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An optimal pricing scheme to improve transmission opportunities for a mobile virtual network operator

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ABSTRACT

Selling spectrum resources to mobile virtual network operators (MVNOs) is popular among primary network operators (POs) for increasing licensed spectrum utilization. An MVNO seeks *transmission opportunities* (TXOPs) on the PO's licensed spectrum channel(s) by spectrum sensing. Since licensed primary users (PUs) often have higher transmission priority, TXOPs available to the MVNO are directly determined by PUs' transmission behaviors. In this paper, we present the first study that uses a pricing scheme to regulate PUs' transmission behaviors so that TXOPs for an MVNO are improved, meanwhile quality-of-service (QoS) of PUs can be guaranteed as well. Our idea is to design a non-uniform pricing scheme that regulates PUs to transmit based on a time-varying non-uniform transmission cost. We model the optimal pricing problem as a hierarchical game where the interaction between the PO and PUs is modeled as a Stackelberg game and the spectrum random access among PUs is modeled as a non-cooperative game. We first solve the non-cooperative game among PUs, as a building block, then we embed its outcome to a Markov Chain Monte Carlo (MCMC) framework to solve the whole optimal pricing problem. We strictly shows that there could be multiple optimal pricing schemes for a PO. Comprehensive simulations confirm the theoretical analysis, and valid the effectiveness of our proposed scheme.

1. Introduction

Due to spectrum scarcity, free spectrum bands, which can be allocated to new mobile service operators as licensed spectrum resources, are becoming limited. In order to satisfy increasing demands on spectrum resources, radio management authorities such as FCC open the spectrum market thus allows licensed spectrum resources owned by licensed operators, often called primary operators (POs), to be sold to emerging operators who have no statically allocated spectrum bands. For example, in US, AT&T hosts 420 Wireless, AirVoice Wireless and Black Wireless as its secondary operators. Similar strategies are also used by Verizon, T-Mobile and Sprint, so as in UK like Vodafone and O2 as well. Those secondary operators are often called mobile virtual network operators (MVNOs) where “virtual” is in the sense that those MVNOs do not really own any spectrum bands. In this way, spectrum scarcity issue is mitigated and market demands is relatively fulfilled.

Not only aiming to satisfy the demands of emerging MVNOs who are unable to have any licensed bands

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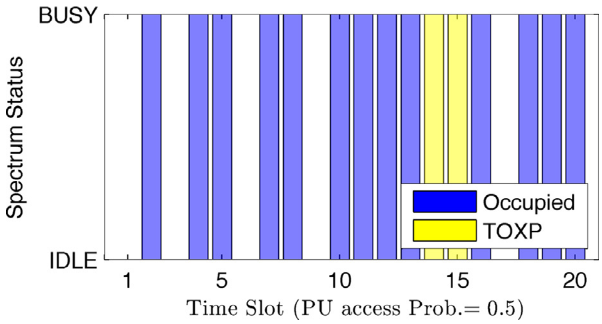


Fig. 1. A motivating example.

anymore, but also because of potential improvement on spectrum utilization of POs, renting licensed spectrum resources to MVNOs becomes popular among POs nowadays. One motivation behind is that POs wish, in future with cognitive radio technology [1], an MVNO will be able to access the spectrum resources cognitively so as to efficiently utilize the idle spectrum left by primary users (PUs). More importantly, POs also expect to gain extra revenues by charging transmissions from MVNOs. Driven by these, one rich line of research works done was to study how well an MVNO can exploit transmission opportunities (TXOPs) [2], and how a PO should charge an MVNO so that to gain maximum revenues [3–7]. In nutshell, existing works focused more on the subjects of TXOP exploitation and TXOP selling.

We observe that those works have an explicit prerequisite that idle spectrum already exist ahead, of which we think such a reason makes existing works less considered the influence of PUs' transmission behaviors to the availability of idle spectrum (i.e., TXOPs). A few prior investigations such as in [8,9] have, nevertheless, shown that PUs' behaviors can largely affect the availability of TXOPs. Clearly, due to unmanaged transmissions from PUs, TXOPs for MVNOs would be unstable and unpredictable. More importantly, as an MVNO will often sign *service-level-agreements* (SLAs) with a PO in reality, if the stability of TXOPs (e.g., a minimum available length) cannot be reached, those idle spectrum cannot be used by the MVNO. Consequently, some idle spectrum resources will be wasted since neither PUs nor the MVNO utilize them eventually.

Here we give a motivating example in Fig. 1 to illustrate the consequence. We define a TXOP to an MVNO as a window containing 2 continuously available time slots. During a time frame with 20 time slots, if a PU accesses the spectrum channel with probability 0.5, randomly there is only 1 TXOP (yellow bars), though there are total 5 idle time slots. This means that $\frac{3}{5} = 60\%$ idle slots are wasted (white bars). The root reason here is the PU's arbitrary transmissions producing many idle spectrum fragments that cannot be used by the MVNO either. Instead, if we can regulate the PU to transmit more regular (e.g. more compactly or more periodically), more continuously available time slots (i.e., TXOPs) may appear. Hence, in addition to TXOP exploitation and TXOP selling issues, we believe that it is equally important to study another line of issues, that may be more critical: how a PO can improve TXOPs for an MVNO, while such a study is still missing.

Our study will be positioned in a typical cellular base-station (BS) scenario as follows. A PO manages a cellular network and in one cell covered by a BS tower, there is a set of PUs, each of which contends to access the spectrum channel like TDMA. Meanwhile, with coexisting to the primary network (i.e., the PO + PUs), an MVNO seeks TXOPs cognitively and pays the PO for any successful occupations of TXOPs. Based on this scenario, our goal is to maximize the total utilization/revenues of the spectrum channel, subject to (1) the *quality-of-service* (QoS) of PUs and (2) the SLAs of the MVNO signed with the PO. The objective accounts for two parts of revenues where the first is the revenues generated by PUs' transmissions whose behaviors directly determine the TXOPs to the MVNO, who only generates the second part of revenues to the PO when the idle spectrum fulfill the SLAs requirement.

It is not a trivial task to generate more TXOPs fulfilling the SLAs requirement as well as to guarantee meanwhile the QoS of PUs because of the following reasons. First of all, PUs' transmissions are hardly to be controlled directly as they are distributed and selfish endpoints. In addition, it is even more difficult to predict their influence to the TXOPs to the MVNO because outcomes of PUs' random access are complex. In order to solve the problem, we use a pricing mechanism, namely *an invisible hand*, to stimulate PUs transmitting more regular. We formulate the pricing problem as a *Stackelberg game* modeling the interactions between the PO and PUs. For PUs, their spectrum random access are further formulated as a *non-cooperative game*. In the hierarchical game, the PO first prescribes an incentive pricing scheme to the PUs, then the PO observes the responses from PUs (i.e., the outcomes of the spectrum random access game among PUs) and measures how the TXOPs for the MVNO will be affected. Through optimization, the PO tries to derive optimal pricing scheme(s) subject to the constraints mentioned above.

To solve the hierarchical game, technically, such a problem is known as *a bilevel programming with equilibrium constraint* (BPEC) problem [10] (The equilibrium constraint here refers to the spectrum random access game among PUs). Theoretical results have shown that a BPEC problem is difficult because in most of cases the response from the followers (i.e., the PUs) is implicit. Consequently, traditional optimization techniques cannot be applied directly to find the optimal solutions for the leader (i.e., the PO). In addition to the inherited challenge as a typical BPEC problem, as to be shown later, our problem has a unique challenge in that the objective function of the leader is a "black-box", because it is impossible to know how many TXOPs can be generated given a pricing scheme analytically. This unique challenge further renders our problem to be a black-box BPEC problem [11], and thus hampers us to design an exact algorithm like gradient-based or approximation-based methods. Hence, we first design an efficient solver for the non-cooperative game among PUs at the lower level, and then use it as a building block to embed in a *Markov Chain Monte Carlo* (MCMC) framework, which holds promise for non-convex and black-box problems, to ultimately solve the optimal pricing problem. Our contributions are summarized as follows.

Table 1
The notations.

Notation table	
$\mathcal{N}$	The set of active PUs with a total number N.
T	The total number of time slots in a frame.
i	The index of the i-th PU.
$-i$	The indices of all PUs other than i.
j	The index of any PU other than i.
c_0	The average transmission cost of PUs.
c_s	The unit price for every slot transmission from an SO.
γ_t	The cost coefficient at time slot t ($\gamma_t \in [\gamma_l, \gamma_u]$).
$\boldsymbol{\gamma}$	The vector of all γ_t during [1, T].
a_{it}	The access probability of PU i at time slot t.
$\mathbf{A}$	The access probability matrix consisting of all a_{it}.
$\mathbf{a}_i$	The row vector consisting of all a_{it} of PU i.
$\hat{\mathbf{a}}_t$	The column vector consisting of all a_{it} at time slot t.
$\boldsymbol{\xi}$	The transmission status vector of all PUs.
p_i	The transmission power of PU i.
h_i	The power gain from PU i at the base station.
SINR_{it}	The signal-to-interference-plus-ratio of PU i at time slot t.
r_{it}	The data rate of PU i at time slot t.
ω_i	The weight coefficient for the utility gain of PU i.
τ	The time window with length ℓ_τ during [1, T].
$\mathbf{S}$	The set of all transmission opportunities during [1, T].
$\mathcal{R}_p$	The revenues from the PUs' transmissions during [1, T].
$\mathcal{R}_s$	The revenues from by the SO transmissions during [1, T].
$\mathcal{R}_T$	The total revenues from by both PUs and SO.

- We present the first study on the influence of PUs' behaviors to TXOPs for an MVNO and propose an optimal pricing scheme to regulate PUs' transmission behaviors so as to improve TXOPs for the MVNO.
- We formulate the optimal pricing problem as a hierarchical game consisting of a Stackelberg game between a PO and PUs, whose responses are modeled as the results of an N-player non-cooperative game among PUs. We prove the existence and uniqueness of the best response of PUs to a given pricing scheme, and there would exist multiple optimal pricing schemes for the PO.
- To solve the hierarchical game, we first develop an efficient solver for the spectrum random access game among PUs, based on which we further develop an MCMC-based algorithm to overcome the unique challenge where the objective function of the PO is a "black-box". To the best of the authors' knowledge, technically, this is also the first attempt to use an MCMC method in a gaming problem.

