Beijing bounce up quickly, beyond the increases of the fundamental economic factors. Therefore, the behavior of speculative investors would exist in the market, similar to the phenomena illustrated in (4.14). Note that system (4.14) is a deterministic nonlinear dynamic system while the real data is a stochastic time series. To match this gap, based on the idea of He and Li (2015), we introduce a noise term in (4.14) as follows

[11] WIND is a leading financial data provider in China, providing accurate and complete data on financial markets and the macroeconomy.

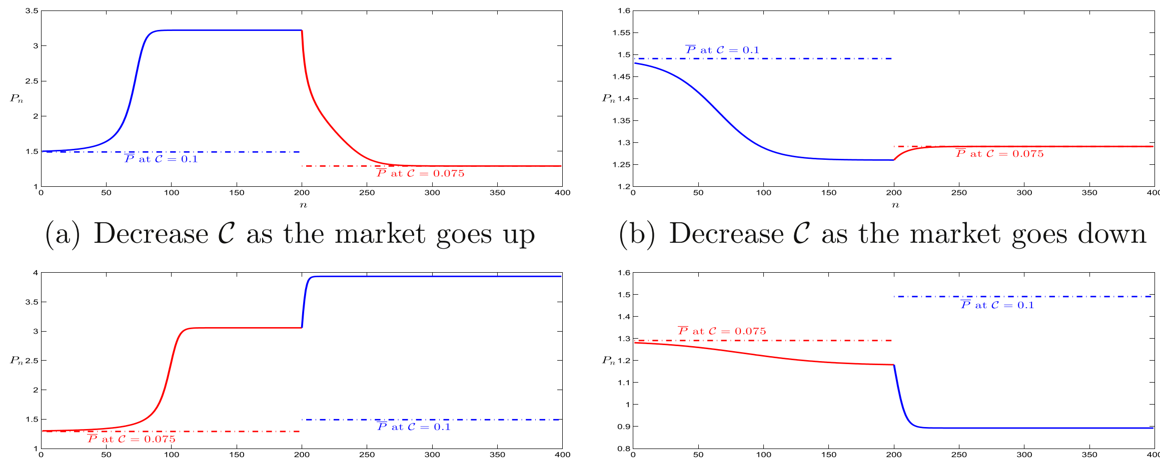

(a) Decrease $\mathcal{C}$ as the market goes up (b) Decrease $\mathcal{C}$ as the market goes down
(c) Increase $\mathcal{C}$ as the market goes up (d) Increase $\mathcal{C}$ as the market goes down

Fig. 5.4. Time series at Y^B=10, k^B=2, $\mu = 0.1, \theta^B = 0.3, \alpha^I = 5, \sigma^I = 0.1$, g=0.5, $\varphi^I = \varphi^B = 1$ and R_f=1.05. (a–b) Decrease $\mathcal{C}$ from 0.1 to 0.075 at f=1.1 and (c–d) increase $\mathcal{C}$ from 0.075 to 0.1 at f=1.4.

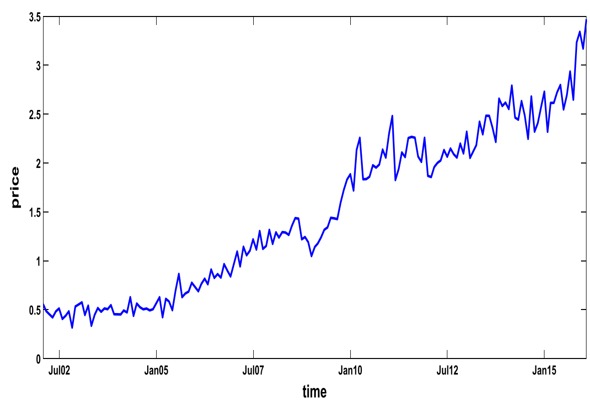


Fig. 5.5. The real housing prices of Beijing from February, 2002 to February, 2016. Here the price unit is RMB 10,000.

$$P_n = G(P_{n-1}) = \frac{m_{1,n}(P_{n-1}) + \sqrt{m_{1,n}^2(P_{n-1}) + 4m_0 m_2}}{2m_0} + \varepsilon_n, \quad (5.1)$$

where $\varepsilon_n \sim \mathcal{N}(0, \sigma_\varepsilon)$ captures a market noise either driven by unexpected news about fundamentals, or representing noise created by noise traders. Thus, the nonlinear dynamic model considered in Section 4 can be treated as the deterministic skeleton of the corresponding stochastic model. The prices observed in the real market are presumably the outcome of the interaction of both nonlinear and stochastic elements.