This paper is organized as follows. In Section 2, we formulate the optimal pricing problem as a hierarchical game. After that, the properties of the hierarchical game will be analyzed in Section 3. In Section 4, we develop efficient algorithms to solve the hierarchical game. Comprehensive numerical simulations will be conducted in Section 5. Related work will be discussed in Section 6 and conclusion is made in Section 7.

2. Problem formulation

In this section, we first formally define our system model and our pricing model, and then formulate the hierarchical game. To facilitate the discussions, we list important notations in Table 1.

2.1. The system model

As mentioned earlier, we investigate a typical cellular BS network scenario, where a BS covers a certain geographical area, known as a *cell*. In our study, we consider that $|\mathcal{N}| = N$ PUs within the cell have data to send to the BS. When transmitting data to the BS, all PUs are synchronized and the time horizon is partitioned into equal-sized time frames, each of which consists of T time slots[1].

In each time slot t, each PU i accesses the spectrum channel with an access probability a_{it}. In order to represent the transmission status of all PUs succinctly, we introduce an N-tuple vector $\boldsymbol{\xi}$ whose entry $\xi_i = 1$ if PU i transmits, $\xi_i = 0$ otherwise. Hence, at time slot t, the probability that PU i is transmitting together with some other active PUs can be calculated as:

$$\Pr\big[\boldsymbol{\xi}, \xi_i = 1\big] = \overbrace{a_{it} \prod_{\substack{\forall \xi_j = 1,\\ j \neq i}} a_{jt}}^{\text{Pr. of active PUs}} \cdot \overbrace{\prod_{\substack{\forall \xi_{j'} = 0,\\ j' \neq i}} (1 - a_{j't})}^{\text{Pr. of inactive PUs}}, \tag{1}$$

The data rate of each PU i depends on the *signal-to-interference-plus-noise-ratio* (SINR). Suppose that PU i transmits with a fixed power level p_i, and the power gain at the BS side is h_i, the SINR of PU i at time slot t will be:

$$\mathsf{SINR}_{it}(\boldsymbol{\xi}) = \frac{p_i h_i}{\mathsf{I}_0 + \sum_{j \neq i} \xi_j p_j h_j}, \tag{2}$$

in which I_0 is the level of background white noise, and the expected data rate of PU i in time slot t, denoted as r_{it}, will be:

$$r_{it} = \sum_{\forall \boldsymbol{\xi}, \xi_i = 1} \Pr[\boldsymbol{\xi}, \xi_i = 1] \cdot \log\big(1 + \mathsf{SINR}_{it}(\boldsymbol{\xi})\big). \tag{3}$$

where we assume that the unlicensed users (e.g., unlicensed users managed by the MVNO) also have ability to avoid collisions with PUs.

In this work, we assume the channel gain is identical during the time frame. We explain the major reasons as follows.

First of all, introducing a time-varying channel gain $\mathtt{h}(t)$ is straightforward, but this will not change the problem property fundamentally rather only modifies the equation of the channel rate r_{it}. However, such a time-varying factor will further complicated the formulation. Therefore, for simplicity, we assume channel gain is a constant during a time frame.

More importantly, the major goal of this work is to verify if the shapes of the desirable pricing schemes would present some regularities and specific forms, as to be shown in the simulation results. If we change the channel gain factor as time-varying, due to the resulted changes on expected gains in Eq. (3), such features may become not very obvious, which hampers us to reveal the insights for the pricing schemes.

It is worth to note that, thanks to the MCMC-based method, our proposed algorithm will also

[1] In a Long-Term Evolution (LTE) system, the duration of each frame is 10ms, which consists of 10 sub-frames or 20 time slots.

work - sampling from a different objective function with a time-varying the channel gain even without any modification. this again shows that the benefits of MCMC-based method on handling tough computational problems.

As mentioned earlier, we investigate a generalized scenario where an MVNO can have a specific requirement to the duration of a TXOP that is necessary for a particular frame structure or to transmit a minimum amount of data. Thus, we define TXOP as below:

Definition 1. A transmission opportunity (TXOP) is a time window τ in a frame whose interference level $I_\tau \le \zeta$ and length $\ell_\tau = \ell$ (ζ and ℓ are two predefined values).

Note that our definition is flexible and can cover different co-existence scenarios. First, if the length requirement is very small, then any available time duration can be a TXOP, which is assumed in many existing studies. Second, if the MVNO accesses spectrum with the overlay model, then we can let $\zeta = 0$, otherwise it becomes an underlay model, in which we let $\zeta > 0$.

Another point is that, in this work, we directly assume that such interference level is known. There are many ways to define this value. For example, it could be negotiated between the primary operator and the MVNO, or be calculated from empirical study based on historical statistics. Another important reason is that, since PUs are basically autonomous users in the primary network, real-time gathering the interference level and applying on the secondary market with the MVNO seems impractical. Moreover, on a spectrum market, we believe that a stable interference level is also desirable to the MVNOs because this value potentially affects their TXOPs, thereby affects the QoS of the secondary network in general.

2.2. The pricing model

Based on the system model above, we define the pricing model, i.e., how the PO charges PUs and sell TXOPs to the MVNO.

For the pricing model to PUs, we use c_0 to denote the *average* transmission cost per time slot. In most of existing studies, the cost per time slot during a frame is fixed. Thus, for a PU, as transmitting at any time the cost is the same, it will have no incentive to consider possibly different transmission costs but purely depend on its transmission demand. To create more TXOPs, we can encourage PUs to change their behaviors by non-uniform pricing schemes. Specifically, we define the cost per time slot as $\gamma_t c_0$, where γ_t is a time-varying cost coefficient for the time slot t. We further define $\boldsymbol{\gamma}$ as the cost vector consisting of all γ_t ($1 \le t \le T$). Since the average cost c_0 should be unchanged, we have:

$$\frac{1}{T}\sum_{t=1}^{T}\gamma_t = 1. \tag{4}$$

This constraint is practically imperative because it restricts the PO to behave rationally by preventing it from advertising any irrational pricing schemes like only lifting/discounting the prices at every time slot. Clearly, if any $\gamma_t = 1$, it reduces to the usual case with a uniform pricing scheme.

For the pricing model to the MVNO, we consider that the PO will charge c_s per time slot during a full TXOP, and the PO will not charge the MVNO if the available time window is shorter than that of a TXOP. For example, if $\ell = 3$ time slots and there is a time window τ with only $\ell_\tau = 2$ time slots, then the PO will not charge the MVNO because the MVNO cannot occupy a fully useful TXOP. With such a pricing model, when designing the optimal pricing scheme for PUs, the PO has to consider how much revenues could be generated from the MVNO.

2.3. The hierarchical game

Given the system model and the pricing model, we formulate the optimal pricing problem as a hierarchical game. The goal of the PO is to fully utilize its spectrum resources so as to maximize its total revenues, subject to a QoS constraint for PUs. The goal of a PU is, on the other hand, trying to transmit more by determining its optimal access probability a_{it} during a frame [1, T] according to its own transmission budget.

For compact and succinct representation, let an $N \times T$ matrix $\mathbf{A}$ be the access strategy matrix whose element a_{it} is the access probability of PU $i \in [1, N]$ at time slot $t \in [1, T]$.

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & \cdots & a_{1,t} & \cdots & a_{1,T} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{i,1} & \cdots & a_{i,t} & \cdots & a_{i,T} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{N,1} & \cdots & a_{N,t} & \cdots & a_{N,T} \end{bmatrix}_{N\times T}. \tag{5}$$

Clearly, the row vector $\mathbf{a}_i$ of $\mathbf{A}$ is the access probability vector of PU i in the whole frame, and the column vector $\hat{\mathbf{a}}_t$ of $\mathbf{A}$ is the access probability vector of all PUs at time slot t.

2.3.1. The spectrum random access game among PUs

For PU i, given a pricing scheme $\boldsymbol{\gamma} c_0$, its utility function $u_{it}(\cdot)$ at time slot t is its transmission gain minus its transmission cost, namely *net utility gain* (NUG):

$$u_{it}(\hat{\mathbf{a}}_t, \gamma_t) = \omega_i r_{it}(\hat{\mathbf{a}}_t) - a_{it}\gamma_t c_0, \tag{6}$$

where ω_i is the gain coefficient. The total NUG of PU i over a frame is the sum of NUG at every time slots:

$$u_i(\mathbf{A}, \boldsymbol{\gamma}) = \sum_{t=1}^{T} u_{it}(\hat{\mathbf{a}}_t, \gamma_t). \tag{7}$$

Each PU i considers to determine an optimal strategy trace $\mathbf{a}_i^*$ for every time slots in the frame by solving the optimization problem below.

$$\begin{aligned} \mathbf{P}_i : \max_{\mathbf{a}_i} & \, u_i(\mathbf{a}_i, \mathbf{A}_{-i}, \boldsymbol{\gamma}) \\ \text{s.t.} & \sum_{t=1}^{T} a_{it}\gamma_t c_0 \le b_i \\ & a_{it} \in [0, 1], \forall t \in [1, T], \end{aligned} \tag{8}$$

where $\mathbf{A}_{-i}$ represents the access probabilities of all PUs other than i, and b_i is the transmission budget limiting the total affordable costs of PU i. In the sequel, we refer the N-player non-cooperative game among PUs as $\{\mathbf{P}_i\}_{i\in\mathcal{N}}$.

2.3.2. The optimal pricing game between PO and PUs

We now consider the optimal pricing problem for the PO. We first represent revenues generated from the PUs and the MVNO by two functions $\mathcal{R}_p(\cdot)$ and $\mathcal{R}_s(\cdot)$, respectively. Revenues from PUs $\mathcal{R}_p(\cdot)$ can be calculated by the sum of the transmission costs paid by all PUs during a frame:

$$\mathcal{R}_p(\mathbf{A}, \boldsymbol{\gamma}) = \sum_{t=1}^{T}\sum_{i\in\mathcal{N}} a_{it}\gamma_t c_0. \tag{9}$$

Revenues from the MVNO $\mathcal{R}_s(\cdot)$ depends on the number of TXOPs during a time frame. To simplify the discussions, hereafter we assume that the duration of a TXOP is an integer multiple of the duration of a time slot. Now, let $\mathbf{S}$ be the set of all available windows (i.e., $\mathsf{I}_\tau < \zeta, \forall\tau\in\mathbf{S}$) during the frame, which depends on the access probability of all PUs (i.e., the matrix $\mathbf{A}$). Hence, $\mathcal{R}_s(\cdot)$ equals to the whole cost for all the TXOPs in $\mathbf{S}$, which can be formally calculated as:

$$\mathcal{R}_s(\mathbf{A}) = c_s \sum_{\tau\in\mathbf{S}} \left\lfloor \frac{\ell_\tau}{\ell} \right\rfloor \cdot \ell, \tag{10}$$

where $\lfloor \frac{\ell_\tau}{\ell} \rfloor$ represents the maximum number of TXOPs that can be used by the MVNO in the idle window τ. For instance, if an idle window $\ell_\tau = 5$ and a TXOP requires $\ell = 2$, then $\lfloor \frac{\ell_\tau}{\ell} \rfloor = \lfloor \frac{5}{2} \rfloor = 2$, meaning that there are at most 2 TXOPs (i.e., 4 time slots) in τ for the MVNO, and the PO can maximally gain revenues of $4c_s$ from this idle window.

Remarks. As mentioned before, this part of revenues (i.e., $\mathcal{R}_s(\cdot)$) is a "black-box" because (1) it depends on the exact form of the access probability matrix $\mathbf{A}$, which is the outcome of the spectrum random access game among PUs, and (2) even if we have the exact form of $\mathbf{A}$, we can only identify the idle window set $\mathbf{S}$ but have to algorithmically calculate the amount of TXOPs according to *Definition 1*. Recall that, the first challenge above inherits from being a BPEC problem, while the second one is the unique challenge of our problem.

With $\mathcal{R}_p(\cdot)$ and $\mathcal{R}_s(\cdot)$, the objective function of the PO is the sum of the two parts:

$$\mathcal{R}_T(\mathbf{A}, \boldsymbol{\gamma}) = \mathcal{R}_p(\mathbf{A}, \boldsymbol{\gamma}) + \mathcal{R}_s(\mathbf{A}). \tag{11}$$

There are some constraints for the PO when designing the optimal pricing scheme. The first constraint has been given in Eq. (4), which maintains the average price to the PUs unchanged. The second constraint for the PO is to guarantee the QoS of PUs, which considers that the average transmission rate of the primary network in a frame should be at least a pre-defined value $\tilde{r}$:

$$\frac{1}{N\cdot T}\sum_{t=1}^{T}\sum_{i\in\mathcal{N}} r_{it} \ge \tilde{r}. \tag{12}$$

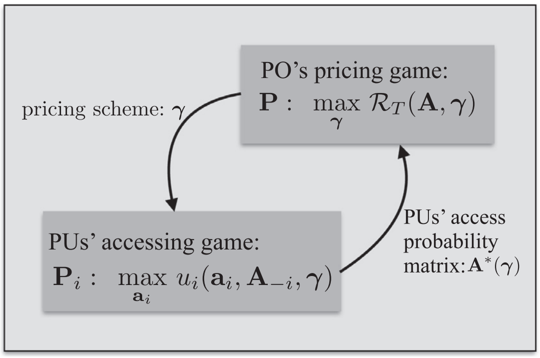


Fig. 2. The structure of the hierarchical game.

Note that other kinds of constraints such as fairness of individual users can be introduced in our formulation. Due to limited space, we only consider the average transmission rate above.

With the above definitions, the optimal pricing scheme problem for the PO (i.e., to find the optimal pricing scheme $\boldsymbol{\gamma}^* c_0$) can be formulated as:

$$\begin{aligned} \mathbf{P}: \ & \max_{\boldsymbol{\gamma}} \mathcal{R}_T(\mathbf{A}^*, \boldsymbol{\gamma}) \\ & \text{s.t.} \frac{1}{T}\sum_{t=1}^{T}\gamma_t = 1 \\ & \quad \frac{1}{N\cdot T}\sum_{t=1}^{T}\sum_{i\in\mathcal{N}} r_{it} \ge \tilde{r} \\ & \quad \mathbf{A}^*(\boldsymbol{\gamma}) = \arg\max\{\mathbf{P}_i\}_{i\in\mathcal{N}} \end{aligned} \tag{13}$$

where the equilibrium constraint appears as the constraint at the last line.

In practice, the PO can further link the values of c_s and c_0 based on its preference to different sets of users. Particularly, the PO can set $c_s < c_0$ if it prefers revenues from the PUs, and can set $c_s > c_0$ when it considers selling to the MVNO is more profitable. The impact of c_s/c_0 will be further investigated in the numerical discussions.

Overall, when the PO solves the problem $\mathbf{P}$ in (13), the best response of the PUs (i.e., $\mathbf{A}^*$) is given by solving the non-cooperative game defined by $\{\mathbf{P}_i\}_{i\in\mathcal{N}}$ in (8). The structure of the hierarchical game formulated in this section is sketched in Fig. 2.

3. Game property analysis

In this section, we first analyze the equilibrium properties of the spectrum random access game $\{\mathbf{P}_i\}_{i\in\mathcal{N}}$ among PUs. After that, we analyze the properties of the optimal pricing game $\mathbf{P}$ between the PO and PUs.

3.1. The spectrum random access game among PUs

According to the system model, the strategy of a PU is its access probability $a_{it}\in[0, 1]$, meaning that all PUs randomize between "transmit" and "back-off". Before we study the property of the randomized strategy of PUs, we first exclude the existence of pure-strategy *Nash* equilibrium by Theorem 1.

Theorem 1. *No pure-strategy Nash equilibrium exists in the spectrum random access game among PUs.*

Proof. We prove the statement by contradiction.

We first assume that the capacity of the spectrum channel is $\bar{n}$, which means that if more than $\bar{n}$ PUs transmit simultaneously, utility gains of PUs will be smaller than the transmission cost. We then assume that there exists a pure-strategy *Nash* equilibrium $\mathbf{A}^*$ where each PU deterministically chooses to either "transmit" or "back-off" (i.e., $\forall a_{it}^* \in \{0, 1\}$) during a frame. At time slot t, we denote the PUs choosing to transmit as $\mathcal{N}_{tx}$ (i.e., $a_{it}^* = 1, \forall i \in \mathcal{N}_{tx}$) and the PUs choosing to back off as $\mathcal{N}_{bo}$ (i.e., $a_{jt}^* = 0, \forall j \in \mathcal{N}_{bo}$). Hence, there are three possible cases as following.

Case $|\mathcal{N}_{tx}| < \bar{n}$, which means that the channel capacity is not saturated. Clearly, any PU $j \in \mathcal{N}_{bo}$ who is back off now, can improve its NUG by unilaterally deviating from $a_{jt}^* = 0$ to $a_{jt}^* = 1$. Thus, the total NUG of PU j will increase during the frame.

Case $|\mathcal{N}_{tx}| > \bar{n}$, which means that the channel usage is over its capacity. In this case, any PU $i \in \mathcal{N}_{tx}$ who is transmitting now, can save its transmission cost by unilaterally deviating from $a_{it}^* = 1$ to $a_{it}^* = 0$ since keeping transmitting will suffer serious interference from other active PUs. Thus, switching to be back-off can improve the NUG as well.

Case $|\mathcal{N}_{tx}| = \bar{n}$, which means that the PUs in $\mathcal{N}_{tx}$ exactly saturate the channel capacity. This strategy profile is unstable because the PUs in $\mathcal{N}_{tx}$ benefit from always successful transmissions while those PUs in $\mathcal{N}_{bo}$ will never transmit to avoid interference. Without any communication and cooperation, this status will never be achieved among selfish PUs because any PU is not willing to back-off while letting others to transmit forever.

Note that, during the whole time, the state machine runs dynamically rather than stays statically. For example, when the channel is not saturated (i.e., in the first case above), for those successfully occupy the opportunity, their NUGs will increase. However, for those fail to transmit, their situation will transfer to the second case in this proof.

Therefore, in general, all PUs' status will transfer among those three cases, which exactly implies that there is no pure strategy *Nash* equilibrium again. Otherwise, every PUs will hold on one deterministic state. And the increase of respectively NUGs is also an expected view locally from individual PUs.

Generally, at any time slot, a PU transmitting deterministically (i.e., choosing a pure strategy) is not an equilibrium strategy. Thus, the pure strategy $\mathbf{A}^*$ is not a *Nash* equilibrium and contradicts the initial assumption. This completes the proof. □

Remarks. For the spectrum random access game among the PUs, the equilibrium will be a mixed strategy, called a *focal* equilibrium of the game [12]. It has been theoretically proved that focal equilibrium will be the first choice a player will tend to use in the absence of communication.

Based on Theorem 1, the optimal solution of the game among PUs will be a *mixed-strategy Nash equilibrium* (MS-NE), meaning that each PU shall transmit probabilistically. Next we show that this MS-NE is unique. The result is established by verifying two properties of the spectrum random access game among PUs: (1) the convexity of the strategy set of a PU during a frame, and (2) the convexity of the payoff function of a PU. Based on these properties we reach the conclusion in Theorem 2.

Lemma 1. *The strategy set of any PU i is a convex set (i.e., condition (1) holds).*

Proof. In a frame, the strategy set of a PU i is to choose its access probability a_{it} for each time slot subject to its transmission budget constraint. Since the transmission budget constraint is a half hyperspace and the intersection operation on every constraints preserves convexity, the possible strategies of a PU form a non-empty convex set. This completes the proof. □

Lemma 2. *The utility function $u_i(\cdot)$ of PU i is a concave-convex function in $\mathbf{a}_i$ and $\mathbf{A}_{-i}$, respectively (i.e., condition (2) holds).*

Proof. We first prove that $u_i(\cdot)$ is concave in $\mathbf{a}_i$, i.e., concave in PU i's strategy profile during a frame. According to the definition of u_i in Eq. (7), entry $a_{it} \in \mathbf{a}_i$ only appears in time slot t during a frame, and $u_i(\cdot, \mathbf{A}_{-i})$ is linear in $\mathbf{a}_i$. For a linear function, it is both convex and concave. Hence, u_i is a concave function in $\mathbf{a}_i$.