To describe the characteristics of the real data, we need to select a set of structural parameters of the model (5.1) through a systematic procedure. In particular, we pay attention to the range of changes in the real prices and their autocorrelations. Autocorrelation is often used as a measure of predictability of price changes in the empirical literature. We should emphasize that econometric analysis, especially estimation of heterogeneous agent models (henceforth, HAMs) is still a challenging task, see for instance, a recent comprehensive review by Chen et al. (2012). Generally speaking, the difficulties of estimation come from the complexity of HAMs, together with (typically) many parameters, which makes verification of identification rather difficult, and thus proving consistency of estimation troublesome. For recent attempts to estimate HAMs, for example, Amilon (2008) and Franke (2009), the identification problem is typically circumvented by focussing on a relatively simple HAM, or by estimating a few key parameters only. Therefore, in this paper, following Li et al. (2010), we select structural parameters of interests in our housing pricing model that minimizes a distance between data based and model based parameters.

By (5.1), we know that in our model, there are six important parameters which are the risk characteristics of investors ($\mathcal{A}^I$), the real demand factor (m_2), the intensity of switching speed (μ), the extrapolative intensity (f), the mean-reverting speed (g) and the development cost ($\mathcal{C}$). If we can determine those six parameters, then the evolution of system (5.1) is known. Note that the six parameters should be determined simultaneously using the real data of housing prices and in addition, the size of the real data is relatively large. Hence, to find the optimal solution of the parameters, a genetic algorithm is adopted to solve the problem.

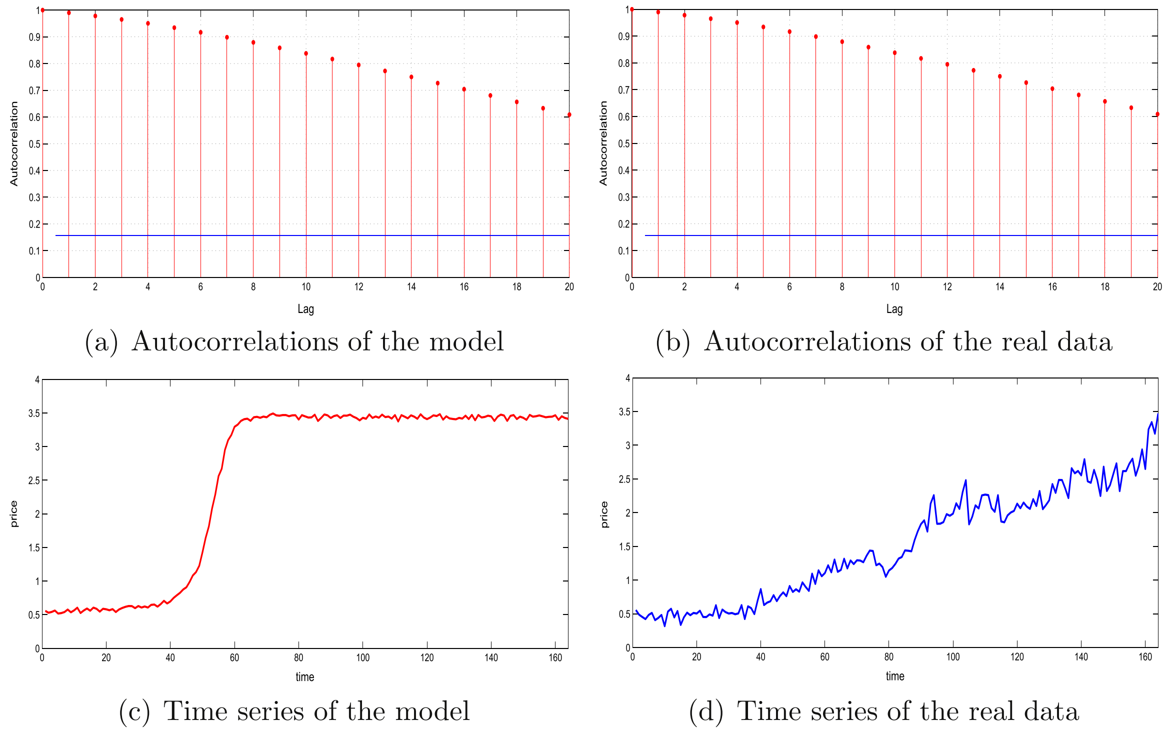
A genetic algorithm is derived from the theory of natural selection. Genetic algorithms are commonly used to generate high-quality solutions to optimization and search problems by relying on bio-inspired operators such as mutation, crossover and selection. In this paper, the six parameters of the model are encoded into chromosome. The differences between numerical features of the time series of the real housing prices, and those generated by the model are functioned into the objective function. More precisely, denote Θ the parameter space including the above six parameters. Let $\theta = (\mathcal{A}^I, m_2, \mu, g, \mathcal{C}, f) \in \Theta$ be the vector of the six parameters, X_M be the simulated time series of the model, X_{BJ} be the real data of the Beijing housing prices, $\widehat{\beta}_M$ be the estimated autocorrelations of the model, and $\widehat{\beta}_{BJ}$ be that of the real Beijing housing prices. Thus, we solve

$$\theta^* \in \underset{\theta \in \Theta}{\operatorname{argmin}}(\|\widehat{\beta}_M - \widehat{\beta}_{BJ}\|^2 + \omega_1|\max(X_M) - \max(X_{BJ})| + \omega_2|\min(X_M) - \min(X_{BJ})|) \quad (5.2)$$

for the standard Euclidian norm $\|\cdot\|$ and $\omega_1, \omega_2 > 0$ are adjustment weights.[12] A genetic algorithm is then exploited to find the optimal parameters (θ^*) to match the characteristics of the real housing prices. From Fig. 5.6, we can see that the selection method of model parameters is very effective. The selected housing pricing model is able to characterize successfully not only the strong autocorrelations, but also the persistently increasing pattern in the Beijing housing prices as well.

From Table 1, we know that this continuously increasing pattern is generated by the extrapolative behavior of investors. In fact, we can see that during the period from February, 2002 to February, 2016, the extrapolative intensity (f) of investors is very strong and is much bigger than the saddle-node and transcritical bifurcation points (f^* and f^{**})

[12] The adjustment weights are used to let the deviations of the autocorrelations and those of the price maximum and minimum at the same level.



(a) Autocorrelations of the model

(b) Autocorrelations of the real data

(c) Time series of the model

(d) Time series of the real data

Fig. 5.6. Autocorrelations and time series of the real data and the simulated data.

both, which means that the benchmark price is unstable and the price will deviate from the benchmark price upwards because its initial price is bigger than $\overline{P}$. In other words, in the real housing market, compared with some fundamental factors, including national income, down payment ratio and development cost, the behavior of investors, including their bounded rationality and heterogeneity, is more important to push the housing price up.

For the continuously increasing trend of the Beijing housing prices, the Beijing government has taken many measures to cool down the Beijing housing market, for example, by increasing the down payment ratio and adding restrictions to home-purchase qualification. However, the consequence of these policies is not obvious, that is, the increasing trend has not changed. From (5.1), we can get some insights on the failure of these policies. In particular, we know that from April, 2010, the government took steps to cool the housing market by increasing the down payment ratio to at least 30% and especially, to 50% for the second house, which was later increased to 60% in 2011 and then 70% in 2013. However, we can see that these measures have little impact on the Beijing housing prices. In fact, using the model (5.1), we can calibrate the parameters corresponding to the situations before and after the housing policy adjustments. That is, we take the data before April, 2010 as the market without increasing the down payment ratio and the data after May, 2010 as the market adjusted by the down payment policies. From Table 1, before the policy change, the extrapolative behavior is very salient, which makes the benchmark price unstable because of $f > f^{**}$. By contrast, after increasing the down payment ratio, the extrapolative intensity is decreased while the mean-reverting speed is increased. It seems that it would cool the market down because the mean-revering behavior has the role of stabilizing the housing price and the extrapolative behavior is weakened. However, we know that all the six parameters, not just one, play a part in the housing market together. Hence, on the whole, the extrapolative and mean-reverting behavior cannot cool the market down because we can see that the extrapolative intensity (f) is still bigger than f^{**}, which means that the housing price is still kept far away from the benchmark price. Similar to the illustration in Section 5.1, the effectiveness of the down payment policy is not obvious.