We next prove that $u_i(\cdot)$ is convex in $\mathbf{A}_{-i}$, i.e., convex in the strategy profiles of PUs other than i. In the definition of u_i given by Eq. (7), any PU j other than i at time slot t (i.e., a_{jt}) could have a transmission status of either $\xi_j = 1$ or $\xi_j = 0$ in the transmission status vector $\boldsymbol{\xi}$. Therefore, the coefficients denoted by co_{ξ_j} of any a_{jt} in the 1st-order derivative of $u_i(\cdot)$ could be either

$$co_{\xi_j=1} = a_{it} \prod_{\substack{\forall \xi_{j''}=1,\\ j''\neq i,j}} a_{j''t} \prod_{\forall \xi_{j'}=0} (1 - a_{j't}) \log(\cdot), \text{ or}$$
$$co_{\xi_j=0} = -a_{it} \prod_{\forall \xi_{j'}=1} a_{j't} \prod_{\substack{\forall \xi_{j''}=0,\\ j''\neq i,j}} (1 - a_{j''t}) \log(\cdot), \forall \boldsymbol{\xi}, \tag{14}$$

where both cases are constant. Thus, $u_i(\cdot)$ is also linear in $\mathbf{A}_{-i}$, which implies that it is convex (and concave) in $\mathbf{A}_{-i}$. This completes the proof. □

Remarks. Lemma 1 and Lemma 2 together ensure that the game among PUs is an N-player concave-convex game, but the equilibrium solution of such a game might not be unique. We prove its uniqueness in the following theorem.

Theorem 2. *The game among PUs possesses a unique MS-NE.*

Proof. To prove the uniqueness, we verify the sufficient condition proposed in [13], which guarantees the uniqueness of MS-NE. Specifically, the sufficient condition states that there should exist an N-element positive vector $\boldsymbol{\lambda}$ such that a linear combination of all PUs' utility functions $g(\mathbf{A}) = \sum_{i\in\mathcal{N}} \lambda_i u_i(\cdot)$ is concave in the access probability matrix $\mathbf{A}$. We next verify this property of the function $g(\mathbf{A})$.

In the function $g(\mathbf{A})$, entry $a_{it} \in \mathbf{A}$ appears in two different parts:

$$g(\mathbf{A}) = \lambda_i u_i(\cdot) + \sum_{j\in\mathcal{N}, j\neq i} \lambda_j u_j(\cdot), \tag{15}$$

where the first part is the utility function of PU i itself, i.e., $u_i(\cdot)$. For this part, we have proven in Lemma 2 that u_i is linear in $\mathbf{a}_i$, thus it is a concave function. The second part is the utility function of any PU j other than i, i.e., $u_j(\cdot)$. From the perspective of PU j, a_{it} is a strategy of PU i in $\mathbf{A}_{-j}$. Since we have also proven in Lemma 2 that any utility function u_i (w.r.t. u_j) is linear in $\mathbf{A}_{-i}$ (w.r.t. $\mathbf{A}_{-j}$), the second part is also linear and thus concave in a_{it}. Clearly, the sum of the two linear terms is also linear in a_{it}, which means that the function $g(\mathbf{A})$ is linear (i.e., concave) in $\mathbf{A}$. Hence, the sufficient condition is satisfied. This completes the proof. □

Remarks. Theorem 2 guarantees the existence of a unique optimal strategy trace $\mathbf{a}_i^*$ of any PU i during a frame and any unilaterally deviating this optimal strategy will lower the NUG of the PU.

3.2. The optimal pricing game between the PO and PUs

We now analyze the properties of the optimal pricing game defined in **P**. The main result is that our optimal pricing game could have *multi-optimality* because it is proven as of a D.C. programming problem. A D.C. programming has an objective function which can be decomposed as *a convex function minus a convex function* (or equivalently, a convex function plus a concave function) [14]. We establish this result based on the following two Lemmas.

Lemma 3. *In the objective function of the PO $\mathcal{R}_T(\cdot)$, the revenue from PUs $\mathcal{R}_p(\cdot)$ is a concave function in the decision variable $\boldsymbol{\gamma}$.*

Proof. Since variable $\boldsymbol{\gamma}$ of $\mathcal{R}_p(\cdot)$ in Eq. (9) is also a parameter affecting the access probability matrix $\mathbf{A}$, we first need to analyze the property of equilibrium solution of the spectrum random access game among PUs defined in $\mathbf{P}_i$.

According to the formulation of $\mathbf{P}_i$, if the strategy profiles of all PUs other than i, i.e., $\mathbf{A}_{-i}$, are fixed, given any pricing scheme $\boldsymbol{\gamma}$, the problem $\mathbf{P}_i$ is a linear programming (LP) that maximizes $u_i(\cdot, \mathbf{A}_{-i}, \boldsymbol{\gamma})$ subject to the transmission resource budget constraint. However, the LP changes in terms of a parameter, i.e., $\boldsymbol{\gamma}$. Such a type of LP problems containing uncertain parameters is called *multi-parametric LP* (MP-LP) problems [15].

According to the property of an MP-LP problem, the optimal solution is a continuous and piecewise affine function of the uncertain parameters. To our problem, this property means that the optimal access probability matrix $\mathbf{A}^*$ is an affine function of $\boldsymbol{\gamma}$, which is both convex and concave. Thus we have the slop value $a_{it}'^* < 0$ (since the access probability decreases with the price coefficient γ_t) and 2nd-order derivative $a_{it}''^* = 0$ (since $\mathbf{A}^*$ is affine).

The entries of the Hessian matrix of the revenues from the PUs $\mathcal{R}_p(\cdot)$ with respect to $\boldsymbol{\gamma}$ are:

$$\nabla^2_{\gamma_t}\mathcal{R}_p = a_{it}''^*\gamma_t c_0 + a_{it}'^*$$
$$\nabla_{\gamma_t\gamma_{t'}}\mathcal{R}_p = 0 \tag{16}$$

Since $a_{it}''^* = 0$ and $a_{it}'^* < 0$, we have:

$$\nabla^2_{\gamma_t}\mathcal{R}_p < 0, \quad \nabla_{\gamma_t\gamma_{t'}}\mathcal{R}_p = 0, \tag{17}$$

This indicates that the Hessian matrix is negative definite, which means that $\mathcal{R}_p$ is concave in $\boldsymbol{\gamma}$. This completes the proof. □

Lemma 4. *In the objective function of the PO $\mathcal{R}_T(\cdot)$, the revenues from the MVNO $\mathcal{R}_s(\cdot)$ is convex in the decision variable $\boldsymbol{\gamma}$.*

Proof. Since $\mathcal{R}_s(\cdot)$ is already a form of a non-negative weighted sum, proving its convexity is equivalent to proving that the set of all available windows, i.e. **S**, mapped from the transmission strategy profile of PUs (i.e., matrix **A**), is convex.

However, **S** is a discrete set as the period length of any τ is an integer number, which renders the ordinary definition of a convex set invalid. We thus use the definition from discrete convexity analysis as developed in [16] to verify the generalized convexity of the set **S**.

In the discrete case, a convex set requires that, given any two distinct points x and x' from that set, the two new points $\lceil \frac{x+x'}{2} \rceil$ and $\lfloor \frac{x+x'}{2} \rfloor$ are still in the same discrete set. This is to say that, given two available windows τ and τ' both from **S** and their middle point $\tilde{\tau} = \frac{\tau+\tau'}{2}$ (may not be defined in **S**), we need to prove that the two new points $\lceil \tilde{\tau} \rceil$ and $\lfloor \tilde{\tau} \rfloor$ are still in **S**.

Since we can always organize the set **S** on a 2-D grid, e.g., by sorting all available windows in an ascending order, the $\lceil \tilde{\tau} \rceil$ and $\lfloor \tilde{\tau} \rfloor$ can always find the closest rounding points on the grid. Therefore, both the two new points are in the set **S**. Given this property, the set **S** is real-value extensible by relaxing Z^+ to R^+. The relaxed set in the real-valued space is also a normal convex set. This completes the proof. □

Theorem 3. *The optimal pricing game is a D.C. programming problem (multi-optimality).*

Proof. According to Lemmas 3 and 4, we know that the objective function of the optimal pricing game problem in **P** is the sum of a concave function (i.e., the revenues from the PUs $\mathcal{R}_p$) and a convex function (i.e., the revenues from the MVNO $\mathcal{R}_s$). Since the objective function can be decomposed as the sum of a concave function and a convex function, it is a D.C. programming problem. This completes the proof. □

Overall, the key properties of our optimal pricing problem are: (1) PUs' best response $\mathbf{A}^*(\boldsymbol{\gamma})$ to any pricing scheme is a unique MS-NE, and (2) there could be multiple optimal pricing schemes $\boldsymbol{\gamma}^*$ for the PO.

4. Algorithms

In this section, we first describe the main idea and then elaborate on the details of the algorithms.

4.1. The main idea

Recall that the optimal pricing game is a black-box BPEC problem because PO's utility function contains a black-box term: the revenues from the MVNO (i.e., $\mathcal{R}_s$). Without a black-box term, traditional methods generally replaced the equilibrium constraint (i.e., the game among PUs) by its KKT-conditions so that the bilevel programming can be reduced to a single-level programming, and solved by branch-and-bound methods, trust-region methods, gradient descent methods, etc. One initial presumption behind those methods is that, after transforming to a single-level programming, the problem will become easier.

However, this is not the case in our problem because even if the spectrum random access game among PUs is replaced with its KKT conditions, the objective function of the transformed single-level programming is still a black-box problem due to the implicit revenue term of the MVNO $\mathcal{R}_s$.

To solve the problem, our idea is to design an efficient solver for the spectrum access game among PUs first, and then embed the solver as a building block into an MCMC framework, targeting on the black-box challenge, to solve the optimal pricing game. An MCMC method will establish a Markov chain of feasible solutions to visit the solution space. With sufficient samples, the trace can restore the whole domain of the objective function. This therefore overcomes the black-box challenge because it only needs to probe the value of the objective function, which can be done algorithmically. More importantly, since our problem has multiple optimal solutions, an MCMC method overcomes locally optimal issues.