Table 1
Optimal parameters.

Time period	$\mathcal{A}^l$	m_2	μ	g	C	f	f^*	f^{**}
2002–2016	1.2362	0.1562	0.1002	0.0884	1.9378	1.1778	0.6709	0.8513
2002–2010	1.0512	0.1937	0.3497	0.0526	1.5714	1.5364	1.0081	1.2107
2010–2016	1.6921	1.8027	0.1925	0.4011	1.7454	0.5328	0.2898	0.3041

Therefore, the housing price cannot regress back to the benchmark level and may even deviate further away.

The overall analysis in this subsection shows that the selection method of model parameters is very effective. By calibrating the structure parameters, we can know the market situation and get some insights of the policy adjustments. This is probably due to the simplicity of the housing pricing model, which makes it possible to identify potential sources and mechanisms of the model in matching some important characteristics in the Beijing housing prices. However, this simplicity also makes the model unable to match too many characteristics, for example, the time series of the real data, illustrated in Fig. 5.6(c), which is beyond the scope of the paper and we leave it as our future work.

6. Conclusion

We study a housing pricing model with heterogeneous agents including house buyers, investors and property developers. Each agent maximizes his/her own objective subject to financial constraints. House buyers want to maximize their consumption utility based on consumption goods and real demands of houses when they are able to make down payments of the houses they buy. Investors make investment decisions in order to maximize their wealth's mean-variance utility based on their speculative expectations. Property developers supply houses to maximize their profit. Without speculative investors, the equilibrium housing price is determined by real cost and demand, which serves as a benchmark price. In contrast, because of the existence of speculative behavior, the equilibrium housing price may persistently deviate from the benchmark level and even explode.

Furthermore, the model produces several bifurcation routes to the deviations of housing prices from the benchmark value, as investors' extrapolative intensity increases. The bifurcations depend on different types of investors' behavior. Investors with only extrapolative strategy give rise to a transcritical bifurcation of the benchmark price, in which the benchmark price loses its stability and one non-benchmark price becomes a stable attractor. In this case, when the extrapolative intensity is large, an upward deviation of housing prices away from the benchmark level can lead to an explosion of the house market. In the whole evolutionary process, the housing price trends are persistent and predictable, which is observed empirically. However, investors with both extrapolative and mean-reverting strategies lead to a saddle-node bifurcation. Although the interaction between heterogeneous investors increases the complexity of the whole system such as the appearance of two attractors, investors' different strategies balance each other and help stabilizing the market to keep the system bounded. The appearance of two attractors makes the housing price evolution depend on its local value. This corresponds to the local characteristics of housing price changes observed in the real housing market. Finally, from the viewpoint of policy implications, the model indicates that the down payment ratio is not a very effective means to adjust the housing market whenever the market is booming or busting, which is illustrated by the real data of the Beijing housing market. But, decreasing the development cost of developers can increase the rationality of the market and decrease the sensitivity of extrapolative investors, which helps the stability of the benchmark price, reviving the market and stabilizing housing prices.

Therefore, by constructing some theoretical models, this paper demonstrates that speculative behavior of investors is an important factor that affects the housing price dynamics and evolution. The results obtained in this paper can provide interesting insights into the generation mechanism on some stylized facts observed in the real market and policy implications. However, in order to make the model parsimonious and to focus on the speculative behavior of investors, we consider a very simple housing market with heterogeneous agents in this paper. The house buyers' real demand is assumed to be based on the current consumption utility rather than on the whole life utility with solvency constraints. Justification and variation of the real demand with financial constraints are of interest. It is also interesting to extend the analysis to a non-smooth model in which investors have short selling constraints. For property developers, it is better that the delay effect of construction is regarded as their decision factor. Those improvements could contribute to a more realistic housing price model. We leave these issues for future research.

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Appendix A: Proofs of propositions

Proof of Proposition 3.1. By (3.3), it is easy to obtain

$$\frac{\partial \mathcal{P}}{\partial m_0} = \frac{-m_1(m_1 + \sqrt{m_1^2 + 4m_0m_2}) - 2m_0m_2}{2m_0^2\sqrt{m_1^2 + 4m_0m_2}} < 0,\ \frac{\partial \mathcal{P}}{\partial m_1} = \frac{\mathcal{P}}{\sqrt{m_1^2 + 4m_0m_2}} > 0,$$

and

$$\frac{\partial \mathcal{P}}{\partial m_2} = \frac{1}{\sqrt{m_1^2 + 4m_0m_2}} > 0.$$

Proof of Proposition 3.2. By $\frac{\partial \mathcal{P}}{\partial m_1} > 0$, we know $\mathcal{P} = \overline{P}$ only at $P_Q^I = R_f\overline{P}$. In addition, if $P_Q^I \gtreqless R_f\overline{P}$, then $P_Q^I \gtreqless R_f\mathcal{P}$ and furthermore, $\mathcal{H}^I \gtreqless 0$. From (3.3), we can obtain

$$\frac{\partial \mathcal{P}}{\partial \mathcal{A}^I} = \frac{\mathcal{P}}{(2m_0\mathcal{P} - m_1)\mathcal{A}^I}\mathcal{H}^I.$$

By $2m_0\mathcal{P} - m_1 > 0$, the other result can be obtained.

Proof of Proposition 4.1. (1) By (4.3) and letting $P_{n-1} = P_n = P^*$, we can know

$$(P^* - \overline{P})(\tilde{\mathfrak{c}}P^* + \tilde{\mathfrak{d}}) = 0, \tag{A.1}$$

where

$$\tilde{\mathfrak{c}} = \frac{1}{C} - \mathcal{A}^I f \quad \text{and} \quad \tilde{\mathfrak{d}} = \frac{\overline{P}}{C}.$$

Therefore, when $f < \tilde{f}^*$, (A.1) has a unique positive solution, which is the benchmark price $\overline{P}$. When $f > \tilde{f}^*$, (A.1) has another positive solution $\tilde{P} = \frac{\overline{P}}{\mathcal{A}^I Cf - 1}$.

(2) By (4.5), we know,

$$P_n = \tilde{G}(P_{n-1}, f) = \frac{\tilde{m}_{1,n} + \sqrt{\tilde{m}_{1,n}^2 + 4m_0m_2}}{2m_0}. \tag{A.2}$$

Then[13]

$$\frac{\partial P_n}{\partial P_{n-1}}\bigg|_{P_{n-1}=\overline{P}} = \widetilde{G}_P(\overline{P}, f) = \frac{\mathcal{A}^I C(R_f + f)}{\mathcal{A}^I C R_f + 2}. \tag{A.3}$$

and

$$\frac{\partial P_n}{\partial P_{n-1}}\bigg|_{P_{n-1}=\widetilde{P}} = \widetilde{G}_P(\widetilde{P}, f) = \frac{\mathcal{A}^I C(R_f + f)}{(\mathcal{A}^I Cf - 1)^2 + \mathcal{A}^I C R_f + 1}. \tag{A.4}$$

Therefore, $\overline{P}$ is stable as $f < f^{**}$ but unstable as $f > f^{**}$ while $\widetilde{P}$ is unstable as $\widetilde{f}^* < f < f^{**}$ but stable as $f > f^{**}$.

(3) At $f = f^{**}$, we know that $\overline{P} = \widetilde{G}(\overline{P}, f^{**})$, $\widetilde{G}_P(\overline{P}, f^{**}) = 1$ and $\widetilde{G}_f(\overline{P}, f^{**}) = 0$. In addition, it is easy to test

(1) Nondegeneracy condition:

$$\widetilde{G}_{PP}(\overline{P}, f^{**}) \neq 0,$$

(2) Transversality condition:

$$\widetilde{G}_{Pf}(\overline{P}, f^{**}) \neq 0.$$

Therefore, by the bifurcation theory,[14] we know that at $(P_{n-1}, f) = (\overline{P}, f^{**})$, (A.2) undergoes a transcritical bifurcation, such that when $f > f^{**}$, $\overline{P}$ loses its stability and $\widetilde{P}$ changes from unstable to stable.