Next, we will first introduce the building block solver for the PUs' random access game, and the elaborate on the MCMC algorithm to solve the whole optimal pricing game.

4.2. The solver for PUs' random access game

We have shown that the game among PUs possesses a unique MS-NE $\mathbf{A}^*(\boldsymbol{\gamma})$ in Theorem 2. During a frame, there are a total N optimization problems to solve, each of which is defined in $\mathbf{P}_i$. Due to the correlations of the decision variables in the formulation, it is impossible to solve the set of optimization problems sequentially because when a PU calculates its $\mathbf{a}_i^*$ all other PUs will also calculate their own optimal access probabilities. This renders $\mathbf{A}_{-i}$ to be not fixed but PU i cannot control them. In order to resolve these correlations, we reformulate the spectrum random access game originally containing N optimization problems to one single optimization problem. The main idea is as following:

Since $\mathbf{a}_i^*$ is an equilibrium strategy for PU i, PU i cannot further improve its NUG by unilateral deviation. This is to say that if there exists another strategy profile $\mathbf{a}_i'$ that could further improve the NUG $u_i(\cdot)$, the current strategy $\mathbf{a}_i$ would not be optimal. Hence, solving the original game can be reformulated as iteratively searching a new strategy profile $\mathbf{a}_i'$ based on the current strategy profile $\mathbf{a}_i$ until the improvement becomes zero. This idea was first proposed in [17] to deal with strategy set correlations among multiple players[2], and is called "NI-reformulation". Formally, at each iteration, we solve the following optimization problem to find a locally optimal access probability matrix $\mathbf{A}'^*$:

$$\max_{\mathbf{A}' \in \mathcal{X}} \Psi(\mathbf{A}, \mathbf{A}'), \tag{18}$$

where

$$\mathcal{X} := X_1 \times X_2 \times \cdots \times X_N \tag{19}$$

is the whole strategy space of all PUs, X_i is the strategy set of PU i, and $\Psi(\cdot)$ is defined as:

$$\Psi(\mathbf{A}, \mathbf{A}') = \sum_{i \in \mathcal{N}} \Big[u_i(\mathbf{a}_i', \mathbf{A}_{-i}) - u_i(\mathbf{a}_i, \mathbf{A}_{-i}) \Big], \tag{20}$$

to represent the sum of improvements of each PU i by unilaterally deviating from its current strategy $\mathbf{a}_i$ to $\mathbf{a}_i' \in \mathbf{A}'$ while keeping $\mathbf{A}_{-i}$ fixed.

Algorithm 1 The PUs' game solver

Input: Maximum iteration number K, Pricing scheme $\boldsymbol{\gamma}$, Precision tolerance ϵ
Output: $\mathbf{A}^*$
1. initialize $\mathbf{A}^0 \sim \mathcal{X}$, tol $\leftarrow$ inf;
2. $\mathbf{A}^k \leftarrow \mathbf{A}^0$;
3. **while** tol $> \epsilon$ or $k \le K$
4. $\quad \mathbf{A}' \leftarrow \arg\max_{\mathbf{A}' \in \mathcal{X}} \Psi(\mathbf{A}^k, \mathbf{A}')$;
5. $\quad$ update $\mathbf{A}^k \leftarrow (1-\tau_k)\mathbf{A}^k + \tau_k \mathbf{A}'$;
6. **end**
7. $\mathbf{A}^* \leftarrow \mathbf{A}^k$;

The algorithm for solving this reformulated problem is a dynamic process iteratively approaching the MS-NE. At the beginning, we initialize a random feasible $\mathbf{A}^k$, and then we search an optimal $\mathbf{A}'^*$ that maximizes the improvement $\Psi(\cdot)$ based on $\mathbf{A}^k$. With $\mathbf{A}'^*$, we move $\mathbf{A}^k$ toward $\mathbf{A}'^*$ with a convex combination. This process is repeated until the improvement approaches zero (in practice: becomes small enough) or a prescribed K iterations finished. The pseudocode is given in Algorithm 1. Convergence of the algorithm in a finite number of iterations is guaranteed provided that $\tau_k \to 0$ and $\sum_{k=0}^{\infty} \tau_k = \infty$ according to the result in [18].

Remarks. It is worth noting that the equilibrium $\mathbf{A}^*$ can be easily achieved in a distributed way as well if all PUs follow a continuous best-response dynamic strategy [19]. It means that communications between PUs are not necessary. Moreover, it is enough to consider converged results for the interest of a PO.

4.3. The solver for the PO's optimal pricing

We now solve the optimal pricing game. One challenge is that there could be multiple local optima in our problem, and any local optimum cannot guarantee its global optimality. In order to find a global optimum, we must find all local optima and compare them. This, however, is unusually hard, especially in high-dimensional cases. More

[2] The correlation on the feasible strategy sets of players means that one player's strategy set is constrained by the strategies applied by other players. Simply put, with correlation on strategy sets, players cannot choose their strategy profiles freely since one's strategy set depend on the moves of other players.

Algorithm 2 Pricing scheme sampling solver

Input: Total number of samples K
Output: $\boldsymbol{\gamma}^*$
1. $k \leftarrow 0$, $\boldsymbol{\gamma}_k \sim \mathrm{dom}\, \mathcal{R}_T$, $\boldsymbol{S} \leftarrow O$;
2. $\mathbf{A}^*(\boldsymbol{\gamma}_k) \leftarrow$ call Algorithm 1 for all $\{\hat{\mathbf{a}}_t^*\}_{t\in[1,T]}$;
3. **while** $k < K$
4. $\quad \boldsymbol{\gamma}_{k+1} \sim \pi(\boldsymbol{\gamma}_k, \boldsymbol{\sigma}^2)$ within $\mathrm{dom}\, \mathcal{R}_T$;
5. $\quad \mathbf{A}^*(\boldsymbol{\gamma}_{k+1}) \leftarrow$ Algorithm 1;
6. $\quad a \sim \min\left\{1, \frac{\mathcal{R}_T(\boldsymbol{\gamma}_{k+1})}{\mathcal{R}_T(\boldsymbol{\gamma}_k)}\right\}$;
7. $\quad$ **if** accepted, $\boldsymbol{\gamma}_k \leftarrow \boldsymbol{\gamma}_{k+1}$, $\mathbf{A}^*(\boldsymbol{\gamma}_k) \leftarrow \mathbf{A}^*(\boldsymbol{\gamma}_{k+1})$,
$\quad\quad \boldsymbol{S} \leftarrow \boldsymbol{S} \cup \boldsymbol{\gamma}_k$;
8. $\quad$ **else** keep $\boldsymbol{\gamma}_k$, $\mathbf{A}^*(\boldsymbol{\gamma}_k)$;
9. $\quad k \leftarrow k+1$;
10. **end**
11. $\boldsymbol{\gamma}^* \leftarrow \arg\max_{\boldsymbol{\gamma}\in\boldsymbol{S}} \mathcal{R}_T$;

essentially, as discussed before, another challenge in our problem is that the PO's utility function is a black-box. No matter if we can have exact solutions for the responses of the PUs (i.e., the access probability matrix $\mathbf{A}$), the revenue part from the MVNO can only be evaluated algorithmically, which impedes us to derive the analytic gradient information.

In order to address these two challenges, we design an MCMC algorithm as introduced in the beginning of this section. Formally, this sampling optimizer works as following:

First, we initialize a pricing scheme $\boldsymbol{\gamma}_0 c_0$ from the domain of $\mathcal{R}_T(\cdot)$ and let it be the sample in the kth step $\boldsymbol{\gamma}_k$ ($k = 0$ at the beginning). Given the value $\boldsymbol{\gamma}_k$, we call Algorithm 1 to calculate the best response $\mathbf{A}^*(\boldsymbol{\gamma}_k)$. Next we randomly generate a new feasible sample $\boldsymbol{\gamma}_{k+1}$ from a proposal distribution[3] $\pi(\cdot)$ based on $\boldsymbol{\gamma}_k$. After that, we calculate an acceptance probability a by choosing the smaller value between 1 and $\frac{\mathcal{R}_T(\boldsymbol{\gamma}_{k+1})}{\mathcal{R}_T(\boldsymbol{\gamma}_k)}$. Based on the acceptance probability a, if accepted, we update both $\boldsymbol{\gamma}_k$ and $\mathbf{A}^*(\boldsymbol{\gamma}_k)$ with $\boldsymbol{\gamma}_{k+1}$ and $\mathbf{A}^*(\boldsymbol{\gamma}_{k+1})$, otherwise we discard the new sample and do re-sampling. The sampling algorithm ends once it collected enough samples K (predefined). Finally, we choose the pricing scheme(s) that maximize $\mathcal{R}_T(\cdot)$ as the optimal solution(s) $\boldsymbol{\gamma}^*$. The pseudo-code of this sampling algorithm is given in Algorithm 2.

Note that integrating Algorithm 1 into the MCMC framework is crucial because it largely reduces the dimensionality of the search space. Specifically, by using the solver to calculate $\mathbf{A}$ each time for a given $\boldsymbol{\gamma}_k$, we reduce the whole solution space from the original $[\mathbf{A}, \boldsymbol{\gamma}]$ whose dimension is $(N+1)\times T$ to a T-tuple vector space $\boldsymbol{\gamma}$. This greatly improves the efficiency of the sampling process.

Additionally, advanced sampling frameworks are available. One could, for example, approximate the objective function better and quicker by dynamically adapting the proposal distribution $\pi(\cdot)$ [20]. Also, maximum-entropy arguments can be used to establish optimality of the proposal in some sense [21]. However, since developing a new sampler is not the main topic of this work, we use the basic one.

4.4. Computational analysis

An MCMC algorithm normally needs many computational resources because of its sampling process. And usually the efficiency of an MCMC algorithm is weaker than a deterministic algorithm, when it is possible to facilitate the problem's structures. As we explained already, however, the spectrum pricing problem formulated in our work has some unique computational challenges, especially the 'black-box' feature, which makes most deterministic algorithms infeasible. Therefore, when exploiting the solution of our problem, our priority in mind is to solve the problem.