Proof of Proposition 4.2. Denoting P_1^*, P_2^* and P_3^* as three solutions of (4.16) and by Vieta's formulas, then we know $P_1^* P_2^* P_3^* = -\frac{d}{a} < 0$ which means (4.16) has a negative real root, denoted as P_3^*. For another two roots, P_1^* and P_2^*, they depend on the discriminant (Δ) of (4.16).

(1) When $\Delta > 0$, P_1^* and P_2^* are nonreal complex conjugate roots.
(2) When $\Delta = 0$, P_1^* and P_2^* are equal real roots.
(3) When $\Delta < 0$, P_1^* and P_2^* are different real roots.

In addition, by $P_1^* + P_2^* + P_3^* = -\frac{b}{a} > 0$, we know when $\Delta \leq 0$, P_1^* and P_2^* are positive.

Proof of Proposition 4.3. (1) By (4.14), we know

$$P_n = G(P_{n-1}, f) = \frac{m_{1,n} + \sqrt{m_{1,n}^2 + 4m_0 m_2}}{2m_0}. \tag{A.5}$$

Then similar to Proposition 4.1, we can prove that when $f < f^{**}$, $\overline{P}$ is a stable steady state.

(2) Now we first prove that $0 < f^* < f^{**}$. Note that f^* is a solution of $\Delta = 0$ where Δ can be written as the following format

$$\Delta = \Delta(f) = \delta_0 f^3 + \delta_1 f^2 + \delta_2 f + \delta_3. \tag{A.6}$$

By the symbolic computation in Matlab R2010a, we can know the discriminant of (A.6) is positive and $\delta_0\delta_3 < 0$. Thus, (A.6) has two nonreal complex conjugate roots and one real positive root which is f^*. Because of $\Delta(0) > 0$ and $\Delta(f^{**}) < 0$, then $0 < f^* < f^{**}$.

(3) When $f < f^*$, the discriminant of (4.16) is positive, that is $\Delta > 0$, and then $\overline{P}$ is a unique positive steady state of (A.5) but at $f = f^*$, two new steady states of (A.5) appear which are $P^* = P_1^* = P_2^* > \overline{P}$ and

$$\frac{\partial P_n}{\partial P_{n-1}}\bigg|_{P_{n-1}=P_1^*, f=f^*} = G_P(P_1^*, f^*) = \frac{N^*}{D^*}, \tag{A.7}$$

where

$$N^* = \mathcal{A}^I P_1^*[R_f + \omega^*(2\omega^* - 1)f^* - (1 - \omega^*)(2\omega^* + 1)g],\ D^* = \sqrt{m_{1,n}^2 + 4m_0 m_2}\,\big|_{P_{n-1}=P_1^*, f=f^*}.$$

and $\omega^* = (1 + \mu(P_1^* - \overline{P})^2)^{-1}$. By the symbolic computation in Matlab R2010a, we can get $G_P(P_1^*, f^*) = 1$.

In addition, by the assumption,

[13] We use subscripts for partial derivatives:

$$\widetilde{G}_P(P^*, f^*) \equiv \frac{\partial \widetilde{G}(P_{n-1}, f)}{\partial P_{n-1}}\bigg|_{P_{n-1}=P^*, f=f^*}.$$

Similarly, $\widetilde{G}_f = \frac{\partial \widetilde{G}}{\partial f}$, $\widetilde{G}_{PP} = \frac{\partial^2 \widetilde{G}}{\partial P_{n-1}^2}$ and $\widetilde{G}_{Pf} = \frac{\partial^2 \widetilde{G}}{\partial P_{n-1}\partial f}$.

[13] We use subscripts for partial derivatives:

$$\widetilde{G}_P(P^*, f^*) \equiv \frac{\partial \widetilde{G}(P_{n-1}, f)}{\partial P_{n-1}}\bigg|_{P_{n-1}=P^*, f=f^*}.$$

Similarly, $\widetilde{G}_f = \frac{\partial \widetilde{G}}{\partial f}$, $\widetilde{G}_{PP} = \frac{\partial^2 \widetilde{G}}{\partial P_{n-1}^2}$ and $\widetilde{G}_{Pf} = \frac{\partial^2 \widetilde{G}}{\partial P_{n-1}\partial f}$.

[13] We use subscripts for partial derivatives:

(Nondegeneracy Condition): $G_{PP}(P_1^*, f^*) < 0,$

and moreover, by $P_1^* = P_2^* > \overline{P}$ at $f = f^*$, it is easy to get

(Transversality Condition): $G_f(P_1^*, f^*) > 0.$

Therefore, we know that at $(P_{n-1}, f) = (P_1^*, f^*)$, (A.5) undergoes a saddle-node bifurcation, such that when $f > f^*$, P_2^* is stable and P_1^* is unstable.

(4) Similar to Proposition 4.1, we can prove that at $(P_{n-1}, f) = (\overline{P}, f^{**})$, (A.5) satisfies the nondegeneracy and transversality conditions of a transcritical bifurcation, that is

$G_{PP}(\overline{P}, f^{**}) \neq 0$ and $G_{Pf}(\overline{P}, f^{**}) \neq 0.$

Therefore, we know that at $(P_{n-1}, f) = (\overline{P}, f^{**})$, (A.5) undergoes a transcritical bifurcation.

References

Adam, K., Kuang, P., Marcet, A., 2011. House price booms and the current account. NBER Macroecon. Annu. 26 (1), 77–122.
Alonso, W., 1964. Location and Land Use. Harvard University Press, Cambridge, MA.
Amilon, H., 2008. Estimation of an adaptive stock market model with heterogeneous agents. J. Empir. Financ. 15 (2), 342–362.
Basak, S., 2005. Asset pricing with heterogeneous beliefs. J. Bank. Financ. 29, 2849–2881.
Bauer, C., De Grauwe, P., Reitz, S., 2009. Exchange rate dynamics in a target zone: a heterogeneous expectations approach. J. Econ. Dyn. Control 33, 329–344.
Bolt, W., Demertzis, M., Diks, C., Hommes, C., van der Leig, M., 2014. Identifying Booms and Busts in House Prices Under Heterogeneous Expectations. Tinbergen Institute Discussion Paper, TI 2014-157/II.
Boswijk, H., Hommes, C., Manzan, S., 2007. Behavioral heterogeneity in stock prices. J. Econ. Dyn. Control 31, 1938–1970.
Brock, W., Hommes, C., 1997. A rational route to randomness. Econometrica 65, 1059–1095.
Brock, W., Hommes, C., 1998. Heterogeneous beliefs and routes to chaos in a simple asset pricing model. J. Econ. Dyn. Control 22, 1235–1274.
Case, K.E., Shiller, R.J., 1989. The efficiency of the market for single-family homes. Am. Econ. Rev. 79 (1), 125–137.
Chen, S.-H., Chang, C.-L., Du, Y.-R., 2012. Agent-based economic models and econometrics. Knowl. Eng. Rev. 27 (2), 187–219.
Chiarella, C., He, X., Zwinkels, R., 2014. Heterogeneous expectations in asset pricing: empirical evidence from the S&P500. J. Econ. Behav. Organ. 105, 1–16.
Chiarella, C., Dieci, R., He, X., 2009. Heterogeneity, market mechanisms and asset price dynamics. In: Hens, T., Schenk-Hoppé, K. (Eds.), Handbook of Financial Markets: Dynamics and Evolution. North-Holland, Amsterdam, pp. 277–344.
Clayton, J., 1998. Further evidence on real estate market efficiency. J. Real. Estate Res. 15, 41–57.
Day, R., Huang, W., 1990. Bulls, bears and market sheep. J. Econ. Behav. Organ. 14, 299–329.
De Grauwe, P., 2012. Booms and busts in economic activity: a behavioral explanation. J. Econ. Behav. Organ. 83, 484–501.
Dieci, R., Westerhoff, F., 2012. A simple model of a speculative housing market. J. Evolut. Econ. 22, 303–329.
Dieci, R., Westerhoff, F., 2013. Modeling house price dynamics with heterogeneous speculators. In: Bischi, G., Chiarella, C., Sushko, I. (Eds.), Global Analysis of Dynamic Models in Economics and Finance. Springer-Verlag, 35–61.
Föllmer, H., Horst, U., Kirman, A., 2005. Equilibria in financial markets with heterogeneous agents: a probabilistic perspective. J. Math. Econ. 41, 123–155.