Regarding the computational efficiency, our algorithm generates a Markov chain in the solution space based on a proposal sampling distribution. With time t increasing, we can collect increasing number of samples (i.e., possible solutions) from the solution space. Those samples are expected to represent the objective function well, so that we can achieve the optimal solutions. Here have two aspects to concern:

The first aspect is whether or not those samples can perfectly represent the objective function, which is the approximation ratio of the algorithm. For this, an important result is that, the approximation ratio of the algorithm could be 1, which means that collected samples will perfectly represent the objective function [22]. Here we refer a classic convergence result:

Theorem 4. *(Convergence Theorem) Suppose that P is irreducible and aperiodic, with stationary distribution π. Then there exist constants $\alpha \in (0, 1)$ and $C > 0$ such that*

$$\max_{x\in\Omega} ||P^t(x,\cdot) - \pi|| \leq C\alpha^t. \tag{21}$$

This theorem directly says that with t increasing, the maximum distance between the generated distribution and the stationary distribution approaches to 0, considering α is a number between (0, 1) and t is a positive integer (could be a positive float number as well in continuous case). Defining the left-hand-side in inequality (21) as $d(t)$, a positive result that we can also have is the distance $d(t)$ decaying exponentially with the sampling time t, and will approaches to 0 when $t \to \infty$.

As we can see, the convergence result is related with the sampling time t, so the second aspect is how long does the sampling process need to converge, called "mixing time". However, such problem is still an open question in this research field, which means that it is quite difficult to quantitatively how long does an MCMC algorithm needs to 100% converge in general except some low dimensional or discrete cases.

In general, to make α^t small it suffices to take t larger than $\frac{-1}{\log\alpha}$. In our work, we exactly took large time referred by the parameter discussed above when sampling, because (1) the dimension of our problem is high and (2) long sampling time helps to cover multi-optimality of the pricing solutions. While it is truly a long time, and potentially a parallel sampling algorithm or a more advanced proposal

[3] Here we choose a multivariate Gaussian distribution $\pi(\boldsymbol{\gamma}_k, \boldsymbol{\sigma}^2)$ whose mean is $\boldsymbol{\gamma}_k$ and covariance matrix $\boldsymbol{\sigma}^2$ is a predefined square matrix.

Table 2
The three different pricing schemes.

	The values of γ_t
$\boldsymbol{\gamma}_{LH}$	$\gamma_{t,t\in[1,5]} = 0.5$ and $\gamma_{t,t\in[6,10]} = 1.5$
$\boldsymbol{\gamma}_{LHL}$	$\gamma_{t,t\in[1,2]} = 0.5$, $\gamma_{t,t\in[3,8]} = 1.5$ and $\gamma_{t,t\in[9,10]} = 0.5$
$\boldsymbol{\gamma}_{ZigZag}$	$\gamma_{t,t\in odd} = 0.5$ and $\gamma_{t,t\in even} = 1.5$
$\boldsymbol{\gamma}_0$	$\gamma_{t,t\in[1,10]} = 1$

distribution can speed up the sampling process. This is indeed one important extension in our future work.

5. Numerical results

To evaluate the performance of the proposed scheme, we develop a comprehensive simulation testbed using MATLAB. In this section, we first explain the simulation settings. We then show the effect of non-uniform pricing schemes to PUs' spectrum access behaviors. After that, we illustrate the characteristics of the optimal pricing scheme. Finally, we test the proposed pricing schemes under different dynamic spectrum access (DSA) protocols.

5.1. Settings

In our experiments, we let the number of PUs $N = 10$, and we consider a typical LTE scenario whose frame consists of $T = 10$ sub-frames. The transmission power of any PU i is $p_i = 10\,\text{dBm}$ and the power gain at the BS $\forall h_i = -30\,\text{dB}$. The background noise $\text{I}_0 = -100\,\text{dBm}$. We let the transmission budget of a PU be $b_i = 0.4Tc_0$, which is not enough to cover the whole frame's transmission, otherwise the problem becomes trivial because a PU has no incentive to prefer cheaper transmission prices. Additionally, the weight coefficient of the transmission rate r_{it} is set to $\omega_i = 5c_0$. For the QoS constraint of PUs in Eq. (12), we set $\tilde{r} = 80\%r_0$, where r_0 is the average transmission rate under a uniform pricing scheme $\boldsymbol{\gamma}_0$.

5.2. The spectrum random access game among PUs

We compare a uniform pricing scheme $\boldsymbol{\gamma}_0$ with three non-uniform pricing schemes: $\boldsymbol{\gamma}_{LH}$, $\boldsymbol{\gamma}_{LHL}$ and $\boldsymbol{\gamma}_{ZigZag}$, shown in Table 2. Given the pricing schemes, the expected transmission numbers (Expected Tx#) as the outcomes of corresponding spectrum random access game among PUs are shown in Fig. 3.

In Fig. 3, we can see that the three non-uniform pricing schemes stimulate PUs to transmit non-uniformly. Specifically, during the low-price periods (e.g., $t \in [1, 5]$ under $\boldsymbol{\gamma}_{LH}$), 5 PUs are expected to transmit, but the Expected Tx# is 2 during the high-price periods (e.g., $t > 5$ under $\boldsymbol{\gamma}_{LH}$). If we assume that a TXOP requires $\ell = 2$ as a minimum length and ζ equivalent to 2 expected PUs' transmissions, then there will be 2 TXOPs with pricing scheme $\boldsymbol{\gamma}_{LH}$ but no TXOP available with uniform pricing scheme $\boldsymbol{\gamma}_0$, since the Expected Tx# of PUs is 3, which is above the threshold $\zeta = 2$.

5.3. The optimal pricing game

We first show *multi-optimality* of the optimal pricing game. We then study the link between the price c_s to c_0.

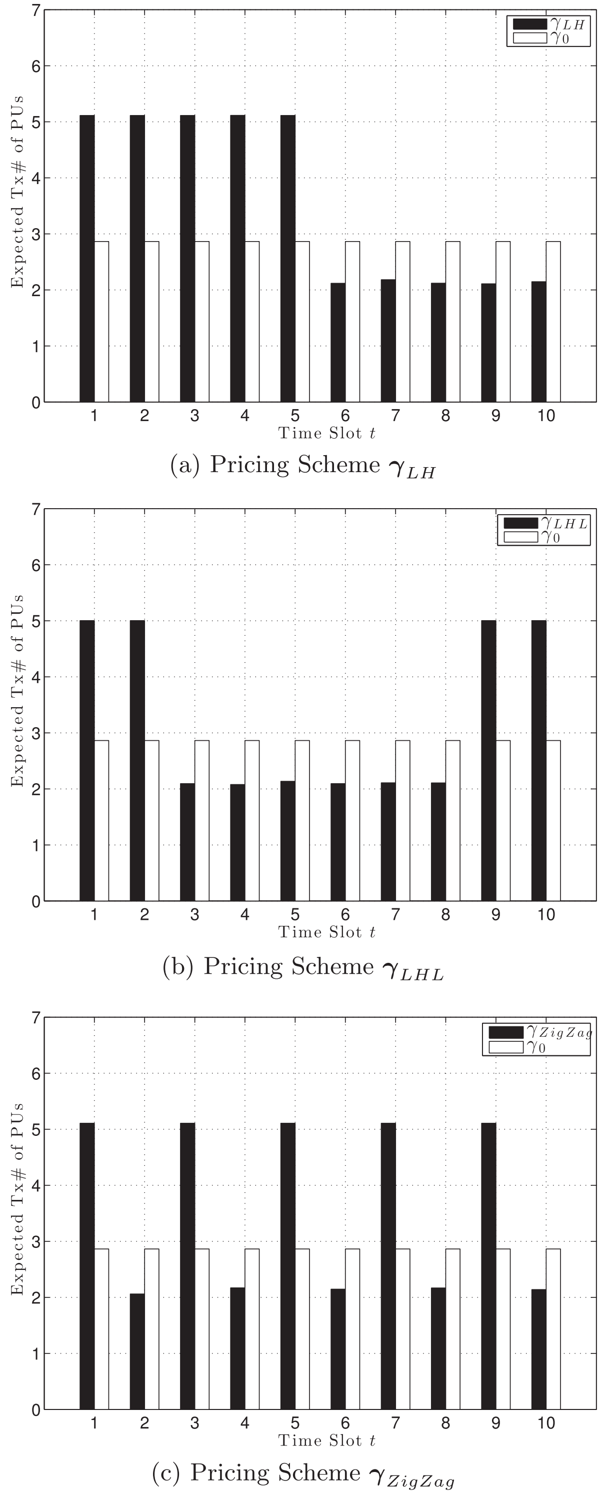
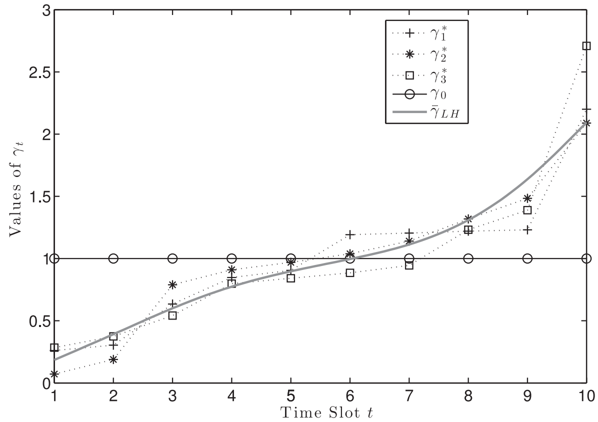
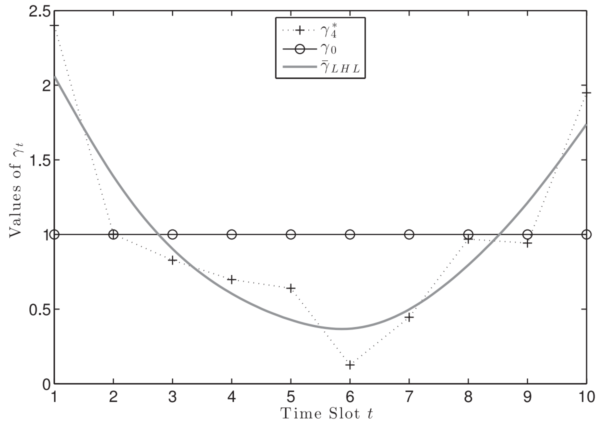


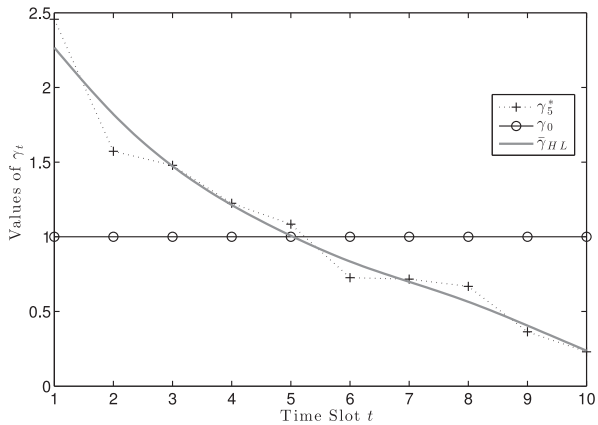
Fig. 3. PUs' behaviors and TXOPs of the MVNO.