Franke, R., 2009. Applying the method of simulated moments to estimate a small agent-based asset pricing model. J. Empir. Financ. 16 (5), 804–815.
Gelain, P., Lansing, K.J., 2014. House prices, expectations, and time-varying fundamentals. J. Empir. Financ. 29, 3–25.
Glaeser, E.L., Gyourko, J., Saiz, A., 2008. Housing supply and housing bubbles. J. Urban Econ. 64, 198–217.
Glaeser, E.L., Gyourko, J., Morales, E., Nathanson, C., 2014. Housing dynamics: an urban approach. J. Urban Econ. 81, 45–56.
Granziera, E., Kozicki, S., 2015. House price dynamics: fundamentals and expectations. J. Econ. Dyn. Control 60, 152–165.
He, X., Westerhoff, F., 2005. Commodity markets, price limiters and speculative price dynamics. J. Econ. Dyn. Control 29, 1577–1596.
He, X., Li, Y., 2015. Testing of a market fraction model and power-law behaviour in the DAX 30. J. Empir. Financ. 31, 1–17.
He, X.-Z., Li, K., Wei, J., Zheng, M., 2009. Market stability switches in a continuous-time financial market with heterogeneous beliefs. Econ. Model. 26, 1432–1442.
Hommes, C., 2006. Heterogeneous agent models in economics and finance. In: Tesfatsion, L., Judd, K.L. (Eds.), Handbook of Computational Economics. Vol. 2, Agent-based Computational Economics. North-Holland, Amsterdam.
Kabundi, A., Schaling, E., Some, M., 2015. Monetary policy and heterogeneous inflation expectations in South Arica. Econ. Model. 45, 109–117.
Kuznetsov, Y., 2004. Elements of Applied Bifurcation Theory, 3rd Edition. Vol. 112 of Applied Mathematical Sciences, Springer.
Li, Y., Donkers, B., Melenberg, B., 2010. Econometric analysis of microscopic simulation models. Quant. Financ. 10 (10), 1187–1201.
Lux, T., 1998. The socio-economic dynamics of speculative markets: interacting agents, chaos, and the fat tails of return distributions. J. Econ. Behav. Organ. 33, 143–165.
Malpezzi, S., Wachter, S., 2005. The role of speculation in real estate cycles. J. Real. Estate Lit. 13, 143–164.
Piazzesi, M., Schneider, M., 2009. Momentum traders in the housing market: survey evidence and a search model. Am. Econ. Rev. 99 (2), 406–411.
Potter, S., 2011. The Failure to Forecast the Great Recession, Liberty Street Economics Blog, Federal Reserve Bank of New York. (November 25).
Roback, J., 1982. Wages, rents, and the quality of life. J. Political Econ. 90 (4), 1257–1278.
Rosen, S., 1979. Wage-based indexes of urban quality of life. In: Mieszkowski, P., Straszheim, M. (Eds.), Current Issues in Urban Economics. Johns Hopkins University Press, Baltimore.
Schindler, F., 2013. Predictability and persistence of the price movements of the S&P/Case-Shiller house price indices. J. Real. Estate Financ. Econ. 46, 44–90.
Schindler, F., Rottke, N., Fuess, R., 2010. Testing the predictability and efficiency of securitized real estate markets. J. Real. Estate Portf. Manag., 159–179.
Serrano, C., Hoesli, M., 2010. Are securitized real estate returns more predictable than stock returns? J. Real. Estate Financ. Econ. 41 (2), 170–192.
Shiller, R.J., 2005. Irrational Exuberance 2nd Edition. University Press, Princeton.
Shiller, R.J., 2008. The Subprime Solution. Princeton University Press, Princeton.
Wang, Y., Xu, F., Hu, A., 2013. Impact of heterogeneous beliefs and short sale constraints on security issuance decisions. Econ. Model. 30, 539–545.