(a) Increasing Values γ_t^* v.s. $\boldsymbol{\gamma}_0$



(b) Convex-Shaped Values γ_t^* v.s. $\boldsymbol{\gamma}_0$



(c) Decreasing Values γ_t^* v.s. $\boldsymbol{\gamma}_0$

Fig. 4. Multiple optimal pricing schemes ($\ell = 4$).

5.3.1. Multi-optimality

We show the optimal non-uniform pricing schemes that maximize $\mathcal{R}_T$ and also plot uniform pricing scheme $\boldsymbol{\gamma}_0$ as a baseline. In order to see the pattern of the non-uniform pricing schemes, we fit each of them with a polynomial function. The results are presented in Fig. 4.

In Fig. 4, five optimal pricing schemes are found by Algorithm 2 and plotted in three sub-figures (symbolized as $\boldsymbol{\gamma}_1^* \sim \boldsymbol{\gamma}_5^*$). This first confirms our theoretical result that a D.C. programming problem may have multiple optimal solutions. More interestingly, in Fig. 4, we can observe that the optimal pricing schemes are either monotonic (i.e., increasing or decreasing plotted as $\bar{\gamma}_{HL}$ and $\bar{\gamma}_{LH}$ in Fig. 4(a) and Fig. 4(c), respectively) or convex-shaped plotted as $\bar{\gamma}_{LHL}$ in Fig. 4(b). In fact, they are equivalent if we consider multiple consecutive frames, since they are all periodic.

Additionally, we can expect that if we lower the length requirement of a TXOP ℓ (e.g., changing $\ell = 4 \to 1$), the number of optimal solutions $\boldsymbol{\gamma}^*$ will increase accordingly. Some of them will oscillate very frequently between high and low γ values in a frame. But note that in the optimal set, those solutions generating longer consecutively available windows (as those presented here) will still be found. This is also the reason why we choose ℓ longer because it enables us to observe clearer the features of the optimal solutions.

In order to gain deeper understanding, we show three important metrics including (1) the total revenue values of the PO, (2) the average transmission rate (QoS) of PUs, and (3) the NUGs of PUs under the optimal pricing schemes (i.e., $\boldsymbol{\gamma}_1^* \sim \boldsymbol{\gamma}_5^*$) and those under $\boldsymbol{\gamma}_0$ as well in Fig. 5.

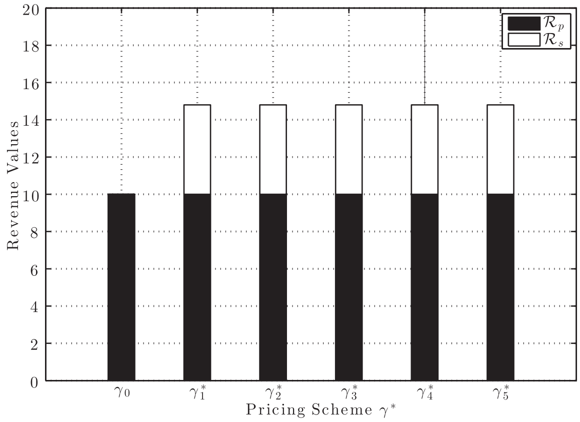
We can observe that the total revenue $\mathcal{R}_T$ under the five optimal pricing schemes is all higher than the revenue with uniform pricing scheme $\boldsymbol{\gamma}_0$ because more TXOPs are generated, which achieves the main goal of our proposed scheme. In Fig. 5(b), we see that the average transmission rates of PUs are lower than the rate under $\boldsymbol{\gamma}_0$. The reason of this phenomenon is because the PO has the QoS constraint in Eq. (12), which allows that the average rate of PUs shall be at least $80\% r_0$. Similarly, in Fig. 5(c), the NUGs of the PUs are also lower than the NUGs under $\boldsymbol{\gamma}_0$. This reveals the importance of the QoS of the PUs because the PO cannot ignore the experiences of PUs who are its major customers.

5.3.2. Different price c_s for the MVNO

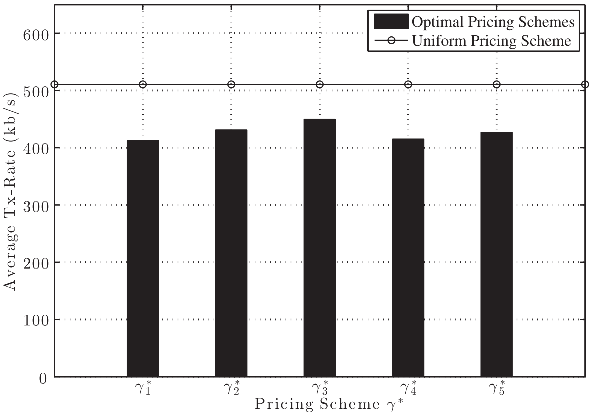
In addition to the price c_0 for the PUs, we are also interested in how the price c_s for charging the MVNO affects the non-uniform pricing schemes. We evaluate different ratios $\frac{c_s}{c_0} \in [0.5, 1.5]$. Since every optimal pricing schemes are globally optimal, we take the monotonic optimal pricing scheme (e.g., an increasing-value one) as an example in the sequel.

First, in Fig. 6, we show three optimal pricing schemes under c_s coupled to c_0 with $\{0.5c_0,\ 1c_0,\ 1.5c_0\}$ fitted by linear functions, respectively. We observe the increase of slopes of the optimal pricing schemes (fitted with dashed, solid, and dash-dotted lines, respectively) when c_s increasing. This indicates that, if possible, the PO prefers to gain more revenues from the MVNO. Intuitively, it conforms the reality because trading the transmission management out is easier and still can yield revenues from secondary operators.

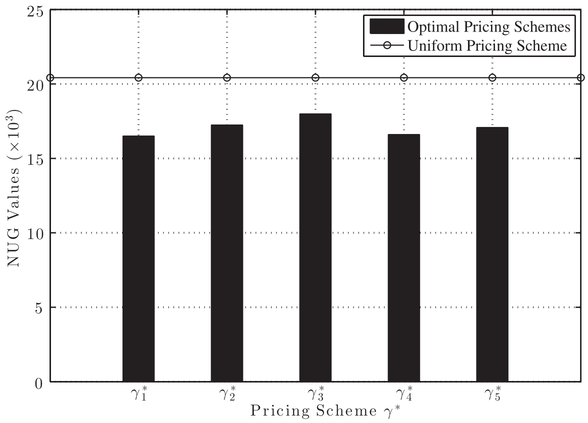
Similarly, we show in Fig. 7 the three metrics to evaluate and understand the optimal pricing schemes when c_s increases.



(a) Revenues of the PO



(b) Avg. Transmission Rates of PUs



(c) Average NUG Values of PUs

Fig. 5. Three metrics under multiple optimal pricing schemes.

Specifically, in Fig. 7(a), we observe that TXOPs as well as the total revenues, increase when c_s approaches c_0, while TXOPs does not increase further when $c_s > c_0$. This is due to the QoS constraint for PUs where no more room to generate more TXOPs. However, since the transmission from the MVNO is valued more (due to larger c_s), the revenues still rise. In Fig. 7(b) and (c), both the average trans-

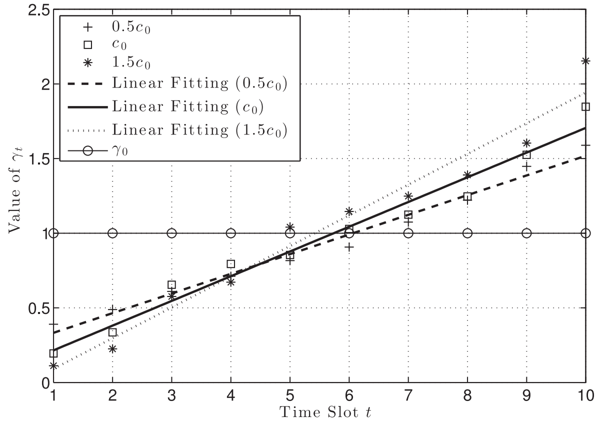


Fig. 6. Pricing schemes with different prices c_s for The MVNO.

mission rate and the NUG values of the PUs decrease with increasing c_s but still guaranteed above $80\% r_0$ as required in Eq. (12).

5.4. Different DSA protocols

We now evaluate the proposed pricing scheme with two different DSA protocols, where the MVNO can use two different access strategies, a best-effort strategy operation (denoted as "BE-SO") and an exponential back-off strategy operation[4] (denoted as "ExpB-SO") to capture free spectrum, respectively. Fig. 8 shows the access of PUs and the MVNO. We can see that the proposed scheme successfully stimulated PUs to access spectrum non-uniformly (dash-line curve fitting by polynomial in red), which generates TXOPs for MVNOs (see yellow and green bars representing transmissions from different MVNOs). We also note that, the "BE-SO" strategy can capture more TXOPs than the "ExpB-SO" because the latter may miss some TXOPs due to longer back-off effects. However, using smarter sensing strategies in literature, those missed TXOPs can be more efficiently utilized as well.

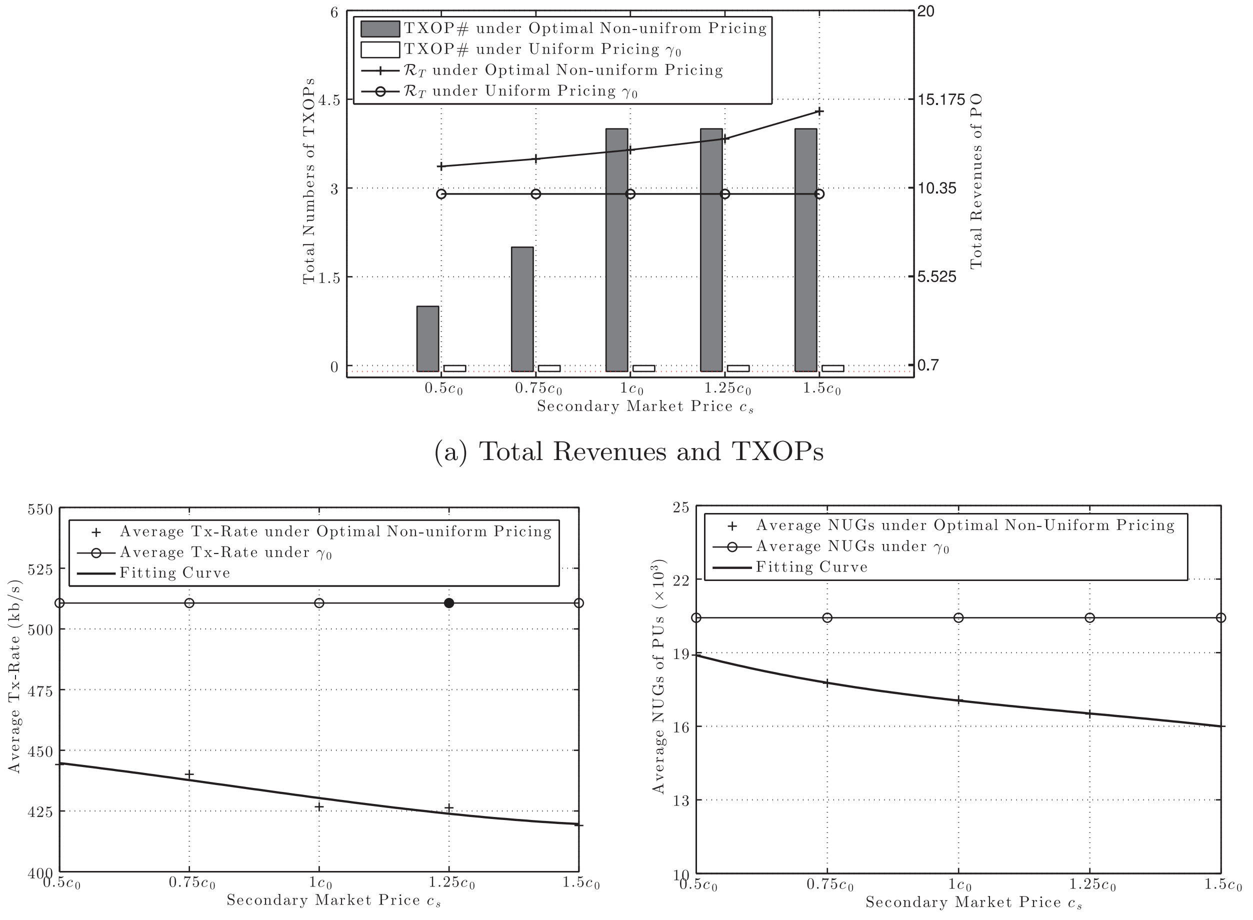
6. Related work

We mainly discuss our differences from two perspectives: spectrum pricing schemes and game-theoretical approaches.

6.1. Spectrum pricing schemes

Pricing schemes have been widely used in wireless network management especially when directly intervening is unrealistic. Spectrum as resources being sold and bought between spectrum owners and customers has been considering as a very effective manner for its efficient utilization. Fruitful results can be found as in [3,5,7,23–29], though numbers of spectrum sellers and buyers

[4] Initially, if the spectrum is busy, the MVNO waits $cw_0 = \frac{T}{4}$ time slots. If the spectrum is busy after back-off, the back-off time will be $k \cdot cw_0, k \in [0, 2^c - 1]$, where c is the number of collisions encountered.



(a) Total Revenues and TXOPs

(b) Average Transmission Rates of PUs

(c) Average NUG Values of PUs

Fig. 7. Three metrics under different secondary market prices.

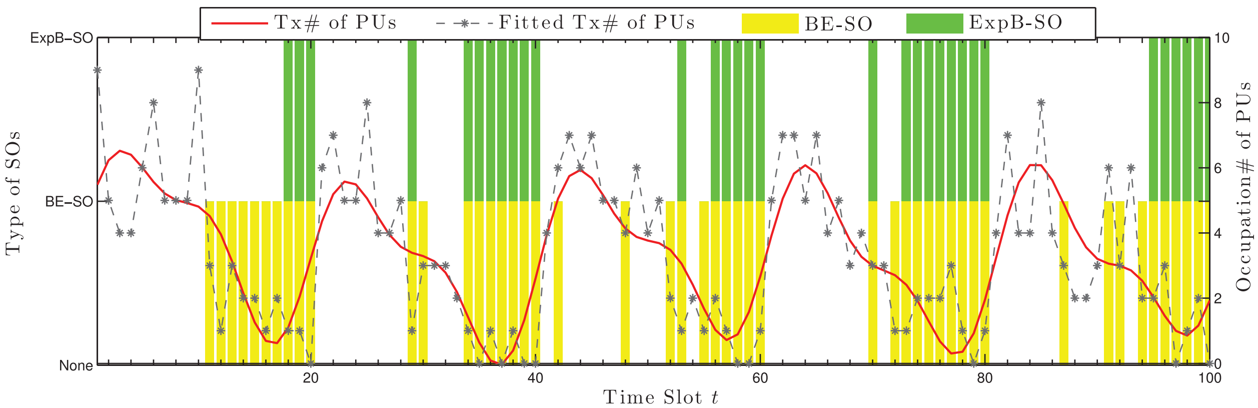


Fig. 8. Transmissions (Tx) in 5 frames (1 frame = 20 slots).

involved are different in their problem scenarios respectively. As being pointed out, most of them focus on designing pricing schemes to charge unlicensed spectrum buyers (in our problem, the MVNO). Authors of [27] studied the coexistence issue of PUs and unlicensed users, however, their goal was to maximize the whole network throughput and assumed unlicensed users cognitively access any idle spectrum. Differently, our goal aims to study the influence of PUs' behaviors ot the TXOPs for an MVNO, and we proposed a pricing scheme applied on licensed users (i.e., PUs) to generate more TXOPs for an MVNO.

Regarding the main idea of the pricing scheme: *non-uniform pricing*, some studies have a similar idea such as in [6,30]. Prices depend on transmission powers of cognitive radio endpoints [30] and in [6], prices depend on the interference generated by the unlicensed users. Their goals, however, are to reduce collision and/or interference rather than to improve the spectrum availability. Again, the present pricing scheme charges non-uniform prices on PUs while improves TXOPs' availability for an MVNO.

Since the availability of TXOPs is directly determined by PUs' behaviors, e.g., the achievable transmission rate of a secondary network [8], this again shows that it is equally important to regulate PUs' behaviors though they have higher priority. Some studies considered cooperation between sellers and buyers [31,32,33]. They proposed cooperative schemes based on revenue rewarding to motivate licensed users to relay traffics from unlicensed users. In this way, licensed users indirectly provide more TXOPs for unlicensed users. Differently, our scheme realizes cooperation in a different way. Specifically, rather than directly forwarding traffics of unlicensed users, we regulate transmissions of PUs themselves via pricing so as to provide more TXOPs to an MVNO. First of all, since PUs still transmit their own traffics, there is no resources consumed for others. More importantly, our scheme also physically separates PUs and an MVNO, but any cooperative schemes as in [[31,32,33], need protocol-level modification for inter-domain communications, which is unrealistic when involving a large number of PUs.

Last but not least, most of existing works consider any idle spectrum as a TXOP, but it may not be practical enough, e.g. an MVNO may require longer consecutively available time slots as its minimum data and this will be claimed in the SLAs with the PO.

6.2. Game-theoretical approaches

We now discuss from the technical point of view.

From the perspective of game models, previous studies either formulated Stackelberg games [34] or non-cooperative games [35]. Our problem, however, contains both game models as a hierarchical game. Furthermore, some previous studies contained both game models, but they often replaced the non-cooperative game with its analytical solutions. This reduced their problems to a single-level optimization problem. Our problem does not allow such a reduction because the outcome of the spectrum random access game has no analytic solutions.

We also note that most of methodologies presented in existing works were restricted to explicit forms. Different from these cases, our game problem has a unique challenge where the whole problem is generally a black-box BPEC problem due to the implicit form in the objective function of the PO and the equilibrium constraint from the spectrum random access game among PUs. This fails exact algorithms like gradient-based and approximation-based methods. Technically, our work showed a first attempt of using an MCMC-based framework to solve a gaming problem.

7. Conclusions

Emergence of secondary operators motivated us to study the influence of PUs' behaviors to TXOPs for MVNOs. In this paper, we proposed a non-uniform pricing scheme applied on PUs to improve TXOPs for MVNOs subject to the QoS constraint for PUs and the SLA constraint for an MVNO. We formulated the optimal pricing problem as a hierarchical game consisting of a PO-PUs Stackelberg game and a spectrum random access game among PUs, which is a challenging BPEC black-box optimization problem. To solve it, we thoroughly studied the property of the problem, and based on the understandings, we designed efficient solver based on an MCMC-based framework, which has not been used in game-theoretical analysis. Finally, we have also conducted extensive simulation experiments, which confirm the effectiveness of the proposed scheme. In future, multiple MVNOs scenarios can be also considered and studied under the general framework here.

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